Compact boson stars and charged black hole in the CP^{2n+1} model

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Abstract

Q-balls are non-topological solitons that appear in certain nonlinear complex scalar field models. The scalar field models with standard kinetic terms and V-shaped potential gives rise to compact Q-balls. When they couple with gravity, (compact) boson stars arise. We study compact Q-balls (-shells) and compact boson stars (boson shell stars) in nonlinear sigma model with CP^N target space. The models with odd integer $N(=2n+1)$ and suitable potential can be parametrized by Nth complex scalar fields and they support compact solutions. For $n=0,1$ the solutions form Q-ball while $2\leq n$ they always have shell structure. We find the $U(1)$ gauged compact Q-balls(-shells) and their gravitating solutions. The interior of the shell-like solutions can be empty space or harbor a black hole or a naked singularity. We discuss basic properties of our solutions of charged black holes.

The gravitating CP^N compact Q-ball

The principal variable X

The action

$$S=\int d^4x\sqrt{-g}\Big[\frac{R}{16\pi G}-\frac{M^2}{2}g^{\mu\nu}\mathrm{Tr}(X^{-1}\partial_\mu X)(X^{-1}\partial_\nu X)-\lambda^2 V\Big]$$

The potential (compacton potential)

$$V(X)=\frac{1}{2}\mathrm{Tr}(1-X_\infty^{-1}X)^{1/2}$$

A set of complex scalar fields u_i $\quad i=1,2,\cdots,N$

$$X=\begin{pmatrix}1_{N\times N} & 0\\ 0 & -1\end{pmatrix}+\frac{2}{\vartheta^2}\begin{pmatrix}-u\otimes u^\dagger & iu\\ iu^\dagger & 1\end{pmatrix}\qquad \vartheta\equiv\sqrt{1+u^\dagger\cdot u}$$

$$S=\int d^4x\sqrt{-g}\Big[\frac{R}{16\pi G}+4M^2g^{\mu\nu}\frac{\partial_\mu u^\dagger\cdot\partial_\nu u}{(1+u^\dagger\cdot u)}-4M^2g^{\mu\nu}\frac{(\partial_\mu u^\dagger\cdot u)\,(u^\dagger\cdot\partial_\nu u)}{(1+u^\dagger\cdot u)^2}-\lambda^2 V\Big]$$

Schwarzschild metric

$$\begin{aligned}ds^2&=g_{\mu\nu}dx^\mu dx^\nu\\&=A^2(r)C(r)dt^2-\frac{1}{C(r)}dr^2-r^2(d\theta^2+\sin^2\theta d\varphi^2)\qquad C(r)=1-\frac{2m(r)}{r}\end{aligned}$$

Einstein equations

$$G_{\mu\nu}=8\pi G T_{\mu\nu}$$

Stress-energy tensor

$$T_{\mu\nu}=2\frac{\partial\mathcal{L}}{\partial g^{\mu\nu}}-g_{\mu\nu}\mathcal{L}$$

The ansatz for CP^{2n+1} model

$$u_{m+n+1}=\sqrt{\frac{4\pi}{2n+1}}f(r)Y_{nm}(\theta,\varphi)e^{i\omega t}$$

The dimensionless coordinates $x^\mu\to\frac{\lambda}{M}x^\mu$

$$(x^0,x^1,x^2,x^3)=(t,r\sin\theta\cos\varphi,r\sin\theta\sin\varphi,r\cos\theta)$$

!

$$|u_{m+n+1}|^2=\frac{4\pi}{2n+1}f(r)^2\sum_{m=-n}^{n}Y^*_{nm}(\theta,\varphi)Y_{nm}(\theta,\varphi)\Rightarrow f(r)^2$$

We get an ordinary differential equation of the variable r

The system

The matter field equation

$$Cf''+\frac{2Cf'}{r}+C'f'+\frac{A'Cf'}{A}-\frac{n(n+1)f}{r^2}+\frac{\omega^2 f(1-f^2)}{A^2C(1+f^2)}-\frac{2Cff'^2}{1+f^2}-\frac{1}{8}\sqrt{1+f^2}=0$$

The Einstein equations

$$A'=\alpha rA\Big(\frac{4\omega^2f^2}{A^2C^2(1+f^2)^2}+\frac{4f'^2}{(1+f^2)^2}\Big)$$

$$C'=\frac{1-C}{r}-\alpha r\Big(\frac{4\omega^2f^2}{A^2C(1+f^2)^2}+\frac{4Cf'^2}{(1+f^2)^2}+\frac{4n(n+1)f^2}{r^2(1+f^2)}+\frac{f}{\sqrt{1+f^2}}\Big)$$

coupling constant $\quad\alpha\equiv 8\pi G\lambda^2$

The (dimensionless) energy density

$$\mathcal{H}=\frac{4}{(1+f^2)^2}\Big(Cf'^2+\frac{\omega^2f^2}{A^2C}+\frac{n(n+1)(1+f^2)f^2}{r^2}\Big)+\frac{f}{\sqrt{1+f^2}}$$

The conserved charge

$$Q=\frac{16\pi\omega}{2n+1}\int r^2dr\frac{f^2}{AC(1+f^2)^2}$$

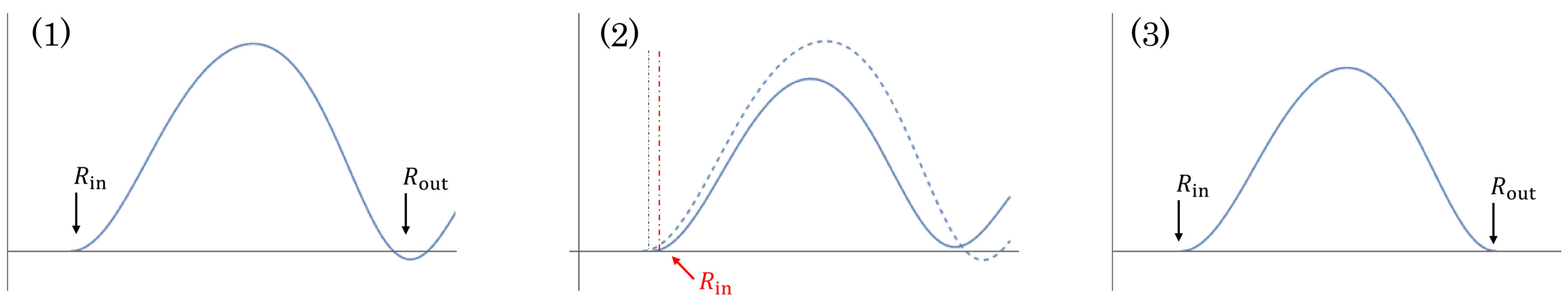
Strategy of the numerical analysis

Shooting method

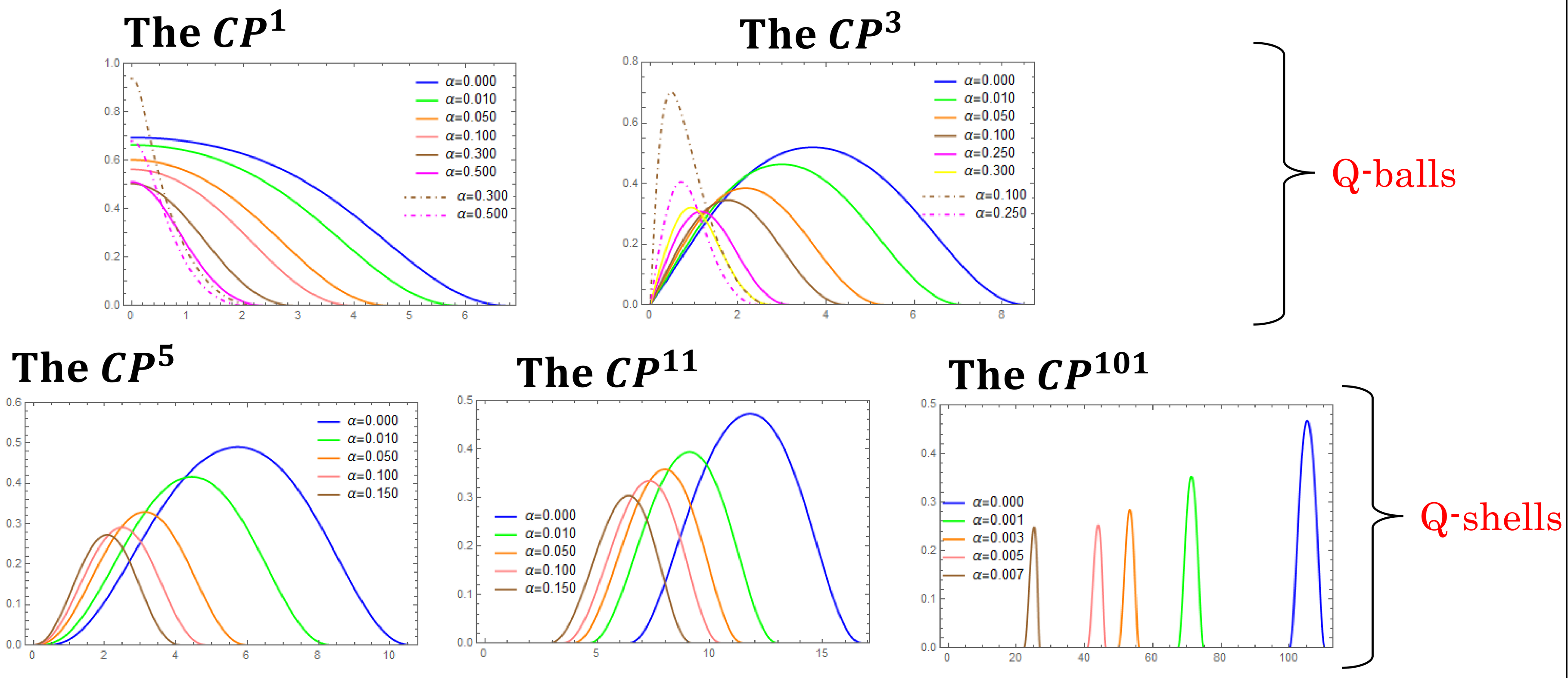
(1) We numerically integrate the radial equation and determinate a value of the radius R_{out} such that $f'(R_{\text{out}})=0$.

(2) A value of the expression $f(R_{\text{out}})$ is used to modify an initial shooting parameter R_{in} according to $f(R_{\text{out}})\to 0$.

(3) The loop is interrupted when $|f(R_{\text{out}})|<10^{-6}$ is satisfied.

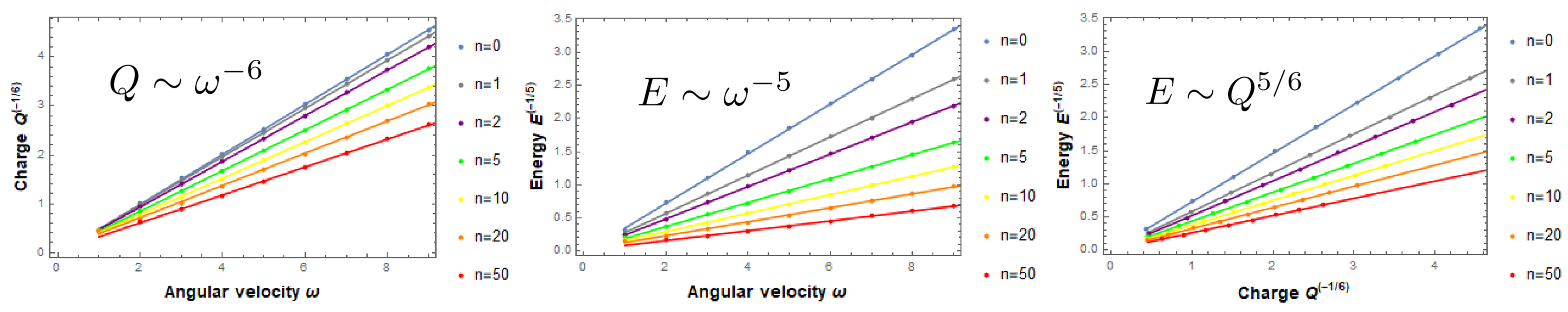


The solutions



The matter function $f(r)$ for the $CP^1, CP^3, CP^5, CP^{11}$ and CP^{101}

The stability



Charge Q vs. Angular velocity ω — Energy E vs. Angular velocity ω — Energy E vs. Charge Q

A general criterion of stability

$$\frac{\omega}{Q}\frac{dQ}{d\omega}<0\qquad \omega\text{: angular velocity}$$

Energy "bound"

$$E\sim Q^{|\alpha|}\ :\ \alpha=\frac{5}{6}<1$$

In the case of black hole

For $n=0,1$ there are no non-trivial solutions

For $n\geq 2$ the shells need not be empty in their interior

Black holes are allowed to exist in the region $0<r_H<R_{\text{in}}$

The boundary condition

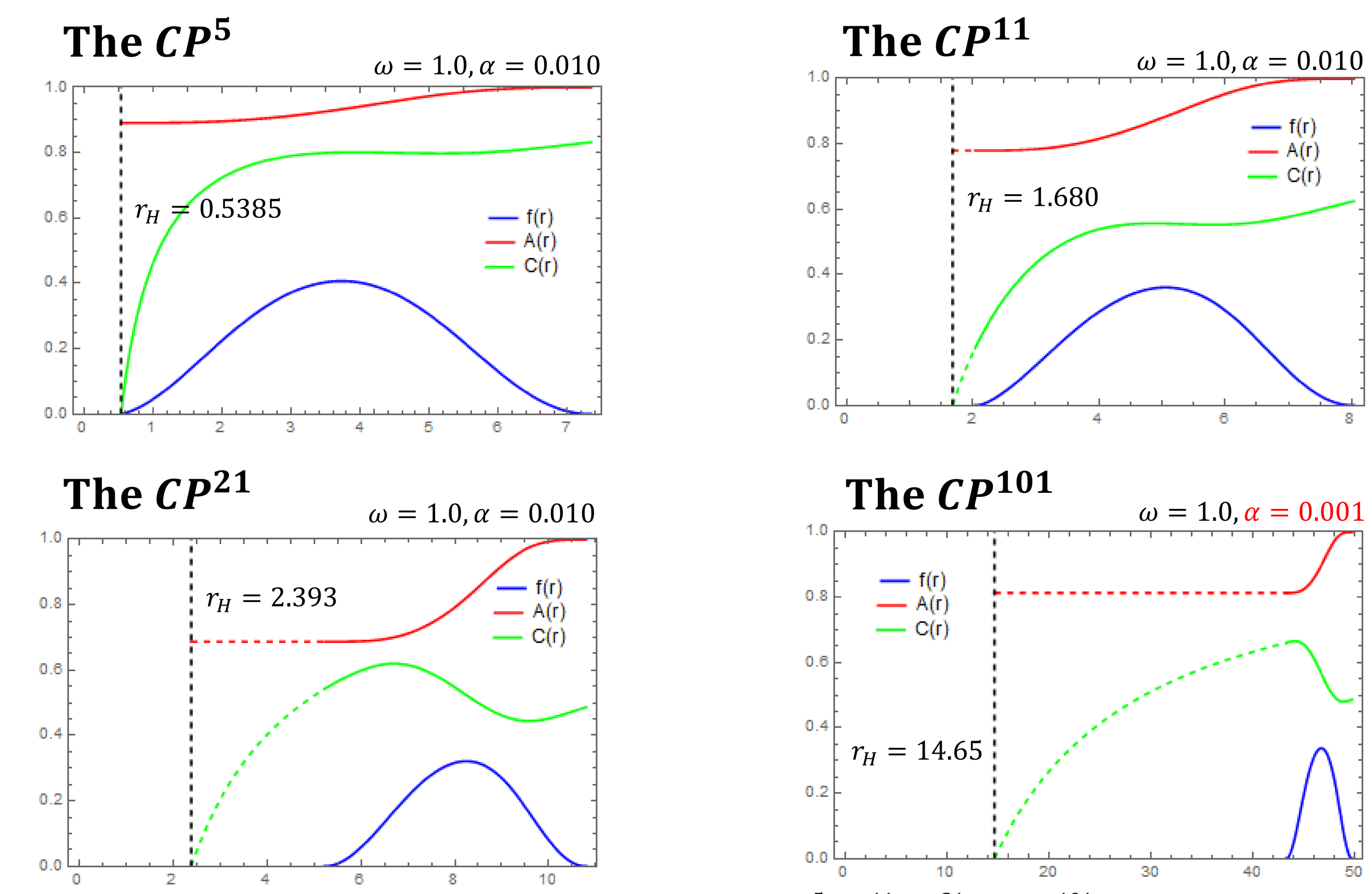
$$f(R_{\text{in}})=0,\quad f'(R_{\text{in}})=0\qquad A(R_{\text{out}})=1$$

$$f(R_{\text{out}})=0,\quad f'(R_{\text{out}})=0\qquad C(r_H)=0\quad\Big(C(r)=1-\frac{r_H}{r}\Big)$$

R_{in} : inner compacton radius $\quad R_{\text{out}}$: outer compacton radius

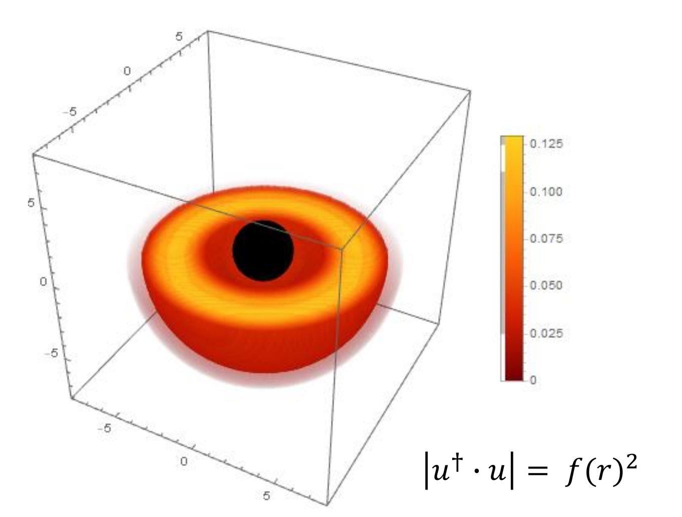
r_H : event horizon

The harbor-type solutions

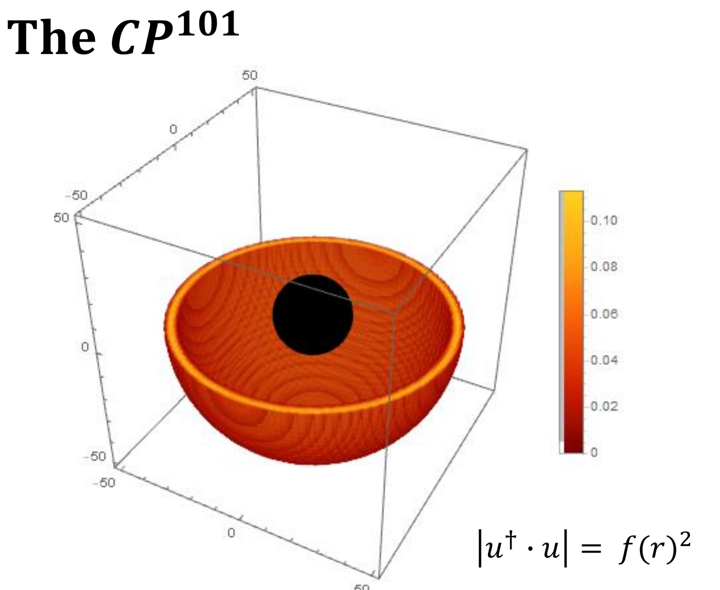


The functions $f(r), A(r)$ and $C(r)$ for the CP^5, CP^{11}, CP^{21} and CP^{101}

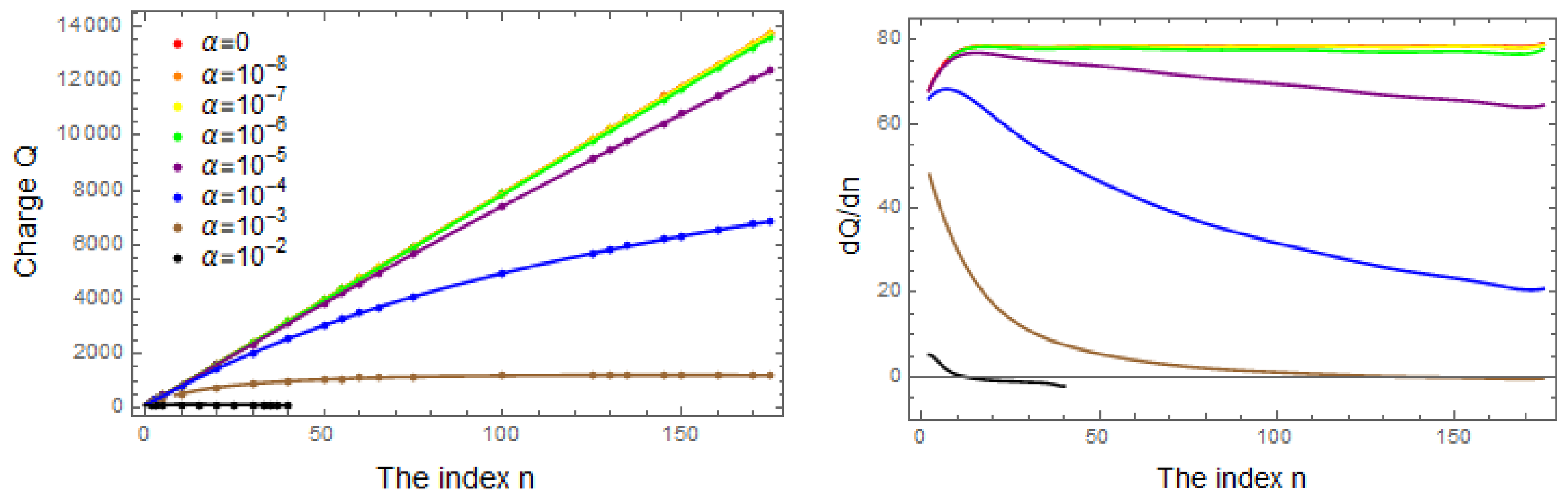
The CP^{11}



The CP^{101}



The relation between charge Q and the number of fields $(2n+1)$



The number of fields can be interpreted as the number of particles

The CP^N gravitating $U(1)$ gauged compact Q-ball(-shell) model

$$S=\int\sqrt{-g}d^4x\Big[\frac{R}{16\pi G}-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+4M^2g^{\mu\nu}\Big[\frac{D_\mu u^\dagger\cdot D_\nu u}{(1+u^\dagger\cdot u)}-\frac{(D_\mu u^\dagger\cdot u)(u^\dagger\cdot D_\nu u)}{(1+u^\dagger\cdot u)^2}\Big]-\lambda^2V\Big]$$

$$D_\mu=\partial_\mu-ieA_\mu$$

The ansatz for CP^{2n+1} model

$$u_{m+n+1}=\sqrt{\frac{4\pi}{2n+1}}f(r)Y_{nm}(\theta,\varphi)e^{i\omega t}\qquad A_\mu dx^\mu=A_0(r)\,dt$$

$$Cf''+C'f'+\frac{A'Cf'}{A}+\frac{2C}{r}f'-\frac{n(n+1)f}{r^2}+\frac{(1-f^2)b^2f}{A^2C(1+f^2)}-\frac{2Cff'^2}{(1+f^2)}-\frac{\lambda^2}{8M^2}\sqrt{1+f^2}=0$$

$$b''+\frac{2rA-A'r^2}{Ar^2}b'+\frac{8e^2M^2}{C}\left(\frac{bf^2}{(1+f^2)^2}\right)=0$$

$$A'=4M^2\kappa Ar\Big[\frac{b^2f^2}{A^2C^2(1+f^2)^2}+\frac{f'^2}{(1+f^2)^2}\Big]$$

$$C'=\frac{1-C}{r}-4M^2\kappa r\Big[\frac{b^2f^2}{A^2C(1+f^2)^2}+\frac{Cf'^2}{(1+f^2)^2}+\frac{n(n+1)f^2}{(1+f^2)r^2}\Big]-\frac{\kappa r}{2A^2e^2}b'^2-\kappa\lambda^2r\frac{f}{\sqrt{1+f^2}}$$

coupling constant κ $\qquad (b(r)=\omega-eA_0(r))$

The (dimensionless) energy density

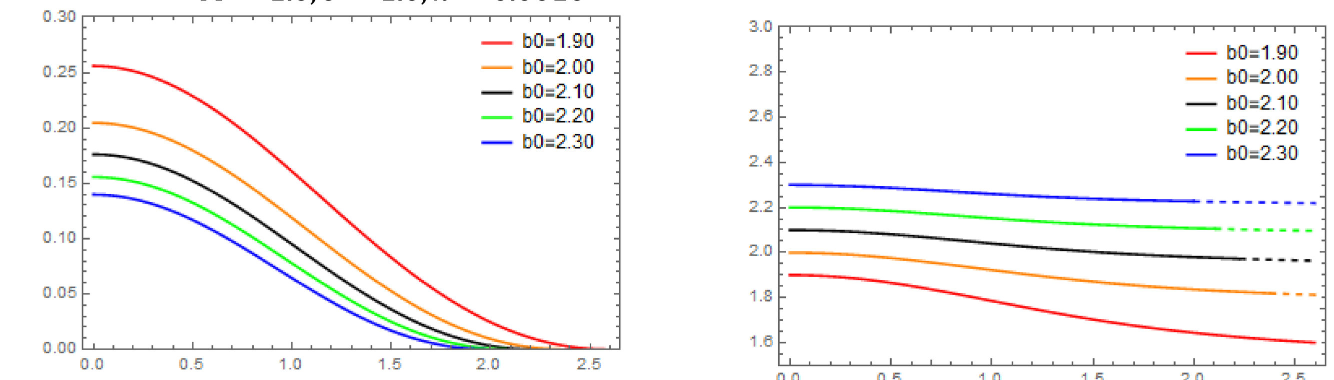
$$\mathcal{H}=\frac{4M^2}{(1+f^2)^2}\left(Cf'^2+\frac{b^2f^2}{A^2C}+\frac{n(n+1)(1+f^2)f^2}{r^2}\right)+\frac{b'^2}{2A^2e^2}+\frac{\lambda^2f}{\sqrt{1+f^2}}$$

The conserved charge

$$Q=\frac{16\pi M^2}{2n+1}\int r^2dr\frac{bf^2}{AC(1+f^2)^2}$$

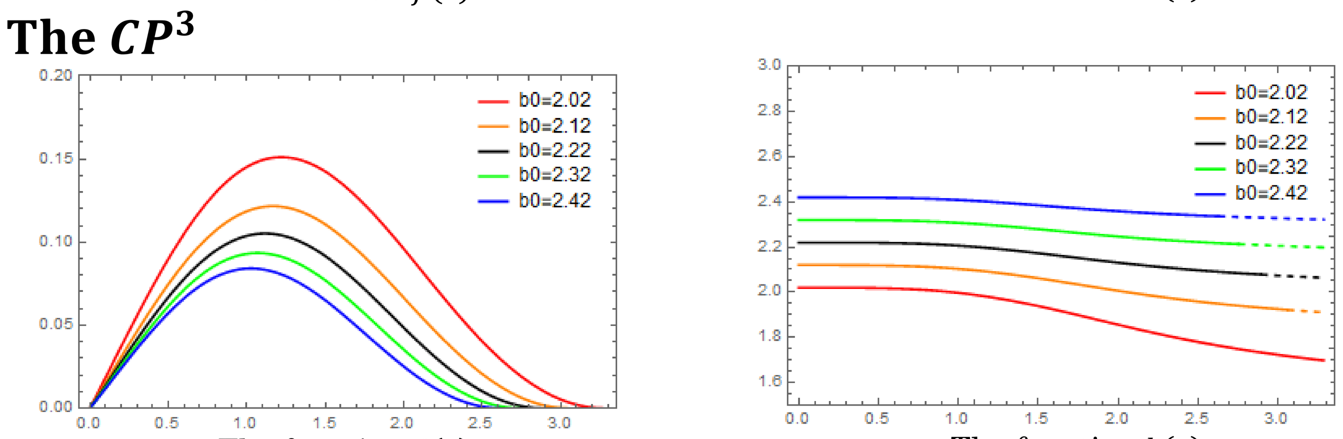
The solutions

The CP^1 $M=1.0, e=1.0, \kappa=0.0010$



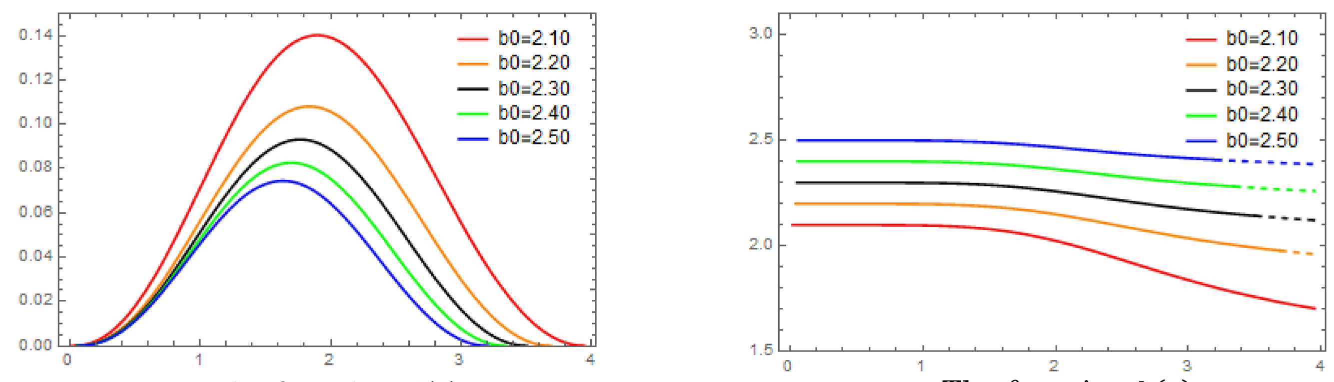
The function $f(r)$ The function $b(r)$

The CP^3



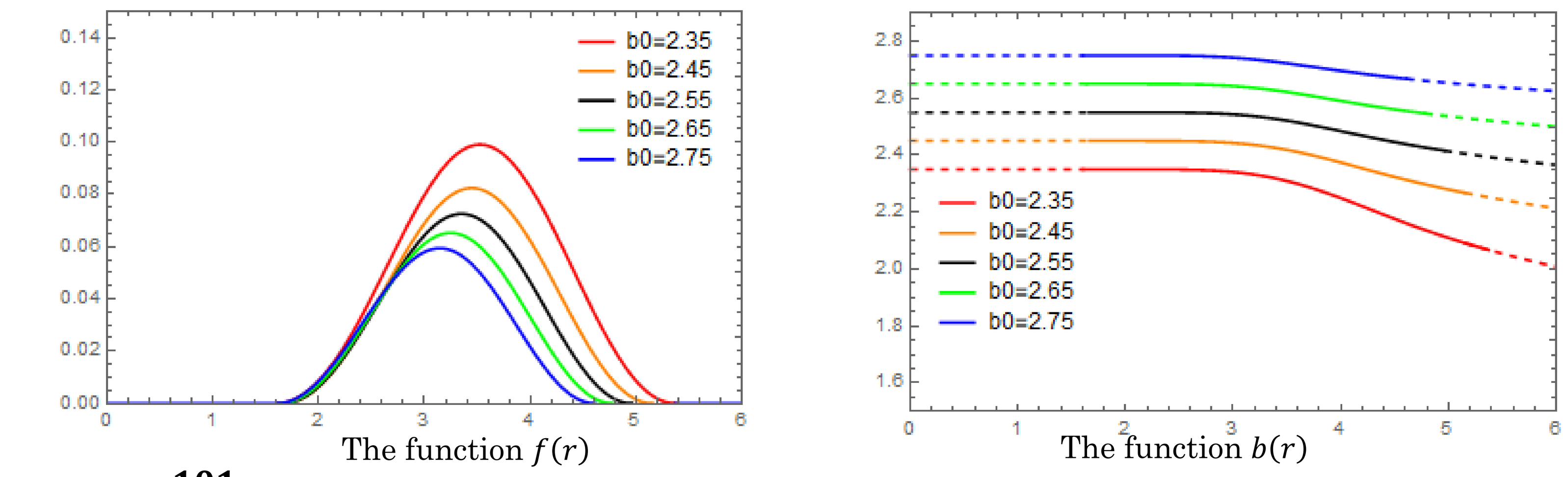
The function $f(r)$ The function $b(r)$

The CP^5



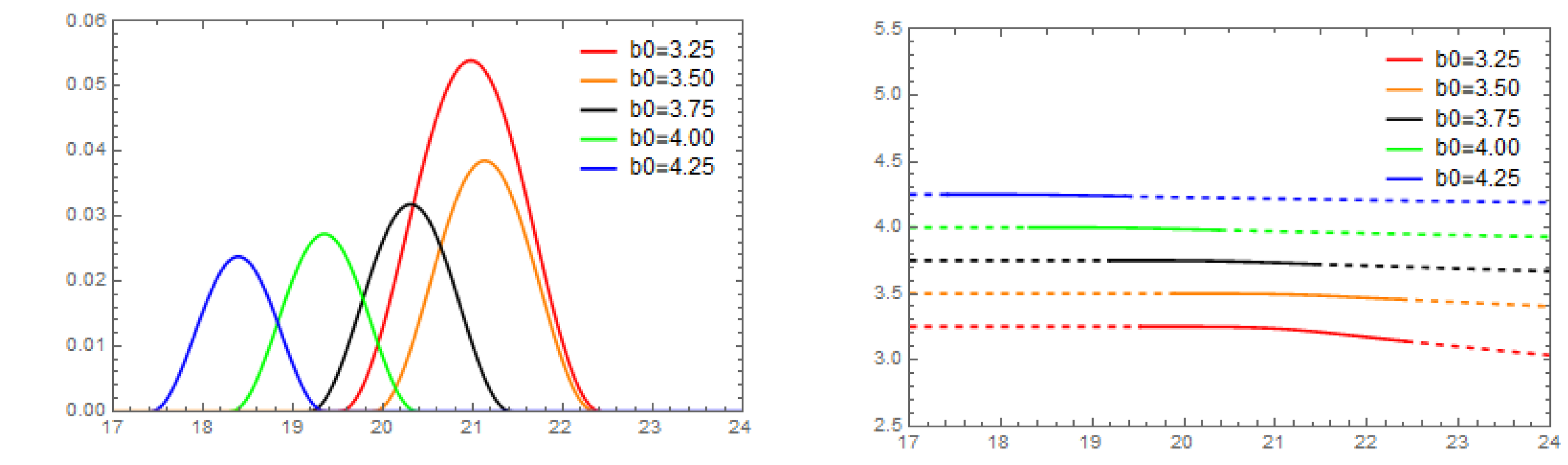
The function $f(r)$ The function $b(r)$

The CP^{11} $M=1.0, e=1.0, \kappa=0.0010$



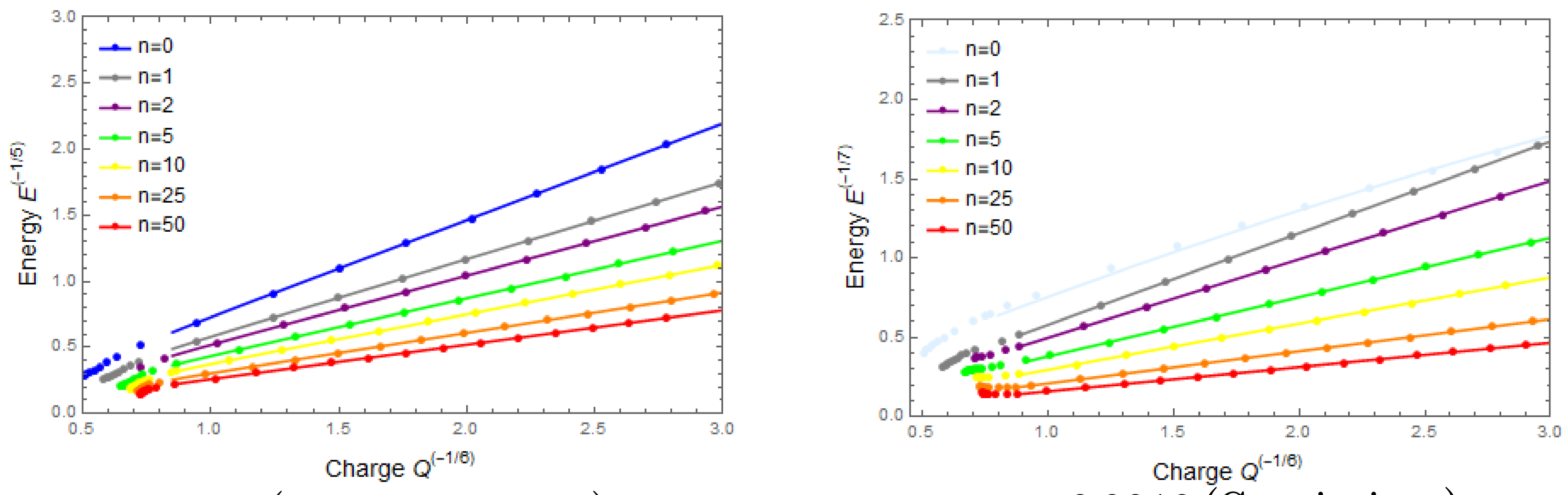
The function $f(r)$ The function $b(r)$

The CP^{101}



The function $f(r)$ The function $b(r)$

The stability



κ = 0 (Non gravitaitng) $\qquad$ κ = 0.0010 (Gravitaitng)

$E\sim Q^{5/6}$ $\qquad$ $E\sim Q^{7/6}$

Stable $\qquad$ Unstable

In the case of black hole

The boundary condition

$$f(R_{\rm in})=0,\quad f'(R_{\rm in})=0\qquad A(R_{\rm in})=A_i,\quad A(R_{\rm out})=1$$

$$f(R_{\rm out})=0,\quad f'(R_{\rm out})=0\qquad C(R_{\rm in})=1-\frac{r_H}{R_{\rm in}}+\frac{r_c}{R_{\rm in}^2}\quad\left(r_c=\frac{\kappa Q_H^2}{2A_i^2e^2}\right)$$

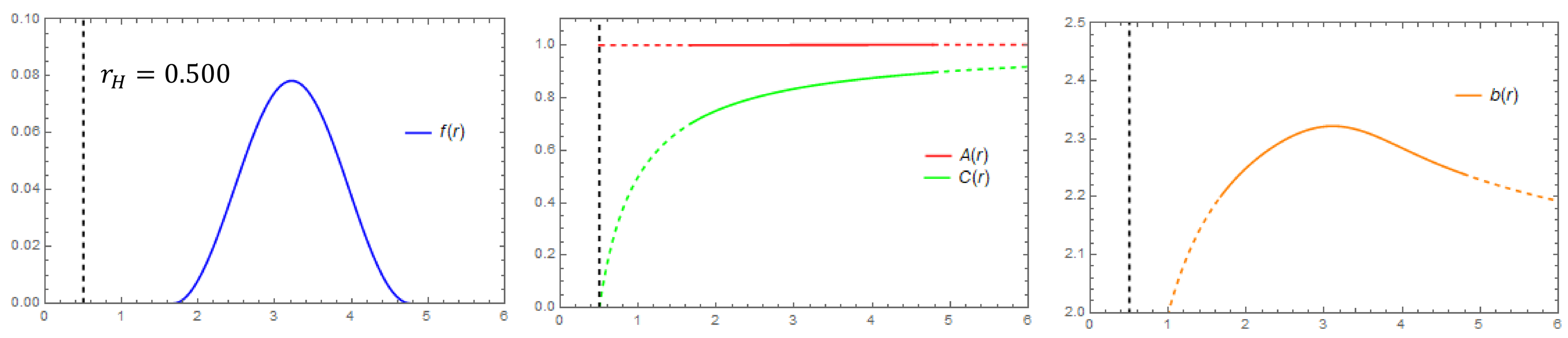
$$b(R_{\rm in})=b_0,\quad b'(R_{\rm in})=b_0-\frac{Q_H}{R_{\rm in}}$$

$R_{\rm in}$: inner compacton radius $\quad$ $R_{\rm out}$: outer compacton radius $\quad$ A_i, b_0 : free parameter

r_H : event horizon $\quad$ Q_H : horizon charge

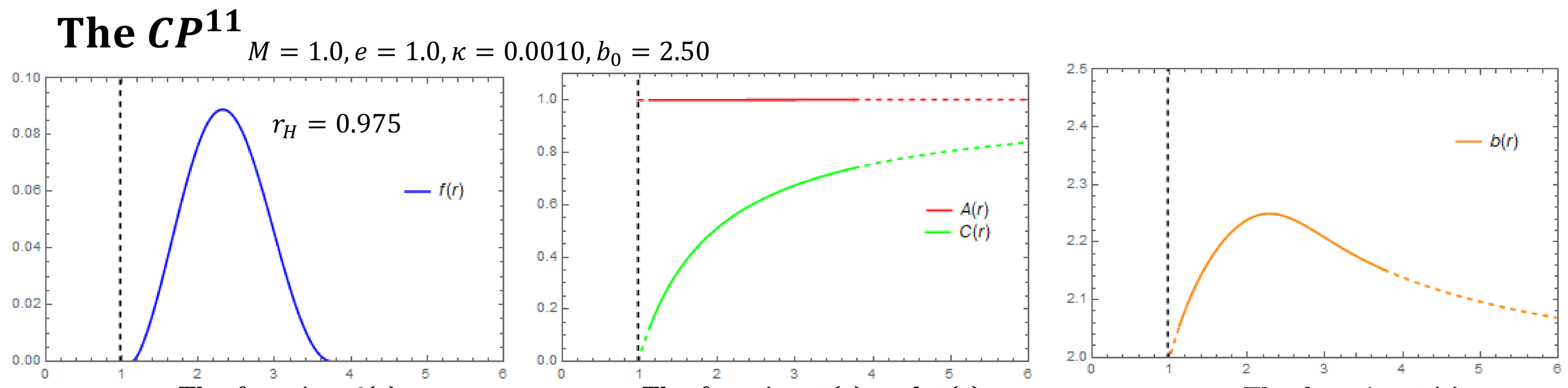
The harbor-type solutions

The CP^{11} $M=1.0, e=1.0, \kappa=0.0010, b_0=2.50$



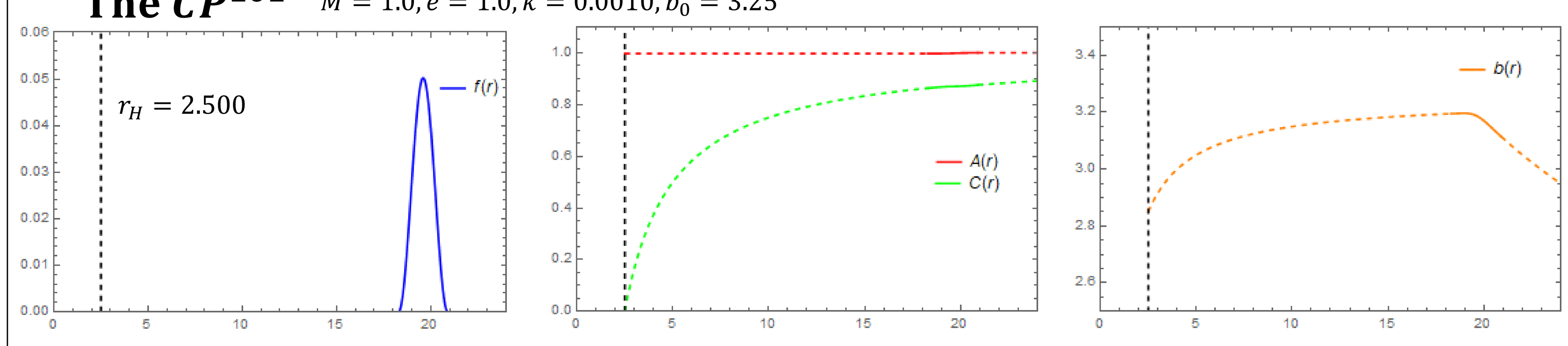
The function $f(r)$ The function $A(r)$ and $C(r)$ The function $b(r)$

The CP^{11} $M=1.0, e=1.0, \kappa=0.0010, b_0=2.50$



The function $f(r)$ The function $A(r)$ and $C(r)$ The function $b(r)$

The CP^{101} $M=1.0, e=1.0, \kappa=0.0010, b_0=3.25$



The function $f(r)$ The function $A(r)$ and $C(r)$ The function $b(r)$

Conclusions

- We have considered the family of CP^{2n+1} nonlinear sigma models coupled with gravity.
- For $n=0,1$ we got the gravitating compact Q-balls, and for $n\geq 2$ found the Q-shell solutions.
- Q-shells can harbor a black hole in the region $0<r_H<R_{\rm in}$.
- There is an upper bound for Q for certain coupling constant α.
- We get the CP^N $U(1)$ gauged Q-balls for $n=0,1$ and Q-shells for $n\geq 2$.
- Our new $U(1)$ gauged solutions satisfy the $E\sim Q^{5/6}$ (non gravitating) or $E\sim Q^{7/6}$ (gravitating).

Reference

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