Supersymmetric Gauge Theories with Matters, Toric Geometries and Random Partitions

Department of Physics, Graduate School of Science, Osaka University Yui NOMA
E-mail: yuhii@het.phys.sci.oska-u.ac.jp

Gauge/Gravity correspondences are fascinating properties suggested from the string theory. Recently, interesting correspondences were found [1] which relate the five-dimensional $SU(N)$ gauge theory compactified on a circle with eight super-charges, a certain toric variety called a local $SU(N)$ geometry and a certain statistical model of partitions which is called a random plane partition model. We generalize the correspondences to the case of the gauge theory with matters [2]. By this work, we can get a deep understanding of geometric engineering and see a microscopic emergence of a geometry.

The low energy effective action for the five-dimensional $SU(N)$ gauge theory with eight super-charges is given by means of Nekrasov's partition function. From the topological A-model string amplitudes on certain toric varieties the instanton part of Nekrasov's partition function is obtained. Such works show us deep relations between gauge theory and string theory. However there are missing part, that is the perturbative part. In [1] it is shown that the missed part is the geometry. By quantizing the geometry we can reproduce the perturbative part of Nekrasov's partition function for the gauge theory with no matters. We generalized such relation to the gauge theory with massive fundamental matters and with a massive adjoint matter [2].

We showed that Nekrasov's partition function for five-dimensional $U(1)$ gauge theory can be expressed as a correlation function of a two-dimensional CFT and as a partition function of a statistical model of partitions. Furthermore we showed that from Nekrasov's partition function for five-dimensional $U(1)$ gauge theory, Nekrasov's partition function for five-dimensional $SU(1)$ gauge theory is obtained by means of embedding N-tuple of partitions to a single partition.

Each box in a partition can be considered as a gravitational quantum. From the ground state of the statistical model of partitions, we can reproduce the polyhedron for the toric variety on which topological string amplitudes gives the instanton part of Nekrasov's partition function. We can understand this fact as follows. By condensing gravitational quanta the geometry emerged. Therefore this is a emergence of geometry.

References

[1] T. Maeda, T. Nakatsu, K. Takasaki and T. Tamakoshi, JHEP **0503**, 056 (2005) [arXiv:hep-th/0412327]. Nucl. Phys. B **715**, 275 (2005) [arXiv:hep-th/0412329]. T. Maeda, T. Nakatsu, Y. Noma and T. Tamakoshi, Nucl. Phys. B **735**, 96 (2006) [arXiv:hep-th/0505083]. T. Maeda and T. Nakatsu, arXiv:hep-th/0601233.

[2] Y. Noma, arXive:hep-th/0604141.