Is Symmetry Breaking into Special Subgroup Special?

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Ref. [1, T.Kugo, N.Y., PTEP(2019),07B06;arXiv:1904.06857]

Purpose of this talk

I will show that symmetry breaking into **special subgroups** is not special in QFT by using **dynamical symmetry breaking** in 4D $SU(N)$ Nambu-Jona-Lasinio (NJL) type models [2, 3, Y.Nambu, G.Jona-Lasinio'61].

Key words:

Lie groups and their {**(non-)Maximal**}

{ **Regular** / **Special** } **Subgroups** and **Little groups**

Motivation for Grand Unification [4, 5, R.Slansky'81;...]

The idea of grand unification has attractive features; e.g.,

- Unification of the SM gauge bosons
- Unification of the SM Weyl fermions
- 4D SM gauge anomaly cancellation
- Charge quantization for quarks and leptons

...

GUT gauge groups (broken to regular subgroups)

4D GUTs based on GUT gauge groups e.g.,

$SU(5)$ [6, H.Georgi,S.L.Glashow'74], $SU(6)$ [7, K.Inoue,A.Kakuto,Y.Nakano'77], $SO(10)$ [8, H.Fritzsch,P.Minkowski'75], E_6 [9, F.Gursey,P.Ramond,P.Sikivie'76].

5D GUTs based on GUT gauge groups e.g.,

$SU(5)$ [10,11, K.Kojima et al.'11], $SU(6)$ [12,13, G.Burdman,Y.Nomura'03], $SO(10)$ [14,15, H.D.Kim,S.Raby'03;...], E_6 [16,17, Y.Kawamura,T.Miura'13], $SO(11)$ [18–22, Y.Hosotani,N.Yamatsu'15;...].

Usually, GUT gauge groups are broken to **regular subgroups**.

Recently, I have proposed "special GUTs" based on GUT groups broken to their **special subgroups** [23–25, N.Y.'17-'18].

Regular and special subgroups [4, 5, 26–28, E.Dynkin'57;...]

Lie group $SU(3)$

Regular subgroup

$SU(2) \times U(1)$

Branching rules

$$\mathbf{3} = (\mathbf{2})(1) \oplus (\mathbf{1})(-2)$$
$$\overline{\mathbf{3}} = (\mathbf{2})(-1) \oplus (\mathbf{1})(2)$$
$$\mathbf{8} = (\mathbf{3})(0) \oplus (\mathbf{2})(3) \oplus (\mathbf{2})(-3) \oplus (\mathbf{1})(0)$$

Special subgroup

$SO(3) \simeq SU(2)$

Branching rules

$$\mathbf{3} = (\mathbf{3})$$
$$\overline{\mathbf{3}} = (\mathbf{3})$$
$$\mathbf{8} = (\mathbf{5}) \oplus (\mathbf{3})$$

$SU(3) \supset SU(2) \times U(1)$ (R) [4, 5, 26–28, E.Dynkin'57;...]

The Gell-Mann matrices (the generators of the $SU(3)$ $\mathbf{3}$ rep.):

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix},$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

$\lambda_j (j = 1, 2, 3)$ and λ_8 are identified with $SU(2)$ and $U(1)$ ones.

$SU(3) \supset SO(3) \simeq SU(2)$ (S) [4, 5, 26–28, E.Dynkin'57;...]

The Gell-Mann matrices (the generators of the $SU(3)$ $\mathbf{3}$ rep.):

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

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$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

$\lambda_j (j = 2, 5, 7)$ can be identified with $SO(3) \simeq SU(2)$ generators.

"Special GUT" [23–25, N.Y.'17-'18]

- $SU(16)$ **special GUT** [23, N.Y.'17]
  A GUT based on $SU(16)$ broken to its maximal special subgroup $SO(10)$ (A branching rule of $SU(16) \supset SO(10)$ $\mathbf{16} = \mathbf{16}$.)

- $SO(32)$ **special GUT** [24, N.Y.'17]
  A GUT base on $SO(32)$ broken to its subgroup $SO(10) \times U(1)$
  Note: It may be related with $SO(32)$ heterotic string.

- $SU(19)$ **special GUT** [25, N.Y.'18]
  A GUT base on $SU(16)$ broken to its subgroup $SO(10) \times SU(3)_F$.
  Note: 3 generations of the SM fermions can be unified into one.

Questions when I talked about special GUTs

- Is it possible to break a GUT group into its special subgroups? (by many people)
- Are there any role for an $SU(16)$ $\mathbf{120}$ (rank-2 anti-symmetric tensor representation) fermion except 4D $SU(16)$ anomaly cancellation? (by Kugo-san)
- Is it possible to break $SU(16)$ into $SO(10)$ by a composite Higgs field? (by Kugo-san)

$\Rightarrow$ I have started to consider whether Lie groups are broken to their special subgroups by dynamical symmetry breaking (fermion pair condensation) with Kugo-san.

Symmetry and its breakings [4, 29, e.g.,L.Michel'80;...]

What should we know to discuss symmetry breaking in QFT?

- What are Lie group and their **subgroups**?
- What are **little groups** of Lie groups for each representation?
- What is invariant action under Lie group transformation?

$\Rightarrow$ Which symmetry is realized on the minimum of effective potential?

E.g., $SU(3)$ case [4, 5, e.g.,R.Slansky'81;N.Y.'15]

- What are **maximal subgroups of** $SU(3)$?

  $SU(2) \times U(1)$(R) and $SO(3) \simeq SU(2)$(S).

- What are **maximal little groups of** $SU(3)$ for each rep.?

  For $SU(3)$ $\mathbf{3}$ case, only $SU(2)$(R).

  For $SU(3)$ $\mathbf{6}$ case, $SU(2)$(R) and $SO(3) \simeq SU(2)$(S).

  For $SU(3)$ $\mathbf{8}$ case, only $SU(2) \times U(1)$(R).

For $SU(3)$ $\mathbf{3}$, $\mathbf{8}$, regular breaking seems to be realized, while for $SU(3)$ $\mathbf{6}$, not only regular but also special breaking does.

Results for $SU(n)$ gauge theories [30, L.-F.Li'74]

Summary: $SU(n)$ symmetry breaking by scalar condensation

Rep. of $SU(n)$	Symmetry on the minimum of potential
Defining rep.	$SU(n) \to SU(n-1)$(R)
Rank-2 Sym.	$SU(n) \to SU(n-1)$(R) or $SO(n)$(S)
Rank-2 Anti-Sym.	$SU(n) \to SU(n-2)$(R) or $USp(2[n/2])$(S)
Adjoint rep.	$SU(n) \to SU(m) \times SU(n-m) \times U(1)$(R)

Almost people think $SU(n)$ is broken to its **regular subgroups**.

However, $SU(n)$ can be broken to its **special subgroups**.

Symmetry breaking mechanism

- **Scalar condensation (Higgs mechanism)**:
  e.g., $SU(n) \to SO(n)$ [30,31, L.-F.Li'74;S.Meljanac,M.Milosevic,S.Pallua'82;...].

- **Fermion pair condensation (dynamical symmetry breaking)**:
  e.g., $SU(n) \to SO(n)$, $E_6 \to F_4, G_2$ [2, 3, 32–34, Y.Nambu,G.Jona-Lasinio'61;L.Susskind'78;M.E.Peskin'80;T.Kugo,J.Sato'94;...].

- **Orbifold boundary condition**: e.g.,
  $SU(n) \to SO(n)$, $E_6 \to F_4$ [35, A.Hebecker,J.March-Rusell'02;...].

We check the discussion of Ref. [34, T.Kugo,J.Sato'94] for dynamical symmetry breaking because we use the same potential analysis.

Symmetry breaking in E_6 GUT [34, T.Kugo,J.Sato'94]

- We consider 4D NJL type model in which the fermion matter $\psi_I (I = 1, 2, \cdots, 27)$ belongs to $\mathbf{27}$ of E_6:

$$\mathcal{L} = \bar{\psi}^I i\bar{\sigma}^\mu \partial_\mu \psi_I + \sum_{p=1}^{2} G_{\mathbf{R}_p} (\psi_I \psi_J)_{\mathbf{R}_p} (\bar{\psi}^I \bar{\psi}^J)_{\overline{\mathbf{R}}_p},$$

where E_6 tensor product $(\mathbf{27} \otimes \mathbf{27})_S = (\overline{\mathbf{351'}}) \oplus (\overline{\mathbf{27}}) =: \mathbf{R}_1 \oplus \mathbf{R}_2$.

- As in usual 4D NJL models, by using the so-called auxiliary field method [36, 37, D.J.Gross,A.Neveu'78;T.Kugo'78], we calculate 1-loop effective potential. So, we introduce two auxiliary complex scalar fields $\Phi_{\overline{\mathbf{351'}}}$ and $\Phi_{\overline{\mathbf{27}}}$. (The detail discussion is omitted.)

Subgroup/little groups of E_6 [4, 5, R.Slansky'81;N.Y.'15]

- What are **maximal subgroups of** E_6?

  $SO(10) \times U(1), SU(6) \times SU(2), SU(3) \times SU(3) \times SU(3)$(R);

  $F_4, SU(3) \times G_2, USp(8), G_2, SU(3)$(S).

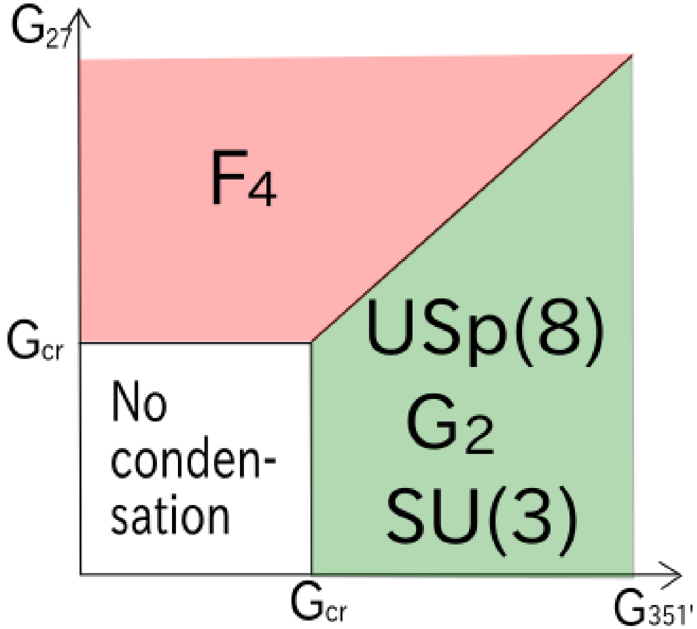
- What are **maximal little groups of** E_6 for $\mathbf{27}$ and $\mathbf{351}'$ reps?

  For $\mathbf{27}$ case, $SO(10)$(R); F_4(S)

  For $\mathbf{351}'$ case,
  $SO(10)$(R); $F_4, USp(8), G_2, SU(3), SU(4) \times SU(2)$(S).

All maximal little groups are **special subgroups** except $SO(10)$.

Results for 4D E_6 NJL model [34, T.Kugo,J.Sato'94]



Note: G_{27} and $G_{351'}$ are NJL-type coupling constants.

In the E_6 NJL model, E_6 is broken to its **special subgroups**.

$SU(n)$ symmetry breaking [1, 33, e.g.,T.Kugo,N.Y.'19]

- We consider 4D NJL type model in which the fermion matter $\psi_I (I = 1, 2, \cdots, n)$ belongs to $\square$ of $SU(n)$.

$$\mathcal{L} = \bar{\psi}^I i\bar{\sigma}^\mu \partial_\mu \psi_I + \sum_{p=1}^{1} G_{\mathbf{R}_p} (\psi_I \psi_J)_{\mathbf{R}_p} (\bar{\psi}^I \bar{\psi}^J)_{\overline{\mathbf{R}}_p},$$

  where $SU(n)$ tensor product $\left(\square \otimes \square\right)_S = \square\square =: \mathbf{R}_1$.
  Note: $\square$, $\square\square$ are Young tableaux.

- As in the 4D E_6 NJL model, we introduce an auxiliary complex scalar field $\Phi_{\square\square}$.

Subgroup/little groups of $SU(n)$ [5, N.Y.'15]

- What are **maximal subgroups of** $SU(n)$?

  $SU(m) \times SU(n-m) \times U(1) (m < n)$(R);

  for even n, $SO(n), USp(n)$(S);

  for odd n, $SO(n)$(S).

  In addition, for specific n, $SU(2), SU(3), \ldots$(S).

- What are **maximal little groups of** $SU(n)$ for $\square\square$ rep?

  For $\square\square$ case, $SU(n-1)$(R); $SO(n)$(S).

Results for 4D $SU(n)$ NJL model [1, e.g.,T.Kugo,N.Y.'19]

Summary for $SU(n)$ $\square$ fermion pair condensation

Representation	Symmetry on the minimum of potential
$\square\square$	$SU(n) \to SO(n)$(S)

In the $SU(n)$ NJL model, $SU(n)$ is always broken to its **special subgroups** $SO(n)$.

Next, we check an $SU(n)$ rank-2 anti-symmetric tensor fermion case because it may realize $SU(16) \to SO(10)$. (The symmetry breaking pattern is needed in special GUTs [23–25, N.Yamatsu,17-18].)

$SU(n)$ symmetry breaking [1, T.Kugo,N.Y.'19]

- We consider 4D NJL type model in which the fermion matter $\psi_I (I = 1, 2, \cdots, n(n-1)/2)$ belongs to $\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$ of $SU(n)$.

$$\mathcal{L} = \bar{\psi}^I i\bar{\sigma}^\mu \partial_\mu \psi_I + \sum_{p=1}^{2} G_{\mathbf{R}_p} (\psi_I \psi_J)_{\mathbf{R}_p} (\bar{\psi}^I \bar{\psi}^J)_{\overline{\mathbf{R}}_p},$$

where $\left(\begin{smallmatrix}\Box\\\Box\end{smallmatrix} \otimes \begin{smallmatrix}\Box\\\Box\end{smallmatrix} \right)_S = \begin{smallmatrix}\Box&\Box\\\Box&\Box\end{smallmatrix} \oplus \begin{smallmatrix}\Box\\\Box\\\Box\\\Box\end{smallmatrix}$; $\mathbf{R}_1 = \begin{smallmatrix}\Box&\Box\\\Box&\Box\end{smallmatrix}$, $\mathbf{R}_2 = \begin{smallmatrix}\Box\\\Box\\\Box\\\Box\end{smallmatrix}$.

- As in the 4D E_6 NJL model, we introduce two auxiliary complex scalar fields $\Phi_{\begin{smallmatrix}\Box&\Box\\\Box&\Box\end{smallmatrix}}$ and $\Phi_{\begin{smallmatrix}\Box\\\Box\\\Box\\\Box\end{smallmatrix}}$.

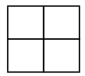
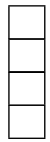
Subgroup/little groups of $SU(n)$ [5, N.Y.'15]

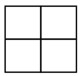
- What are **maximal subgroups of** $SU(n)$?

  $SU(m) \times SU(n-m) \times U(1) (m < n)$(R);

  for even n, $SO(n), USp(n)$(S); for odd n, $SO(n)$(S).

  In addition, for specific n, $SU(2), SU(3), ...$(S).

- What are **maximal little groups of** $SU(n)$ for  and  ?

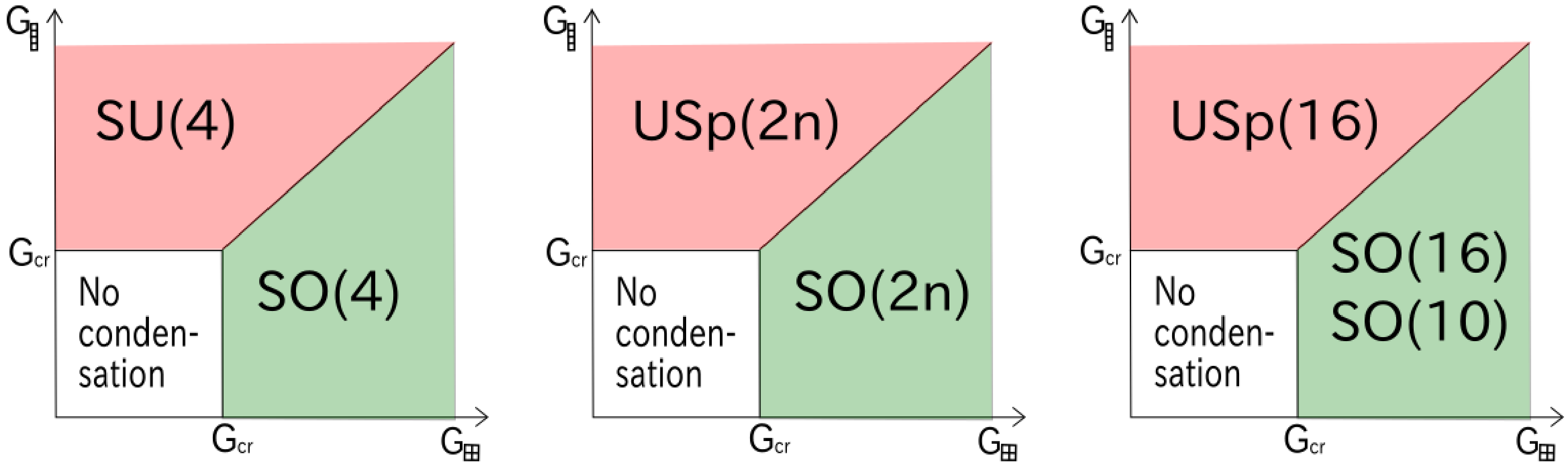
  For  case, $SU(2) \times SU(n-2)$(R); $SO(n), USp(2[n/2]), ...$(S)

  For  case, $SU(4) \times SU(n-4)$(R); $USp(2[n/2]), ...$(S).

Results for 4D $SU(n)$ NJL model [1, T.Kugo,N.Y.'19]

$SU(N)$ symmetry breaking for even $N = 2n(n \geq 2)$ cases:

$n = 2$ case $\quad$ $n \geq 3(n \neq 8)$ cases $\quad$ $n = 8$ case

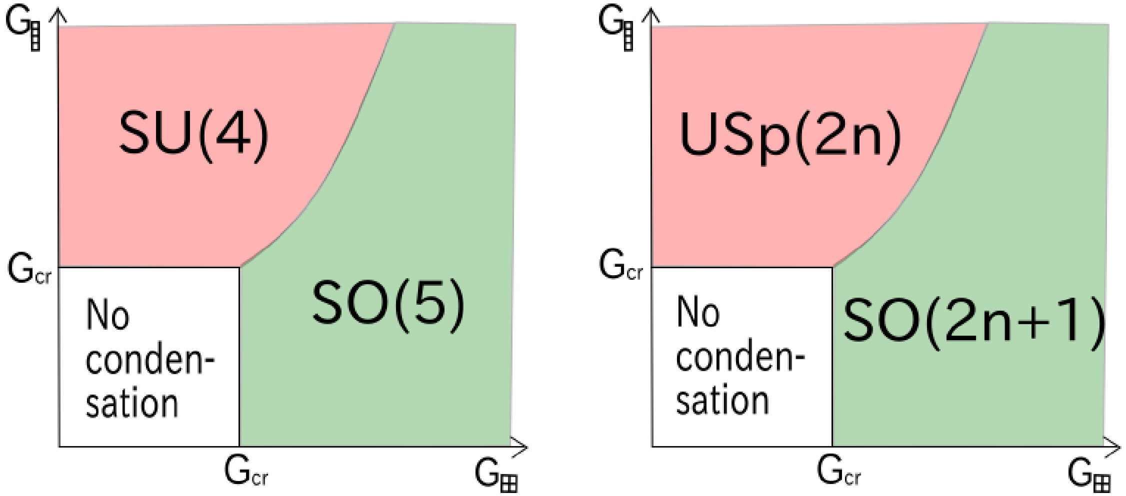


Note: $G_{\boxplus}$ and $G_{\begin{smallmatrix}\Box\\\Box\\\Box\\\Box\end{smallmatrix}}$ are NJL-type coupling constants.

Results for 4D $SU(n)$ NJL model [1, T.Kugo,N.Y.'19]

$SU(N)$ symmetry breaking for odd $N = 2n + 1 (n \geq 2)$ cases:

$n = 2$ case $\qquad$ $n \geq 3$ cases



Note: $G_{\boxplus}$ and $G_{\begin{smallmatrix}\square\\\square\\\square\\\square\end{smallmatrix}}$ are NJL-type coupling constants.

Summary

I have showed that symmetry breaking into **special** subgroups is not special, by using **dynamical symmetry breaking** in 4D $SU(N)$ NJL type models.

4D NJL type model for $SU(n)\,\begin{smallmatrix}\square\\\square\end{smallmatrix}$ fermion cases

- $SU(n)$ is broken to its special subgroups except $n = 5$ case.

$\Rightarrow$ symmetry breaking into **special** subgroups is not special.

$\Rightarrow$ To construct unified models beyond the SM, we must discuss symmetry breaking into **Regular** AND **Special** subgroups.

Several comments

- The maximal little groups for $SU(n)$ defining and adjoint scalar fields are only **Regular** subgroups. They are exceptional cases.

$\Rightarrow$ It is better not to apply the same results for scalar fields in other representations, composite scalar cases, etc.

- For a scalar field in an irreducible rep. of a group symmetry is broken to one of its maximal little groups (as Michel's conjecture [29, L.Michel'80] says).

  For multi scalar fields, symmetry can be broken to its non-maximal little groups.

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