String interpretation for finite N Yang-Mills in two-dimensions [1]

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Summary : We discussed the equivalence between a string theory and the YM2 with $SU(N)$ gauge group for finite N. We find a sector in the partition function which can be interpreted as a sum of covering maps from closed string world-sheets to the target space, whose covering number is less than N. This gives an asymptotic expansion of $1/N$ whose large N limit becomes the chiral sector defined by D. Gross and W. Taylor. The residual part of the partition function provides the non-perturbative corrections to the perturbative expansion. We also pointed out the correspondence between the finite N YM2 and $c = 0$ matrix model with matter.

The partition function of Yang-Mills theory on 2D manifold with genus G and area A is [2]

$$Z_{YM}(G, A, N) = \sum_{n=1}^{\infty} \sum_{R \in Y_n^N} (dim R)^{2-2G} e^{-\frac{A}{2N} C_2(R)}$$

which has an asymptotic $1/N$ expansion and can be translated into a string theory

$$Z_{string} = \sum_{n=1}^{\infty} \sum_{i,t,h=0}^{\infty} \sum_{\nu \in \Sigma(G,n,i)} \frac{g_s^{2g-2}}{|S_\nu|} e^{-\frac{nA}{2}} \frac{A^{i+t+h}}{i!t!h!} \left(\frac{n(n-1)}{2} \right)^t \left(\frac{n}{2} \right)^h .$$

Here we are interested in the finite N corrections to it. We simply divide the partition function;

$$\begin{aligned} Z_{YM}(G, A, N) &= \left[\sum_{n=1}^{N-1} \sum_{R \in Y_n} + \sum_{n=N}^{\infty} \sum_{R \in Y_n^N} \right] (dim R)^{2-2G} e^{-\frac{A}{2N} C_2(R)} \\ &= Z^{pert}(G, A, N) + Z^{res}(G, A, N). \end{aligned}$$

Z_{pert} turns out to be Z_{string} in the large N limit. The residual part provides the non-perturbative corrections

$$F_{res} = \ln Z_{res} = \sum_{n=N}^{\infty} e^{-\frac{nA}{2}} \sum_{i,t,h} N^{-i-2t-2h} \cdots = e^{-\frac{A}{2\pi\alpha' g_s}} \sum_{j=1} N^{-j} \cdots .$$

which indicates the existence of D-branes (D1-branes) and open string on them. Another interesting obsevation which supports the idea that the finite N effect might introduce open string degrees of freedom is as follows. Finite N YM2 action ($G = 0$) can be written in terms of the profile function as

$$S_{YM} = \frac{1}{4} \int dx dy f_k''(x) f_k''(y) \gamma_\epsilon(x-y) + \frac{A}{4\epsilon^2} \int dx f_k''(x) \left(\frac{x^2}{2} + \frac{x^3}{3N\epsilon} \right) - \int dx f_k''(x) \gamma_\epsilon(x + \epsilon N)$$

where the last term is the finite N correction. It is ammusing to compare this with the effective action of $c = 0$ matrix model with fundamental matter which describes the open string degrees of freedom

$$S_{eff} = -2g_s N^2 \int dx dy \rho(x) \rho(y) \ln(x-y) + N^2 \int dx \rho(x) V(x) + g_s N N_f \int dx \rho(x) \ln(x-m).$$

However, a precise understanding of the relation between these two is yet to be uncovered.

[1] This report is based on the collabolations with S. Matsuura and K. Ohta (RIKEN): hep-th/0404204, /0406191.

[2] Here we do not consider the large N chiral factorization.