Non-invertible symmetries in axion electrodynamics

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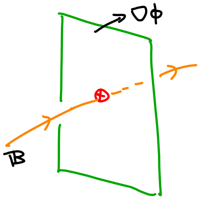
Based on RY, arXiv:2212.05001

Overview

Message: axion electrodynamics has **non-invertible 1-form** global symmetry



- We find non-invertible 1-form global symmetry by EOM of photon.
- We discuss current algebra of non-invertible global symmetries

  from the viewpoint of an effect in axion electrodynamics

Contents

Global symmetries

Conventional definition of symmetry

Symmetry = invariance of action

$$S[g \cdot \Phi] = S[\Phi]$$

under change of variables $\Phi \to \Phi' = g \cdot \Phi$ with a group $g \in G$,

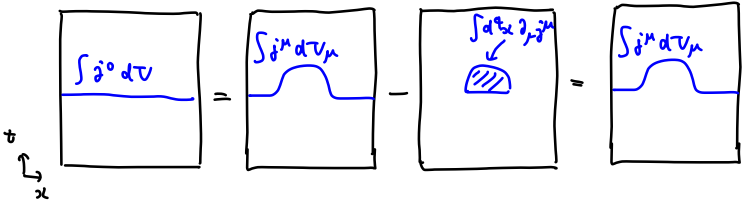
Importance of symmetries

- Prediction of new particles using representation
- Constraints on interactions
- Nambu-Goldstone theorem

Symmetry generators are topological

Symmetry generators are topological

- Continuous symmetries: $\partial_\mu j^\mu = 0 \Rightarrow U = \exp\left(i\alpha \int d^3\boldsymbol{x} j^0\right)$ is topological



- Discrete (internal) symmetries: $\frac{\partial U}{\partial t} \propto [U, H] = 0$, $\frac{\partial U}{\partial x^i} \propto [U, P_i] = 0$

  $\Rightarrow U$ is topological

Generalized global symmetry

Symmetry = existence of topological object

Generalized global symmetries (1/2) [Gaiotto, Kapustin, Seiberg, Willett '14]

Higher p-form symmetry

Existence of $(D-p-1)$-dim. topological object acting on p-dim. object

D: spacetime dim.

$$e^{i\alpha\int_S E\cdot dS} = e^{i\alpha q} \quad q$$

Example: 1-form symmetry in Maxwell theory

- Electric flux $\int \boldsymbol{E}\cdot d\boldsymbol{S}$ is topological due to Gauss law
- Charged object $=$ source of elec. flux: Wilson loop (1d) [detail]

Dim. of symmetry generators can be lower than 3d (in $(3+1)$d)

Generalized global symmetries (2/2) [Gaiotto, Kapustin, Seiberg, Willett '14]

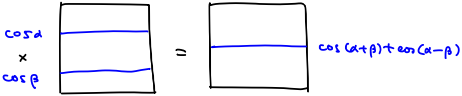
Non-invertible symmetry [e.g., Bhardwaj & Tachikawa '17]

Existence of topological object that is not associated with group

- Existence of topological object do not require invertibility
- Cf. Ordinary symmetries are invertible $U^{-1}U = UU^{-1} = 1$.

Example: **Sum** of phase factors can be non-invertible

- cosine $\cos\alpha = \frac{1}{2}(e^{i\alpha} + e^{-i\alpha})$: non-invertible (except for $\alpha = 0, \pi$)



cf. phase factor $e^{i\theta}$ is invertible $e^{-i\theta}e^{i\theta} = 1$

Developments of generalized global symmetries

1. Characterization of previously known phases by global symmetries

   [Gaiotto, Kapustin, Seiberg, Willett '14]

   - Confined phase of $SU(N)$ Yang-Mills theory $=$ unbroken phase of global 1-form symmetry
   - Topologically ordered phase $=$ broken phase of global 1-form symmetry

2. New phases by global symmetries

   - Spontaneous CP symmetry breaking phase of $SU(N)$ Yang-Mills theory at $\theta = \pi$

     [Gaiotto, Kapustin, Komargodski, Seiberg '17]

3. New understanding of anomalies

   - Chiral anomaly $\to$ non-invertible symmetry [Choi, Lam, Shao, '22; Córdova & Ohmori, '22]

Recent theme of generalized global symmetries (personal view)

How universal are generalized global symmetries?

This talk: existence of non-invertible higher-form symmetry in axion electrodynamics

Contents

Non-invertible 0-form symmetry in axion electrodynamics

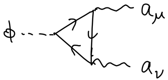
review based on Choi, Lam, Shao '22; Córdova & Ohmori '22

Message: Non-invertible symmetries exist in a realistic model.

Axion electrodynamics: axion ϕ + photon a_μ + topological coupling

[Wilczek '87]

$$\frac{1}{4\pi^2}\phi \boldsymbol{E}\cdot\boldsymbol{B} = \frac{1}{32\pi^2}\phi\epsilon^{\mu\nu\rho\sigma} f_{\mu\nu} f_{\rho\sigma}$$



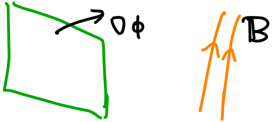
Features

1. Simple and ubiquitous

   $\phi =$ QCD axion, inflaton, dark matter, π^0 meson, quasi particle in topological matter,...

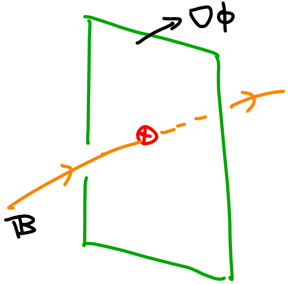
2. Topological coupling determined by chiral anomaly

3. Existence of extended objects, e.g., axionic domain walls, magnetic flux,...



Topological coupling $\rightarrow$ peculiar effects for extended objects

Sikivie effect [Sikivie '84]



- Intersection of domain wall and magnetic flux $\to$ induced charge
- Gauss law $\nabla \cdot \boldsymbol{E} = \frac{1}{4\pi^2} \nabla \phi \cdot \boldsymbol{B}$

Generalized global symmetries behind physics of extended objects?

Action

Massless axion + photon + topological coupling

$$S = -\int d^4x \left(\frac{v^2}{2} |\partial_\mu \phi|^2 + \frac{1}{4e^2} |f_{\mu\nu}|^2 + \frac{1}{32\pi^2} \phi \epsilon^{\mu\nu\rho\sigma} f_{\mu\nu} f_{\rho\sigma} \right)$$

- Axion: 2π periodic pseudo scalar $\phi + 2\pi \sim \phi$

  (axion as Nambu-Goldstone boson of $U(1)$ symmetry)

  2π periodicity $=$ gauge symmetry (redundancy)

- Photon a_μ: $U(1)$ gauge field, Dirac quantization condition

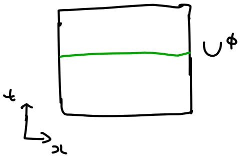
  $\int_{\mathcal{S}} f_{\mu\nu} dS^{\mu\nu} = \int_{\mathcal{S}} \boldsymbol{B} \cdot d\boldsymbol{S} \in 2\pi\mathbb{Z}$

Non-invertible symmetries can be found by EOM of axion and photon

EOM of axion $= U(1)$ shift symmetry of axion...? (1/2)

If photon is absent, we have ordinary $U(1)$ symmetry

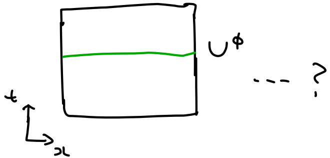
- EOM as cons. law: $v^2\partial_\mu(\partial^\mu\phi) = 0 \quad \to j^\mu = v^2\partial^\mu\phi$ is conserved.
- **Unitary** operator $U^\phi = \exp\left(i\alpha \int_{\mathcal{V}} v^2\partial^\mu\phi \, dV_\mu\right)$



- $U(1)$ phase rotation $U^\phi e^{i\phi}(U^\phi)^\dagger = e^{i\alpha}e^{i\phi}$

If the axion-photon coupling $\phi f_{\mu\nu}\tilde{f}^{\mu\nu}$ is present, ...

EOM of axion $= U(1)$ shift symmetry of axion...? (2/2)



Conservation law is deformed

- EOM: $\partial_\mu(v^2\partial^\mu\phi - \frac{1}{8\pi^2}a_\nu\tilde{f}^{\mu\nu}) = 0$, cons. current: $j^\mu = v^2\partial^\mu\phi - \frac{1}{8\pi^2}a_\nu\tilde{f}^{\mu\nu}$
- However, **unitary** operator

$$\exp\left(\frac{i\alpha}{8\pi^2}\int_{\mathcal{V}} a_\nu\tilde{f}^{\mu\nu}dV_\mu\right) \times e^{-i\alpha\int_{\mathcal{V}} v^2\partial^\mu\phi\, dV_\mu}$$

  is only gauge invariant if $e^{i\alpha} = 1$ (Dirac quantization condition) [detail]

- No conventional global symmetry: $e^{i\phi} \to e^{i\alpha}e^{i\phi} = e^{i\phi}$

  Just a consequence of chiral anomaly

Shift symmetry with a rational number, e.g., $\alpha = \frac{2\pi}{p}$ $(p \in \mathbb{Z})$

is still valid at the expense of **unitarity**!

Modification using $U(1)$ Chern-Simons theory [Choi, Lam, Shao, '22; Córdova & Ohmori, '22]

$\exp\left(\frac{i}{4\pi p}\int_{\mathcal{V}} dV_\mu a_\nu \tilde{f}^{\mu\nu}\right)$ has a gauge invariant expression!

Modification

$$\exp\left(\frac{i}{4\pi p}\int_{\mathcal{V}} dV_\mu a_\nu \tilde{f}^{\mu\nu}\right) \to \int \mathcal{D}c_\mu \exp\left(\frac{i}{4\pi}\int_{\mathcal{V}} dV_\mu(-p\, c_\nu \tilde{d}^{\mu\nu} + 2c_\nu \tilde{f}^{\mu\nu})\right)$$

Essentially $\frac{1}{p}x^2 \to -py^2 + 2xy$

- c_μ: auxiliary $U(1)$ gauge field, field strength $d_{\mu\nu} = \partial_\mu c_\nu - \partial_\nu c_\mu$,

  Dirac quant. cond. $\int dS^{\mu\nu} d_{\mu\nu} \in 2\pi\mathbb{Z}$
- Consistent with Dirac quantization condition: $p \in \mathbb{Z}$ in numerator
- Original one: naive expression after eliminating c_μ by its EOM $pd_{\mu\nu} = f_{\mu\nu}$

  with trivial Dirac quant. cond. $\int_{\mathcal{S}} f_{\mu\nu} dS^{\mu\nu} = 0$ (mod p)
- Sum (path integral) of phase factors: **unitarity** is lost

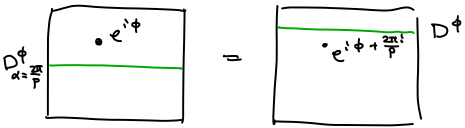
We can construct symmetry generator by this modification.

Non-invertible 0-form symmetry [Choi, Lam, Shao, '22; Córdova & Ohmori, '22]

Symmetry generator

$$D^{\phi}_{\alpha=\frac{2\pi}{p}} = \int \mathcal{D}c_\mu \exp\left(-\frac{i}{4\pi}\int_{\mathcal{V}} dV_\mu (p\, c_\nu \tilde{d}^{\mu\nu} - 2c_\nu \tilde{f}^{\mu\nu})\right) \times e^{\frac{2\pi i}{p}\int_{\mathcal{V}} dV_\mu \partial^\mu \phi}$$

- Symmetry generator is gauge invariant
- Topological: EOM of axion
- 0-form symmetry: acting on 0-dim. axion operator $e^{i\phi} \to e^{i\phi + \frac{2\pi i}{p}}$



- Non-invertible (sum of phase factors)

How about EOM of photon?

Contents

Non-invertible 1-form symmetry in axion electrodynamics

RY [2212.05001] (see also Choi, Lam, Shao [2212.04499])

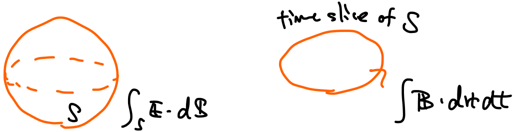
Message: Non-invertible **higher-form** symmetry exists in a realistic model.

EOM of photon = conservation of electric flux = symmetry? (1/2)

If the axion is absent, ...

- EOM as cons. law: $\partial_\mu(\frac{1}{e^2} f^{\mu\nu}) = 0$, $\quad j^{\mu\nu} = \frac{1}{e^2} f^{\mu\nu}$ is conserved
- **Unitary** operator $U^a = \exp\left(i\beta \int_{\mathcal{S}} \frac{1}{e^2} \tilde{f}_{\mu\nu} dS^{\mu\nu}\right)$

  $\int_{\mathcal{S}} \tilde{f}_{\mu\nu} dS^{\mu\nu} = \int \boldsymbol{E} \cdot d\boldsymbol{S}$ (Gauss law) or $\int \boldsymbol{B} \cdot d\boldsymbol{r} dt$ (Ampère law)



- $U(1)$ 1-form symmetry: phase rotation on Wilson loop

If the axion-photon coupling $\phi f_{\mu\nu} \tilde{f}^{\mu\nu}$ is present, ...

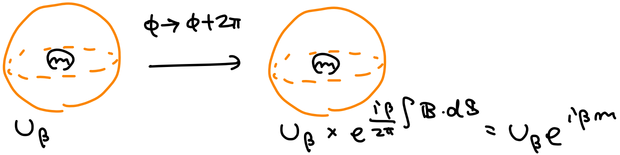
EOM of photon = conservation of electric flux = symmetry? (2/2)

Conservation law is deformed.

- EOM: $\partial_\mu(\frac{1}{e^2} f^{\mu\nu} - \frac{1}{4\pi^2}\phi\tilde{f}^{\mu\nu}) = 0$, cons. current $j^{\mu\nu} = \frac{1}{e^2} f^{\mu\nu} - \frac{1}{4\pi^2}\phi\tilde{f}^{\mu\nu}$
- However, **unitary** operator

$$U_\beta = \exp\left(\frac{i\beta}{4\pi^2}\int_{\mathcal{S}} \phi f_{\mu\nu} dS^{\mu\nu}\right) \times e^{i\beta\int_{\mathcal{S}} \frac{1}{e^2}\tilde{f}_{\mu\nu} dS^{\mu\nu}}$$

is only invariant under $\phi \to \phi + 2\pi$ if $e^{i\beta} = 1$



- No symmetry?

What we find: symmetry with a rational number, e.g., $\beta = \frac{2\pi}{q}$ $(q \in \mathbb{Z})$

is still valid at the expense of **unitarity**!

Modification using topological field theory [Choi, Lam, Shao, '22; RY '22]

$\exp\left(\frac{i}{2\pi q}\int_{\mathcal{S}} \phi f_{\mu\nu} dS^{\mu\nu}\right)$ has a gauge invariant expression!

Modification

$$\exp\left(\frac{i}{2\pi q}\int_{\mathcal{S}} \phi f_{\mu\nu} dS^{\mu\nu}\right)$$
$$\to \int \mathcal{D}\chi \mathcal{D}u_\mu \exp\left(-\frac{i}{2\pi}\int_{\mathcal{S}} dS^{\mu\nu}(q\chi w_{\mu\nu} - \chi f_{\mu\nu} - \phi w_{\mu\nu})\right)$$

Essentially $\frac{1}{q}xy \to -qab + ax + by$

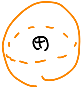
- Introducing auxiliary fields
  - χ: 2π periodic scalar
  - u_μ: $U(1)$ gauge field, field strength $w_{\mu\nu} = \partial_\mu u_\nu - \partial_\nu u_\mu$,
    Dirac quant. cond. $\int_{\mathcal{S}} w_{\mu\nu} dS^{\mu\nu} \in 2\pi\mathbb{Z}$
- 2π periodicity is preserved.
- Original one: naive expression with trivial Dirac quant. cond.
- Sum (path integral) of phase factors: **unitarity** is lost

Non-invertible 1-form symmetry [Choi, Lam, Shao, '22; RY '22]

Symmetry generator

$$D^a_{\beta=\frac{2\pi}{q}} = \int \mathcal{D}\chi\mathcal{D}u \exp\left(\frac{i}{2\pi}\int_{\mathcal{S}} dS^{\mu\nu}\left(-q\chi w_{\mu\nu} + \chi f_{\mu\nu} + \phi w_{\mu\nu}\right)\right)\cdot e^{i\beta\int_{\mathcal{S}}\tilde{f}dS}$$

- Symmetry generator is gauge invariant
- Topological: EOM of photon
- 1-form symmetry: phase rotation on Wilson loop

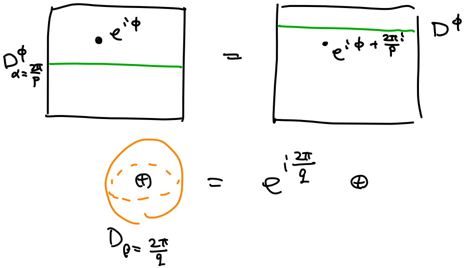


$= e^{i\frac{2\pi}{q}} \oplus$

$D_{\beta=\frac{2\pi}{q}}$

- Non-invertible (sum of phase factors)

Summary of non-invertible symmetries in axion electrodynamics



There are two non-invertible symmetries in axion electrodynamics

- EOM of axion $\rightarrow$ non-invertible 0-form symmetry
- EOM of photon $\rightarrow$ non-invertible 1-form symmetry

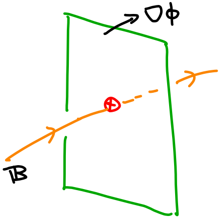
Question: How about current algebra for these symmetries?

- Ordinary symmetries: current algebra = group theory
- Non-invertible symmetries: current algebra = ?? (category theory?)

Let us discuss current algebra from Sikivie effect.

Current algebra of non-invertible symmetries from Sikivie effect

Sikivie effect [Sikivie '84]



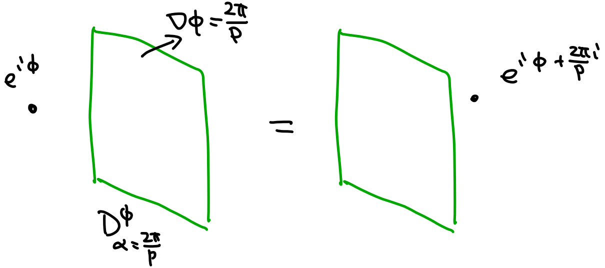
Induced charge on intersection of axionic domain wall & $\boldsymbol{B}$

- Modification of Gauss law $\nabla \cdot \boldsymbol{E} = \frac{1}{4\pi^2} \nabla\phi \cdot \boldsymbol{B}$

Current algebra from Sikivie effect?

Non-inv. 0- & 1-form symmetry generators = axionic domain wall & magnetic flux

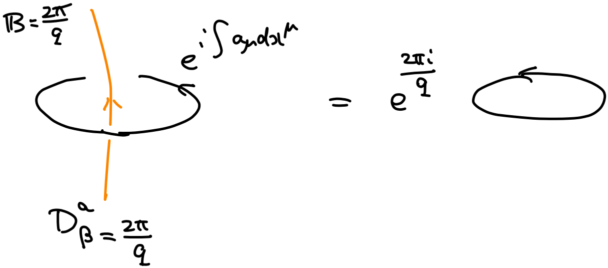
Non-inv. 0-form symmetry generator $D^{\phi}_{\alpha=\frac{2\pi}{p}}$ = axionic domain wall



Rough explanation

- $(2+1)$-dim. object
- Generating shift of axion by $\frac{2\pi}{p}$

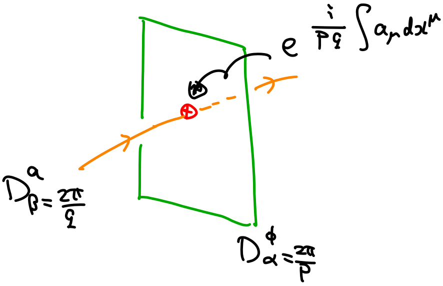
Non-inv. 1-form symmetry generator $D^a_{\beta=\frac{2\pi}{q}}$ = static magnetic flux tube



- $(1+1)$-dim. object
- Generating Aharonov-Bohm phase of Wilson loop by $\frac{2\pi}{q}$

Expectation: intersection of D^ϕ_α & D^a_β gives Sikivie effect

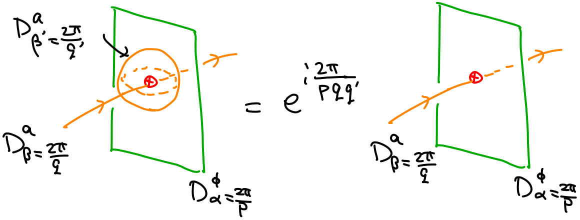
Sikivie effect by D^{ϕ}_{α} & D^{a}_{β} [Choi, Lam, Shao, '22; RY '22]



- Wilson loop $e^{\frac{i}{pq}\int a_\mu dx^\mu}$ with fractional charge $\frac{1}{pq}$ is induced
- Detail
  - Intersection $\to$ shift transf. $\phi \to \phi + \frac{2\pi}{p}$ of axion-photon coupling in D^a_β:
    $D^a_\beta \to e^{\frac{i}{pq}\int_{\mathcal{C}} a_\mu dx^\mu} D^a_\beta$
  - Fractional charge coincides with Gauss law $\frac{1}{4\pi^2}\nabla\phi \cdot \boldsymbol{B} = \frac{1}{pq}$

Fractional charge can be captured by another 1-form sym. generator.

Intersection of D^ϕ_α, D^a_β, $D^a_{\beta'}$ = fractional phase rotation [RY '22]



Rough derivation

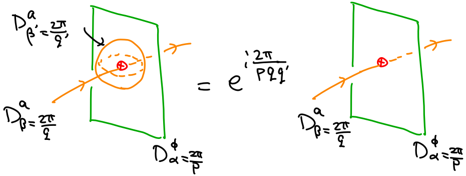
- Detection of electric flux from fractional charge by $D^a_{\beta'}$

Mixed 't Hooft anomaly? [cf. Kaidi Nardoni, Zafrir, Zheng, '23]

- Symmetry generators D^ϕ_α & D^a_β become source of D^a_β

  $\rightarrow$ conservation of D^a_β is violated by symmetry generators: $\langle D^a_\beta D^\phi_\alpha D^a_{\beta'} \rangle \propto e^{\frac{2\pi i}{pqq'}}$

Summary: axion electrodynamics has non-invertible 1-form symmetry



- Non-invertible 1-form symmetry is associated with EOM of photon
- Gauge invariant symmetry generator is constructed by path integral of phase factors
- Current algebra of symmetry generators are derived from the viewpoint of Sikivie effect

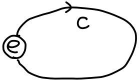
Future work

- Mathematical structure of current algebra?
- New examples of non-invertible symmetries?

Dirac quantization (1/2)

Electric & and magnetic charges should be quantized in $U(1)$ gauge theories

1. Quantization of electric charge



$e^{iq_e \int_C a_\mu dx^\mu}$

- Periodicity of $U(1)$ $\rightarrow$ gauge parameter can have a winding number $\oint \partial_\mu \lambda dx^\mu = 2\pi$

  (Insertion of Dirac string)

- Wilson loop should be invariant under gauge transf. with a winding number $a_\mu \rightarrow a_\mu + \partial_\mu \lambda$

  $e^{iq_e \int_{\mathcal{C}} a_\mu dx^\mu} \rightarrow e^{iq_e \int_{\mathcal{C}} \partial_\mu \lambda dx^\mu} \cdot e^{iq_e \int_{\mathcal{C}} a_\mu dx^\mu} = e^{2\pi i q_e} \cdot e^{iq_e \int_{\mathcal{C}} a_\mu dx^\mu}$

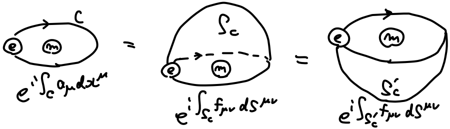
- Electric charge q_e must be quantized $q_e \in \mathbb{Z}$

Periodicity of $U(1)$ is important. If the gauge group is $\mathbb{R}$, $q_e \in \mathbb{R}$ due to $\oint \partial_\mu \lambda dx^\mu = 0$. [e.g., Banks & Seiberg '10].

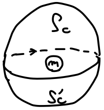
Dirac quantization (2/2)

2. quantization of magnetic charge q_m

- Stokes theorem for Wilson loop: some choices of surfaces



- One choice of should be equal to another



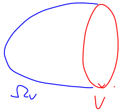
$$e^{i\int_{S_c \cup \bar{S}_c'} f_{\mu\nu} dS^{\mu\nu}} = e^{2\pi i q_m} = 1$$

- Magnetic charge must be quantized $q_m \in \mathbb{Z}$. [back]

Gauge invariance of $\exp\left(\frac{i\alpha}{8\pi^2}\int_{\mathcal{V}} dV_\mu a_\nu \tilde{f}^{\mu\nu}\right)$ (1/2)

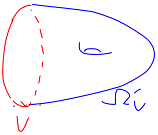
1. To make integrand of unitary operator gauge invariant, we use Stokes theorem

$$\exp\left(\frac{i\alpha}{8\pi^2}\int_{\mathcal{V}} dV_\mu a_\nu \tilde{f}^{\mu\nu}\right) = \exp\left(\frac{i\alpha}{8\pi^2}\int_{\Omega_{\mathcal{V}}} f_{\mu\nu}\tilde{f}^{\mu\nu} d^4x\right)$$



2. We have chosen an auxiliary 4-dim. space $\Omega_{\mathcal{V}}$.

3. Another choice $\Omega'_{\mathcal{V}}$ is possible:



Gauge invariance of $\exp\left(\frac{i\alpha}{8\pi^2}\int_{\mathcal{V}} dV_\mu a_\nu \tilde{f}^{\mu\nu}\right)$ (2/2)

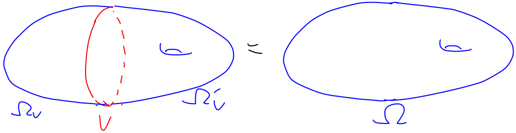
3. Two choices should be identical

$$\exp\left(\frac{i\alpha}{8\pi^2}\int_{\Omega_{\mathcal{V}}} f_{\mu\nu}\tilde{f}^{\mu\nu} d^4x\right) = \exp\left(\frac{i\alpha}{8\pi^2}\int_{\Omega'_{\mathcal{V}}} f_{\mu\nu}\tilde{f}^{\mu\nu} d^4x\right),$$

   Therefore, we have the constraint

$$\exp\left(\frac{i\alpha}{8\pi^2}\int_{\Omega} f_{\mu\nu}\tilde{f}^{\mu\nu} d^4x\right) = 1$$

   for $\Omega = \Omega_{\mathcal{V}} \cup \bar{\Omega}'_{\mathcal{V}}$



4. Dirac quantization requires $\int_\Omega f_{\mu\nu}\tilde{f}^{\mu\nu} d^4x \in 2\cdot(2\pi)^2\mathbb{Z}$ for closed Ω

5. Therefore $\exp(i\alpha) = 1$.

[back]

Derivation of 1-form symmetry transformation 0/4

Symmetry transformation in terms of correlation function

$$\langle e^{i\alpha \int_{\mathcal{S}} \frac{1}{e^2} \tilde{f}_{\mu\nu} dS^{\mu\nu}} e^{i \int_{\mathcal{C}} a_\mu dx^\mu} \rangle = \int \mathcal{D}a e^{iS + i\alpha \int_{\mathcal{S}} \frac{1}{e^2} \tilde{f}_{\mu\nu} dS^{\mu\nu} + i \int_{\mathcal{C}} a_\mu dx^\mu}$$

We want to eliminate $\int_{\mathcal{S}} \tilde{f}_{\mu\nu} dS^{\mu\nu}$ by field redefinition.

Two problems:

1. Action ($\int f_{\mu\nu} f^{\mu\nu}$) is 2nd. order derivative, but $\int_{\mathcal{S}} \tilde{f}_{\mu\nu} dS^{\mu\nu}$ is 1st order deriv.
2. Action is 4-dim integral, but $\int_{\mathcal{S}} \tilde{f}_{\mu\nu} dS^{\mu\nu}$ is 2-dim. integral

Derivation of 1-form symmetry transf. 1/4

We can rewrite $\int_{\mathcal{S}} \tilde{f}_{\mu\nu} dS^{\mu\nu}$ as follows:

Gauss-Stokes theorem

$$\int_{\mathcal{S}} \tilde{f}_{\mu\nu} dS^{\mu\nu} = \int_{\mathcal{V}_{\mathcal{S}}} \partial_\mu f^{\mu\nu} dV_\nu$$

$\partial \mathcal{V}_{\mathcal{S}} = \mathcal{S}$, dV_ν: volume element

- Tensorial expression of $\int_{\mathcal{S}} \boldsymbol{E} \cdot d\boldsymbol{S} = \int_{\mathcal{V}_{\mathcal{S}}} \nabla \cdot \boldsymbol{E} dV$

The right-hand side can be rewritten as 4-dim integral

Derivation of 1-form symmetry transf. 2/4

By using delta function, we have

4-dim integral

$$\int_{\mathcal{V}_{\mathcal{S}}} \partial_\mu f^{\mu\nu} dV_\nu = \int d^4x\, \partial_\mu f^{\mu\nu}\, \delta_\nu(\mathcal{V}_{\mathcal{S}}), \quad \text{where} \quad \delta_\nu(\mathcal{V}_{\mathcal{S}}) = \int_{\mathcal{V}_{\mathcal{S}}} dV_\nu(y)\delta^4(x-y)$$

Derivation:

$$\begin{aligned}\int_{\mathcal{V}_{\mathcal{S}}} \partial_\mu f^{\mu\nu}(y) dV_\nu(y) &= \int d^4x \int_{\mathcal{V}_{\mathcal{S}}} \delta^4(x-y)\partial_\mu f^{\mu\nu}(x) dV_\nu(y) \\ &= \int d^4x \partial_\mu f^{\mu\nu}(x) \left(\int_{\mathcal{V}_{\mathcal{S}}} \delta^4(x-y) dV_\nu(y) \right)\end{aligned}$$

Two problems have been solved. Symmetry generator can be absorbed into the action,

$$\frac{1}{e^2}\int (-\tfrac{1}{4}|f_{\mu\nu}|^2 + \alpha\partial_\mu f^{\mu\nu}\delta_\nu(\mathcal{V}_{\mathcal{S}})) = S[a_\mu - \alpha\delta_\mu(\mathcal{V}_{\mathcal{S}})]$$

(UV divergence is renormalized.)

Derivation of 1-form symmetry transf. 3/4

By the field redefinition $a_\mu - \alpha\delta_\mu(\mathcal{V}_\mathcal{S}) \to a_\mu$,

Symmetry generator can be eliminated

$$\langle e^{i\alpha\int_\mathcal{S}\frac{1}{e^2}\tilde{f}_{\mu\nu}dS^{\mu\nu}} e^{i\int_\mathcal{C} a_\mu dx^\mu}\rangle = \int \mathcal{D}a e^{iS + i\alpha\int_\mathcal{S}\frac{1}{e^2}\tilde{f}_{\mu\nu}dS^{\mu\nu} + i\int_\mathcal{C} a_\mu dx^\mu}$$
$$= e^{i\alpha\int_\mathcal{C}\delta_\mu(\mathcal{V}_\mathcal{S})dx^\mu}\langle e^{i\int_\mathcal{C} a_\mu dx^\mu}\rangle$$

What is $\int_\mathcal{C}\delta_\mu(\mathcal{V}_\mathcal{S})dx^\mu$ in the right-hand side?

Derivation of 1-form symmetry transf. 4/4

Linking number between $\mathcal{S}$ & $\mathcal{C}$

$$\int_{\mathcal{C}} \delta_\mu(\mathcal{V}_{\mathcal{S}}) dx^\mu = \mathrm{Link}\,(\mathcal{S}, \mathcal{C}) \in \mathbb{Z}$$

- Integral = intersection number between $\mathcal{V}_{\mathcal{S}}$ & $\mathcal{C}$ = linking number between $\mathcal{S}$ & $\mathcal{C}$

We arrive at

$U(1)$ symmetry transformation

$$\langle e^{i\alpha \int_{\mathcal{S}} \frac{1}{e^2} \tilde{f}_{\mu\nu} dS^{\mu\nu}} e^{i \int_{\mathcal{C}} a_\mu dx^\mu} \rangle = e^{i\alpha\, \mathrm{Link}\,(\mathcal{S},\mathcal{C})} \langle e^{i \int_{\mathcal{C}} a_\mu dx^\mu} \rangle$$

[back]

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