

Extra dimension の物理



基礎 橋 基

Randall-Sundrum 以後

§1. Intro.

§2. Impact of the RS model

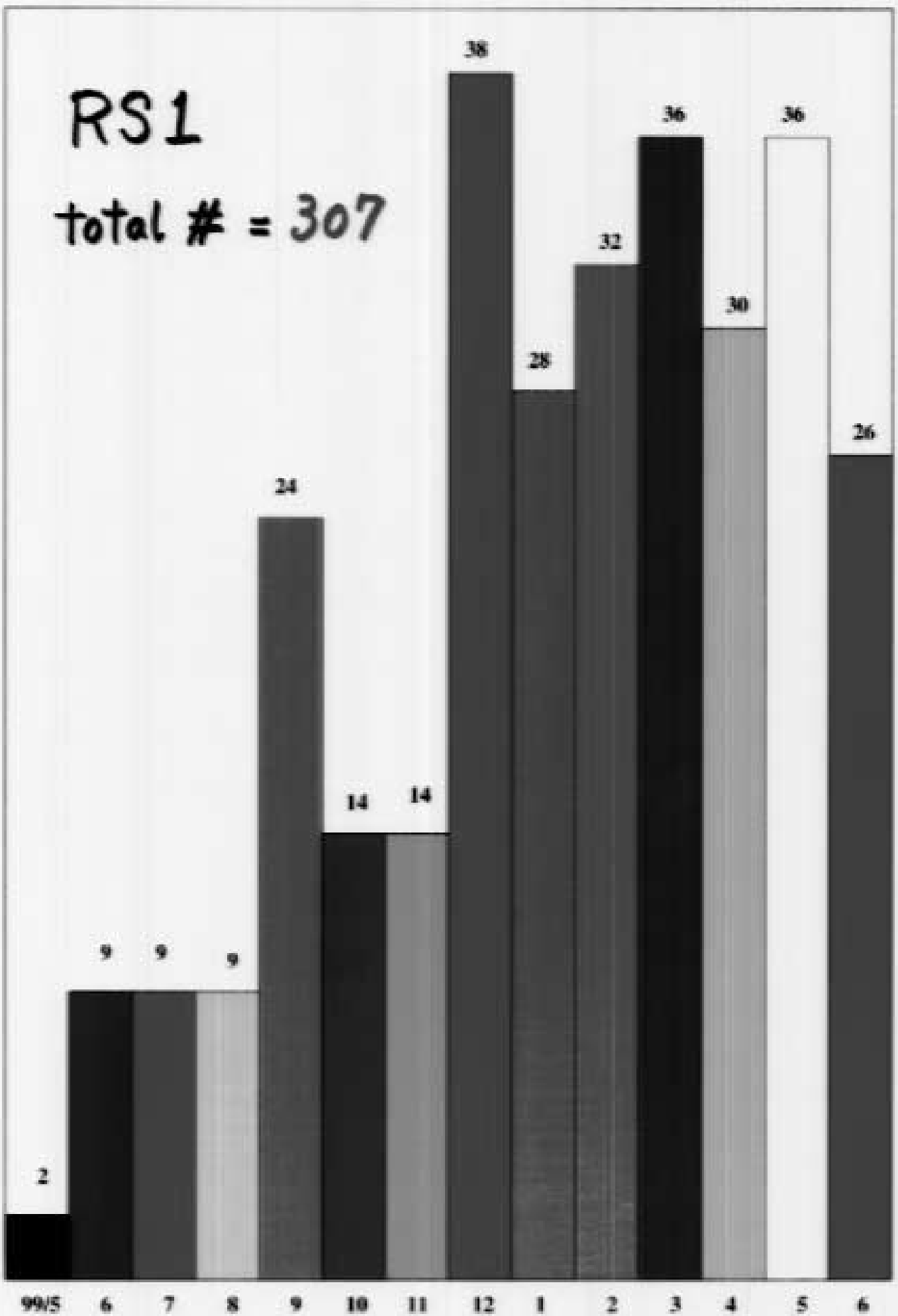
§3. Universal Aspects of
Bound State Graviton
(Localized Gravity)

§4. Toward the Cosmological
Constant Problem

§5. Outro.

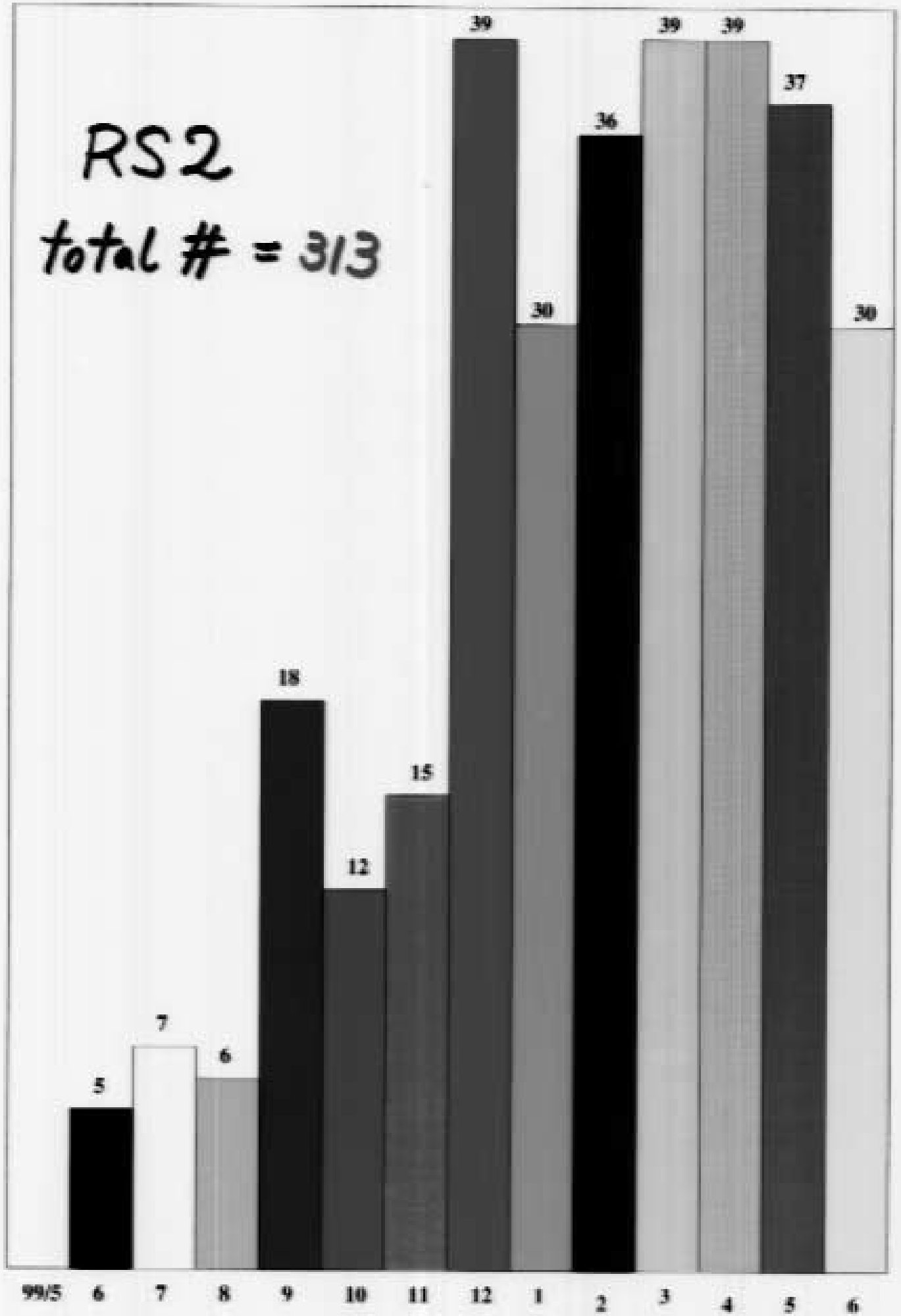
RS1

total # = 307



RS2

total # = 313



§2. Impact of the RS model

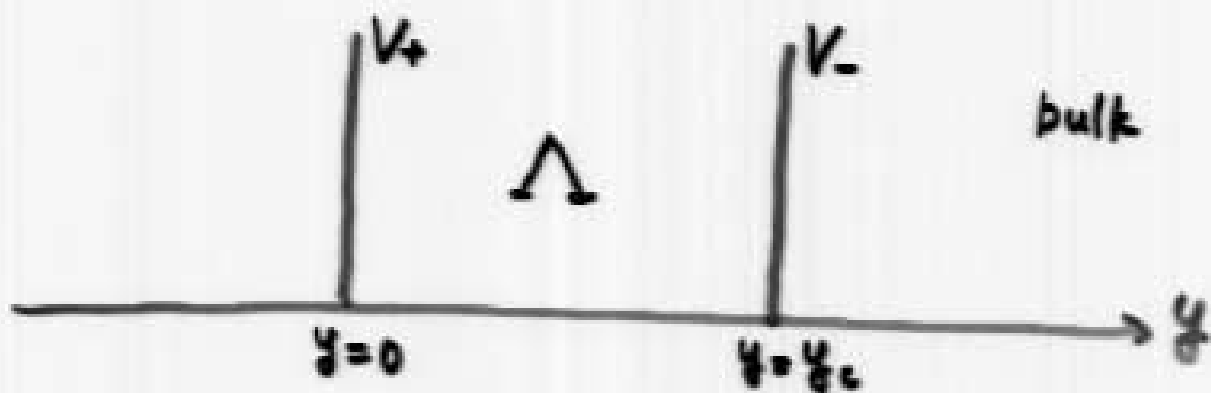
(PRL 83 (99) 3370)

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$$S = \int d^5x \int dy \sqrt{g} \{ 2M^3 R - \Lambda \}$$

↖ 5-d Planck

$$- \int d^4x \left[\sqrt{g_+^{(4)}} V_+ + \sqrt{g_-^{(4)}} V_- \right]$$



warped geometry (4-d Poincaré inv.)

$$ds^2 = e^{-2k|y|} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$$

$$k \equiv \left(\frac{-\Lambda}{24M^3} \right)^{1/2} \rightarrow \boxed{\Lambda < 0} \quad \begin{array}{l} \text{bulk is} \\ \text{AdS} \end{array}$$

★ $V_+ = -V_- = -\frac{\Lambda}{k}$: Fine tuning!

4-d Planck scale

$$M_{\text{Pl}}^2 \equiv 2M^3 \int_0^{y_c} dy e^{-2ky} = \frac{M^3}{k} (1 - e^{-2ky_c})$$



Bound State Graviton

(PRL 83 ('99) 4690)

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$y_c \rightarrow \infty$ (Infinite extra dimension)

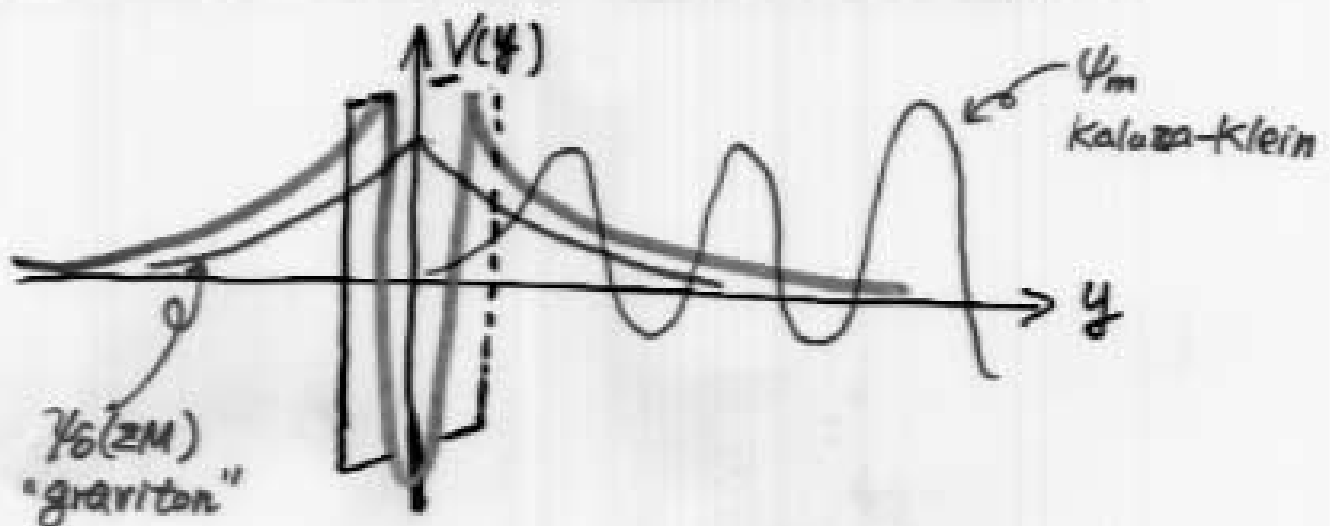
ゆびき

$$ds^2 = [e^{-2k|y|} \eta_{\mu\nu} + h_{\mu\nu}(x, y)] dx^\mu dx^\nu + dy^2$$

$$h_{\mu\nu}(x, y) = \psi(y) e^{i p \cdot x} \quad \text{with } p^2 = m^2$$

linearized eq.

$$[-\partial_y^2 + V(y)] \psi(y) = m^2 \psi(y)$$



★ Static Potential (alternative to compactification)

$$V(r) = G \frac{m_1 m_2}{r} + \int_0^\infty dm \frac{G}{k} \frac{m_1 m_2 e^{-mr}}{r} \frac{m}{k}$$

$$= G \frac{m_1 m_2}{r} \left(1 + \underbrace{\frac{1}{k^2 r^2}}_{\text{small!}} \right)$$



Recent developments

3'

- some extension of the RS model
 - e.g.) multi-brane configuration
 - Hatanaka et al. (next talk!)
 - Kogan et al. ph/9912552
 - Gregory - Rubakov - Sibiryakov
th/0002072
- stabilization
(Goldberger-Wise, PRL 83 (99)4922)
including bulk scalar
- Cosmology (Csaki et al. ph/9911406)
- AdS/CFT, Holographic RG
(Verlinde², Verlinde²-de Boer, Verlinde)
9912018 9912012 9912058
⋮

問いたいこと

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① Warped geometry と

Bound state graviton の関係は？

(normalizable zero mode
と KK mode の decoupling)

② Fine-tuning

$$V_+ = -V_- = -\frac{\Lambda}{k}$$

③ SUGRA, String へ埋めこめられか？

以下

{ ① についての詳しい議論
②, ③ についての部分的な解答

を与える。

§3. Universal Aspects of Localized Gravity on Thick Branes (Csaki et al. hep-th/0001033)

- "Smearing" the RS solution
(Domain Walls)
- General Structures
of Localized Gravity
- Resonant modes

- Adding scalar field(s)
- Intersecting branes
(DW junctions)

setup

along
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$$S = \int d^5x \left[\sqrt{g} (\chi^{-2} R + \Lambda(r)) \right.$$

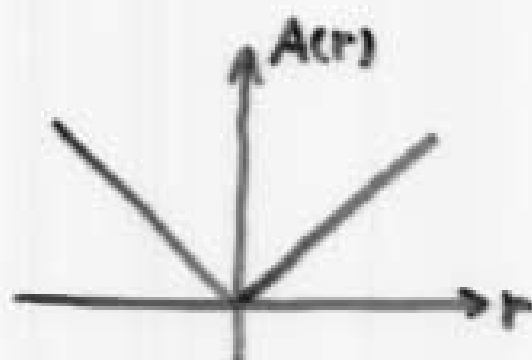
bulk
"c.c."

$$+ \sqrt{g_{(4)}} V(r)]$$

"brane tension"

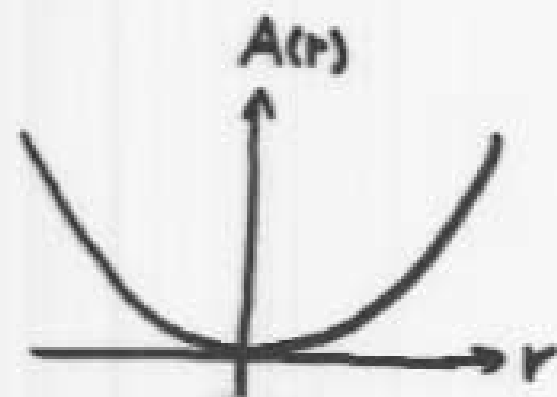
$$ds^2 = e^{-A(r)} \eta_{\mu\nu} dx^\mu dx^\nu - dr^2$$

$$\Rightarrow \begin{cases} \Delta(r) = -3\chi^{-2} A'(r) \\ V(r) = 3\chi^{-2} A''(r) \end{cases}$$



RS solution

$$A(r) = 2k|r|$$



Thick brane

$$A(r) = 2 \frac{k}{\mu} \log \cosh(\mu r)$$

thickness

(with $\mu \sim k$)

$$\star |z| \approx \frac{e^{k|z|} - 1}{k}$$

$$\Rightarrow ds^2 = e^{-A(z)} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2)$$

conformally flat form

Universal equation for

gravitational fluctuation

o Dams
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$$ds^2 = e^{-A(z)} \left((\eta_{\mu\nu} + h_{\mu\nu}(x, z)) dx^\mu dx^\nu - dz_i dz_i \right)$$

$$\partial_\mu h^{\mu\nu} = h_\mu{}^\mu = 0 \quad (\text{RS gauge})$$

△ (Wald, "General Relativity")

$$\text{For } g_{MN} \equiv e^{-A} \tilde{g}_{MN}$$

$$G_{MN} = \tilde{G}_{MN} + \frac{d-2}{2} \left[\frac{1}{2} \tilde{\nabla}_M A \tilde{\nabla}_N A + \tilde{\nabla}_M \tilde{\nabla}_N A \right. \\ \left. - \tilde{g}_{MN} \left(\tilde{\nabla}_K \tilde{\nabla}^K A - \frac{d-3}{4} \tilde{\nabla}_K A \tilde{\nabla}^K A \right) \right]$$

↑
Einstein tensor

EM tensor

$$T_{MN} = \frac{1}{2} (\Lambda g_{MN} + V g_{\mu\nu} \delta_\mu^M \delta_\nu^N)$$

★ Linearized graviton eq. ($\delta G_{MN} = \kappa^2 \delta T_{MN}$)

$$-\frac{1}{2} \partial^K \partial_K h_{MN} + \frac{d-2}{4} \partial^K A \partial_K h_{MN} = 0$$

$$\begin{cases} h_{MN} \equiv e^{\frac{\kappa}{4} A} \tilde{h}_{MN} \\ \tilde{h}_{\mu\nu}(x, z) \equiv \check{h}_{\mu\nu}(x) \Psi(z) \quad (\square_x \check{h} = m^2 \check{h}) \\ \partial^\kappa \partial_\kappa = -\square_x - \nabla_z^2 \end{cases}$$



$$-\nabla_z^2 \psi + \underbrace{\left[\frac{(d-2)^2}{16} \nabla_z A \cdot \nabla_z A - \frac{d-2}{4} \nabla_z^2 A \right]}_{V(z)} \psi = m^2 \psi \quad (*)$$

graviton "Schrödinger eq."

4-d Planck scale



$$S \sim \int d^d x \kappa^2 \sqrt{g} R$$

$$\downarrow g_{MN} = e^{-A} \tilde{g}_{MN}$$

$$= \int d^d x \int d^n z \kappa^2 e^{-\frac{d}{2} A} \sqrt{\tilde{g}} \cdot e^A \tilde{R}$$

$$= M_*^{d-2} \int d^n z e^{-\frac{d-2}{2} A(z)} \cdot \int d^d x \sqrt{\tilde{g}} \tilde{R}$$

$$\Rightarrow M_{Pl}^2 = M_*^{d-2} \int d^n z e^{-\frac{d-2}{2} A(z)}$$

4-d graviton

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$$(\star) \quad Q^\dagger Q \Psi(\mathbf{z}) = m^2 \Psi(\mathbf{z})$$

$$Q \equiv \nabla_{\mathbf{z}} + \frac{d-2}{4} \nabla_{\mathbf{z}} A, \quad Q^\dagger \equiv -\nabla_{\mathbf{z}} + \frac{d-2}{4} \nabla_{\mathbf{z}} A$$

$$\hat{\Psi}_0(\mathbf{z}) \sim e^{-\frac{d-2}{4} A(\mathbf{z})}$$

ZM wave function (massless graviton)

normalization cond.

$$\int d^n \mathbf{z} e^{-\frac{d-2}{2} A(\mathbf{z})} < \infty$$

same as nonvanishing Newton's const.

Asymptotic behavior of the potential

$$V(\mathbf{z}) \begin{cases} > 0 & (|\mathbf{z}| \rightarrow \infty) & \text{normalizable} \\ < 0 & \text{"} & \text{not normalizable} \\ = 0 & \text{"} & ? \end{cases}$$

(N.B.)

$A(\mathbf{z}) \rightarrow \text{const} \quad (|\mathbf{z}| \rightarrow \infty)$
asymptotic Minkowski;

not
normalizable

GRS model

d=5 case

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$$-\frac{d^2\psi}{dz^2} + \left[\frac{9}{16}A'^2 - \frac{3}{4}A'' \right] \psi = m^2 \psi(z)$$

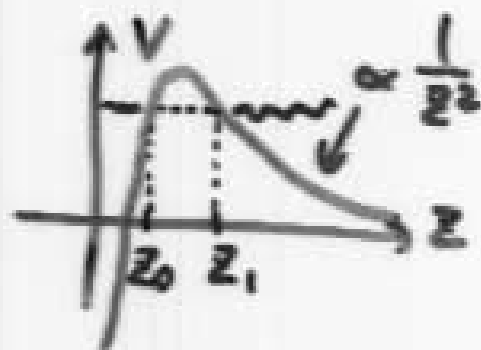
$$\psi_0(z) = e^{-\frac{3}{4}A(z)} : \text{zero mode}$$

Asymptotic behavior

At $|z| \rightarrow \infty$, $\psi_0(z) \sim |z|^{-\alpha}$ とすると

$$\boxed{\alpha > \frac{1}{2}} \leftarrow \psi_0 \text{ is normalizable!}$$

$$\Rightarrow V(z) \rightarrow \frac{\alpha(\alpha+1)}{z^2}$$



★ この時 KK mode はどうか？

直観 Transition Probability (WKB)

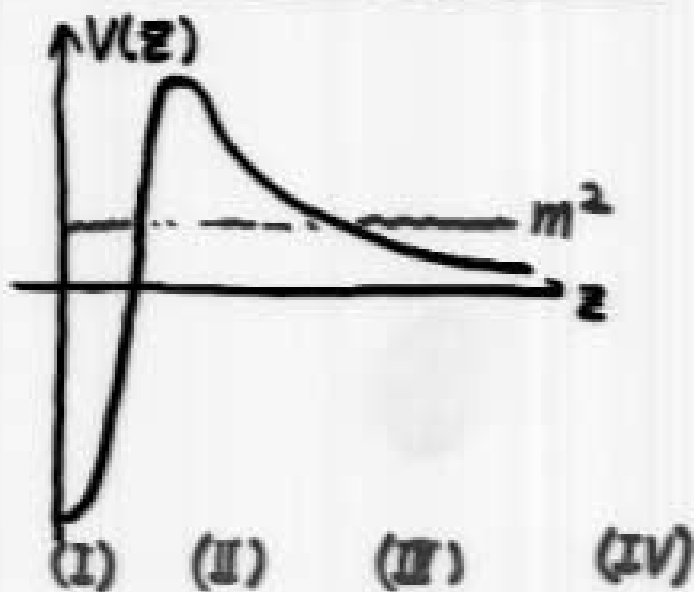
$$T(m) \sim \exp \left[-2 \int_{z_0(m)}^{z_1(m)} dz \sqrt{V(z) - m^2} \right]$$

decoupling cond. $T(m=0) = 0$ //

$$\hookrightarrow V(z) \sim \frac{1}{z^2} \text{ at large } z!$$

もうひとつマジメな議論

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- (I) $z \ll \frac{1}{k}$
- (II) $\frac{1}{k} \ll z \ll \frac{1}{m}$
- (III) $\frac{1}{k} \ll z \sim \frac{1}{m}$
- (IV) $z \gg \frac{1}{m}$

(II) ~ (IV) では

$$-\frac{d^2 \psi_m}{dz^2} + \frac{\alpha(\alpha+1)}{z^2} \psi_m = m^2 \psi_m(z)$$

$$\rightarrow \psi_m(z) = a_m \sqrt{z} N_{\alpha+\frac{1}{2}}(mz) + b_m \sqrt{z} J_{\alpha+\frac{1}{2}}(mz)$$

領域 (II) 及 (IV) での漸近形を (III) で match させることで、係数 a_m, b_m の漸近形が決定。

$$\psi_m(0) \sim \left(\frac{m}{k}\right)^{\alpha-1} \quad \text{原点での KK mode の overlapping}$$

⊙

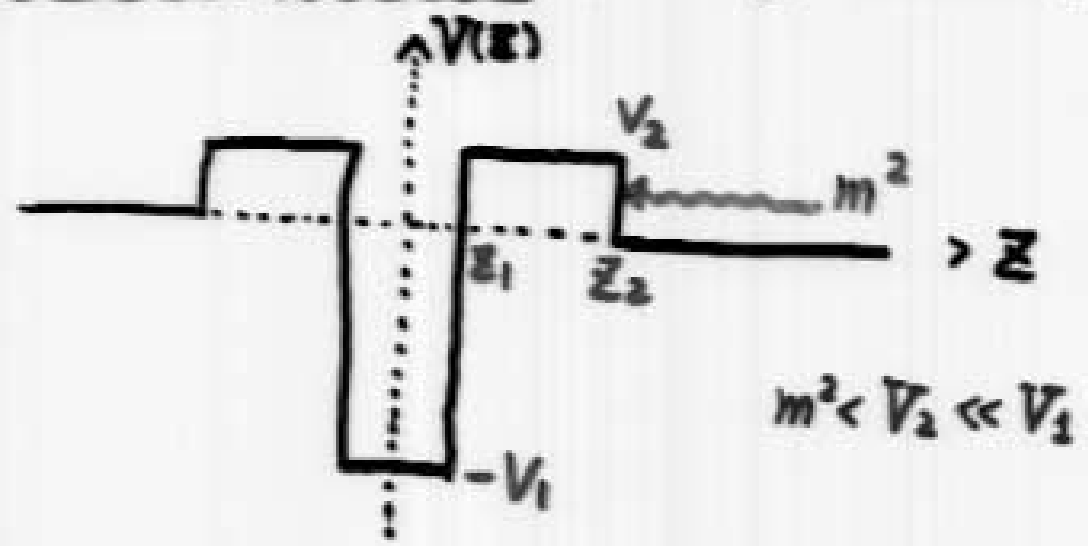
$$V(r) \equiv \frac{GM_1 M_2}{r} + \frac{1}{M_*^2} \int_0^\infty dm \frac{M_1 M_2 e^{-2\alpha r}}{r} |\psi_m(0)|^2$$

$$= \frac{GM_1 M_2}{r} \left(1 + \frac{G}{(kr)^{2\alpha-1}} \right) \quad \left(\begin{array}{l} \text{* RS1d} \\ \alpha = \frac{3}{2} \end{array} \right)$$

Resonant mode (M. Gremm, th/99/2060) ①

$|\psi_m(0)|^2$ が大きくなる一つの可能性

Volcano box potential (A toy model)

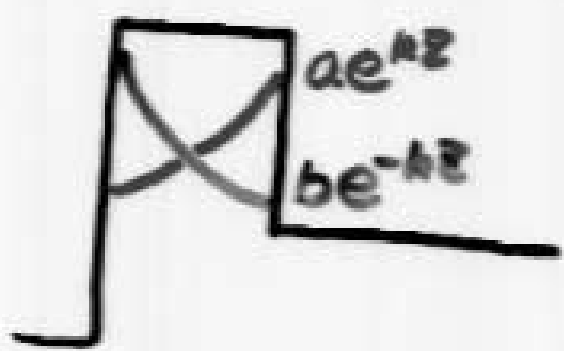


KK mode w.f.

$$\psi(z) = \begin{cases} \cos k_1 z & |z| \leq z_1 \\ a e^{k_2 z} + b e^{-k_2 z} & z_1 \leq |z| \leq z_2 \\ c \cos k_3 z + d \sin k_3 z & |z| \geq z_2 \end{cases}$$

$$k_1 = \sqrt{m^2 + V_1}, \quad k_2 = \sqrt{V_2 - m^2}, \quad k_3 = \sqrt{m^2}$$

★ Resonant mode \rightarrow $a=0!$



$$\cos k_1 z_1 - \frac{k_1}{k_2} \sin k_1 z_1 = 0$$

$$V_1 z_1 \cong k, \quad V_2 \cong k^2$$

0/20/20
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$$\Rightarrow \boxed{1 - \frac{V_1 z_1}{\sqrt{V_2 - m^2}} = 0}$$

However, in the RS case

$$V(z) = \frac{15k^2}{4(k|z|+1)^2} - 3k\delta(z)$$

$$\Rightarrow V_1 z_1 = 3k \quad V_2 = \frac{15}{4}k^2$$

これはどんな $m^2 > 0$ でも上を満たせる!!

(N.B.)

GRS model

$$Q(z) = \begin{cases} e^{-kz} & z \leq |z_c| \\ e^{-kz_c} & z \geq |z_c| \end{cases}$$

no localized graviton!

but quasi-localization can occur!
(resonance)

Other topics

• $d > 5$

$$\psi_m(0) \sim \left(\frac{m}{k}\right)^{\alpha-d+4}$$

• Adding scalar field

(\rightarrow next section)

• Intersecting branes

(Domain Wall junctions)

Csaki-Shirman

th/9908186

Nelson

th/9909001

Carroll-Hellerman-Trodden

th/9911083

Nihei

th/0005014

§4. Toward the Cosmological Constant Problem

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Refs

- ① Alkani-Hamed et al. th/0001197
- Kachru et al. th/0001206
- Nilles et al. th/0002164
- Witten Ph/0002297
- Csaki et al. th/0004133
- Polchinski-Bousso th/0004134
- de Alwis et al. th/0004125
- Henry Tye et al. th/0006068
- Kakushadze th/0005217, 0006059
- Krause th/0006226

A small cosmological constant
from a large extra dimension

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ここでやろうとする事

$$\lambda = \mathcal{O}(M^2) + \underbrace{\mathcal{O}(T_{\text{br.}})}_{\substack{\text{SM loops} \\ \parallel \\ \text{Brane tension}}} + \cancel{\mathcal{O}\left(\frac{T_{\text{br.}}^2}{M^2}\right)}_{\substack{\text{quantum} \\ \text{gravity}}} + \dots$$

T_{br} の値に依らず flat な空間を与える

(N.B.)

In the RS case,

$$V_+ = -V_- = \frac{\Lambda}{k} \Rightarrow \text{flat 空間}$$

A model ($y \rightarrow -y$)

$$\mathcal{L} = \int d^4x dy \sqrt{g} \left(\frac{R}{2x^2} - \frac{3}{2} (\nabla_\mu \phi)^2 \right)$$

↑ bulk scalar

$$- \int d^4x \sqrt{-\det g_4} e^{x\phi} \mathcal{L}_{\text{SM}}(\underbrace{H, g_{\mu\nu}^{(0)}}_{\text{SM field}}) e^{x\phi(0)}$$

$$x^2 = M_*^{-3}$$

↑ SM field

SM quantum loop effect

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$$S_{\text{brane}} \rightarrow \Gamma_{\text{eff}}^{\text{SM}}(H, g_{\mu\nu}^{(0)} e^{2\chi\phi})$$

$$\Gamma_{\text{eff}}^{\text{SM}} = - \int d^4x \sqrt{g_4} V_{\text{eff}}(H) e^{2\chi\phi}$$

warped geometry

$$dS^2 = a^2(y) \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$$

A set of eqs. of motion

$$\bullet \left. \frac{\partial V_{\text{eff}}}{\partial H} \right|_{H_0} = 0 \rightarrow V_{\text{eff}}(H_0) \equiv V_{\text{ext.}}$$

$$\left\{ \begin{array}{l} \frac{a'}{a^2} = \frac{\chi^2}{4} \tilde{\phi}'^2 \\ \tilde{\phi}'' + 4 \frac{a'}{a} \tilde{\phi}' = 0 \\ \frac{a''}{a} = - \frac{3\chi^2}{4} \tilde{\phi}'^2 \end{array} \right. \quad \tilde{\phi} = \phi + \frac{1}{2\chi} \ln\left(\frac{V_{\text{ext.}}}{M_*^4}\right)$$

bulk の方程式

Solution

$$ds^2 = \left(1 - \frac{2M_4}{3} e^{2x\tilde{\phi}_0} |y|\right)^{3/2} dx^2 + dy^2$$

$$\hat{\phi} = \tilde{\phi}_0 - \frac{1}{2x} \ln\left(1 - \frac{2M_4}{3} e^{2x\tilde{\phi}_0} |y|\right)$$

$$\phi = \phi_0 - \frac{1}{2x} \ln\left(1 - \frac{2V_{\text{ext}}}{3M_4^3} e^{2x\phi_0} |y|\right)$$

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A 4-d flat space with any V_{ext} !

An important feature

"Singularity"

At

$$y_s = \frac{3}{2M_4} e^{-2x\tilde{\phi}_0} = \frac{3M_4^3}{2V_{\text{ext}}} e^{-2x\phi_0}$$

この点で metric は vanish, $\hat{\phi}$ は発散.

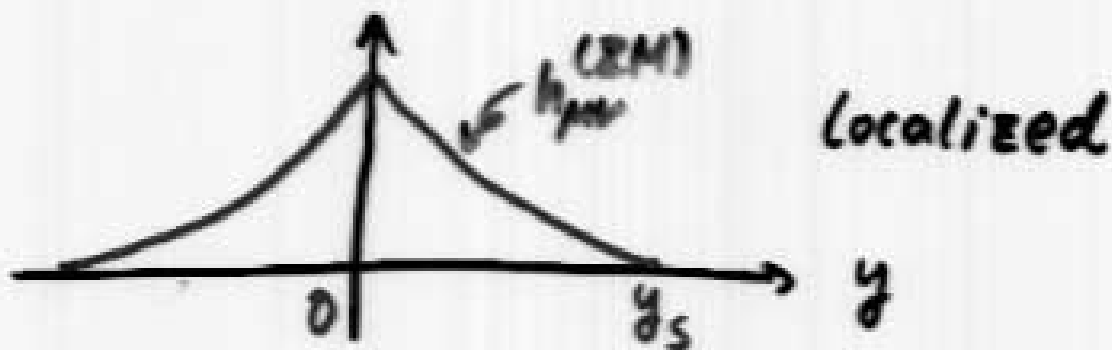
解釈

① "End of the World" (~~Big Bang~~
Big Crunch)

② 古典的記述の破綻(?)

ZM graviton

$$h_{\mu\nu}^{(ZM)} \sim \sqrt{1 - \frac{2M_*}{3} e^{2\chi\phi_0} |y|}$$



4-d Planck

$$M_{Pl}^2 \equiv M_*^3 \int_0^{y_s} a^2(y) dy = \frac{M_*^6}{\text{Vert.}} e^{-2\chi\phi_0}$$

size of y_s

$$y_s = \frac{3}{2} \frac{M_{Pl}^2}{M_*^3} = 1 \text{ mm} \quad \text{for } M_* = 10^8 \text{ GeV}$$

flat space	No fine-tune	fine-tune
localization	O.K.	
singularity	有	無

(Csaki et al. +h/0004133)

私、橘基はこの度

結婚することになりました！