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YITP workshop ``Strings and Fields 2019'', Kyoto

$T\overline{T}$-deformation and Holography

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Main subjects in this talk

1) $T\overline{T}$ -deformation of 2D QFT, especially 2D CFT

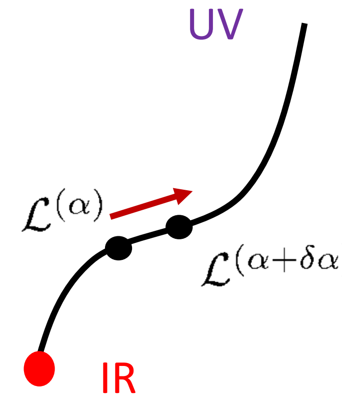
2) Gravity duals for $T\overline{T}$-deformed 2D CFT

For a nice review, see Y. Jiang, 1904.13376.

0. Introduction

The basics on the $T\overline{T}$-deformation (5 slides)

What is $T\overline{T}$-deformation?



For $T\overline{T}$-deformed CFT

Assume the set of 2D QFTs described by Lagrangian.

Consider a trajectory in the theory space parametrized by α

and denote the Lagrangian at each point of the trajectory by $\mathcal{L}^{(\alpha)}$.

The flow for theories on the trajectory is triggered by an irrelevant operator "$\det T_{\mu\nu}^{(\alpha)}$".

$$\begin{aligned} \mathcal{L}^{(\alpha+\delta\alpha)} &= \mathcal{L}^{(\alpha)} + \frac{\delta\alpha}{4} \det T_{\mu\nu}^{(\alpha)} \\ &\equiv \mathcal{L}^{(\alpha)} - \delta\alpha \, \mathrm{T}\overline{\mathrm{T}}^{(\alpha)} \end{aligned}$$

$\delta\alpha$: coupling constant
with dimension (length)^2

Here the $\mathrm{T}\overline{\mathrm{T}}$-operator (as composite operator) is given by

$$\mathrm{T}\overline{\mathrm{T}} \equiv -\frac{1}{4} \det T_{\mu\nu} = T\overline{T} - \Theta^2 \qquad \left(T \equiv T_{zz}, \quad \overline{T} \equiv T_{\bar{z}\bar{z}}, \quad \Theta \equiv T_{z\bar{z}}\right)$$

NOTE: the undeformed theory $\mathcal{L}^{(0)}$ may be a general 2D QFT.

Factorization of expectation value

$$\langle \mathrm{T\overline{T}} \rangle = \langle T \rangle \langle \overline{T} \rangle - \langle \Theta \rangle^2$$

[A. B. Zamolodchikov, hep-th/0401146]

This factorization is valid for stationary states under the following assumptions:

1. Local translational and rotational invariance (L)

   The existence of local $T_{\mu\nu}$ and $T_{\mu\nu} = T_{\nu\mu}$

2. Global translational invariance (G)

   $\langle \mathcal{O}_i(z) \rangle$ does not depend on z (for any local field $\mathcal{O}_i(z)$).

3. Infinite separations (G) There should exist at least one direction,

   such that for any $\mathcal{O}_i$ and $\mathcal{O}_j$, $\lim_{t \to \infty} \langle \mathcal{O}_i(z + et) \mathcal{O}_j(z') \rangle = \langle \mathcal{O}_i \rangle \langle \mathcal{O}_j \rangle$

   Note: Assumps. 2 & 3 ➡ 2D space is infinite plane or infinitely long cylinder

4. CFT limit at short distances (L) To make definition of $T\overline{T}$-op. unambiguous.

Application of the factorization

Let us consider a 2D QFT on an infinitely long cylinder.

$(\tau : \text{time}, \ x : \text{compactified with a period } L)$

Note: $\langle T \rangle, \ \langle \overline{T} \rangle$ vanish on infinite plane.

The factorization enables us to compute the expectation value of $T\overline{T}$- op.

With an arbitrary non-degenerate eigenstate of the energy $|n\rangle$, such that

$$H|n\rangle = E_n|n\rangle$$

one obtains that

[A. B. Zamolodchikov, hep-th/0401146]

$$\begin{aligned} \langle n|\mathrm{T}\overline{\mathrm{T}}|n\rangle &= \langle n|T|n\rangle\langle n|\overline{T}|n\rangle - \langle n|\Theta|n\rangle^2 \\ &= -\frac{1}{4}\left(\langle n|T_{\tau\tau}|n\rangle\langle n|T_{xx}|n\rangle - \langle n|T_{\tau x}|n\rangle^2\right) \end{aligned}$$

With the physical meaning of the stress tensor,

$$\langle T_{\tau\tau} \rangle = \frac{E_n}{L}, \qquad \langle T_{xx} \rangle = \frac{\partial E_n}{\partial L}, \qquad \langle T_{\tau x} \rangle = \frac{iP_n}{L},$$

(energy density) (pressure) (momentum density)

$T\overline{T}$- flow equation

$$\frac{\mathrm{d}S^{(\alpha)}}{\mathrm{d}\alpha} = -\int \mathrm{d}^2x\, (\mathrm{T}\overline{\mathrm{T}})^{(\alpha)}$$

[Smirnov -Zamolodchikov, 1608.05499]

[Cavaglia-Negro-Szecsenyi -Tateo, 1608.05534]

$$\frac{\partial E_n}{\partial \alpha} = -L\langle n|\mathrm{T}\overline{\mathrm{T}}|n\rangle$$

$$\left((\because) \quad \frac{\partial}{\partial \alpha}\langle n|H|n\rangle = \int \mathrm{d}x\, \langle n|\mathrm{T}\overline{\mathrm{T}}|n\rangle = L\langle n|\mathrm{T}\overline{\mathrm{T}}|n\rangle \right)$$

$$4\frac{\partial E_n}{\partial \alpha} = E_n \frac{\partial E_n}{\partial L} + \frac{P_n^2}{L}$$

A forced inviscid Burgers eq.

By solving the Burgers eq., the spectrum of the $T\overline{T}$-deformed system is computed exactly.

- One has to know the original spectrum E_n → CFT_2, Integrable QFT_2 (IQFT_2).
- Even if the original spectrum is unknown, the deformation effect itself can be examined.

  → \`\`Integrable'' deformation
- The deformed action can also be obtained.

  EX a free massless scalar → Nambu-Goto action (with static gauge)

A comment on $T\overline{T}$-deformation of 2D IQFT

Indeed, the $T\overline{T}$-deformation is really integrable deformation of relativistic IQFT_2 .

In relativistic IQFT_2 , it is well known that the *N*-body S-matrix is factorized to the product of the 2-body S-matrices,

$$S(p_1, p_2, \ldots, p_N) = \prod_{i<j} S(p_i, p_j)$$

➡ Quantum Integrability

Then the 2-body S-matrix can be determined from the assumptions, Lorentz symmetry, unitarity, crossing symmetry, Yang-Baxter eq. (S-matrix bootstrap)

$$S(\theta) \sim S(\theta) f(\theta)$$

up to the CDD factor $f(\theta)$

[Castilejo-Dalitz-Dyson, Phys. Rev. 101 (1956) 453]

FACT

The $T\overline{T}$-deformation deforms only the CDD factor. [Mussardo-Simon, hep-th/9903072]

➡ The quantum integrability is preserved (integrable deformation).

Plan of this talk

1. $T\overline{T}$-deformation of CFT_2 (5 slides)

   The spectrum of $T\overline{T}$-deformed CFT on a infinitely long cylinder.

   The behavior of entropy

2. Gravity duals for $T\overline{T}$-deformed CFT_2 (6 + 4 slides)

   i) Positive sign: RG flow from Little String Theory (LST) to Sch. AdS BH

   [Giveon-Itzhaki-Kutasov, 1701.05576]

   ii) Negative sign: cut-off AdS [McGough-Mezei-Verlinde, 1611.03470]

3. Summary and outlook

1. $T\overline{T}$-deformation of 2D CFT

[Smirnov-Zamolodchikov, 1608.05499]

[Cavaglia-Negro-Szecsenyi-Tateo, 1608.05534]

Consider $T\overline{T}$-deformation of CFT on an infinitely long cylinder with period *L*.

Then let us take the CFT data given by

$$E_n(L) = \frac{2\pi}{L}\left(\Delta_n + \overline{\Delta}_n - \frac{c}{12}\right), \qquad P_n(L) = \frac{2\pi}{L}(\Delta_n - \overline{\Delta}_n)$$

By solving the Burgers eq., the energy spectrum of the $T\overline{T}$-deformed CFT is

$$E_n(L,\alpha) = \frac{2\pi}{L}\left(\frac{1}{b}\right)\left[-1 + \sqrt{1 + 2bM_n + b^2 J_n^2}\right]$$

$$M_n \equiv \Delta_n + \overline{\Delta}_n - \frac{c}{12}, \quad J_n \equiv \Delta_n - \overline{\Delta}_n, \quad b \equiv \frac{\pi\alpha}{L^2}$$ (dimensionless)

When $\alpha = 0$, the original spectrum is reproduced.

Here we should be careful for the signature of the coupling α .

i) $\alpha > 0 \quad (b > 0)$: ``good'' sign (positive case),

ii) $\alpha < 0 \quad (b < 0)$: ``bad'' sign (negative case)

This terminology was introduced in [Giveon-Itzhaki-Kutasov, 1701.05576]

Why is the signature so significant?

You may wonder why the signature of the coupling should be significant.

Consider a Φ^4 theory in four dimensions, for example.

Then deform this system by adding a term to the original action,

$$-\alpha \int d^4x \, \Phi^6 \quad .$$

If $\alpha > 0$, then the potential is still bounded and the vacuum is stable.

But, if $\alpha < 0$, then the potential is not bounded any more
and the vacuum becomes unstable.

Thus, the signature of irrelevant perturbation is significant to physics.

For simplicity, let us see the ground state i.e., $\Delta_n = \overline{\Delta}_n = 0$

The ground-state energy is given by

$$E_0(L,\alpha) = \frac{2\pi}{L}\left(\frac{1}{b}\right)\left[-1+\sqrt{1-\frac{bc}{6}}\right]$$

When $\alpha > 0$, the following condition should be satisfied,

$$c \le \frac{6}{b}$$

So the large *c* limit might appear to be problematic in looking for the gravity dual.

A possible resolution [Giveon-Itzhaki-Kutasov, 1701.05576]

`t Hooft like limit: $c \to \infty, \quad b \to 0, \quad bc : \text{fixed} \ < 6$

Then the large c limit is possible while avoiding the imaginary part in energy.

Entropy

The case with $\Delta_n = \overline{\Delta}_n$

($T\overline{T}$-deformation preserves the modular invariance)

The entropy of the deformed system still can be described by the Cardy formula:

$$S(E_n) \cong 2\pi\sqrt{\frac{c}{3}M_n}$$

(at high energy)

This entropy can be evaluated as

$$S(\mathcal{E}) \cong 2\pi\sqrt{\frac{c}{6}(2\mathcal{E} + b\mathcal{E}^2)}$$

$$\mathcal{E} \equiv \frac{EL}{2\pi}$$

This is valid for $\mathcal{E} \gg 1$. For $1 \ll \mathcal{E} \ll 1/b$, the entropy of the original CFT is recovered.

On the other hand, for $\mathcal{E} \gg 1/b$

$$S \cong 2\pi\sqrt{\frac{bc}{6}}\,\mathcal{E} = \sqrt{\frac{\pi c\alpha}{6}}\,E$$

[Giveon-Itzhaki-Kutasov, 1701.05576]

The resulting entropy is proportional to *E* and this is a Hagedorn entropy $S = \beta_H E$

➡ Little String Theory (LST) (at high energy), the usual AdS (at low energy)

Another resolution?

It seems likely that there is no problem for the large c limit for the case with $\alpha < 0$.

But in this case, the energies of the highly excited states become imaginary,

$$E_n(L,\alpha) = \frac{2\pi}{L}\left(\frac{1}{|b|}\right)\left[1 - \sqrt{1 - 2|b|M_n}\,\right] \qquad b \equiv \frac{\pi\alpha}{L^2}$$

So, it is rather necessary to introduce a cut-off for the energy to put the upper bound.

$$E < E_{\max} \equiv \frac{2L}{|\alpha|}$$

The entropy also has the upper bound,

$$S < S_{\max} \equiv L\sqrt{\frac{2\pi c}{3|\alpha|}}$$

The associated gravity dual → Cut-off AdS [McGough-Mezei-Verlinde, 1611.03470]

2. Gravity duals of $T\overline{T}$-deformation of 2D CFT

i) Gravity dual for the positive sign [Giveon-Itzhaki-Kutasov, 1701.05576]

ii) Gravity dual for the negative sign [McGough-Mezei-Verlinde, 1611.03470]

i) Gravity dual for the positive sign

[Giveon-Itzhaki-Kutasov, 1701.05576]

Starting point:

AdS$_3$ string with NS-NS *B*-field (solvable!)

[Giveon-Kutasov-Seiberg, hep-th/9806194]
[Giveon-Seiberg, hep-th/9903219]

The full 10D background is $\mathrm{AdS}_3 \times \mathrm{S}^3 \times T^4$,

which is realized as a near-horizon limit of *k* NS5-branes and *p* F-strings on $R^{1,4} \times S^1 \times T^4$

	R_t	r (R^4)	S^3 (R^4)	S^1	T^4			
$k \times$NS5-brane	○			○	○	○	○	○
$p \times$F-string	○			○				

$R_t, \; r, \; S^1$ describe AdS$_3$ in a near-horizon limit of the above configuration.

This is the standard setup for AdS$_3$/CFT$_2$. $R_{\mathrm{AdS}} = \sqrt{k}\,\ell_s$

A relevant perspective of this duality

At large *p*, the boundary CFT has the form of a symmetric product,

$$\mathcal{M}^p/S_p \quad , \qquad S_p \text{ : symmetric group}$$

[Argurio-Giveon-Shomer, hep-th/0009242]

[Giveon-Kutasov, 1510.08872]

Each $\mathcal{M}$ is a CFT with central charge $c_{\mathcal{M}} = 6k$.

Thus, the total central charge is $c_{\rm tot} = p\, c_{\mathcal{M}} = 6\, p\, k$.

Roughly speaking, $\mathcal{M}$ can be regarded as the CFT associated with a single F-string. The above structure relies on the fact at large *p* the interaction between the *p* strings in the background goes to zero,

$$g_s^2 \sim 1/p$$

Two types of $T\overline{T}$-like operators

[Giveon-Itzhaki-Kutasov, 1701.05576]

According to the product structure, one may consider two types of $T\overline{T}$-like operators:

1) Double Trace

$$\left(\sum_{i=1}^{p} T_i\right)\left(\sum_{i=1}^{p} \overline{T}_i\right)$$

This corresponds to the usual $T\overline{T}$-deformation.

This typically leads to a non-local deformation of the string world-sheet.

2) Single Trace

$$\sum_{i=1}^{p} T_i\overline{T}_i$$

This leads to a current-current deformation of the string world-sheet.

$J\bar{J}$-deformation of the world-sheet (well known)

In the following, we will consider a gravity dual for the single trace.

What is the gravity dual?

From the behavior of energy and entropy, the gravity dual should describe an RG flow,

UV region: Little String Theory (LST) → IR region: the usual AdS3

The associated gravity solution was already constructed [Giveon-Kutasov-Pelc, hep-th/9907178]

In the following, we are interested in the finite temperature version of that,

[Hyun, hep-th/9704005]

$$\mathrm{d}s^2 = -\frac{f_T}{f_1}(\mathrm{d}x^0)^2 + \frac{1}{f_1}(\mathrm{d}x^1)^2 + \frac{f_5}{f_T}\mathrm{d}r^2,$$

$$\mathrm{e}^{2(\Phi-\Phi_0)} = \frac{f_1}{f_5}, \qquad \text{and} \quad H_3\text{-flux}$$

+ 7D internal space

(e.g., S^3 x T^4)

$$f_1 \equiv 1 + \frac{r_1^2}{r^2}, \quad f_5 \equiv \frac{k\,\ell_s^2}{r^2}, \quad f_T \equiv 1 - \frac{r_0^2}{r^2}, \quad r_1^2 \equiv 8\,p\,k^{3/2}\,\ell\,G_3$$

NOTE: r_0 is associated with temperature.

The x^1-direction is compactified on a circle, $x^1 \sim x^1 + L$.

The behavior of this solution

- The $r \to 0$ limit corresponds to the IR limit and the solution becomes 3D Schwarzschild AdS black hole.

- The $r \to \infty$ limit corresponds to the UV limit and Little String Theory is realized.

The Bekenstein-Hawking entropy is given by

$$S_{\rm BH} = \frac{L\sqrt{f_1}}{4G_3\, f_5} = \frac{L\, r_0}{4G_3\, k\, \ell_s^2}\sqrt{r_0^2 + r_1^2}$$

Here let us suppose the following relation:

$$r_0^2 = \frac{16\pi^2\, G_3\, k^{3/2}\, \ell_s^3\, \mathcal{E}}{L^2}$$

NOTE: zero temperature ($r_0 = 0$) corresponds to the ground state ($\mathcal{E} = 0$).

Then the entropy can be rewritten as

$$S_{\rm BH} = 2\pi\sqrt{2p\,k\,\mathcal{E} + \frac{4k\pi^2\ell_s^2\mathcal{E}^2}{L^2}}$$

On the other hand, the Cardy formula is still valid at large *p* .

Remember the product structure of the boundary CFT at large *p*, $\mathcal{M}^p/S_p$.

Then, one can argue that $T\overline{T}$-deformation acts on each $\mathcal{M}$, and the entropy is

$$S_{\mathcal{M}}(\mathcal{E}) = 2\pi\sqrt{\frac{c_{\mathcal{M}}}{6}(2\mathcal{E} + b\mathcal{E}^2)} = 2\pi\sqrt{2k\,\mathcal{E} + k\,b\,\mathcal{E}^2}$$

and the total entropy is

$$S(\mathcal{E}) = p\,S_{\mathcal{M}}(\mathcal{E}/p) = 2\pi\sqrt{2p\,k\,\mathcal{E} + k\,b\,\mathcal{E}^2}$$

Thus, one can see the exact agreement by employing the relation

$$\alpha = \frac{4\pi\ell_s^2}{p}$$

The deformation parameter is α' !

ii) Gravity dual for the negative sign

[McGough-Mezei-Verlinde, 1611.03470]

The bulk theory (3D Einstein gravity + negative cosmological const.)

$$S_{3D} = -\frac{1}{16\pi G}\int_M d^3x\,\sqrt{g}\left(R+\frac{2}{\ell^2}\right) - \frac{1}{8\pi G}\int_{\partial M} d^2x\,\sqrt{h}\left(K-\frac{1}{\ell}\right)$$

(ℓ : AdS radius)

The metric of the Euclidean BTZ black hole with mass *M* and angular momentum *J*,

$$ds^2 = f(r)^2 dt^2 + \frac{1}{f(r)^2}dr^2 + r^2(d\phi - i\omega(r)dt)^2\,,$$

$$f(r)^2 = \left(\frac{r}{\ell}\right)^2 - 8GM + \frac{16G^2J^2}{r^2}\,,\quad \omega(r) = \frac{4GJ}{r^2}\,,$$

The difference is the location of the boundary: $\partial M = B = \{r = r_c\}$

→ cut-off AdS

Computation of boundary energy

[McGough-Mezei-Verlinde, 1611.03470]

[Kraus-Liu-Marolf, 1801.02714]

The quasi-local energy

$$E^{(\mathrm{grav})} = \frac{1}{2\pi} \int_0^{2\pi} \mathrm{d}\phi \, \sqrt{g_{\phi\phi}} \, u^i u^j T_{ij}$$

c.f., [Brown-York, gr-qc/9209012]
[Brown-Creighton-Mann, gr-qc/9405007]

where the boundary energy-momentum tensor

$$T_{ij} = \frac{1}{4G}(K_{ij} - K g_{ij} + g_{ij})$$

K_{ij} : extrinsic curvature

and the unit normal vector

$$u^t = \frac{1}{f(r_c)}, \qquad u^\phi = \frac{i\omega(r_c)}{f(r_c)}$$

It is useful to introduce the proper size of the spatial circle on the boundary,

$$L \equiv \int_0^{2\pi} \mathrm{d}\phi \, \sqrt{g_{\phi\phi}} \Big|_{r=r_c} = 2\pi r_c$$

The energy is evaluated as follows:

$$
\begin{aligned}
E^{(\mathrm{grav})} &= \frac{L}{8\pi G\ell}\left(1-\frac{\ell}{r_c}f(r_c)\right) \\
&= \frac{L}{8\pi G\ell}\left(1-\sqrt{1-\frac{8G\ell^2 M}{r_c^2}+\frac{16G^2\ell^2 J^2}{r_c^4}}\right)
\end{aligned}
$$

Thus, by employing the relation listed below,

	Gravity		CFT
Deformation Parameter	$16\pi G\ell$	$\equiv$	$-\alpha$
Cylinder Radius	$2\pi r_c$	$\equiv$	L
Mass	$M\ell$	$\equiv$	M_n
Angular Momentum	J	$\equiv$	J_n

one can see the exact agreement with the $T\overline{T}$-deformed CFT!

$$
E^{(\mathrm{grav})} = E^{(\mathrm{cft})}
$$

Other supports

Speed of propagation [McGough, Mezei and Verlinde, 1611.03470]
[Kraus-Liu-Marolf, 1801.02714]

Correlation functions [Kraus-Liu-Marolf, 1801.02714] [Aharony-Vaknin, 1803.00100]
and more

Entanglement Entropy in the cut-off AdS [Nishida-kun's talk]

[Chakraborty-Giveon-Itzhaki-Kutasov, 1805.06286] [Donnelly-Shyam, 1806.07444]
[Chen-Chen-Hao, 1807.08293] [Park, 1812.00545] [Sun-Sun, 1901.08796] [Caputa-Datta-Shyam, 1902.10893]
[Banerjee-Bhattacharyya-Chakraborty, 1904.00716] [Ota, 1904.06930] [Jeong-Kim-Nishida, 1906.03894]
[Murdia-Nomura-Rath-Salzetta, 1907.12603] [Chen-Chen-Zhang, 1907.12110] [He-Shu, 1907.12603]
and more

The Ryu-Takayanagi formula works well even for $T\overline{T}$-deformed case.

3. Summary and Outlook

Summary

$T\overline{T}$ -deformation has an intriguing property and has led to a lot of progress.

As an application, one may discuss holographic duals for $T\overline{T}$-deformed CFT_2.

There exist two gravity duals, depending on the sign of deformation parameter

Positive sign: RG flow: LST to AdS (single trace) [Giveon-Itzhaki-Kutasov, 1701.05576]

Negative sign: cut-off AdS (double trace) [McGough-Mezei-Verlinde, 1611.03470]

Is the bulk cut-off a mirage?

Dirichlet b.c. at finite bulk radius = mixed b.c. at infinity

[Faulkner-Liu-Rangamani, 1010.4036]
[Heemskerk-Polchinski, 1010.1264]
[Balasubramanian-Guica-Lawrence, 1211.1729]

The mixed b.c. can explain holographic duals for both signs (for the double trace case).

[Guica-Monten, 1906.11251]

Outlook

There are a lot of other directions.

- $T\overline{T}$-deformation in general dims.

Some proposals

$$\left(\det T\right)^{1/(D-1)} \quad , \quad T^{ij}T_{ij} - \frac{1}{D-1}T_i^iT_j^j$$

Problem in regularization

[Cardy, 1801.06895]
[Bonelli-Doround-Zhu, 1804.10967]

[M. Taylor, 1805.10287]
[T. Hartman-Kruthoff-Shaghoulian-Tajdini, 1807.11401]

- $J\overline{T}$ -deformation of 2D CFT and holography

[Guica, 1710.08415] [Bzowksi-Guica, 1803.09753]
[Apolo-Song, 1806.10127, 1907.03745]
[Chakraborty-Giveon-Kutasov, 1806.09667, 1905.00051]
[Aharony-Datta-Giveon-Jiang-Kutasov, 1808.08978] and more.

- $T\overline{T}$-deformation of non-Lorentz invariant theory (with non-symmetric energy-momentum tensor, $T_{\mu\nu} \neq T_{\nu\mu}$)

[Cardy, 1809.07849]

- $T\overline{T}$-deformation with SUSY

[Baggio-Sfondrini-Tartaglino Mazzucchelli-Walsh, 1811.00533]
[Jiang-Sfondrini-Tartaglino Mazzucchelli, 1904.04760]
[Chang-Ferko-Sethi-Sfondrini-Tartaglino Mazzucchelli, 1906.00467]

- $T\overline{T}$-deformation of dS/dS [Gorbenko-Silverstein-Torroba, 1811.07965]

- Relation between $T\overline{T}$-deformation for single trace ($\alpha > 0$) and Yang-Baxter deformation

  [Araujo-O Colgain-Sakatani-Sheikh Jabbari-Yavartanoo, 1811.03050] [Borsato-Wulff, 1812.07287]

- $T\overline{T}$-deformation of Jackiw-Teitelboim gravity Okumura-kun's talk

  [Dubovsky-Gorbenko-Mirbabayi, 1706.06604] (flat) [Ishii-Okumura-Sakamoto-KY, 1906.03865] (AdS)

  A gravitational perturbation can be seen as a $T\overline{T}$-deformation of the matter sector.

Thank you!