タキオン凝縮と弦の場の理論

髙橋 智彦 (奈女理)

§ Introduction

§ Bosonic D-brane

- A. Sen, "Descent Relations Among Bosonic D-branes", hep-th/9902105

§ String Field Theory

§ Tachyon Condensation in SFT

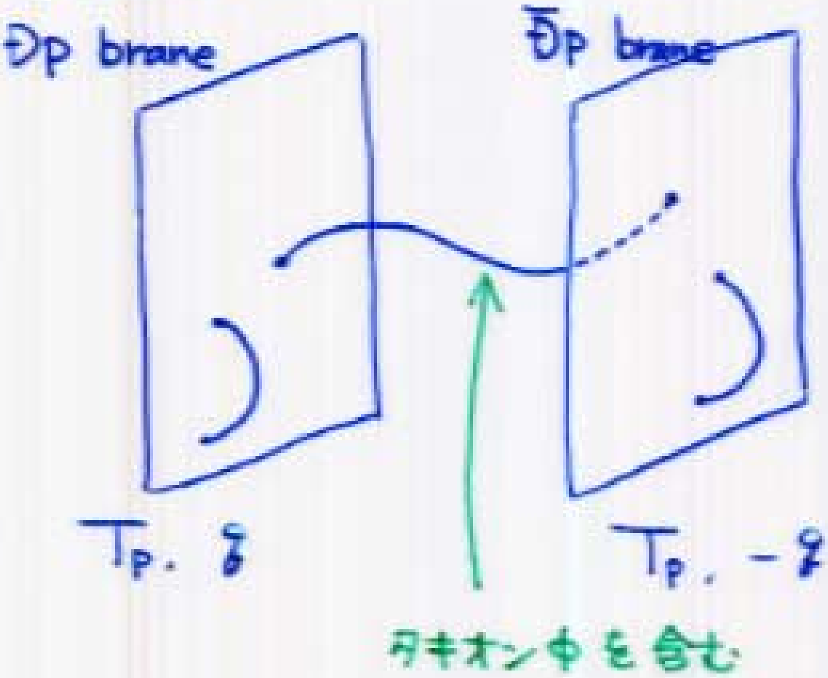
- A. Sen, "Universality of the Tachyon Potential", hep-th/9911116
- A. Sen, B. Zwiebach, "Tachyon Condensation in String Field Theory", hep-th/9912249

§ D-branes as Non-commutative Solitons

- R. Gopakumar, S. Minwalla, A. Strominger, "Noncommutative Solitons", hep-th/0003160
- J. Harvey, P. Kraus, F. Larsen, E. Martinec, "D-branes and Strings as Noncommutative Solitons", hep-th/0005031
- E. Witten, "Noncommutative Tachyons And String Field Theory", hep-th/0006071

§ Introduction

Type II



Dp-$\bar{D}p$ の RR charge $= g + (-g) = 0$

Dp-$\bar{D}p$ の 質量 $= T_p + T_p = 2T_p$

$\Downarrow$

Dp と $\bar{D}p$ は 対消滅をして

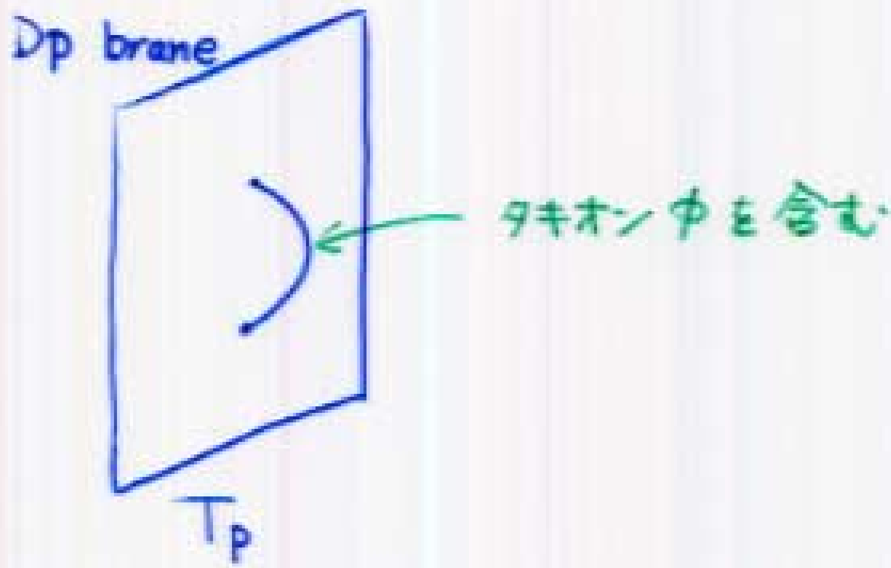
Brane がない真空（閉弦ヒモの真空）になる.

Sen's conjecture

$$\boxed{2T_p + V(\phi_0) = 0}$$

$V(\phi)$: タキオンのポテンシャル.

Bosonic string



$\Downarrow$

タキオンが凝縮して、Dp brane がない真空になる.

Sen's conjecture

$$\boxed{T_p + V(\phi_0) = 0}$$

Sen's conjecture

$$\boxed{T_p + V(\Phi_0) = 0}$$

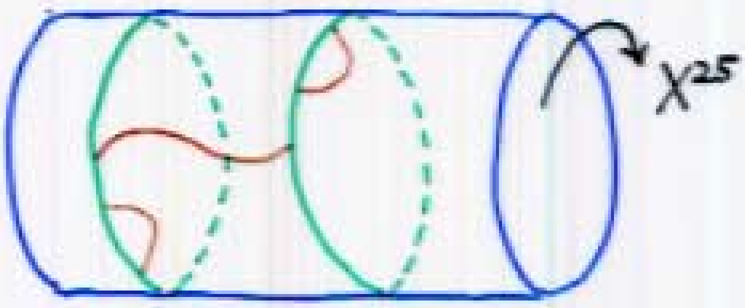
「弦の場の理論 (SFT) を使った定量的テストから
この conjecture が非常に支持できる.」

Off-shell 解析

前半の話

§2 Bosonic D-brane

$S^1 \ R=\frac{1}{2}$

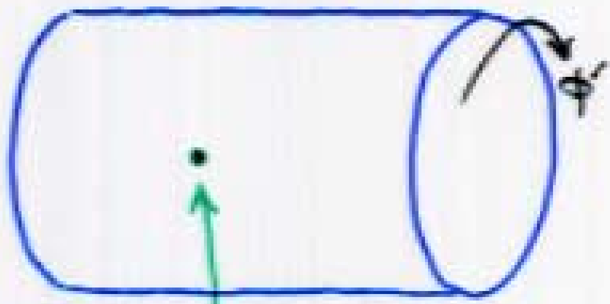


Dstring

Dstring + wilson line $\frac{1}{2}$

タキオンが凝縮 ($\alpha=1$)

$S^1 \ R=\frac{1}{2}$

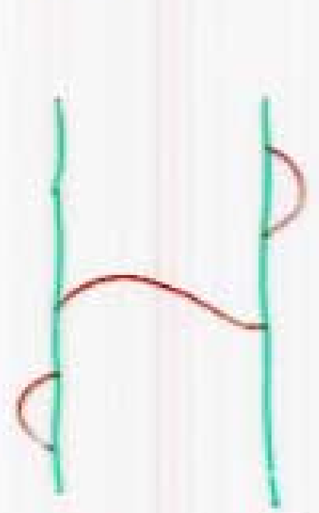


D particle

=

$\boxed{\text{CFTとして二つの理論は等価}}$

$\downarrow R \to \infty$ ← [同じ marginal operator が対応] → $\downarrow R \to \infty$



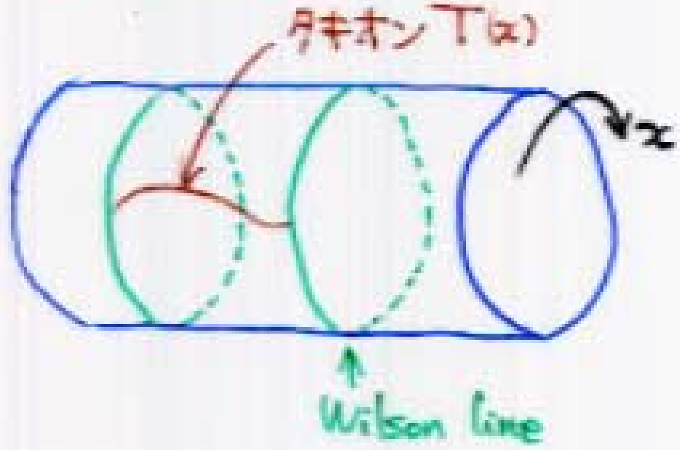
Dstring Dstring

タキオンが凝縮

=

D particle

• D string, D string + Wilson line $\frac{1}{2}$



T(x) のモード展開

$$T(x)=\sum_{n\in\mathbb{Z}} T_{n+\frac{1}{2}}\, e^{i(n+\frac{1}{2})\frac{x}{R_c}}$$

$(R_c=\frac{1}{2})$

$T_{n+\frac{1}{2}}$ の質量 $m_{n+\frac{1}{2}}$

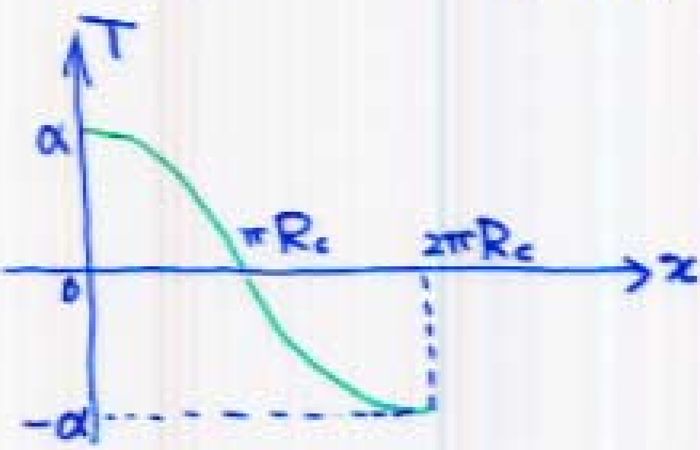
$$m^2_{n+\frac{1}{2}}=\frac{(n+\frac{1}{2})^2}{R_c^2}-1 \qquad (\alpha'=1)$$

$\boxed{T_{\pm\frac{1}{2}} \text{ は massless} \longleftrightarrow \text{marginal operator}}$

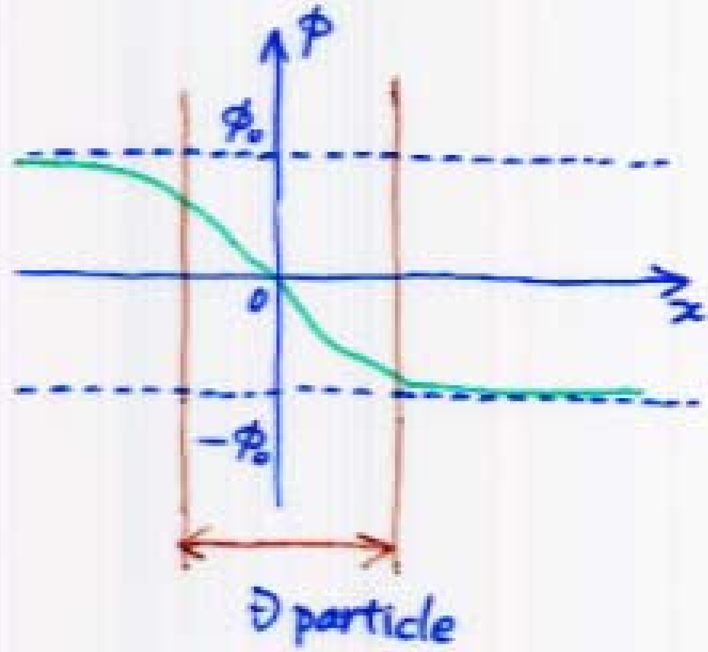
$T_{\frac{1}{2}}+T_{-\frac{1}{2}}=\alpha,\quad T_{\frac{1}{2}}-T_{-\frac{1}{2}}=0,\quad T_r=0\quad (|r|>\frac{1}{2})$

とすると、

$$T(x)=\alpha\cos\left(\frac{x}{2R_c}\right)=\alpha\cos x.$$



S^1 上のタキオン Kink.



$$\int_{-\infty}^{\infty} dx \left(2T_1 + V(\phi(x)) \right) \sim T_0 \quad \text{(有限)}$$

$\therefore$

$$\boxed{2T_1 + V(\phi_0) = 0}$$

であるべき.

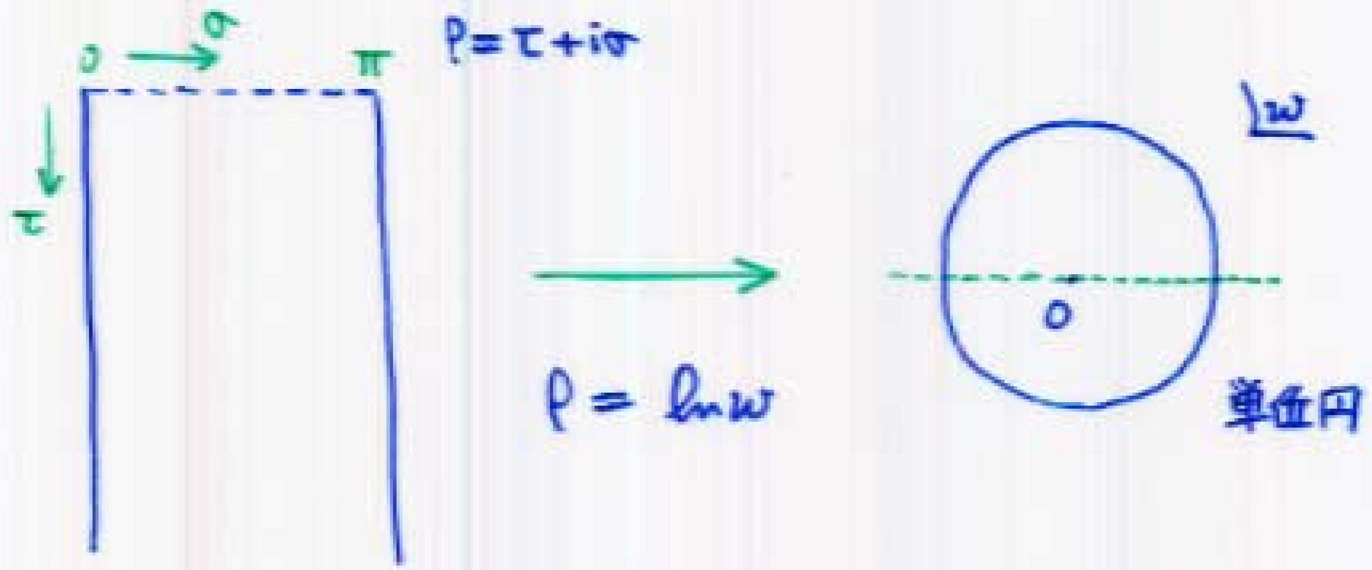
§ String Field Theory

String Field

$$|\Psi\rangle = \phi(x)\, c_1 |0\rangle + A_\mu(x)\, \alpha^\mu_{-1}\, c_1 \underbrace{|0\rangle}_{\text{SL(2,R) vacuum}} + \cdots$$

$$\boxed{\text{String field } \Psi \longleftrightarrow \text{CFTの状態空間}\ \mathcal{H}\ \text{のある状態}\ (\#\text{ghost} = 1)}$$

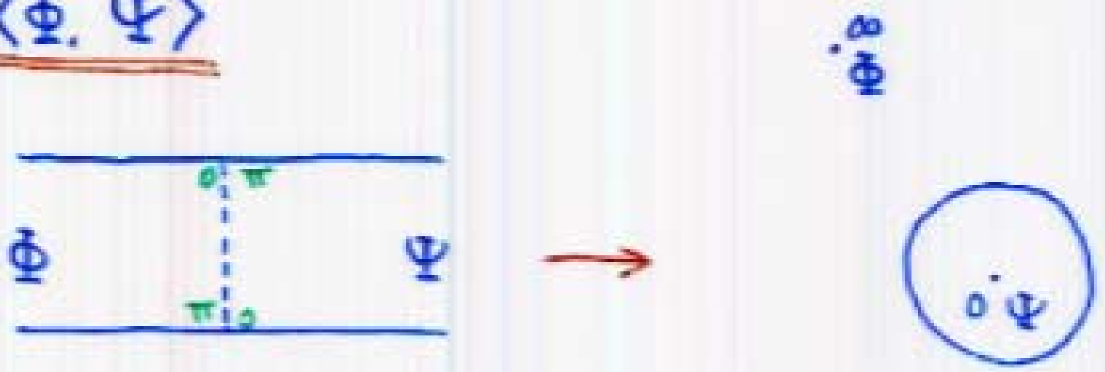
$$|\Psi\rangle = \Psi(w=0)|0\rangle$$



Action

$$S = -\frac{1}{g^2}\left(\frac{1}{2}\langle\Psi, Q_B\Psi\rangle + \frac{1}{3}\langle\Psi,\Psi,\Psi\rangle\right)$$

$\langle\Phi, \Psi\rangle$



$$\langle\Phi, \Psi\rangle = \langle I\circ\Phi(0)\ \Psi(0)\rangle$$

z平面での
CFTの相関関数

$I(z) = -1/z$

$\langle\Phi, \Psi, \Lambda\rangle$

$$\langle\Phi, \Psi, \Lambda\rangle = \langle h_1\circ\Phi(0)\ h_2\circ\Psi(0)\ h_3\circ\Lambda(0)\rangle$$

$$\langle \Psi, \Psi, \Psi \rangle = \langle V_3 | \, |\Psi\rangle_1 \, |\Psi\rangle_2 \, |\Psi\rangle_3$$

$$\langle V_3 | \in \mathcal{A}_1^* \otimes \mathcal{A}_2^* \otimes \mathcal{A}_3^*$$

$$\langle V_3 | = {}_3\langle 0| \, {}_2\langle 0| \, {}_1\langle 0| \, e^{\frac{1}{2}\sum_{\substack{m,n\geq 0 \\ r,s=1,2,3}} \bar{N}^{rs}_{mn} \alpha^{(r)}_{-m} \alpha^{(s)}_{-n}} \times (\text{ghost})$$

$\bar{N}^{rs}_{mn}$: Neumann 係数.

$$\bar{N}^{rr}_{00} = -\frac{1}{2} \ln\left(\frac{3^3}{4^2}\right)$$

$$\bar{N}^{11}_{11} = -\frac{5}{27}, \quad \bar{N}^{12}_{11} = \frac{16}{27}, \quad \bar{N}^{13}_{11} = \frac{16}{27}$$

$\vdots$

§ Tachyon Condensation in SFT

tachyon potential $V(T)$

$$|T\rangle = T(0)|0\rangle \in \mathcal{A}$$

$$V(T) = -S(T) \Big/ \int d^{26}x = T_{25} f(T)$$

$\therefore$

$$\boxed{f(T) = 2\pi^2 \left(\frac{1}{2}\langle T, Q_B T\rangle + \frac{1}{3}\langle T, T, T\rangle \right)}$$

Sen's conjecture

$$\boxed{1 + f(T_0) = 0}$$

universality (background indepedence)

$$\mathcal{A} = \mathcal{A}_1 \oplus \mathcal{A}_2$$

$\mathcal{A}_1 : \{ L_n^{(m)}, c_n, b_n \} \times |0\rangle$

$\mathcal{A}_2 : \{ L_n^{(m)}, c_n, b_n \} \times \underset{h>0 \text{ primary}}{\phi(0)|0\rangle}$

$|\phi_1\rangle \in \mathcal{A}_1$, $|\phi_2\rangle \in \mathcal{A}_2$ とすると

- $\langle \phi_1, Q_B \phi_2 \rangle = 0$
- $\langle \phi_1, \phi_1, \phi_2 \rangle = 0$

$\therefore$ $f(T)$ の中に $\mathcal{A}_2$ 成分は 2次以上の項でしか現れない.

$$\boxed{\begin{aligned} & f(T) \sim \phi_2\phi_2 + \phi\phi_2\phi_2 \\ & \frac{\partial f}{\partial \phi_2} \sim \phi_2 + \phi\phi_2 = 0 \quad \Rightarrow \quad \phi_2 = 0 \end{aligned}}$$

$\Downarrow$

$f(T)$ は $\mathcal{A}_1$ のみに依存する.
(up to factor)

$\|$

universality

$f(T_0)$ の評価

$$|T\rangle = a\,c_1|0\rangle + b\,L_{-1}c_1|0\rangle + c\,c_{-1}|0\rangle + d\,L_{-2}c_1|0\rangle + \cdots$$

$(a, b, c, d, \cdots = \text{const.})$

$f(T)$ は無限個の変数の potential.

そこで

level truncation

$|T\rangle$ を level N までの状態で近似する.

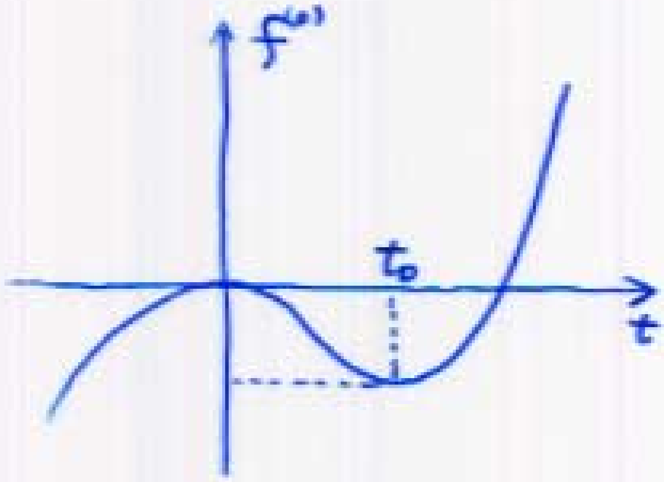
$\langle T, T, T\rangle$ の level の和 $\leqq 2N$.

level 0 近似

$$|T\rangle = t\, c_1 |0\rangle$$

$$f^{(0)}(t) = 2\pi^2\left(-\frac{1}{2}t^2 + \frac{1}{3}\left(\frac{4}{3\sqrt{3}}\right)^{-3} t^3\right)$$

$$\therefore\ t_0 = \left(\frac{4}{3\sqrt{3}}\right)^3$$



$$f^{(0)}(t_0) = \underline{-0.684}$$

"Sen's conjecture" の値の 約70% を与える.

level 2 近似

$$|T\rangle = t\,c_1|0\rangle + u\,c_{-1}|0\rangle + v\,\frac{1}{\sqrt{13}}L_{-2}c_1|0\rangle$$

$$\boxed{f^{(2)}(T_c) = \underline{-0.949}}$$

level 4 近似

$$|T\rangle = t\,c_1|0\rangle + u\,c_{-1}|0\rangle + v\,\frac{1}{\sqrt{13}}L_{-2}c_1|0\rangle$$
$$+ A\,L_{-4}c_1|0\rangle + B\,L_{-2}L_{-2}c_1|0\rangle + C\,c_{-3}|0\rangle$$
$$+ D\,b_{-3}c_{-1}c_1|0\rangle + E\,b_{-2}c_{-2}c_1|0\rangle + F\,L_{-2}c_{-1}|0\rangle$$

$$\boxed{f^{(4)}(T_c) = \underline{-0.9864}}$$

"Sen's conjecture" は 99% 正しい.

Moeller & Taylor

level 10 近似

$|T\rangle = \cdots$ 252項

$\langle T.T.T\rangle = \cdots$ 138.202項!

$$\boxed{f^{(20)}(T_0) = -0.99912}$$

$$\boxed{99.91\% \text{ 正しい}}$$

N. Berkovits. A. Sen. B. Zwiebach

"Tachyon Condensation in Superstring Field Theory"

hep-th/0002211

Super SFT. level 3 85% ok

non-poly open. Tachyon Kink 95% ok

⋮

J. Harvey. P. Kraus

"D-branes as unstable lumps in bosonic open string field theory"

hep-th/0002117

§ D branes as Noncommutative Solitons

$$S = \frac{1}{g^2}\int d^2z \left(\partial_z\phi\, \partial_{\bar z}\phi + V(\phi)\right)$$

$$\phi * \phi = e^{\frac{\theta}{2}(\partial_z \partial_{\bar z'} - \partial_{z'}\partial_{\bar z})} \phi(z,\bar z)\phi(z',\bar z')\Big|_{z'\to z}$$

$z \to \sqrt{\theta}\, z$ と変数変換して. $\theta \to \infty$ の極限をとると

$$S = \frac{\theta}{g^2}\int d^2z\, V(\phi)$$

$$\phi * \phi = e^{\frac{1}{2}(\partial_z \partial_{\bar z'} - \partial_{z'}\partial_{\bar z})} \phi(z,\bar z)\phi(z',\bar z')\Big|_{z'\to z}$$

$$\begin{cases} \phi_0 : & \phi_0 * \phi_0 = \phi_0 \\ \lambda : & \dfrac{dV(\lambda)}{d\lambda} = 0. \end{cases}$$ とすると.

$$V(\lambda\phi_0) = V(\lambda)\cdot\phi_0$$

$\therefore$ $\boxed{\phi = \lambda\phi_0 \text{ は 古典解.}}$

$$\phi(z,\bar{z}) = \frac{1}{(2\pi)^2}\int d^2k\, \tilde{\phi}(k)\, e^{-i(k_q x + k_p y)}$$

$(z = x + iy)$

$\phi(z,\bar{z})$ に $\mathcal{O}_\phi(\hat{p},\hat{q})$ を対応させる

$$\mathcal{O}_\phi(\hat{p},\hat{q}) = \frac{1}{(2\pi)^2}\int d^2k\, \tilde{\phi}(k)\, e^{-i(k_q \hat{q} + k_p \hat{p})}$$

$([\hat{q},\hat{p}] = i)$

このとき

- $\frac{1}{2\pi}\int d^2z\, \phi(z,\bar{z}) = \mathrm{Tr}_{\mathcal{H}}\, \mathcal{O}_\phi(\hat{p},\hat{q})$
- $\mathcal{O}_f \cdot \mathcal{O}_g = \mathcal{O}_{f*g}$

$$\phi_0 * \phi_0 = \phi_0 \longleftrightarrow \mathcal{O}_{\phi_0}^2 = \mathcal{O}_{\phi_0}$$

$\boxed{\phi_0 \text{ は } \mathcal{H} \text{ における projection operator に対応}}$

$$a = \frac{1}{\sqrt{2}}(\hat{q} + i\hat{p}), \quad a^{\dagger} = \frac{1}{\sqrt{2}}(\hat{q} - i\hat{p})$$

$$[a, a^{\dagger}] = 1$$

$$|n\rangle = \frac{1}{\sqrt{n!}}(a^{\dagger})^n |0\rangle$$

Projection operator

$$P = \sum_n c_n |n\rangle\langle n|$$

$U^{\dagger}PU$ も projection operator だから

Noncommutative Soliton は $U(\infty)$ の対称性をもつ.

Tachyon の有効理論

$$S = \frac{C}{g_s}\int d^{26}x\sqrt{g}\left(f(t)\,g_{\mu\nu}\,\partial^\mu t\,\partial^\nu t - V(t) + \cdots\right)$$

$(C = g_s T_{25})$

$$\Big\downarrow\ \theta\to\infty$$

$$S = -\frac{C}{G_s}\int d^{26}x\sqrt{G}\,V(t)$$

◎ $t = t_* \phi_0(z)$ を代入　　($|0\rangle\langle 0|$ に対応する解)

$$S = -\frac{C\,V(t_*)}{G_s}\int d^{24}x\int d^2z\sqrt{G}\,\phi_0(z)$$

$$= -\frac{2\pi\theta\,C\,V(t_*)}{G_s}\int d^{24}x\sqrt{G}$$

◎ $V(t_*) = 1$ (Sen's conjecture) と $G_s = \dfrac{g_s\sqrt{G}}{2\pi\alpha' B\sqrt{g}}$ を代入

$$S = -(2\pi)^2\alpha'\frac{C}{g_s}\int d^{24}x\sqrt{g}$$

$$= -T_{23}\int d^{24}x\sqrt{g}\qquad (T_{23} = (2\pi)^2\alpha' T_{25})$$

$$\boxed{t = t_*\phi_0 \text{ は D23 brane に対応している.}}$$

$\theta \to \infty$ の極限で

~~$(B \to \infty)$~~

$$|\Psi\rangle = |\Psi_0\rangle \otimes e^{ip\underset{\sim}{X}(w=0)}|0\rangle$$

非可換な方向 (2p次元)

$$\hat{\mathcal{A}} = \hat{\mathcal{A}}_0 \otimes \hat{\mathcal{A}}_1$$

$$\left(|\Phi * \Psi\rangle = |\Phi_0 * \Psi_0\rangle \otimes |\Phi_1 * \Psi_1\rangle \right)$$

$B = tB_0$, $X^i = Y^i/\sqrt{t}$, $t \to \infty$

$$\underset{\hat{\mathcal{A}}_1}{e^{ipY}(\tau)} \cdot \underset{\hat{\mathcal{A}}_1}{e^{iqY}(\tau')} \sim e^{-\frac{i}{2}\theta^{ij}p_iq_j}\, \underset{\hat{\mathcal{A}}_1}{e^{i(p+q)Y}(\tau')}$$

$$\underset{\hat{\mathcal{A}}_0}{\partial^n Y(\tau)} \cdot \underset{\hat{\mathcal{A}}_1}{e^{iqY}(\tau')} \sim \frac{1}{t}\frac{1}{(\tau-\tau')^n}\, e^{iqY} \underset{\sim}{\to 0}$$

◎ $\mathcal{A}_1$ を $\underset{\sim}{\mathcal{H}}$ 上の operator で表現する.

$\{|0\rangle, |1\rangle, |2\rangle, \cdots\}$

($\mathcal{A}_1$ を $\infty \times \infty$ 行列 ($U(\infty)$ 行列) とみなす.)

◎ $\mathcal{H} = \underset{\sim}{V} \oplus \underset{\sim}{W}$

W: $N-V$

V: N次元部分空間 $\{|0\rangle, |1\rangle, \cdots, |N-1\rangle\}$

projection operator

$$P_N = |0\rangle\langle 0| + |1\rangle\langle 1| + \cdots + |N-1\rangle\langle N-1|$$

$$\left(\begin{array}{l} P_N V = V \ , \quad P_N W = 0 \\ P_N^2 = P_N \end{array} \right)$$

$$\boxed{|\Psi_0\rangle = \underbrace{|T_0\rangle}_{\text{D25 brane が消滅する解}} \otimes (1 - P_N)}$$

は、String Field の運動方程式

$$Q_B \Psi + \Psi * \Psi = 0$$

の解である.

- $Q_B T_0 + T_0 * T_0 = 0$
- $[Q_B, P_N] \to 0 \quad (\theta \to \infty)$
- $(1 - P_N)^2 = 1 - P_N$

$\Psi = \Psi_0 + \Psi'$ と展開する

Ψ' が VV open string のとき

$$S = \int \left(\Psi' * Q_B \Psi' + \frac{2}{3} \Psi' * \Psi' * \Psi' \right)$$

◎ $|\Psi'\rangle \in \mathcal{A}_0 \otimes M_N$

↑ complex $N \times N$ 行列

≀

CP factor

◎また $|\Psi'\rangle$ は可換な方向だけ運動量をもつ

$\Rightarrow$ Ψ': N 枚の D(25−2p) brane 上の open string

(古典解で brane U(N) sym. が見えている)

Ψ' が WW open string のとき

$$S = \int \left(\Psi' * Q_B' \Psi' + \frac{2}{3} \Psi' * \Psi' * \Psi' \right)$$

$$Q_B' \phi = Q_B \phi + T_0 * \phi + (-)^{|\phi|} \phi * T_0$$

Sen's conjecture より Ψ' に物理的モードはない

VW open string についても同様

§ まとめ.

◎ Sen's conjecture

$$T_p + V(\Phi_0) = 0$$

◎ SFTの解析より. Sen's conjecture は
99.9% 正しい. (off-shell)

◎ $\theta \to \infty$ の極限で

$$D25 \longrightarrow N枚\ D(25-2p)$$

が解析できる.
このとき SFT が役立つ (?)

課題

◎ $1 + f(T_0) = 0$ は 100% 正しいのか?

◎ さらに非摂動論的解析が SFT に可能か?

Closed String
True Vacuum
⋮