

# Fuzzy $\text{CP}^2$ or $\text{S}^2$ — which is the true vacuum ?

Takehiro Azuma (KEK)

azumat@post.kek.jp

This poster session introduces our work [1], in which we have discussed the bosonic matrix model as a toy model that has the curved-space classical solution:

$$S = N \text{tr} \left( -\frac{1}{4} \sum_{\mu, \nu=1}^8 [A_\mu, A_\nu]^2 + \frac{2i\alpha}{3} \sum_{\mu, \nu, \rho=1}^8 f_{\mu\nu\rho} A_\mu A_\nu A_\rho \right)$$

This matrix model accommodates the following two classical solutions. One is the fuzzy  $\text{S}^2$  manifold spanned in the 123 direction. The other is the more interesting four-dimensional fuzzy  $\text{CP}^2$  manifold. To scrutinize the properties of these two fuzzy manifolds, we have studied this matrix model numerically via the heat bath algorithm of the Monte Carlo simulation. The results are recapitulated as follows.

We have found that this matrix model undergoes the first-order phase transition at a certain critical point  $\alpha_{\text{cr}}$ , as we change the coefficient  $\alpha$  of the cubic term in the action. This phase transition is similar to that of the three-dimensional bosonic model studied in our previous work [2]. At  $\alpha < \alpha_{\text{cr}}$ , the effect of the cubic term is negated and the model behaves like the pure Yang-Mills model ( $\alpha = 0$ ). On the other hand, at  $\alpha > \alpha_{\text{cr}}$ , the above-mentioned classical solutions retain the (meta)stability due to the meager quantum effect. In addition, we have elucidated that the observables calculated numerically are close to the one-loop perturbative calculation. The critical point is also close to the value predicted from the one-loop effective action. In this sense, we proclaim that this matrix model has the “one-loop dominance”.

The one-loop dominance plays a pivotal role in determining the true vacuum of this matrix model. By the comparison of the one-loop effective action, we have concluded that the two-dimensional fuzzy  $\text{S}^2$  is the true vacuum of this model, and that the four-dimensional fuzzy  $\text{CP}^2$  is a metastable state.

This poster session is based on the collaboration with Subrata Bal, Keiichi Nagao and Jun Nishimura.

## References

[1] T. Azuma, S. Bal, K. Nagao and J. Nishimura, “Dynamical aspects of the fuzzy  $\text{CP}^2$  in the large  $N$  reduced model with a cubic term,” [hep-th/0405277](#).

[2] T. Azuma, S. Bal, K. Nagao and J. Nishimura, “Nonperturbative studies of fuzzy spheres in a matrix model with the Chern-Simons term,” *JHEP* **0405**, 005 (2004) [[hep-th/0401038](#)].