Noncommutative Geometry and String Theory

Mitsuhiro Kato

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0. Introduction

Classical gravity $\Leftrightarrow$ Riemannian geometry

Quantum gravity $\Leftrightarrow$?

String theory as a candidate of quantum theory of gravity describes a new kind of geometry for the *spacetime* as well as *branes* in it, which differ from the "point-like" geometry.

$\Rightarrow$ **Noncommutative geometry (NCG)**

But what's NCG? . . . still to be developed.

Plan of this talk

1. Noncommutative geometry
2. Stringy geometry
3. D-branes
4. Open string in the background B field
5. NC open string (NCOS) theory
6. D-brane as NC soliton
7. NC geometric formulation: An attempt
8. Conclusion

1. Noncommutative geometry

Basic idea

Manifold $\mathcal{M} \longleftarrow C^\infty(\mathcal{M})$ Complex functions on $\mathcal{M}$

Gelfand-Naimark

$C^\infty(\mathcal{M})$: commutative *-algebra

$$\begin{aligned} (f+g)(x) &= f(x)+g(x) \\ (fg)(x) &= f(x)g(x) \quad \leftarrow \text{commutative} \end{aligned}$$

Generalization:

general *-algebra $\mathcal{A} \longrightarrow$ Extended "geometry"

Noncommutative geometry

Example: torus T^2

$$L^2(\mathsf{T}^2) \quad \ni \quad f(x_1, x_2) = \sum_{n_1,n_2} c_{n_1,n_2} e^{in_i x^i}$$

Fourier series

Algebra of functions is generated by
$u_1 = e^{ix_1}, u_2 = e^{ix_2}$ which satisfy $u_1 u_2 = u_2 u_1$.

NC torus T^2_θ

$$f = \sum_{n_1,n_2} c_{n_1,n_2} U_1^{n_1} U_2^{n_2}$$

with $U_1 U_2 = e^{2\pi i\theta} U_2 U_1$

A realization: Schwarz space $S(\mathbf{R})$
For $\xi(s)$ a square-integrable function on $\mathbf{R}$,

$$\begin{aligned} (V_1\xi)(s) &= e^{2\pi i\gamma s}\xi(s) \\ (V_2\xi)(s) &= \xi(s+1) \end{aligned}$$

where $V_1 V_2 = e^{-2\pi i\gamma} V_2 V_1$

Note:

NCG includes the case where functions themselves are commutative but differential algebra is non-commutative.

<u>2-points space</u> $\quad Y = \{1, 2\}$

$$\mathcal{A} = \mathbf{C} \oplus \mathbf{C} \qquad \ni \qquad (f_1, f_2)$$

$\Omega^*(\mathcal{A})$

$$f = \begin{pmatrix} f_1 & \\ & f_2 \end{pmatrix} \qquad df = \mu \begin{pmatrix} & f_2 - f_1 \\ f_1 - f_2 & \end{pmatrix}$$

Distance between 2-points:

$$d(1, 2) = \sup_{f \in \mathcal{A}} \{ |f_1 - f_2| ; \|df\| \leq 1 \} = \mu^{-1}$$

...Connes' standard model
lattice gauge theory
etc.

2. Stringy geometry

classical size of string $\sim O(l_s)$
quantum size $\langle (X - \langle X \rangle)^2 \rangle \propto \sum_{n=1}^{\infty} \frac{1}{n} \rightarrow$ diverge

Minimal length

$$\langle (X(\sigma + \epsilon) - X(\sigma))^2 \rangle \sim l_s^2$$

Stringy uncertainty relation

(Veneziano, Gross,...)

$$\Delta X \gtrsim \frac{\hbar}{\Delta p} + \frac{\alpha'}{\hbar} \Delta p \gtrsim \alpha'$$

Space-time uncertainty (Yoneya)

$$\left.\begin{array}{ccc} \Delta E & \sim & T \Delta X \\ \Delta t \, \Delta E & \gtrsim & \hbar \end{array}\right\} \rightarrow \Delta t \, \Delta X \gtrsim \frac{\alpha'}{c}$$

T-duality (Kikkawa-Yamasaki, Sakai-Senda)
torus compactification

$$R \leftrightarrow \frac{\alpha'}{R}$$

self-dual point $R \sim \sqrt{\alpha'}$ . . . symmetry enhancement

Noncommutativity

(Itoh-MK-Kunitomo-Sakamoto)

twisted string on orbifold

$$X^i(\sigma + \pi) = U^{ij} X^j(\sigma) + \pi n^a e^i_a$$

$$[x^i, Q^j] = -i\pi(1-U)^{-1}_{ij}$$

where

$$\begin{aligned} x^i &= x^i_L + x^i_R \\ Q^i &= x^i_L - x^i_R + B^{ij}(x^j_L + x^j_R) \\ p^i &= p^i_L + p^i_R + B^{ij}(p^j_L - p^j_R) \\ L^i &= p^i_L - p^i_R \end{aligned}$$

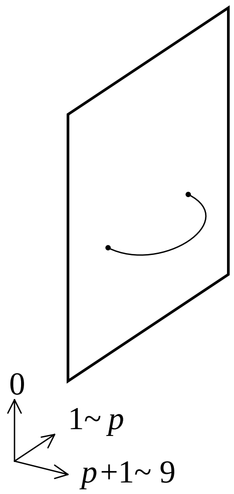
String field theory

(Kaku-Kikkawa,Witten,HIKKO)

$\Phi[X(\sigma)]$ string field $\to$ noncommutative algebra

3. D-branes

Dp-brane



$$\partial_\sigma X^\mu\big|_{\text{boundary}} = 0 \quad \mu = 0, 1, \cdots, p$$
$$X^i\big|_{\text{boundary}} = 0 \quad i = p+1, \cdots, 9$$

massless bosonic excitation of open string

$A_\mu(x)$: $p+1$ dim. gauge fields
$X^i(x)$: collective coordinates
in transverse directions

low energy effective action

$$S = \int d^{p+1}x \,\text{tr}\left\{ -\frac{1}{4g_s l_s^{p-3}} F_{\mu\nu}F^{\mu\nu} - \frac{1}{4g_s l_s^{p+1}} D_\mu X^i D^\mu X^i + \frac{1}{4g_s l_s^{p+5}} [X^i, X^j]^2 \right\}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$$
$$D_\mu X^i = \partial_\mu X^i - i[A_\mu, X^i]$$

g_s : string coupling constant

Noncommutativity of Dp-brane

$[X^i, X^j]$	$[\pi^i, \pi^j]$
$\updownarrow$	$\updownarrow$
D$(p+2)$-brane charge	B_{ij} background
$[D_\mu, D_\nu]$	$[x^\mu, x^\nu]$
$\updownarrow$	$\updownarrow$
D$(p-2)$-brane charge	$B_{\mu\nu}$ background

Worldvolume noncommutativity

static eq. of motion for X^i

$$[X^i, [X^i, X^j]] = 0$$

trivial solution

$[X^i, X^j] = 0 \quad \rightarrow \quad$ simultaneous diagonalization

D$(p+2)$-brane solution

$[X^i, X^j] = if \quad \rightarrow \quad$ extended in {ij}-direction

$$X^i(x^\mu) \leftrightarrow X^i(x^\mu, x^i, x^j)$$

Chern-Simons coupling to RR gauge field

$$S_{\text{C-S}} \sim \int_{\mathcal{M}_{p+1}} \Big[C^{(p+1)} + \mathcal{F} C^{(p-1)} + \mathcal{F}^2 C^{(p-3)} + \cdots$$
$$+ i_X i_X C^{(p+3)} + (i_X i_X)^2 C^{(p+5)} + \cdots \Big]$$

where

$$\begin{aligned} i_X C^{(n+1)} &= X^j C_{j\nu_1\cdots\nu_n} dx^{\nu_1} \wedge \cdots \wedge dx^{\nu_n} \\ \mathcal{F}_{\mu\nu} &= F_{\mu\nu} + B_{\mu\nu} \end{aligned}$$

Noncommutative torus

torus compactification ($x_j = x_j + 2\pi R_j$)

$$U_i X_j U_i^{-1} = X_j + 2\pi R_j \mathbf{1}$$

$$U_1 U_2 = e^{2\pi i \theta} U_2 U_1$$

A representation

$$\begin{aligned} U_i &= e^{i\sigma_i} e^{2\pi\theta\epsilon_{ij}\partial_j} \\ X_i &= -i 2\pi R_i (\partial_i - i A_i(\tilde{U})) \end{aligned}$$

where

$$[\sigma_i, \sigma_j] = 2\pi i \theta \epsilon_{ij} \quad [\partial_i, \sigma_j] = \delta_{ij} \quad [\partial_i, \partial_j] = 0$$

$$\tilde{U}_i = e^{i\sigma_i} \quad U_i \tilde{U}_j = \tilde{U}_j U_i \quad \tilde{U}_1 \tilde{U}_2 = e^{-2\pi i \theta} \tilde{U}_2 \tilde{U}_1$$

$A_i \leftarrow$ gauge field on NC torus

$$\frac{1}{4} \text{tr}([X^i, X^j]^2) \propto -\frac{1}{4} \text{tr} F_{ij} F_{ij}$$

where $F_{ij} = \partial_i A_j - \partial_j A_i - i[A_i, A_j]$

Take

$$\begin{aligned} X_1 &= -i2\pi l_s \partial_1 \\ X_2 &= -i2\pi l_s (\partial_2 - if\sigma_1) \end{aligned}$$

$$[X_1, X_2] = -i(2\pi l_s)^2 f$$

Boundary state on the torus MK-Kuroki

$$|\sigma_i)^r = \sum_{\{w^i \in \mathbf{Z}\}} e^{i(\sigma_2 + 2\pi r) w^2/n} e^{i\sigma_1 w^1}$$

$$\times \left| 2\pi l_s \left(w^1, (1 + 2\pi f\theta) w^2/n - f\sigma_1 \right) \right\rangle$$

$$\begin{aligned} \left(q^1 + (2\pi l_s)^2 f p_2 \right) |\sigma_i)^r &= X^1 |\sigma_i)^r \\ q^2 \, |\sigma_i)^r &= X^2 |\sigma_i)^r \end{aligned}$$

$$\begin{aligned} \left(p_1 - \frac{\theta}{2\pi\alpha'} q^2 + (1 + 2\pi f\theta)\pi_1 \right) |\sigma_i)^r &= 0 \\ (p_2 + \pi_2) \, |\sigma_i)^r &= 0 \end{aligned}$$

where

$$\pi_1 = -\frac{\sigma_1}{2\pi l_s} \qquad \pi_2 = -\frac{\sigma_2}{(1 + 2\pi f\theta) 2\pi l_s}$$

cf.

$$(q - X)\delta(q - X) = 0 \qquad -i(\partial_q + \partial_X)\delta(q - X) = 0$$

4. Open string in the background B field

$$S = \frac{-1}{4\pi\alpha'}\int d\tau d\sigma \left(g_{\mu\nu}\partial_a X^\mu \partial^a X^\nu - 2\pi\alpha' B_{\mu\nu}\epsilon^{ab}\partial_a X^\mu \partial_b X^\nu\right)$$

eq. of motion:

$$(\partial_\tau^2 - \partial_\sigma^2)X^\mu = 0$$

boundary condition:

$$\left(g_{\mu\nu}\partial_\sigma X^\nu + 2\pi\alpha' B_{\mu\nu}\partial_\tau X^\nu\right)\Big|_{\sigma=0,\pi} = 0$$

Canonical conjugate momentum:

$$P_\mu = \frac{1}{2\pi\alpha'}\left(g_{\mu\nu}\partial_\tau X^\nu + 2\pi\alpha' B_{\mu\nu}\partial_\sigma X^\nu\right)$$

$\longrightarrow$ Constraint at the boundary

$$G_{\mu\nu}\left(\partial_\sigma X^\nu - \theta^{\nu\lambda}P_\lambda\right)\Big|_{\sigma=0,\pi} = 0$$

where

$$G_{\mu\nu} = g_{\mu\nu} - (2\pi\alpha')^2(Bg^{-1}B)_{\mu\nu}$$
$$\theta^{\mu\nu} = -(2\pi\alpha')^2\left(\frac{1}{g+2\pi\alpha' B}B\frac{1}{g-2\pi\alpha' B}\right)^{\mu\nu}$$

$\longrightarrow$ **Noncommutativity of string end point**

$$
\begin{aligned}
[X^\mu(\tau,0), X^\nu(\tau,0)] &= i\theta^{\mu\nu} \\
[X^\mu(\tau,\pi), X^\nu(\tau,\pi)] &= -i\theta^{\mu\nu} \\
[X^\mu(\tau,\sigma), X^\nu(\tau,\sigma')] &= 0 \quad \text{otherwise}
\end{aligned}
$$

Chu-Ho, Ardalan-Arfaei-SheikhJabbari

Another derivation Seiberg-Witten

$$
\langle X^\mu(z) X^\nu(z') \rangle = -\alpha' \Big\{ g^{\mu\nu} \ln|z-z'| - g^{\mu\nu} \ln|z-\bar{z}'|
$$

$$
+G^{\mu\nu} \ln|z-\bar{z}'|^2 + \frac{1}{2\pi\alpha'} \theta^{\mu\nu} \ln \frac{z-\bar{z}'}{\bar{z}-z'} + \text{const} \Big\}
$$

Take $\begin{cases} z & \rightarrow & \tau \\ z' & \rightarrow & \tau' \end{cases}$ on real axis

$$
\langle X^\mu(\tau) X^\nu(\tau') \rangle = -\alpha' G^{\mu\nu} \ln(\tau-\tau')^2 + \frac{i}{2} \theta^{\mu\nu} \epsilon(\tau-\tau')
$$

$\downarrow$ point splitting

$$
[X^\mu(\tau), X^\nu(\tau)] = i\theta^{\mu\nu}
$$

$$e^{ip\cdot X}(\tau)\, e^{iq\cdot X}(\tau') \sim$$

$$(\tau - \tau')^{2\alpha' G^{\mu\nu} p_\mu q_\nu}\; e^{-\frac{i}{2}\theta^{\mu\nu} p_\mu q_\nu}\; e^{i(p+q)\cdot X}(\tau') + \cdots$$

$\alpha' \to 0$ (G, θ fixed) limit

$$e^{ip\cdot X}(\tau)\, e^{iq\cdot X}(\tau') \sim e^{ip\cdot X} * e^{iq\cdot X}(\tau')$$

where

$$\begin{aligned} f(x) * g(x) &= e^{\frac{i}{2}\theta^{\mu\nu}\frac{\partial}{\partial\xi^\mu}\frac{\partial}{\partial\eta^\nu}} f(x+\xi)\, g(x+\eta)\Big|_{\xi=\eta=0} \\ &= f\, g + \frac{i}{2}\theta^{\mu\nu}\partial_\mu f\, \partial_\nu g + O(\theta^2) \end{aligned}$$

Moyal product

Effective action for NC gauge field $\widehat{A}$

$$S_{\text{eff}} = \frac{(\alpha')^{\frac{3-p}{2}}}{4(2\pi)^{p-2} G_s} \int \sqrt{G} G^{\mu\lambda} G^{\nu\rho}\, \widehat{F}_{\mu\nu} * \widehat{F}_{\lambda\rho}$$

where

$$\begin{aligned} \widehat{F}_{\mu\nu} &= \partial_\mu \widehat{A}_\nu - \partial_\nu \widehat{A}_\mu - i(\widehat{A}_\mu * \widehat{A}_\nu - \widehat{A}_\nu * \widehat{A}_\mu) \\ G_s &= g_s \left(\frac{\det(g + 2\pi\alpha' B)}{\det g} \right)^{1/2} \end{aligned}$$

open string with Pauli-Villars regularization
$\longrightarrow$ ordinary gauge theory $(F+B)$

open string with point-splitting regularization
$\longrightarrow$ NC gauge theory $(\widehat{F},\theta)$

They give same S-matrix

$\downarrow$

Should be related by " field redefinition "

$$\widehat{A}_\mu = A_\mu - \frac{1}{4}\theta^{\nu\lambda}\{A_\nu\,,\,\partial_\lambda A_\mu + F_{\lambda\mu}\} + O(\theta^2)$$

Seiberg-Witten map

For slowly varying field: DBI action

$$\mathcal{L}_{\rm DBI} = \frac{1}{g_s(2\pi)^p(\alpha')^{\frac{p+1}{2}}}\sqrt{\det\Big(g+2\pi\alpha'(B+F)\Big)}$$

$$\widehat{\mathcal{L}}_{\rm DBI} = \frac{1}{G_s(2\pi)^p(\alpha')^{\frac{p+1}{2}}}\sqrt{\det\Big(G+2\pi\alpha'\widehat{F}\Big)}$$

Seiberg-Witten map for constant F

$$\widehat{F} = B\frac{1}{B+F}F \qquad \theta = \frac{1}{B}$$

☑ With derivative term $\longrightarrow$ Terashima & Okawa's talks

5. NC open string (NCOS) theory

Seiberg-Susskind-Toumbas (hep-th/0005040)
Gopakumar-Maldacena-Minwalla-Strominger (hep-th/0005048)

- Open string theory on the D-brane
- With space/time noncommutativity
- Decoupled from the bulk closed string
- Dual to the NCYM

Strings in the background time-like B field

Consider purely electric case

$$B_{ij} = F_{ij} = 0, \qquad B_{01} \neq 0$$

We choose $(\mu, \nu = 0, 1)$

$$g_{\mu\nu} = g \begin{pmatrix} -1 & \\ & 1 \end{pmatrix}, \quad B_{\mu\nu} = \begin{pmatrix} & E \\ -E & \end{pmatrix}$$

It is convenient to define

$$\tilde{E} = \frac{2\pi\alpha'}{g} E$$

Then

$$\begin{aligned} G_{\mu\nu} &= \frac{g^2 - (2\pi\alpha' E)^2}{g} \begin{pmatrix} -1 & \\ & 1 \end{pmatrix} \equiv G \begin{pmatrix} -1 & \\ & 1 \end{pmatrix} \\ \theta^{\mu\nu} &= \frac{(2\pi\alpha')^2 E}{g^2 - (2\pi\alpha' E)^2} \begin{pmatrix} & 1 \\ -1 & \end{pmatrix} \equiv \theta \begin{pmatrix} & 1 \\ -1 & \end{pmatrix} \\ G_s &= g_s \left(1 - \left(\frac{2\pi\alpha' E}{g} \right)^2 \right)^{1/2} \end{aligned}$$

$$\begin{aligned} G &= g(1 - \tilde{E}^2) \\ \theta &= \frac{2\pi\alpha'}{g} \frac{\tilde{E}}{1 - \tilde{E}^2} \\ G_s &= g_s (1 - \tilde{E}^2)^{1/2} \end{aligned}$$

Since $\tilde{E} < 1$, $\alpha' G^{-1}$ cannot be taken to 0 while keeping θ finite

$$\theta = 2\pi\alpha' G^{-1} \tilde{E}$$

$\rightarrow$ no NC field theory limit

$$\langle X^\mu(\tau) X^\nu(\tau') \rangle = -\alpha' G^{\mu\nu} \ln(\tau - \tau')^2 + \frac{i}{2} \theta^{\mu\nu} \epsilon(\tau - \tau')$$

Effective tension

$$\frac{1}{2\pi\alpha'_{\rm eff}} = \frac{1}{2\pi\alpha'}\frac{G_{11}}{g} = \frac{1-\tilde{E}^2}{2\pi\alpha'} \quad \xrightarrow{|\tilde{E}|\to 1} 0$$

dipole: $\quad \ominus\!\!\longrightarrow\!\!\oplus \qquad \longrightarrow E$

Strings cannot get off the branes

Rescale $G_{\mu\nu}$ to $\eta_{\mu\nu}$

$$g_{ij} = \delta_{ij} \quad i,j \neq 0,1$$

$$g_{00} = -g_{11} = -g = \frac{-1}{1-\tilde{E}^2}$$

Consider a process:

open strings $\quad p_0^{(i)2} = p_1^{(i)2} + p_2^{(i)2} + \cdots + m^{(i)2}$

$\Downarrow$

closed string $\quad \frac{1}{g}p_0^2 = \frac{1}{g}p_1^2 + p_2^2 + \cdots + m^2$

Since $g = \frac{1}{1-\tilde{E}^2} \to \infty$ as $\tilde{E} \to 1$, such a dispersion relation cannot be satisfied unless p_0 is infinite.

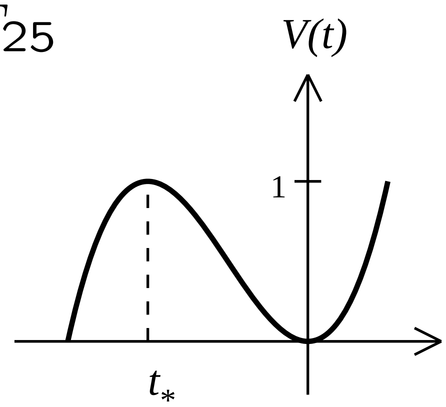
6. D-brane as NC soliton

Harvey-Kraus-Larsen-Martinec (hep-th/0005031)

Effective tachyon action

$$S = \frac{C}{g_s} \int d^{26}x \sqrt{g} \left\{ \frac{1}{2} f(t) g^{\mu\nu} \partial_\mu t \partial_\nu t + \cdots - V(t) \right\}$$

$$C = g_s T_{25}$$



Turn on B_{12}

$$S = \frac{C}{G_s} \int d^{26}x \sqrt{G} \left\{ \frac{1}{2} f(t) g^{\mu\nu} \partial_\mu t \partial_\nu t + \cdots - V(t) \right\}_*$$

Scaling argument + constant solution in commutative direction lead to eq. of motion in large NC limit:

$$\frac{dV(t)}{dt} = 0$$

Utilizing NC soliton:

$$\phi * \phi = \phi$$

(Gopakmar-Minwalla-Strominger)

Simplest solution is

$$t = t_* \phi_0 \quad \text{where} \quad \phi_0 = 2e^{-(x_1^2 + x_2^2)/\theta}$$

$\rightarrow$ D23-brane

Evaluating tension of the soliton

$$\begin{aligned} S &= -\frac{C}{G_s} \int d^{26}x \sqrt{G} V(t) \\ &= -\frac{C 2\pi\theta V(t_*)}{G_s} \int d^{24}x \sqrt{G} \\ &= -(2\pi)^2 \alpha' \frac{C}{g_s} \int d^{24}x \sqrt{g} \end{aligned}$$

where

$$G_s = \frac{g_s \sqrt{G}}{2\pi\alpha' B \sqrt{g}} \quad \text{and} \quad \theta = 1/B$$

are used.

Thus tension of this soliton is

$$(2\pi)^2 \alpha' \frac{C}{g_s} = (2\pi)^2 \alpha' T_{25} = T_{23}$$

In general for bosonic Dp-brane

$$T_p = (2\pi)^{25-p} (\alpha')^{(25-p)/2} T_{25}$$

7. NC geometric formulation: An attempt

Hirano-MK ('97)

Coordinates of NC space: X^i ($i = 1 \cdots 8$)

Arbitrary deformation: $\delta X^i = \epsilon^i$

($\supset$ general coordinate trf.)

Topological type theory: $S_0 = 0$

Taking a gauge fixing cond.: $P_{ijkl}[X^k, X^l] = 0$

where $P_{ijkl} = \frac{1}{4}(\delta_{ik}\delta_{jl} - \frac{1}{2}T_{ijkl})$,
T_{ijkl} is $Spin(7)$ inv. tensor (related to **Octonion**)

Gauge fixed action via BRST

$$
\begin{aligned}
\delta X^i &= \psi^i \\
\delta \psi^i &= [X^i, \phi] \\
\delta \chi_{ij} &= i b_{ij} \\
\delta b_{ij} &= -i[\chi_{ij}, \phi] \\
\delta \phi &= 0 \\
\delta \bar{\phi} &= 2\eta \\
\delta \eta &= \frac{1}{2}[\phi, \bar{\phi}]
\end{aligned}
$$

$$
S = -i\delta \left\{ i\chi^{ij} P_{ijkl}[X^k, X^l] - \frac{1}{4} b_{ij}\chi^{ij} \right.
$$

$$
\left. -\frac{i}{2}\bar{\phi}\left([X_i, \psi^i] - \frac{1}{2}[\phi, \eta]\right) \right\}
$$

Identifying

$$\begin{aligned} A^i &= X^i \\ A^0 &= \frac{1}{2}(\phi+\bar{\phi}) \\ A^9 &= \frac{1}{2}(\phi-\bar{\phi}) \end{aligned} \qquad \begin{aligned} \lambda_+^i &= \psi^i \\ \lambda_-^a &= 2\chi^{8a} \quad (a=1\cdots 7) \\ \lambda_-^8 &= \eta \end{aligned}$$

$$\Psi = (\lambda_+, \lambda_-, 0, 0)^T$$

The action can be written as (integrating out b_{ij})

$$S = \mathrm{tr}\left\{-\frac{1}{4}[A_\mu, A_\nu]^2 - \frac{1}{2}\bar{\Psi}\Gamma^\mu[A_\mu, \Psi]\right\}$$

$$+\frac{1}{8}T_{ijkl}\,\mathrm{tr}\left([X^i, X^j][X^k, X^l]\right)$$

- IIB matrix model + extra term
- 9+1 signature automatically obtained

- b_{ij} is quite similar to the world sheet $b_{\mu\nu}$ discussed by Yoneya (hep-th/0004074)

8. Conclusion

- Nonperturbative analysis can be simplified in the large NC limit.

- NCG is essential ingredient in describing D-branes nonperturbatively.

- Other background (RR gauge field, . . .).

- Curved branes
  (D-branes on Group manifold, . . .).

- Unified view in terms of NCG.
  Importance of Morita equivalence and K-theory.
  (Criteria for the algebras describing "same" geometry.)

- General covariance (automorphism of algebra).

- Background independence.