『Extra dimensionの物理』

基研　橘 基

Randall-Sundrum 以後

§1. Intro.

§2. Impact of the RS model

§3. Universal Aspects of
Bound State Graviton
(Localized Gravity)

§4. Toward the Cosmological
Constant Problem

§5. Outro.

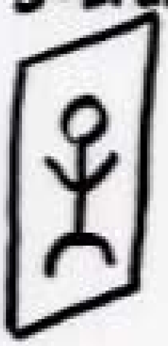
§1. Intro.

Extra spacetime dimensions

$\Longrightarrow$ OLD IDEA!
(Kaluza-Klein, String ⋯)

従来⋯

$\boxed{\text{E.S.D. must be "tiny"}}$



But recently

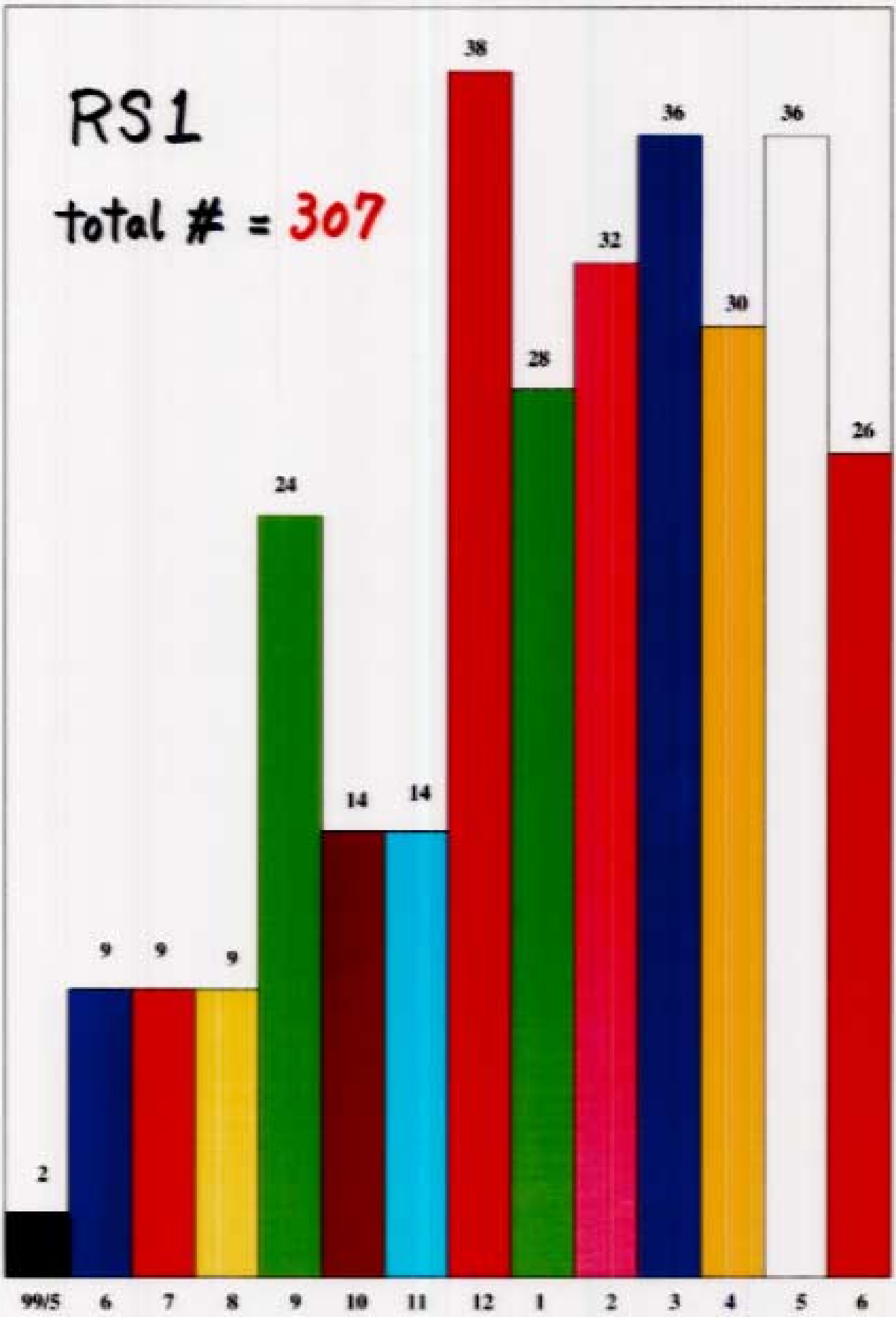
"Brane World"

LARGE EXTRA DIMENSION!
(~mm)

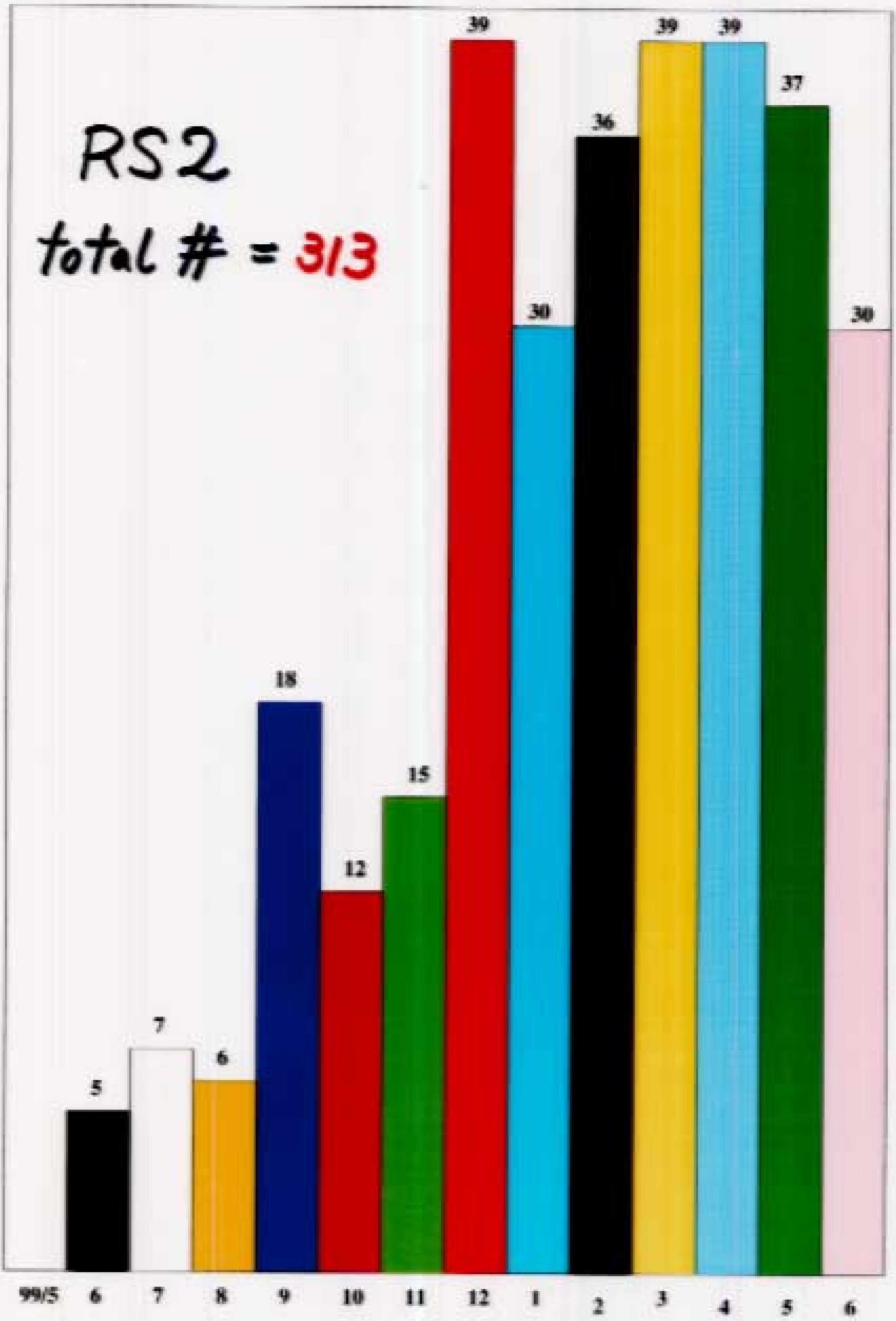
(Arkani-Hamed, Dimopoulos & Dvali '98)

$$\underbrace{M_{Pl}^2}_{\text{4-d Planck}} = \underbrace{M_*^{n+2}}_{\text{(4+n)-d}} \underbrace{R^n}_{\text{size of extra dim.}}$$

A Solution to Hierarchy Problem!



RS1
total # = 307
2
9
9
9
24
14
14
38
28
32
36
30
36
26
99/5
6
7
8
9
10
11
12
1
2
3
4
5
6


RS2
total # = 313
5
7
6
18
12
15
39
30
36
39
39
37
30
99/5
6
7
8
9
10
11
12
1
2
3
4
5
6

§2. Impact of the RS model

(PRL 83('99)3370)

$$S = \int d^4x \int dy \sqrt{g}\,\{2M^3R - \Lambda\}$$

5-d Planck

$$- \int d^4x \left[\sqrt{g_4^{(+)}}\,V_+ + \sqrt{g_4^{(-)}}\,V_-\right]$$

<u>warped geometry</u> (4-d Poincaré inv.)

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$$

$$k \equiv \left(\frac{-\Lambda}{24M^3}\right)^{1/2} \Longrightarrow \boxed{\Lambda<0}$$

bulkは AdS

★ $$\boxed{V_+ = -V_- = -\frac{\Lambda}{k}}$$: Fine tuning!

<u>4-d Planck scale</u>

$$M_{Pl}^2 \equiv 2M^3\int_0^{y_c} dy\, e^{-2ky} = \frac{M^3}{k}\left(1-e^{-2ky_c}\right)$$

Bound State Graviton

(PRL 83 ('99) 4690)

$y_c \to \infty$ (Infinite extra dimension)

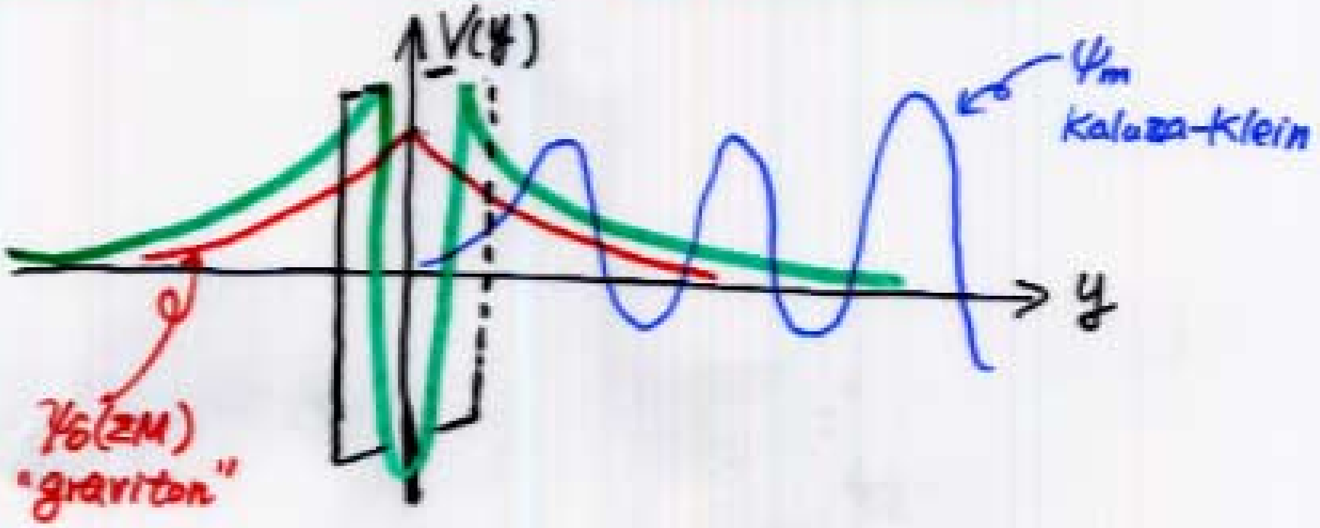
ゆらぎ

$$ds^2 = \left[e^{-2k|y|}\eta_{\mu\nu} + h_{\mu\nu}(x,y)\right]dx^\mu dx^\nu + dy^2$$

$$h_{\mu\nu}(x,y) = \psi(y)\, e^{ip\cdot x} \quad \text{with} \quad p^2 = m^2$$

linearized eq.

$$\left[-\partial_y^2 + V(y)\right]\psi(y) = m^2\,\psi(y)$$



★ Static Potential (alternative to compactification)

$$V(r) = G\frac{m_1 m_2}{r} + \int_0^\infty dm\,\frac{G}{k}\,\frac{m_1 m_2 e^{-mr}}{r}\,\frac{m}{k}$$

$$= G\frac{m_1 m_2}{r}\Big(1 + \underbrace{\frac{1}{k^2 r^2}}_{\text{small !}}\Big)$$

Recent developments

- some extention of the RS model

  e.g.) multi-brane configuration

  - Hatanaka et al. (next talk!)
  - Kogan et al. ph/9912552
  - Gregory - Rubakov - Sibiryakov th/0002072

- stabilization

  (Goldberger-Wise, PRL 83 ('99) 4922)

  including bulk scalar

- Cosmology (Csaki et al. ph/9911406)

- AdS/CFT, Holographic RG

  (Verlinde^2 9912018, Verlinde^2-de Boer 9912012, Verlinde 9912058)

⋮

問いかけたいこと

① Warped geometry と Bound state graviton の関係は？

(normalizable zero mode と KK mode の decoupling)

② Fine-tuning

$$V_+ = -V_- = -\frac{\Lambda}{k}$$

③ SUGRA, String へ埋めこめられるか？

以下

{ ①についての詳しい議論
②, ③についての部分的な解答

を与える.

§3. Universal Aspects of Localized Gravity on Thick Branes

(Csaki et al. hep-th/0001033)

- "Smearing" the RS solution (Domain Wall)
- General Structures of Localized Gravity
- Resonant modes
- Adding scalar field(s)
- Intersecting branes (DW junctions)

setup

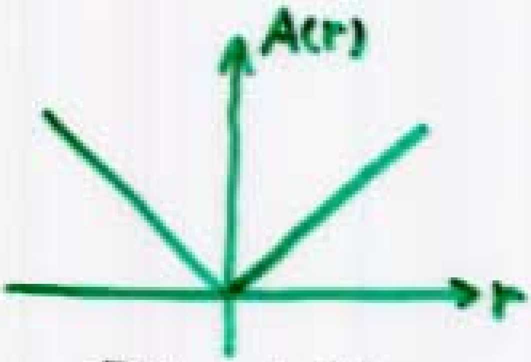
$$S = \int d^5x \left[\sqrt{g} \left(\varkappa^{-2} R + \Lambda(r) \right) + \sqrt{g_{(4)}}\, V(r) \right]$$

bulk "c.c."

"brane tension"

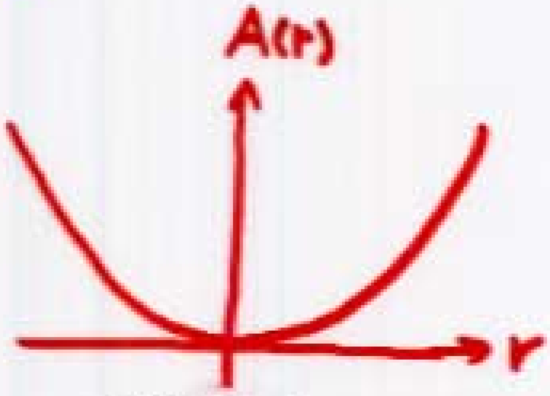
$$ds^2 = e^{-A(r)} \eta_{\mu\nu} dx^\mu dx^\nu - dr^2$$

$$\Longrightarrow \begin{cases} \Lambda(r) = -3\varkappa^{-2} A'(r) \\ V(r) = 3\varkappa^{-2} A''(r) \end{cases}$$



RS solution

$A(r) = 2k|r|$



Thick brane

$$A(r) = 2\frac{k}{\underbrace{\mu}_{\text{thickness}}} \log\cosh(\mu r) \xrightarrow{\mu \gg k} A(r) = 2k|r|$$

(以下 $\mu \sim k$)

$$\bigstar\ |z| = \frac{e^{k|r|} - 1}{k}$$

$$\Longrightarrow ds^2 = e^{-A(z)} \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2 \right)$$

conformally flat form

Universal equation for gravitational fluctuation

$$ds^2 = e^{-A(z)}\left((\eta_{\mu\nu} + h_{\mu\nu}(x,z))dx^\mu dx^\nu - dz_i dz_i\right)$$

$$\partial_\mu h^{\mu\nu} = h_\mu{}^\mu = 0 \quad \text{(RS gauge)}$$

公式 (Wald, "General Relativity")

For $g_{MN} \equiv e^{-A}\tilde{g}_{MN}$

$$G_{MN} = \tilde{G}_{MN} + \frac{d-2}{2}\left[\frac{1}{2}\tilde{\nabla}_M A \tilde{\nabla}_N A + \tilde{\nabla}_M \tilde{\nabla}_N A - \tilde{g}_{MN}\left(\tilde{\nabla}_K \tilde{\nabla}^K A - \frac{d-3}{4}\tilde{\nabla}_K A \tilde{\nabla}^K A\right)\right]$$

Einstein tensor

EM tensor

$$T_{MN} = \frac{1}{2}\left(\Lambda g_{MN} + V g_{\mu\nu}\delta_\mu^M \delta_\nu^N\right)$$

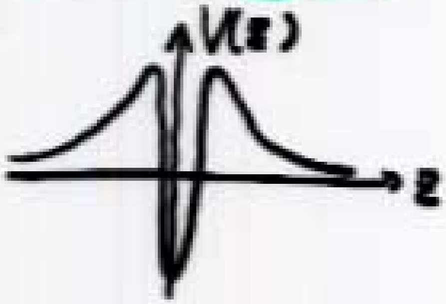
★ Linearized graviton eq. $(\delta G_{MN} = \varkappa^2 \delta T_{MN})$

$$\boxed{-\frac{1}{2}\partial^K \partial_K h_{MN} + \frac{d-2}{4}\partial^K A \partial_K h_{MN} = 0}$$

$$\begin{cases} h_{MN} \equiv e^{\frac{d-2}{4}A}\,\tilde{h}_{MN} \\ \tilde{h}_{\mu\nu}(x,z) \equiv \check{h}_{\mu\nu}(x)\,\psi(z) \quad (\Box_x \check{h} = m^2 \check{h}) \\ \partial^K\partial_K = -\Box_x - \nabla_z^2 \end{cases}$$

$$-\nabla_z^2\psi + \underbrace{\left[\frac{(d-2)^2}{16}\nabla_z A\cdot\nabla_z A - \frac{d-2}{4}\nabla_z^2 A\right]}_{V(z)}\psi = m^2\psi \qquad (*)$$

graviton "Schrödinger eq."



4-d Planck scale

$$S \sim \int d^d x\, \kappa^{-2}\sqrt{g}\, R$$

$$\Downarrow\ g_{MN} = e^{-A}\tilde{g}_{MN}$$

$$= \int d^4x \int d^n z\, \kappa^{-2} e^{-\frac{d}{2}}\sqrt{\tilde{g}}\cdot e^{A}\tilde{R}$$

$$= M_*^{d-2}\int d^n z\, e^{-\frac{d-2}{2}A(z)}\cdot\int d^4x\sqrt{\tilde{g}}\,\tilde{R}$$

$$\Longrightarrow \boxed{M_{Pl}^2 = M_*^{d-2}\int d^n z\, e^{-\frac{d-2}{2}A(z)}}$$

4-d graviton

(☆) $Q^\dagger Q \psi(z) = m^2 \psi(z)$

$Q \equiv \nabla_z + \frac{d-2}{4} \nabla_z A\,, \quad Q^\dagger \equiv -\nabla_z + \frac{d-2}{4} \nabla_z A$

$$\boxed{\hat{\psi}_0(z) \sim e^{-\frac{d-2}{4} A(z)}}$$

ZM wave function (massless graviton)

normalization cond.

$$\boxed{\int d^n z \, e^{-\frac{d-2}{2} A(z)} < \infty}$$

same as nonvanishing Newton's const.

Asymptotic behavior of the potential

$$V(z) \begin{cases} > 0 & (|z| \to \infty) & \text{normalizable} \\ < 0 & \quad〃 & \text{not normalizable} \\ = 0 & \quad〃 & ? \end{cases}$$

(N.B.

$A(z) \to \text{const} \ (|z| \to \infty)$ not normalizable

asymptotic Minkowski

GRS model)

$d=5$ case

$$-\frac{d^2\psi}{dz^2}+\left[\frac{9}{16}A'^2-\frac{3}{4}A''\right]\psi = m^2\psi(z)$$

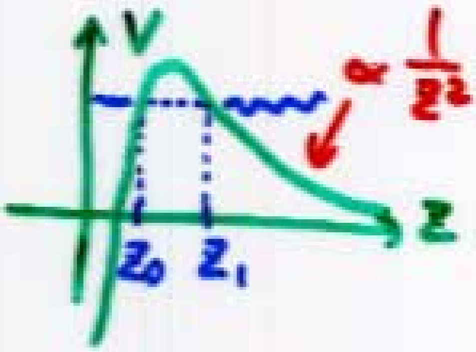
$$\psi_0(z) = e^{-\frac{3}{4}A(z)}$$: zero mode

Asymptotic behavior

At $|z|\to\infty$, $\psi_0(z)\sim|z|^{-\alpha}$ とすると

$\boxed{\alpha>\frac{1}{2}}$ ← ψ_0 is normalizable!

$$\Longrightarrow \quad V(z)\longrightarrow\frac{\alpha(\alpha+1)}{z^2}$$



★ この時 KK mode はどうか？

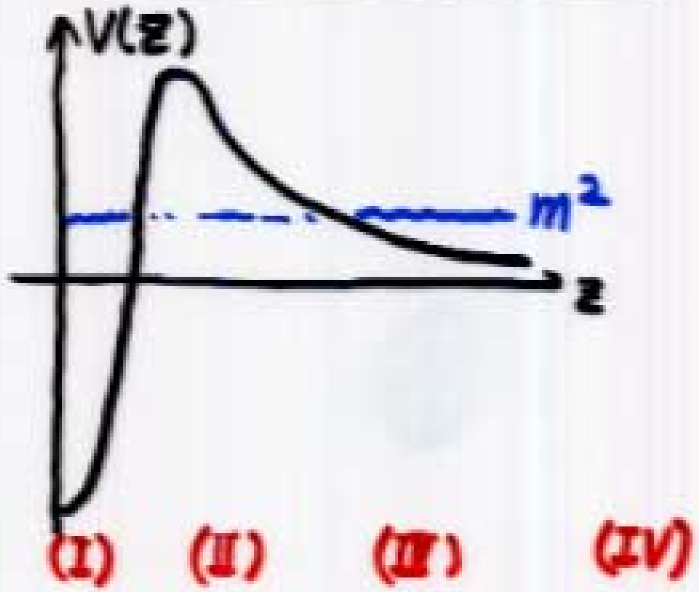
直観 Transition Probability (WKB)

$$T(m)\sim\exp\left[-2\int_{z_0(m)}^{z_1(m)}dz\sqrt{V(z)-m^2}\right]$$

decoupling cond. $T(m=0)=0$ 〃

$\longrightarrow$ $V(z)\sim\frac{1}{z^2}$ at large z!

もうちょっとマジメな議論



(I) $z \ll \frac{1}{k}$

(II) $\frac{1}{k} \ll z \ll \frac{1}{m}$

(III) $\frac{1}{k} \ll z \sim \frac{1}{m}$

(IV) $z \gg \frac{1}{m}$

(II) ~ (IV) では

$$-\frac{d^2\psi_m}{dz^2} + \frac{\alpha(\alpha+1)}{z^2}\psi_m = m^2\psi_m(z)$$

$$\longrightarrow \boxed{\psi_m(z) = a_m\sqrt{z}\,N_{\alpha+\frac{1}{2}}(mz) + b_m\sqrt{z}\,J_{\alpha+\frac{1}{2}}(mz)}$$

領域 (II) 及び (IV) での漸近形を (III) で match させることで、係数 a_m, b_m の漸近形が決定。

$$\boxed{\psi_m(0) \sim \left(\frac{m}{k}\right)^{\alpha-1}}$$

原点での KK mode の overlapping

∴

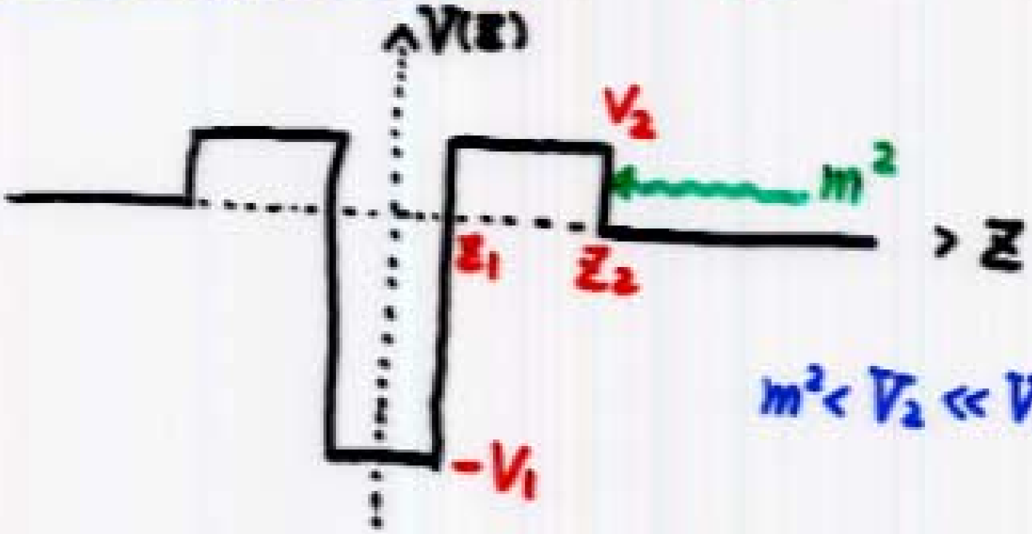
$$V(r) = \frac{GM_1M_2}{r} + \frac{1}{M_*^3}\int_0^\infty dm\,\frac{M_1M_2\,e^{-mr}}{r}|\psi_m(0)|^2$$

$$= \frac{GM_1M_2}{r}\left(1 + \frac{C}{(kr)^{2\alpha-1}}\right) \quad \left(\begin{array}{c} \text{※ RS1d} \\ \alpha = \frac{3}{2} \end{array}\right)$$

Resonant mode (M. Gremm, th/9912060)

$|\psi_m(0)|^2$ が大きくなる一つの可能性

Volcano box potential (A toy model)



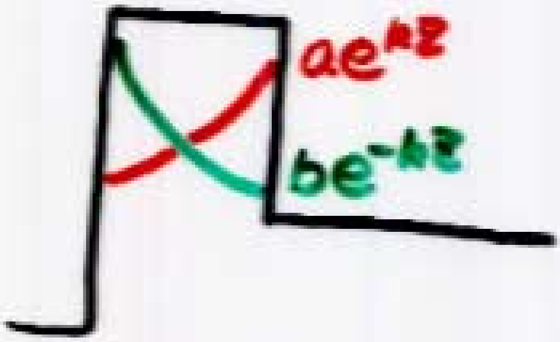
$m^2 < V_2 \ll V_1$

KK mode w.f.

$$\psi(z) = \begin{cases} \cos k_1 z & |z| \le z_1 \\ a e^{k_2 z} + b e^{-k_2 z} & z_1 \le |z| \le z_2 \\ c \cos k_3 z + d \sin k_3 z & |z| \ge z_2 \end{cases}$$

$k_1 = \sqrt{m^2 + V_1}$, $k_2 = \sqrt{V_2 - m^2}$, $k_3 = \sqrt{m^2}$

★ Resonant mode $\longrightarrow$ $\boxed{a = 0 \, !}$



$$\cos k_1 z_1 - \frac{k_1}{k_2} \sin k_1 z_1 = 0$$

//

$$V_1 z_1 \cong k, \quad V_2 \cong k^2$$

$$\Longrightarrow \boxed{1 - \frac{V_1 z_1}{\sqrt{V_2 - m^2}} = 0}$$

However, in the RS case

$$V(z) = \frac{15k^2}{4(k|z|+1)^2} - 3k\,\delta(z)$$

$$\Rightarrow V_1 z_1 = 3k \quad V_2 = \frac{15}{4}k^2$$

これは どんな $m^2>0$ でも 上を 満たせない!

(N.B.)

GRS model

$$a(z) = \begin{cases} e^{-kz} & z \leq |z_c| \\ e^{-kz_c} & z \geq |z_c| \end{cases}$$

no localized graviton!

but quasi-localization can occur!
(resonance)

Other topics

- $d > 5$

$$\psi_m(0) \sim \left(\frac{m}{k}\right)^{\alpha-d+4}$$

- Adding scalar field

(→ next section)

- Intersecting branes

(Domain Wall junctions)

Csaki-Shirman th/9908186
Nelson th/9909001
Carroll-Hellerman-Trodden th/9911083
Nihei th/0005014

§4. Toward the Cosmological Constant Problem

Refs

Arkani-Hamed et al. th/0001197

Kachru et al. th/0001206

Nilles et al. th/0002164

Witten ph/0002297

Csaki et al. th/0004133

Polchinski-Bousso th/0004134

de Alwis et al. th/0004125

Henry Tye et al. th/0006068

Kakushadze th/0005217, 0006059

Krause th/0006226

A small cosmological constant from a large extra dimension

ここでやろうとする事

$$\lambda = \mathcal{O}(M^4) + \underbrace{\mathcal{O}(T_{br})}_{\substack{\text{SM loops} \\ \parallel \\ \text{Brane tension}}} + \underbrace{\cancel{\mathcal{O}\left(\frac{T_{br}^2}{M^4}\right)}}_{\substack{\text{quantum} \\ \text{gravity}}} + \cdots$$

T_{br}の値に依らずflatな空間を与える

(N.B.
In the RS case,
$$V_+ = -V_- = \frac{\Lambda}{k} \Rightarrow \text{flat空間}$$
)

A model ($y \to -y$)

$$S = \int d^4x\, dy \sqrt{g} \left(\frac{R}{2\kappa^2} - \frac{3}{2} (\nabla \underbrace{\phi}_{\text{bulk scalar}})^2 \right)$$

$$- \int d^4x \sqrt{-\det g_*}\, e^{\kappa\phi} \mathcal{L}_{SM}(\underbrace{H}_{\text{SM field}}, g_{\mu\nu}(0) e^{\kappa\phi(0)})$$

$\kappa^2 = M_*^{-3}$

SM quatum loop effect

$$S_{\text{brane}} \longrightarrow \Gamma_{\text{eff}}^{SM}(H, g_{\mu\nu}{}^{(0)} e^{\chi\phi(0)})$$

$$\Gamma_{\text{eff}}^{SM} = -\int d^4x \sqrt{g_4}\, V_{\text{eff}}(H)\, e^{2\chi\phi}$$

warped geometry

$$ds^2 = a^2(y)\,\eta_{\mu\nu}\,dx^\mu dx^\nu + dy^2$$

A set of eqs. of motion

- $\left.\dfrac{\partial V_{\text{eff}}}{\partial H}\right|_{H_0} = 0 \longrightarrow V_{\text{eff}}(H_0) \equiv V_{\text{ext.}}$

$$\begin{cases} \dfrac{a'^2}{a^2} = \dfrac{\chi^2}{4}\tilde{\phi}'^2 \\ \tilde{\phi}'' + 4\dfrac{a'}{a}\tilde{\phi}' = 0 \\ \dfrac{a''}{a} = -\dfrac{3\chi^2}{4}\tilde{\phi}'^2 \end{cases} \qquad \tilde{\phi} = \phi + \frac{1}{2\chi}\ln\left(\frac{V_{\text{ext.}}}{M_*^4}\right)$$

bulkの方程式

Solution

$$\boxed{\begin{aligned} ds^2 &= \left(1-\frac{2M_*}{3}e^{2\kappa\tilde{\varphi}_0}|y|\right)^{1/2}dx^2+dy^2 \\ \tilde{\varphi} &= \tilde{\varphi}_0-\frac{1}{2\kappa}\ln\left(1-\frac{2M_*}{3}e^{2\kappa\tilde{\varphi}_0}|y|\right) \\ \phi &= \phi_0-\frac{1}{2\kappa}\ln\left(1-\frac{2V_{ext}}{3M_*^3}e^{2\kappa\phi_0}|y|\right)\end{aligned}}$$

A 4-d flat space with any $V_{ext.}$!

An important feature

"Singularity"

At

$$\boxed{y_s=\frac{3}{2M_*}e^{-2\kappa\tilde{\varphi}_0}=\frac{3M_*^3}{2V_{ext.}}e^{-2\kappa\phi_0}}$$

この点で metric は vanish, $\tilde{\varphi}$ は発散.

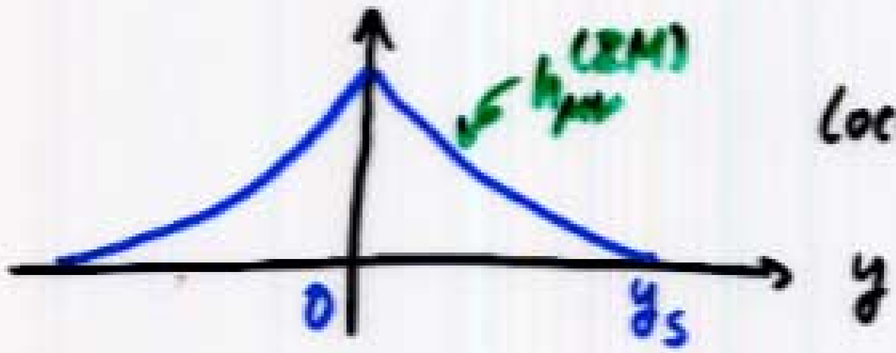
解釈

① "End of the World" (Big Bang / Big Crunch)

② 古典的記述の破綻(?)

ZM graviton

$$h_{\mu\nu}^{(ZM)} \sim \sqrt{1-\frac{2M_*}{3}e^{2\kappa\hat{\phi}_0}|y|}$$



localized

4-d Planck

$$M_{Pl}^2 \equiv M_*^3\int_0^{y_s} a^2(y)\,dy = \frac{M_*^6}{V_{ext.}}e^{-2\kappa\phi_0}$$

$\boxed{\text{Size of } y_s}$

$$y_s = \frac{3}{2}\frac{M_{Pl}^2}{M_*^3} = 1\ \text{mm} \quad \text{for } M_* = 10^8\ \text{GeV}$$

flat space	No fine-tune	fine-tune
localization	O.K.	
singularity	有	無

(Csaki et al. th/0004133)

§5. Outro.

私、橘 基はこの度

結婚することになりました！