Chiral Symmetry Breaking in Brane Models

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1. Introduction

AdS/CFT [Maldacena (1997)]

Weakly coupled IIB String Theory on $AdS_5 \times S^5$

$\Leftarrow$ dual $\Rightarrow$ Strongly coupled 4-D $\mathcal{N} = 4$ Super Yang-Mills

We can analyze non-perturbative aspects of QCD by AdS/CFT.

(holographic QCD)

SχSB was discussed in many holographic models :

D3/D7 [Babington et al. (2003)], D4/D6 [Kruczenski et al. (2003)], $\cdots$

$\longrightarrow$ Chiral symmetry = Rotational symmetry

D4/D8-$\overline{\text{D8}}$ [Sakai-Sugimoto (2004)], $\cdots$

$\longrightarrow$ Chiral symmetry = Gauge symmetry

I'd like to discuss SχSB in general intersecting Dq/Dp model.

2. General setup: Dq/Dp model

Configuration of the Dq/Dp model

	t	$x^{1,\cdots,r}$	$x^{r+1,\cdots,q-1}$	x^q	$x^{q+1,\cdots,q+p-r}$	$x^{q+p-r+1,\cdots,9}$
N_c Dq	∘	∘	∘	∘	−	−
a Dp	∘	∘	−	−	∘	−

There are

- a r-dim. intersection
- a compact x^q direction $\rightarrow$ imposing SUSY breaking B.C.
- a $(9-q-p+r)$-dim. transverse space (assume $q+p-r<9$)

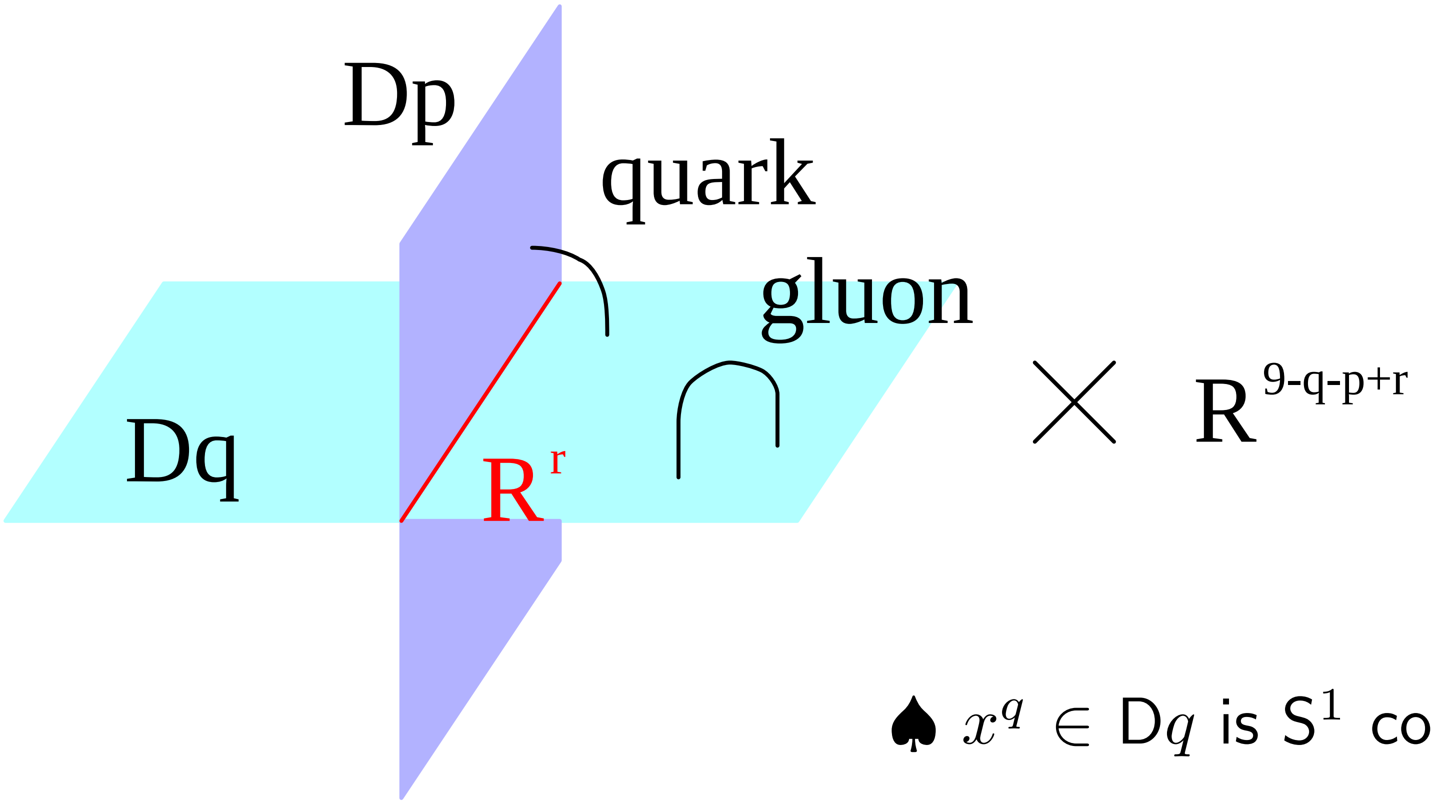
The theory localized at the intersection is QCD-like $\longrightarrow$ " QCD$_{r+1}$ "

Global symmetry of QCD$_{r+1}$

$$\mathrm{SO}(1,r)\times\mathrm{SO}(9-q-p+r)\times\mathrm{U}(1)$$

3. Chiral symmetry from rotational symmetry

The theory localized at the intersection is QCD_{r+1}



♠ $x^q \in \mathrm{D}q$ is S^1 compactified

Rotational symmetry in $\mathbf{R}^{9-q-p+r}$ $\Longleftrightarrow$ Chiral symmetry of QCD_{r+1}

Seperation of Dq and Dp in $\mathbf{R}^{9-q-p+r}$ $\Longleftrightarrow$ Quark mass ($\rightarrow \chi$SB)

4. Representative configurations of QCD_{r+1}

There are many configurations dual to QCD_{r+1}

$\longrightarrow$ classified by the sets of $(q+p, r)$

		0	1	2	3	4	5	6	7	8	9	a^{NS}	$(q+p, r)$
color	D2	○	○	○	—	—	—	—	—	—	—		
flavor	D2	○	○	—	○	—	—	—	—	—	—	$-\frac{1}{4}$	$(4,1)$
	D4	○	○	—	○	○	○	—	—	—	—	0	$(6,1)$
	D6	○	○	—	○	○	○	○	○	—	—	$\frac{1}{4}$	$(8,1)$
color	D3	○	○	○	○	—	—	—	—	—	—		
flavor	D3	○	○	○	—	○	—	—	—	—	—	$-\frac{1}{4}$	$(6,2)$
	D5	○	○	○	—	○	○	○	—	—	—	0	$(8,2)$
	D7	○	○	○	—	○	○	○	○	○	—	$\frac{1}{4}$	$(10,2)$
color	D4	○	○	○	○	○	—	—	—	—	—		
flavor	D4	○	○	○	○	—	○	—	—	—	—	$-\frac{1}{4}$	$(8,3)$
	D6	○	○	○	○	—	○	○	○	—	—	0	$(10,3)$

Concentrate on $a^{\mathrm{NS}} = 0$ configurations.

5. Chiral symmetries in QCD_{r+1}

Rotational symmetry $\mathrm{SO}(9-q-p+r)$ in $\mathbf{R}^{9-q-p+r}$ space

$\Longrightarrow$ Chiral symmetry of QCD_{r+1}

· $\underline{\mathrm{QCD}_4}$

$$\mathrm{SO}(2)_{89} \times \mathrm{U}(1) \sim \underline{\mathrm{U}(1)_V \times \mathrm{U}(1)_A}$$

$\longrightarrow$ Abelian chiral symmetry

· $\underline{\mathrm{QCD}_3}$

$$\mathrm{SO}(3)_{789} \times \mathrm{U}(1) \sim \underline{\mathrm{SU}(2)} \times \mathrm{U}(1)_V$$

$\longrightarrow$ Physical meaning is not clear !

· $\underline{\mathrm{QCD}_2}$

$$\mathrm{SO}(4)_{6789} \times \mathrm{U}(1) \sim \underline{\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R} \times \mathrm{U}(1)_V$$

$\longrightarrow$ Non-Abelian chiral symmetry

6. Supergravity analysis

Study Dq/Dp model with

- Near horizon limit

  N_c Dq-branes $\rightarrow$ " background geometry "

  (classical SUGRA is valid : $1 \ll \lambda_{q+1} \left(\frac{U_{KK}}{\alpha'}\right)^{q-3} \ll N_c^{\frac{4}{7-q}}$)

- Probe approximation $(N_c \gg N_f = 1)$

  a Dp-brane $\rightarrow$ " probe " (not affects Dq background)

We can analyze the dynamics of the Dp-brane in the Dq background.

Ansatz

$$x^{r+1,\cdots,q} = \text{const.}\ , \quad r = r(\lambda)\ , \quad \theta^a = \text{const.}$$

- r, θ^a $\cdots$ radial and angular coordinates of $\mathbf{R}^{9-q-p+r}$ space
- λ $\cdots\cdots$ radial coordinate of $\mathbf{R}^{p-r}$ space

7. Probe Dp-brane dynamics

Effective action of the probe Dp-brane

$$S_{\mathrm{D}p} = -\tilde{T}_p V_{p-r-1} \int d^{r+1}x \int d\lambda \, \rho^{\alpha} \left(1 + \frac{U_{KK}^{7-q}}{4\rho^{7-q}} \right)^{\beta} \lambda^{p-r-1} \sqrt{1 + (r')^2} \, ,$$

where

$$\alpha = \frac{1}{4}(7-q)(4+2r-q-p) \, , \quad \beta = \frac{1}{2}(4+2r-q-p) + \frac{2(p-r)}{7-q} \, .$$

Equation of motion for $r(\lambda)$

$$\frac{d}{d\lambda} \left[\rho^{\alpha} \left(1 + \frac{U_{KK}^{7-q}}{4\rho^{7-q}} \right)^{\beta} \frac{\lambda^{p-r-1} \; r'}{\sqrt{1 + (r')^2}} \right] = \frac{\partial}{\partial r} \left[\rho^{\alpha} \left(1 + \frac{U_{KK}^{7-q}}{4\rho^{7-q}} \right)^{\beta} \right] \lambda^{p-r-1} \sqrt{1 + (r')^2} \, .$$

8. AdS/CFT dictionary

Asymptotic behavior of $r(\lambda)$ is

$$r(\lambda) \sim r_\infty + c\lambda^{-(p-r-2)} \quad (\text{ for } a^{\mathrm{NS}} = 0 \text{ configurations}).$$

AdS/CFT dictionary

$$r_\infty \longleftrightarrow \text{Quark mass} \quad m_q$$

$$c \longleftrightarrow \text{Quark condensate} \quad \langle \bar{\psi}\psi \rangle$$

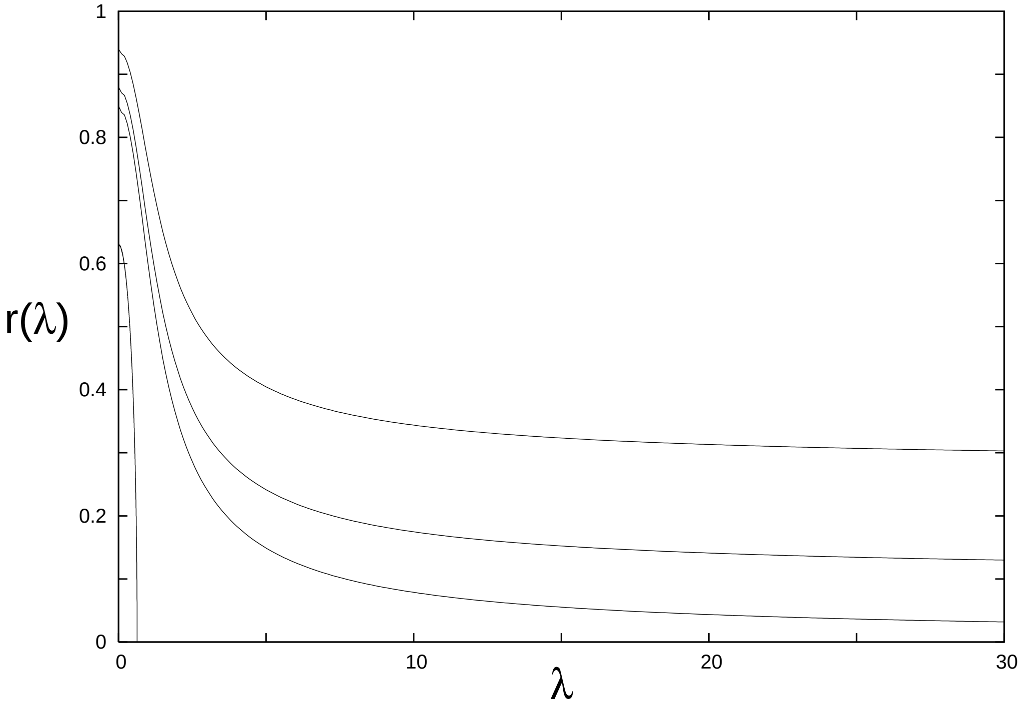
The relation between r_∞, c and m_q, $\langle \bar{\psi}\psi \rangle$ are

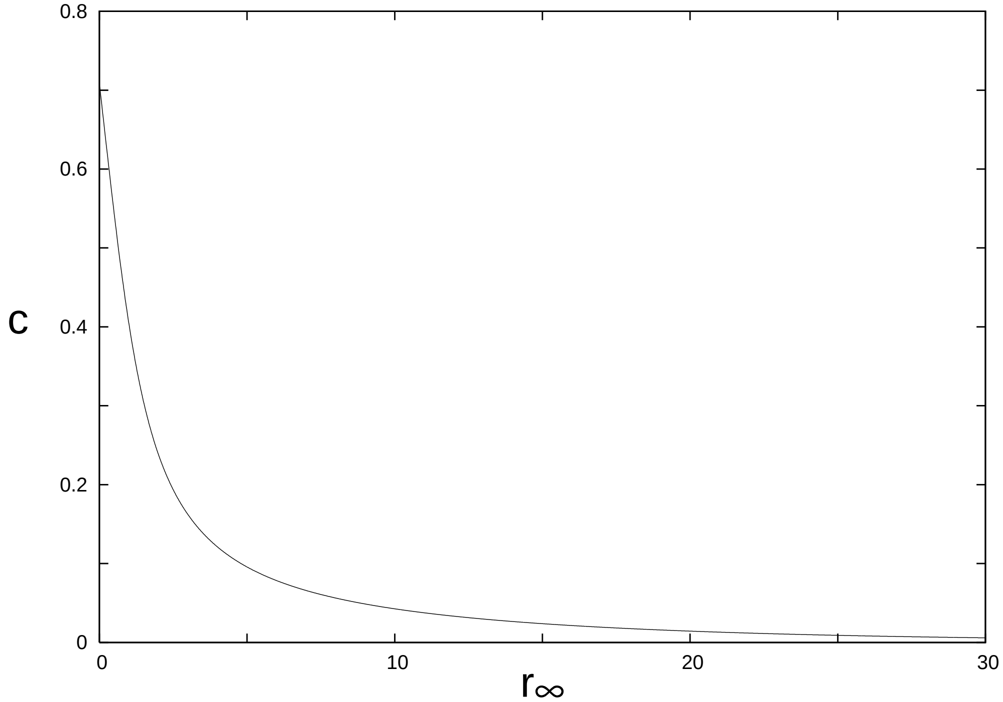
$$m_q = \frac{U_{KK}\, r_\infty}{2\pi\ell_s^2}\,, \quad \langle \bar{\psi}\psi \rangle = -2\pi\ell_s^2\, \tilde{T}_p V_{p-r-1} U_{KK}^{\alpha+p-r-1}\, c\,.$$

9. Numerical solutions (i) D4/D6 with r=3 (QCD_4)

[Kruczenski et al. (2003)]

D6 embedding breaks the rotational symmetry !





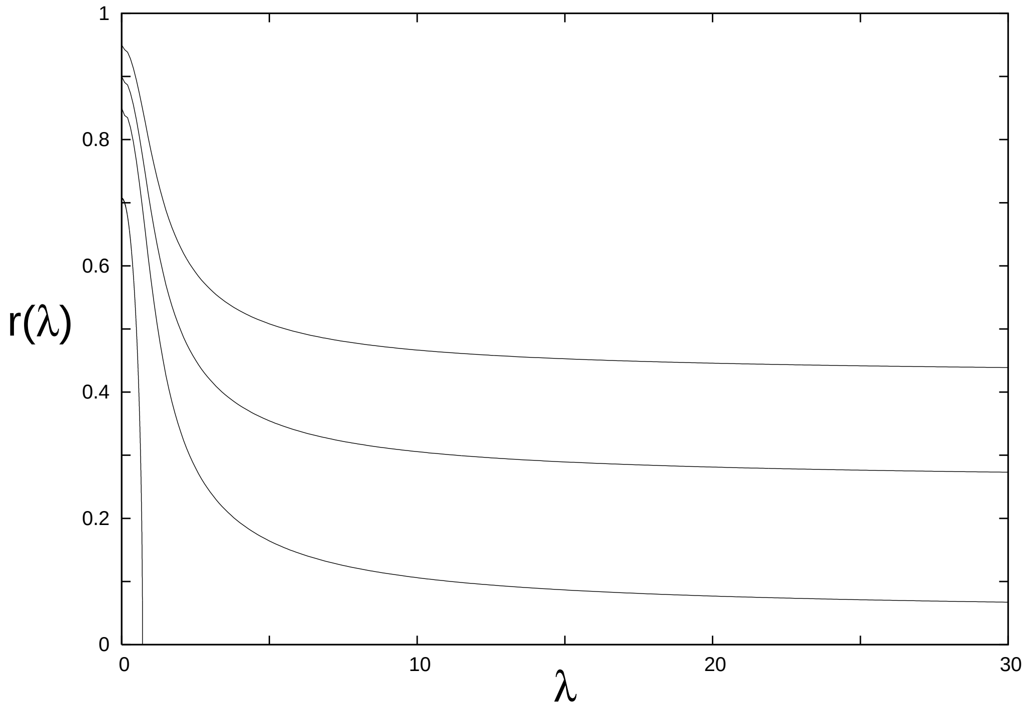
Gravity side : Non-zero c even for $r_\infty = 0$

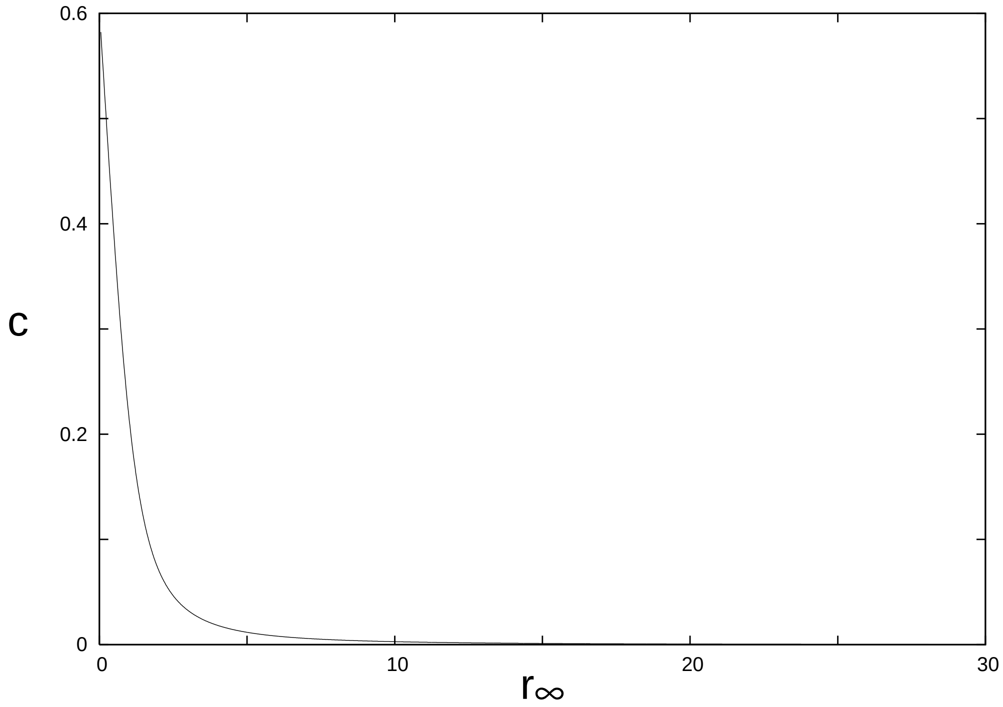
$\Longleftrightarrow$ QCD_4 side : Spontaneous chiral symmetry breaking

$$\mathrm{SO}(2)_{89} \times \mathrm{U}(1) \sim \mathrm{U}(1)_V \times \mathrm{U}(1)_A \rightarrow \mathrm{U}(1)_V$$

9. Numerical solutions (ii) D3/D5 with r=2 (QCD_3)

D5 embedding breaks the rotational symmetry !



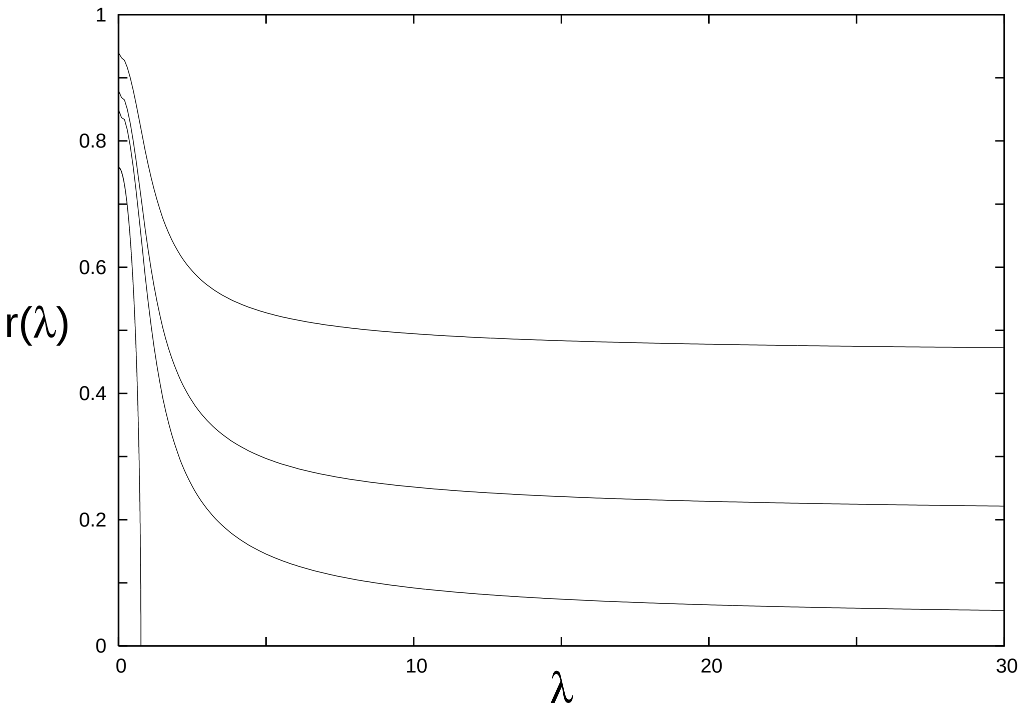


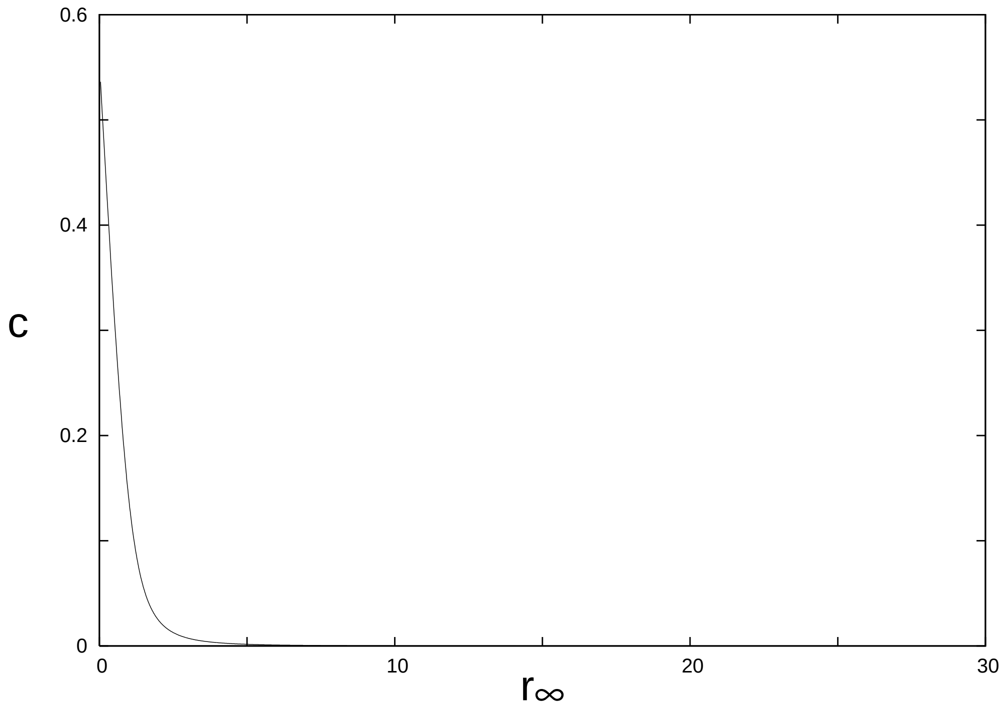
Gravity side : Non-zero c even for $r_\infty = 0$

$\Longleftrightarrow$ QCD_3 side : ???

9. Numerical solutions (iii) D2/D4 with r=1 (QCD_2)

D4 embedding breaks the rotational symmetry !





Gravity side : Non-zero c even for $r_\infty = 0$

$\Longleftrightarrow$ QCD_2 side : Spontaneous chiral symmetry breaking

$$\mathrm{SO}(4)_{6789} \sim \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \to \mathrm{SU}(2)_V$$

10. NG bosons as fluctuations around the embedding

There will be $(8-q-p+r)$ NG bosons associated with the SχSB.

Fluctuations around the vaccum embedding

$$x^{r+1,\cdots,q} = 0\ , \quad r = r_{\mathsf{vac}} + \delta r\ , \quad \theta^a = 0 + \delta\theta^a$$

$$\Longrightarrow \quad S_{\mathsf{D}_p} = S_{\mathsf{vac}} + S_{\delta r} + S_{\delta\theta}$$

For simplicity we assume that ...

$\delta\theta^a$ depends only on the coordinates of the intersection : $\delta\theta^a = \delta\theta^a(x^\mu)$

"Pion" effective action at quadratic order

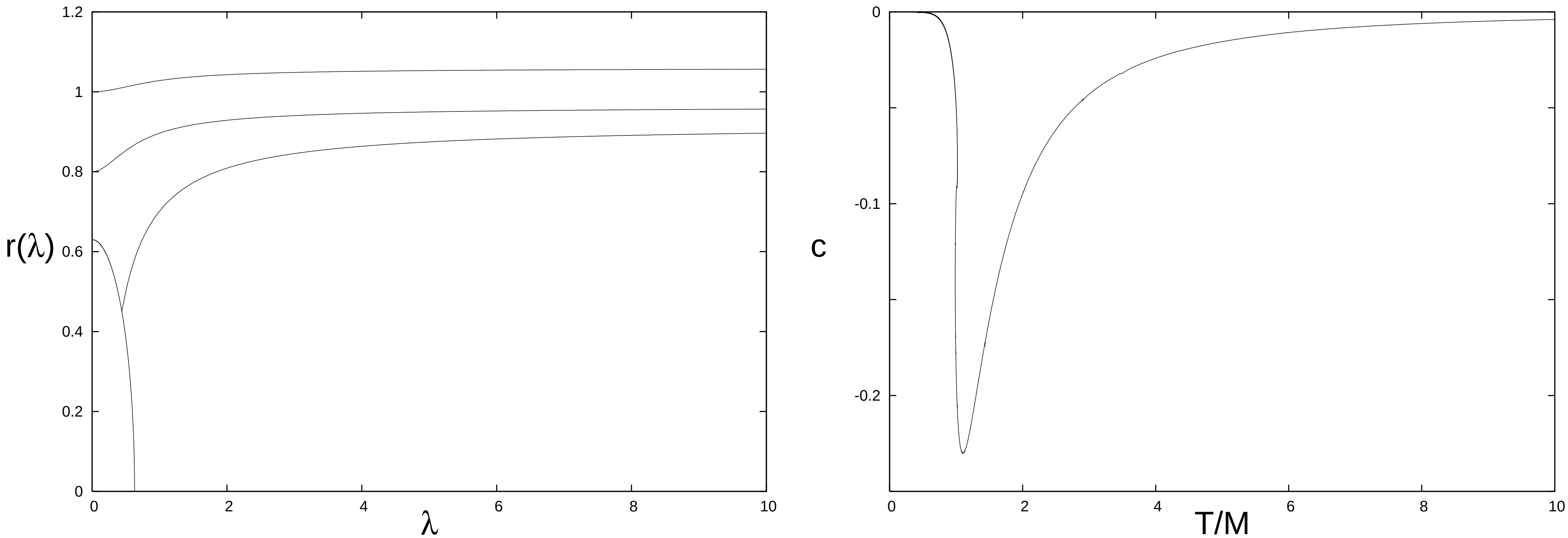
$$S_{\delta\theta} = -f_{\delta\theta}^2 \int d^{r+1}x\ \frac{1}{2}\gamma_{ab}\partial_\mu(\delta\theta^a)\partial^\mu(\delta\theta^b)\ .$$

There appear $(8-q-p+r)$ massless NG bosons $\delta\theta^a$!

11. Finite temp. analysis (i) D4/D6 with r=3 (QCD$_4$)

[Kruczenski et al. (2003)]

D6 embedding breaks the rotational symmetry !



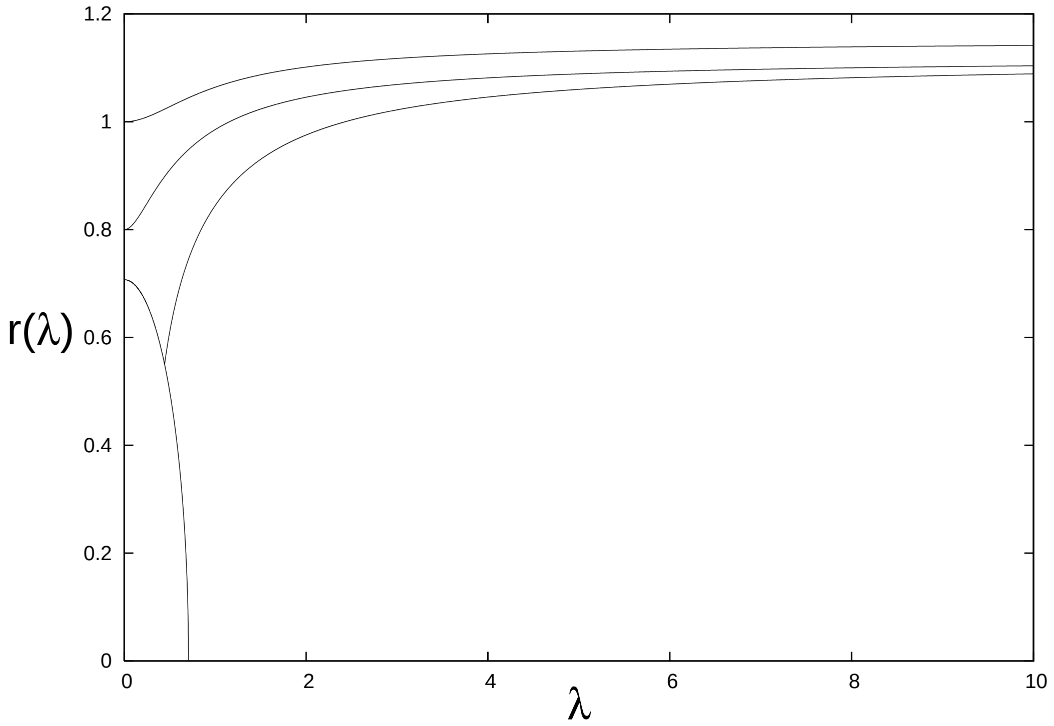
Difference from zero-temp.

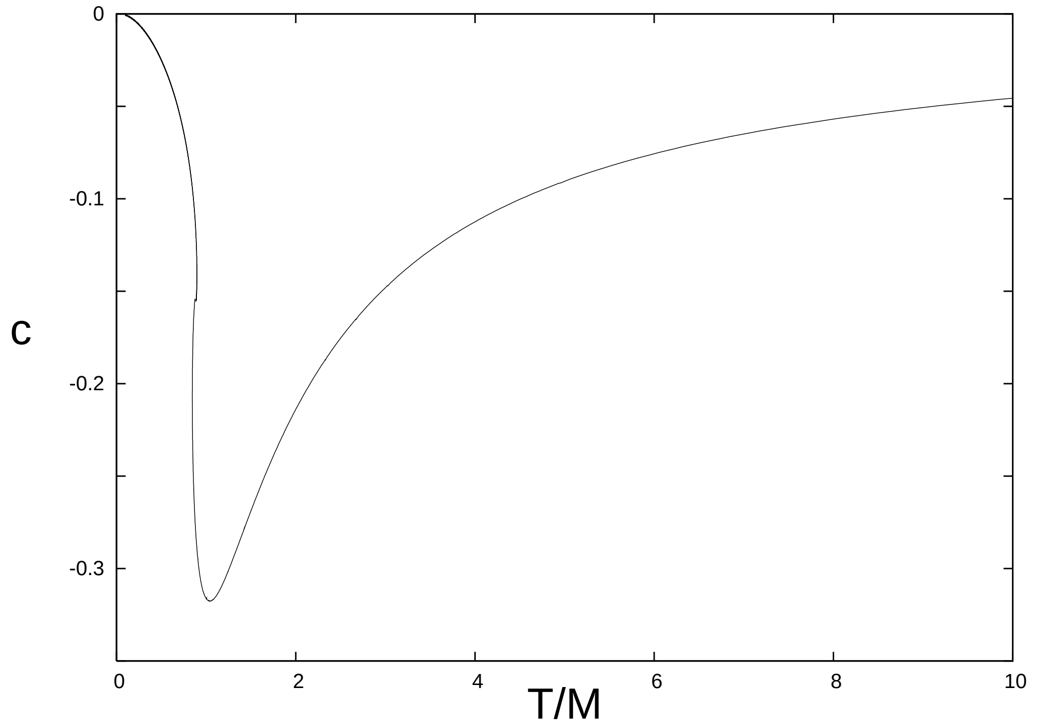
Gravity side : $c = 0$ for $T \to \infty$ $\left(T = \bar{M}/\sqrt{r_\infty}\right)$

$\Longleftrightarrow$ QCD$_4$ side : Chiral symmetry restore at $T \to \infty$

11. Finite temp. analysis (ii) D3/D5 with r=2 (QCD_3)

D5 embedding breaks the rotational symmetry !





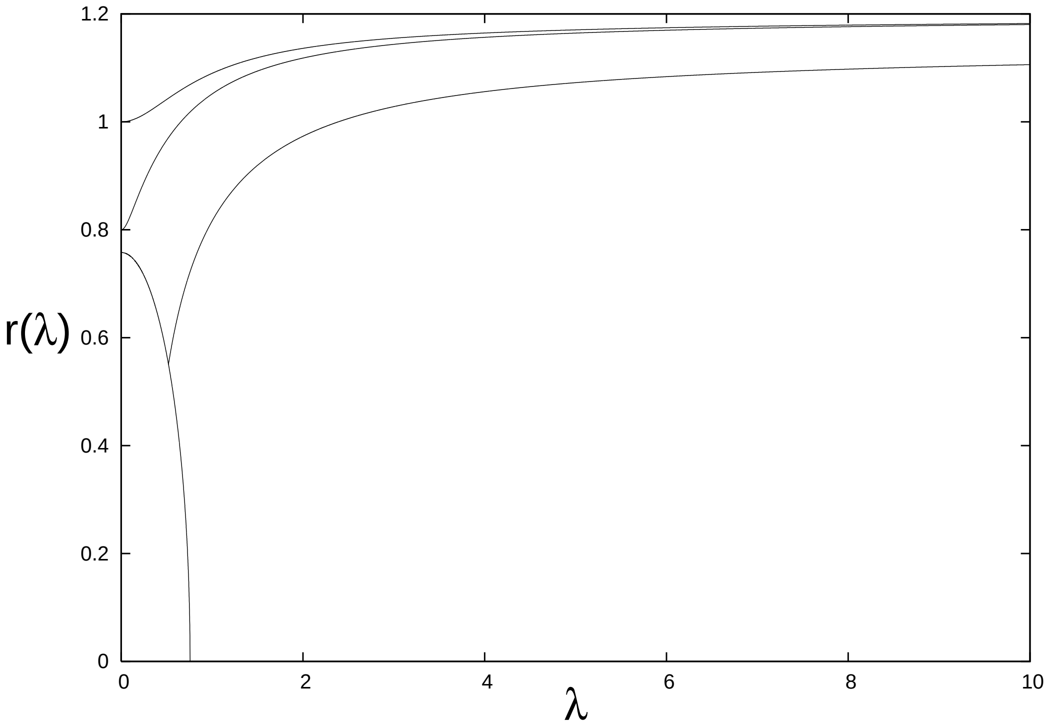
Difference from zero-temp.

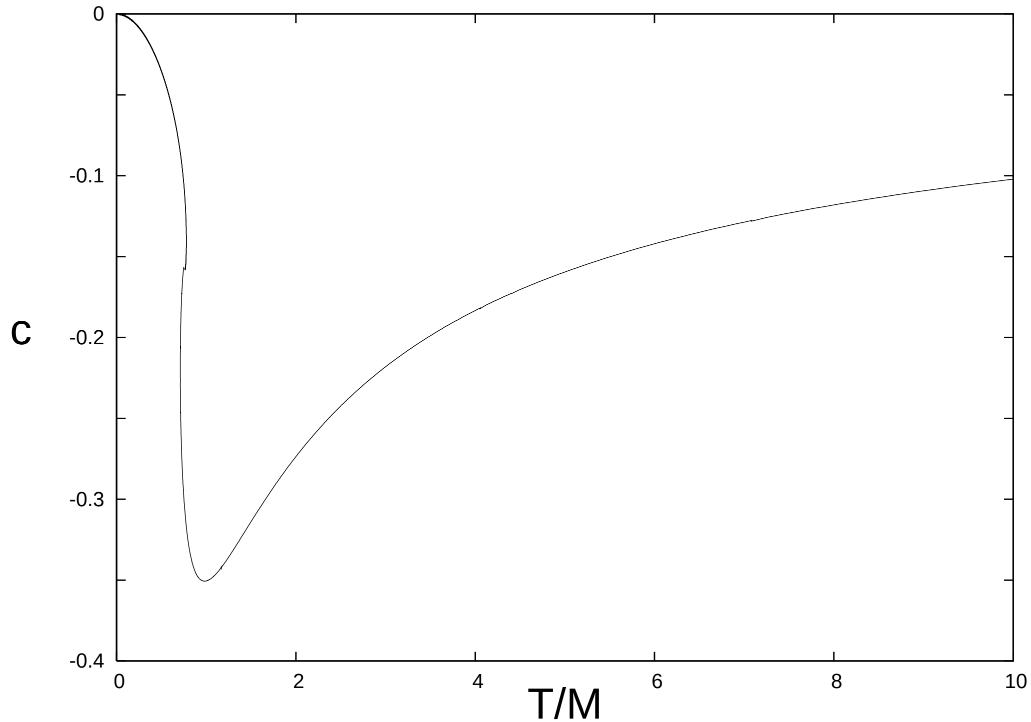
Gravity side : $c = 0$ for $T \to \infty$ $\left(T = \bar{M}/r_\infty\right)$

$\Longleftrightarrow$ QCD_3 side : ???

11. Finite temp. analysis (iii) D2/D4 with r=1 (QCD_2)

D4 embedding breaks the rotational symmetry !





Difference from zero-temp.

Gravity side : $c = 0$ for $T \to \infty$ $\left(T = \bar{M}/\sqrt{r_\infty^3}\right)$

$\Longleftrightarrow$ QCD_2 side : Chiral symmetry restore at $T \to \infty$

12. Summary

We discussed the χSB in the Dq/Dp model by AdS/CFT.

- Rotational symmetry in $\mathbf{R}^{9-q-p+r}$ can be interpreted as chiral symmetry in QCD$_{r+1}$. This chiral symmetry is Non-Abelian for QCD$_2$ and Abelian for QCD$_4$.
- We found SχSB in QCD$_{2,4}$ ($\langle\bar{\psi}\psi\rangle \neq 0$ even for $m_q = 0$) and $(8 - q - p + r)$ NG bosons associated with this SχSB (Physical meaning is not clear for QCD$_3$).
- We also discussed theory at finite temperature and found chiral symmetry restoration at $T \to \infty$.
- It is interesting to introduce chemical potential in the Dq/Dp system.