Constructing a Toy Alternative to Inflation Model from Quantum Creation of the Universe

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Introduction

- Inflation: most-accepted theory of early universe, resolves flatness problem, horizon problem; predicts a near scale-invariant power spectrum, verified by observations.
- Potential Puzzles: trans-Planckian problem, past-incompleteness problem, initial condition problem.
- Could we explain the puzzles within the framework of inflationary cosmology, or with some new ideas?
- Alternative to Inflation Theories: May provide different viewpoints on the physics of very early universe, and helps to understand inflation better.
- In this work, we present a toy alternative to inflation models, and verify it in both conceptual and experimental aspects.

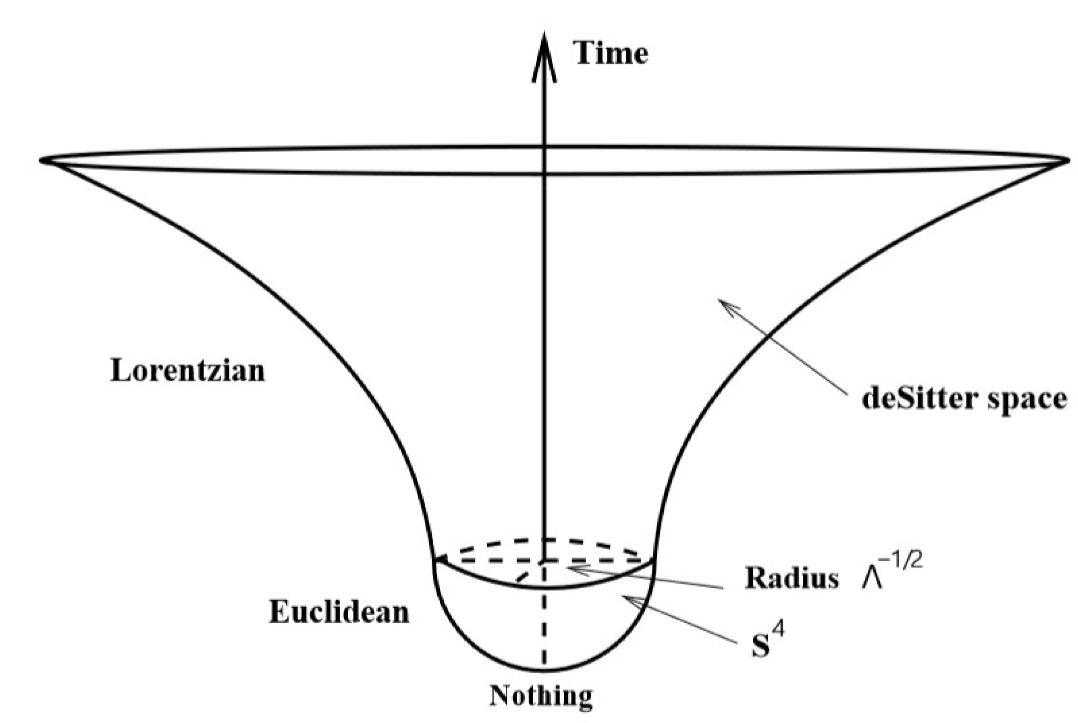
Motivations

- Our model is inspired by the work [1].
- First principle in Quantum Gravity region: the Euclidean Quantum Gravity(EQG) formalism. Advantage: predicts the relative probability of certain configuration of the universe.
- Boundary Condition: no-boundary proposal from Hawking. Can intepret the initial singularity. The horizon problem also disappears.
- UV complete action: Horava-Lifshitz(HL) theory. Supposed to solve the flatness problem, and generate near scale invariant power spectrum.

Introduction to EQG Formalism

- The spacetime should allow an ADM decomposition:
$$ds^2 = -N^2dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$
Here the lapse function N and shift vector N^i can be set to 1 and 0 by the symmetry of time reparametrization and spatial diffeomorphism respectively. In this case h_{ij} uniquely describes the geometry.
- The universe starts in the quantum gravity(QG) region(or the Euclidean region since time is imaginary), during which the state of the universe is described by its wave function $\Psi[h_{ij}, \phi]$, defined as the path integral over possible compact configurations where geometry and matter are described by h_{ij} and ϕ:
$$\Psi[h_{ij}, \phi] = \int d[g_{\mu\nu}]d[\phi]e^{-\hat{I}}$$
$\hat{I}$ is Euclidean action from a Wick rotation $\tau = it$.
- The relative possibility of a universe in a configuration $[h_{ij}, \phi]$ is
$$\mathcal{P}[h_{ij}, \phi] \propto |\Psi[h_{ij}, \phi]|^2$$
Flatness problem can be explained if the model prefers a flat universe(i.e. universes with smaller curvature parameter Ω_k has larger relative probability).
- The starting point of path integral 1 is $a(0) = 0$ from no-boundary proposal, and the ending spacetime point is the quantum tunneling, where the universe tunnels from the Euclidean spacetime to the Lorentzian spacetime and time becomes real.
- After the quantum tunneling, the dynamics of the universe is predicted by the standard cosmology.
- The evolution of the universe is shown in the following figure. The figure is from [2]

An Illustrative Figure of EQG Formalism



Background Cosmology

- We apply spectator field approximation throughout the work. "Spectator" means gravity is just a background and do not receive back-reaction from the scalar field. Besides, there is no metric perturbation.
- The dynamics of background is determined by the HL gravity. The action is
$$I_g = \frac{1}{2}\int Ndtd^3x\sqrt{h}(K_{ij}K^{ij} - \lambda K^2 - 2\Lambda + R + L_{z=3})$$
$$L_{z=3} = c_1 D_i R_{jk} D^i R^{jk} + c_2 D_i R D^i R + c_3 R_i^j R_j^k R_k^i + c_4 R R_i^j R_j^i + c_5 R^3$$
- Euclidean action of universe reaches its minimum when the universe is in the maximum symmetric configuration [3], so the most probable geometric configuration of the universe is that of FRW type:
$$ds^2 = -dt^2 + a^2(t)\left(\frac{d\bar{r}^2}{1-\bar{r}^2} + \bar{r}^2 d\Omega^2\right)$$
- WKB approximation is applied, which states that the most probable trajectory of the universe is described by the classical equation of motion(EoM):
$$\frac{3\lambda - 1}{2}(2\partial_t H + 3H^2) = \frac{\gamma}{a^6} - \frac{1}{a^2} + \Lambda, H \equiv \frac{\partial_t a}{a} \qquad (1)$$
- Integrate Eq.1 and we have(set $\Lambda = 0$ for convenience):
$$\tilde{\lambda}H^2 = \frac{C}{a^3} - \frac{\gamma}{a^6} - \frac{3}{a^2} + \Lambda \qquad (2)$$
where $\tilde{\lambda} \equiv \frac{3}{2}(3\lambda - 1)$. C is effectively a dark matter term.
- In Euclidean spacetime the EoM becomes
$$\tilde{\lambda}\mathcal{H}^2 = -\frac{C}{a^3} + \frac{\gamma}{a^6} + \frac{3}{a^2}, \mathcal{H} \equiv \frac{\partial_\tau a}{a} \qquad (3)$$
Initial condition of Eq.3 is $a(0) = 0$, and the quantum tunneling condition is $\mathcal{H} = 0$.

Explanation of Flatness Problem

- Condition 1: a subdominant curvature component at quantum tunneling: $\frac{3/a(\tau_T)^2}{C/a(\tau_T)^3} \ll 1$. This gives the constraint $\frac{\gamma}{C^4} \ll 1$.
- Condition 2: predict a small enough curvature parameter $\Omega_K \equiv \frac{1}{a^2H^2}$ at the beginning of standard cosmology. This gives the constraint $\frac{\gamma}{C} \gg 1$.

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Perturbation

- Under spectator field approximation, only the matter section generates scalar perturbation. The most general action of scalar field compatible with HL theory is

$$I_m = \int dtd^3x\frac{1}{2}a^3\left[(\partial_t\phi)^2 - \sum_{n=0}^{3}(-1)^n\frac{\lambda_{2,n}}{M^{2n-2}}\Delta^n \star \phi^2\right]$$

- Expand the perturbation as a summation of normalized spherical harmonics:

$$\phi(t,\vec{x}) = \bar{\phi}(t) + \delta\phi(t,\vec{x}) = \bar{\phi}(t) + \sum_k f_k(t)Q_k(\vec{x})$$

the quadratic Euclidean action for wavelength k is:

$$\hat{I}_{m,2k} = \int d\tau\frac{a^3}{2}\left[f_k'^2 + \omega_k^2 f_k^2\right]$$

$$\omega_k^2 \equiv \sum_{n=0}^{3}\frac{\lambda_{2,n}}{M^{2n-2}}(k^2-1)^n \approx \sum_{n=0}^{3}\frac{\lambda_{2,n}}{M^2}\left(\frac{k}{M}\right)^{2n} \tag{4}$$

- Classical EoM for perturbation mode is

$$f_k'' + 3\mathcal{H}f_k' - \omega^2 f_k = 0 \tag{5}$$

for later convenience, we shall introduce $g_k \equiv \frac{f_k'}{f_k}$, 5 then becomes:

$$g_k' + g_k^2 + 3\mathcal{H}g_k - \omega_k^2 = 0 \tag{6}$$

The initial condition of Eq.5 is uniquely determined by the asymptotic behavior of $a(\tau)$

$$f_k(\tau) = J_0(i\omega_k\tau), \tau \to 0$$

- The wave function of perturbation up to quadratic order is

$$\Psi_m[f_k] = \int d[f_k]e^{\frac{1}{2}a(\tau_T)^3 g_k(\tau_T)f_k^2}$$

which gives two-point correlation function of f_k at quantum tunneling event:

$$\langle f_k f_k\rangle = \frac{\int[df_k]f_k f_k e^{-\frac{1}{2}a(\tau_T)^3 g_k(\tau_T)f_k^2}}{\int[df_k]e^{-\frac{1}{2}a(\tau_T)^3 g_k(\tau_T)f_k^2}} = a(\tau_T)^{-3}g_k^{-1}(\tau_T)$$

where τ_T is the time of quantum tunneling.

- Finally we find that near the quantum tunneling event, Eq.6 has asymptotic solution $g_k(\tau) \sim \omega_k$. The two-point correlation function is then related to the model parameter by

$$\langle f_k f_k\rangle = a(\tau_T)^{-3}\omega_k^{-1} = \frac{C}{\gamma\omega_k} \tag{7}$$

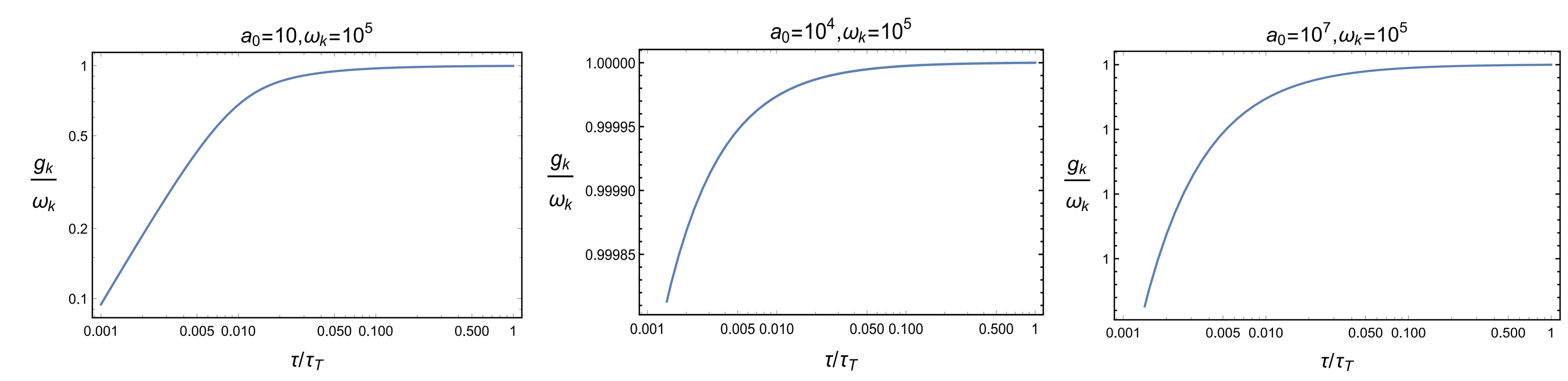
- Compare Eq.7 with the definition of power spectrum

$$\langle f_{k_1} f_{k_2}\rangle \equiv (2\pi)^3\delta(\vec{k_1}+\vec{k_2})\frac{2\pi^2}{k^3}P_\phi(k) \tag{8}$$

we find that the dominance of $\frac{\lambda_{2,3}k^6}{M^8}$ in Eq.4 can help to generate a near scale-invariant power spectrum. The rest terms may contribute to the tilt.

Numerical Study

- We take the parameter in gravity section as $C = 5\times10^{51}l_p$, $\gamma = 2\times10^{48}l_p^4$ and $\lambda = \frac{5}{9}$. These parameter give $a(\tau_T) = 7.4\times10^{48}l_p$.
- We check the conditions for evading flatness problem. For condition 1, $\frac{3/a(\tau_T)^2}{C/a(\tau_T)^3} = 4.4\times10^{-3}$ is relatively small. For condition 2, the curvature parameter at the beginning of standard cosmology is $|\Omega_K|(t_r) = 3.4\times10^{-18}$, which is much larger compared to that predicted by standard Big Bang cosmology.
- We numerically solve Eq.3 and Eq.6 and plot the evolution of $\frac{g_k(\tau)}{\omega_k}$



the results support our approximation $g_k(\tau_T) \sim \omega_k$.

Outlook

- A further investigation without spectator approximation.
- Find other mechanisms to replace C.
- Could metric perturbation of HL gravity alone give the feature?
- Spectrum index and non-Gaussianity.

Reference

[1] Sebastian F. Bramberger, Andrew Coates, João Magueijo, Shinji Mukohyama, Ryo Namba, and Yota Watanabe. Solving the flatness problem with an anisotropic instanton in Hořava-Lifshitz gravity. *Phys. Rev.*, D97(4):043512, 2018.

[2] Saswat Sarangi and S. H. Henry Tye. A Note on the quantum creation of universes. 2006.

[3] Sidney R. Coleman and Frank De Luccia. Gravitational Effects on and of Vacuum Decay. *Phys. Rev.*, D21:3305, 1980.