Noncommutative Gauge Theory on Fuzzy Four-Sphere and Matrix Model

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IIB matrix model has a flat noncommutative background as a classical solution, and the model around the background gives a noncommutative Yang-Mills theory on the background. It is an interesting problem to study noncommutative gauge theories on curved manifolds. (Two-dimensional sphere and torus are considered in [1].) Here we consider the following matrix model based on [2],

$$S = -\frac{1}{g^2} Tr \left(\frac{1}{4}[A_\mu, A_\nu][A_\mu, A_\nu] + \frac{\lambda}{5}\epsilon^{\mu\nu\lambda\rho\sigma} A_\mu A_\nu A_\lambda A_\rho A_\sigma \right). \tag{1}$$

This model has a fuzzy four-sphere as a classical solution. The fuzzy four-sphere is considered in [3], and the coordinates satisfy the following relation

$$\epsilon^{\mu\nu\lambda\rho\sigma} \hat{x}_\mu \hat{x}_\nu \hat{x}_\lambda \hat{x}_\rho = (8n+16)\alpha^3 \hat{x}_\sigma. \tag{2}$$

The commutation relation of the coordinates does not close in this case. Therefore we have to introduce extra matrices $\hat{w}_{\mu\nu}$ which are given by

$$\hat{w}_{\mu\nu} = \frac{i}{2\alpha}[\hat{x}_\mu, \hat{x}_\nu]. \tag{3}$$

It can be shown that $\hat{w}$ satisfy a fuzzy two-sphere algebra on the north pole and the geometry which is described by (2) is not S^4 but $S^4 \times S^2$ [4]. Since the matrix model given in (1) has the fuzzy four-sphere as a classical solution, the expansion around it leads to a noncommutative gauge theory on it. Due to the existence of the extra degrees of freedom which are originated from the fuzzy two-sphere, fields on the fuzzy four-sphere have higher spins.

References

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