Hyperbolic Field Space and Swampland Conjecture for DBI Scalar

Speaker: Yun-Long Zhang

Yukawa Institute for Theoretical Physics, Kyoto University

based on [arXiv:1905.10950]
by Shuntaro Mizuno(Hachinohe), Shinji Mukohyama(YITP), Shi Pi(IPMU), Y. -L. Zhang

(August 21, 2019)

Outline I

1. Refined Swampland conjecture
   - Overview in the cosmological background
2. Two-field Model with Hyperbolic Field Space
   - Attractor Behavior: From Two-Field to DBI
3. Swampland Conjecture for Non-canonical Kinetic Terms
   - DBI scalar and $P(X,\varphi)$ theory
4. Summary

De Sitter swampland conjecture

- For a theory coupled to gravity with the potential V of scalar fields,

$$L = -\frac{1}{2}\mathcal{G}_{IJ}(\phi^K)\, g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J + V(\phi^I), \tag{1}$$

a necessary condition for the existence of a UV completion is

$$|\nabla V| \geq c\, V\,. \tag{2}$$

I, J: indexes of scalar fields. [Obied-Ooguri-Spodyneiko-Vafa 1806.08362]

- The refined de Sitter swampland conjecture

$$|\nabla V| \geq c\, V, \quad \text{or} \quad \min\left(\nabla_I\nabla_J V\right) \leq -c'\, V\,, \tag{3}$$

where c' is another $\mathcal{O}(1)$ positive constant, and $\min\left(\nabla_I\nabla_J V\right)$ is the minimum eigenvalue of the Hessian of the potential in the local orthonormal frame. [Ooguri-Palti-Shiu-Vafa, 1810.05506]

How about theory with non-canonical kinetic term

- The two-dimensional field model

$$I = \int d^4x\sqrt{-g}\Big\{-\frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - \frac{1}{2}e^{2\beta\chi}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\chi,\varphi)\Big\}$$
$$V(\chi,\varphi) = T(\varphi)\left[\cosh(2\beta\chi) - 1\right] + U(\varphi)\,, \tag{4}$$

- How about the one field DBI scalar model?

$$I_{\text{eff}} = \int d^4x\sqrt{-g}\left\{T(\varphi)\left[-\sqrt{1-\frac{2X}{T(\varphi)}}+1\right] - U(\varphi)\right\}\,,$$
$$X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\,, \quad T(\varphi) = \frac{\varphi^4}{\lambda} \tag{5}$$

One approach see: 1812.07670 by M. -S. Seo.

Covariant entropy bound in FRW background

- The applicability of the covariant entropy bound $S \leq \pi/H^2$, a sufficient condition

$$\left|\frac{\dot{H}}{H^2}\right| \lesssim c_1 \quad \text{and} \quad \frac{\min m^2_{\rm scalar}}{H^2} \gtrsim -c_2\,, \tag{6}$$

  $\min m^2_{\rm scalar}$ is the lowest among squared masses of perturbation modes of the scalar fields, and $c_{1,2}$ are positive numbers of $\mathcal{O}(1)$.
- For example, the number of particle species N below the cutoff of an effective field theory is roughly given by $N \sim n(\phi)e^{b\phi}$ with $\frac{dn}{d\phi} > 0$.
- The ansatz for the entropy of the towers of light particles in an accelerating universe $S_{\rm tower}(N, R) \sim N^{\delta_1} R^{\delta_2}$, where $R \sim 1/H$ is the radius of the apparent horizon, H is the Hubble expansion rate and $\delta_{1,2}$ are positive numbers of $\mathcal{O}(1)$.

Refined Swampland Conjecture in FRW background

- As a result, one obtains $S \sim S_{\rm tower}(N,R)$ and consider that $N = n(\phi)e^{b\phi} \sim \left(\frac{1}{H}\right)^{\frac{2-\delta_2}{\delta_1}}$ one obtains $\ln n(\phi) \sim -b\phi - \frac{2-\delta_2}{2\delta_1}\ln H^2$.
- Under the condition $\left|\frac{\dot H}{H^2}\right| \lesssim c_1$ and $\frac{\min m^2_{\rm scalar}}{H^2} \gtrsim -c_2$, from $\frac{dn}{d\phi} > 0$

$$\left|\frac{1}{H^2}\frac{d(H^2)}{d\phi}\right| \gtrsim c_0\,, \quad c_0 \equiv \frac{2b\delta_1}{2-\delta_2}\,. \tag{7}$$

- It is therefore concluded that in FRW background

$$\left|\frac{1}{H^2}\frac{d(H^2)}{d\phi}\right| \gtrsim c_0\,, \quad \text{or} \quad \left|\frac{\dot H}{H^2}\right| \gtrsim c_1\,, \quad \text{or} \quad \frac{\min m^2_{\rm scalar}}{H^2} \lesssim -c_2\,, \tag{8}$$

For slow-roll models with canonical kinetic terms, the conjecture reduces to $|\nabla V| \geq c\,V$, or $\min\left(\nabla_I \nabla_J V\right) \leq -c'\,V$, where $c \equiv \min(c_0, \sqrt{2c_1})$ and $c' \equiv c_2/3$ are still of $\mathcal{O}(1)$.

Two-field Model with Hyperbolic Field Space

- We consider a two-dimensional hyperbolic field space,

$$\mathcal{G}_{IJ}(\phi^K)d\phi^I d\phi^J = d\chi^2 + e^{2\beta\chi}d\varphi^2\,, \tag{9}$$

  where β is a positive constant.
- The action of the scalar fields $\{\phi^I\} = \{\chi, \varphi\}$ is then given by

$$I = \int d^4x\sqrt{-g}\Big\{ -\frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - \frac{1}{2}e^{2\beta\chi}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - T(\varphi)\left[\cosh(2\beta\chi)-1\right] - U(\varphi)\Big\}\,, \tag{10}$$

  where $T(\varphi) \equiv A(\varphi)$ and $U(\varphi) \equiv A(\varphi) + B(\varphi)$.

Attractor Behavior of χ

- The equation of motion for χ leads

$$\Box\chi + 2\beta e^{2\beta\chi}X - 2\beta T(\varphi)\sinh(2\beta\chi) = 0\,, \tag{11}$$

where $X \equiv -g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi/2$.

- If β^2 is large then χ has a heavy mass, with $\Box\chi$ dropped, i.e.

$$2\beta e^{2\beta\chi}X - 2\beta T(\varphi)\sinh(2\beta\chi) \simeq 0\,, \tag{12}$$

- It is easily solved with respect to χ as

$$\chi \simeq \frac{1}{2\beta}\ln\gamma\,, \quad \gamma \equiv \left(1 - \frac{2X}{T(\varphi)}\right)^{-1/2}. \tag{13}$$

Attractor Behavior: From Two Field to DBI

- The two-dimensional field space

$$I = \int d^4x\sqrt{-g}\Big\{ -\frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - \frac{1}{2}e^{2\beta\chi}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - T(\varphi)\left[\cosh(2\beta\chi)-1\right] - U(\varphi)\Big\}\,, \tag{14}$$

- It is reduced to an effective one-dimensional field space spanned by φ with the effective action

$$I_{\rm eff} = \int d^4x\sqrt{-g}\left\{ T(\varphi)\left[-\gamma^{-1}+1\right] - U(\varphi)\right\}\,,$$

$$X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\,, \quad \gamma \equiv \left(1-\frac{2X}{T(\varphi)}\right)^{-1/2} = e^{2\beta\chi}. \tag{15}$$

Attractor Behavior of the two field system

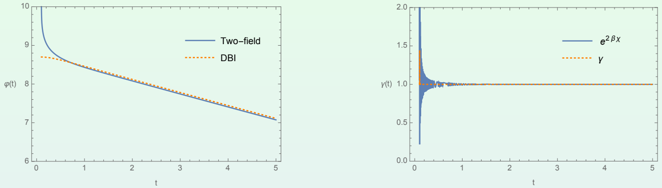

Figure: Non-relativistic attractor ($\gamma = 1$) with the parameter choice $U(\varphi) = 1 + 0.1\varphi^2$, $T(\varphi) \equiv \varphi^4/\lambda$, $\beta = 20$ and $\lambda = 0.5$.

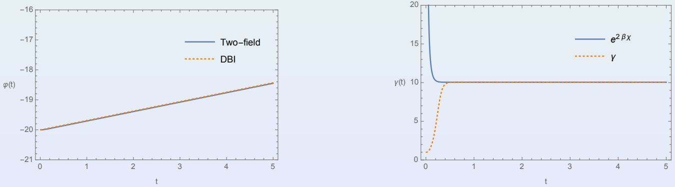

Figure: Relativistic attractor ($\gamma = 10$) with the parameter choice $U(\varphi) = 7.5\varphi^2$, $T(\varphi) \equiv 1/\lambda$, $\beta = 20$ and $\lambda = 10$.

Equations of motion in the FRW background

- "Two-field" denotes that we evaluate $\varphi(t)$ based on the full equations of motion from the two-field action (14)

$$\ddot{\chi} + 3H\dot{\chi} - \beta e^{2\beta\chi}\dot{\varphi}^2 + 2\beta T(\varphi)\sinh(2\beta\chi) = 0\,,$$
$$\ddot{\varphi} + 3H\dot{\varphi} + 2\beta\dot{\chi}\dot{\varphi} + \frac{T'(\varphi)}{e^{2\beta\chi}}\left[\cosh(2\beta\chi) - 1\right] + \frac{U'(\varphi)}{e^{2\beta\chi}} = 0\,,$$
$$3H^2 = \frac{1}{2}(\dot{\chi}^2 + e^{2\beta\chi}\dot{\phi}^2) + T(\varphi)\left[\cosh(2\beta\chi) - 1\right] + U(\varphi)\,, \qquad (16)$$

- "DBI" denotes that we evaluate $\varphi(t)$ based on the equation of motion for the single-field DBI model

$$\gamma^2\ddot{\varphi} + 3H\dot{\varphi} - \frac{T'(\varphi)}{2}\frac{(\gamma-1)^2(\gamma+2)}{\gamma} + \frac{U'(\varphi)}{\gamma} = 0\,,$$
$$3H^2 = \frac{\gamma^2}{(\gamma+1)}\dot{\varphi}^2 + U(\varphi), \qquad \gamma = \left(1 - \frac{\dot{\varphi}^2}{T(\varphi)}\right)^{-1/2}. \qquad (17)$$

Geodesic Distance in the Field Space

- It is given by integrating

$$d\phi = \sqrt{\mathcal{G}_{IJ}(\phi^K) d\phi^I d\phi^J} = \sqrt{d\chi^2 + e^{2\beta\chi} d\varphi^2} \,. \tag{18}$$

- For large enough β^2, by using the attractor behavior $\chi \simeq \frac{1}{2\beta} \ln \gamma$, this is reduced to

$$d\phi \simeq \left(\frac{\dot{\gamma}^2}{4\beta^2\gamma^2\dot{\varphi}^2} + \gamma \right)^{1/2} d\varphi \simeq \sqrt{\gamma} d\varphi \,, \tag{19}$$

- We have used the fact that the evolution of φ is well described by the single-field model and thus $\dot{\gamma}^2/(\gamma^2\dot{\varphi}^2)$ remains finite in the $\beta^2 \to \infty$ limit. Thus, the first inequality $\left| \frac{1}{H^2} \frac{d(H^2)}{d\phi} \right| \gtrsim c_0$ can be rewritten as

$$\frac{1}{\sqrt{\gamma}} \left| \frac{1}{H^2} \frac{d(H^2)}{d\varphi} \right| \gtrsim c_0 \,. \tag{20}$$

Swampland conjecture for DBI scalar

- The effective single-field DBI action

$$I_{\rm DBI} = \int d^4x\sqrt{-g}\left\{T(\varphi)\left[-\sqrt{1-\frac{2X}{T(\varphi)}}+1\right]-U(\varphi)\right\}. \tag{21}$$

- The swampland conjecture is written as

$$\frac{1}{\sqrt{\gamma}}\left|\frac{1}{H^2}\frac{d(H^2)}{d\varphi}\right| \gtrsim c_0\,, \quad \text{or} \quad \left|\frac{\dot{H}}{H^2}\right| \gtrsim c_1\,, \quad \text{or} \quad \frac{\Omega}{H^2} \lesssim -c_2\,. \tag{22}$$

- Where $\gamma \equiv 1/\sqrt{1-2X/T}$, $X \equiv -g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi/2$ and

$$\Omega = \frac{1}{\gamma^3}U'' + \frac{(\gamma-1)^2}{2\gamma^4}T'' - \frac{[(\gamma+3)(\gamma-1)T' - 2\gamma U']^2}{16\gamma^4 T}\,. \tag{23}$$

Squared masses of scalar perturbation modes

- The quadratic part of the action

$$I^{(2)} = \frac{1}{2}\int dta^3 \left[\dot{Y}^{\mathrm{T}}\mathcal{K}\dot{Y} + \dot{Y}^{\mathrm{T}}\mathcal{M}Y + Y^{\mathrm{T}}\mathcal{M}^{\mathrm{T}}\dot{Y} - Y^{\mathrm{T}}\left(-\mathcal{K}\frac{\vec{\nabla}^2}{a^2} + \mathcal{V}\right)Y\right], \tag{24}$$

- where $Y = \begin{pmatrix} \delta\varphi \\ \delta\chi \end{pmatrix}$, $\mathcal{K} = \begin{pmatrix} \gamma & 0 \\ 0 & 1 \end{pmatrix}$,

$$\mathcal{M} = \begin{pmatrix} 0 & \beta\gamma\dot{\varphi} \\ -\beta\gamma\dot{\varphi} & 0 \end{pmatrix}, \quad \mathcal{V} = \begin{pmatrix} \mathcal{V}_{11} & \mathcal{V}_{12} \\ \mathcal{V}_{12} & \mathcal{V}_{22} \end{pmatrix} + \mathcal{O}(M_{\mathrm{Pl}}^{-2}), \tag{25}$$

and

$$\mathcal{V}_{11} = \frac{(\gamma-1)^2}{2\gamma}T'' + U'', \quad \mathcal{V}_{12} = \frac{(\gamma+3)(\gamma-1)}{2\gamma}T'\beta - U'\beta, \quad \mathcal{V}_{22} = \frac{4T}{\gamma}\beta^2. \tag{26}$$

Squared masses of scalar perturbation modes

- The squared masses m^2_{scalar} for scalar perturbations are obtained by solving the following second-order algebraic equation for m^2,

$$\det\left[m^2\mathcal{K} - 2im\mathcal{M} - \mathcal{V}\right] = 0\,. \tag{27}$$

- There are two independent solutions $m^2 = m^2_{\pm}$, where

$$\begin{aligned} m^2_+ &= 4T(\varphi)\gamma\beta^2 + \mathcal{O}(\beta^0)\,, \\ m^2_- &= \Omega + \mathcal{O}(\beta^{-2}) + \mathcal{O}(M_{\rm Pl}^{-2})\,, \end{aligned} \tag{28}$$

and

$$\Omega = \frac{1}{\gamma^3}U'' + \frac{(\gamma-1)^2}{2\gamma^4}T'' - \frac{[(\gamma+3)(\gamma-1)T' - 2\gamma U']^2}{16\gamma^4 T}\,. \tag{29}$$

De Sitter swampland conjecture for the $P(X,\varphi)$ theory

- The total two field action is

$$I = \int d^4x\sqrt{-g}\left\{-\frac{Z^2}{2}g^{\mu\nu}\partial_\mu\hat{\chi}\partial_\nu\hat{\chi} + P_{,\hat{\chi}}(\hat{\chi},\varphi)(X-\hat{\chi}) + P(\hat{\chi},\varphi)\right\}. \quad (30)$$

- De Sitter swampland conjecture for the $P(X,\varphi)$ theory,

$$\frac{1}{\sqrt{P_{,X}(X,\varphi)}}\left|\frac{1}{H^2}\frac{d(H^2)}{d\varphi}\right| \gtrsim c_0\,, \text{ or } \left|\frac{\dot{H}}{H^2}\right| \gtrsim c_1\,, \text{ or } \frac{m_-^2}{H^2} \lesssim -c_2\,. \quad (31)$$

- Slow modes $\sim e^{\pm m_- t}$ with $m_-^2 = \mathcal{O}(Z^0)$. As a special case, we can choose $P(X,\varphi)$ as the Lagrangian in the DBI action, $P(X,\varphi) = T(\varphi)\left[-\sqrt{1-\frac{2X}{T(\varphi)}}+1\right] - U(\varphi)$ and $P_{,X}(X,\varphi) = 1/\sqrt{1-\frac{2X}{T(\varphi)}} \equiv \gamma$.

Summary

- We study a model of two scalar fields with a hyperbolic field space and show that it reduces to a single-field Dirac-Born-Infeld (DBI) model in the limit where the field space becomes infinitely curved.
- Apply the de Sitter swampland conjecture to the two-field model and take the same limit. All quantities appearing in the swampland conjecture remain well-defined within the single-field DBI model.
- The condition derived in this way can be considered as the de Sitter swampland conjecture for a DBI scalar field.
- We propose an extension of the de Sitter swampland conjecture to a more general scalar field in $P(X, \varphi)$ theory

$$\frac{1}{\sqrt{P_{,X}(X,\varphi)}}\left|\frac{1}{H^2}\frac{d(H^2)}{d\varphi}\right| \gtrsim c_0\,, \text{ or } \left|\frac{\dot{H}}{H^2}\right| \gtrsim c_1\,, \text{ or } \frac{m_-^2}{H^2} \lesssim -c_2\,.$$