Calculable One-Loop Contributions to S and T Parameters in the Gauge-Higgs Unification

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Introduction

Gauge-Higgs unification is very interesting scenario because Higgs mass is calculable and predictive regardless of the nonrenormalizable theory

There are various explicit calculations of Higgs mass:

- 5D QED on S^1 @1-loop Hatanaka-Inami-Lim
- 5D Non-Abelian gauge theory on S^1/Z_2 @1-loop Gersdorff-Irges-Quiros
- 6D Non-Abelian gauge theory on T^2 @1-loop Antoniadis-Benakli-Quiros
- 6D Scalar QED on S^2 @1-loop Lim-Maru-Hasegawa
- 5D QED on S^1 @2-loop Maru-Yamashita

The reason for finiteness is the following

In the gauge-Higgs unification, Higgs is identified with extra components of the higher dimensional gauge field

$$A_M = \left(A_\mu , A_5 \right)$$

Higgs

The local mass term immediately is forbidden by the higher dimensional local gauge invariance

$$\frac{1}{2} m^2 A_5^2 \quad \Longleftarrow \quad A_5 \to A_5 + \partial_5 \alpha \left(x, y \right)$$

No local counter term

Finite mass is generated @1-loop

$$m_{A_5}^2 \sim \frac{g_4^2}{16\pi^2 R^2}$$

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A_5: Higgs

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Question:

Is there any other finite physical observable?

If there is, natural to guess in the gauge-Higgs sector of the SM

One of the candidates: S, T (& U) parameters

$$S : \left(H^{\dagger} W^{3}_{\mu\nu} H \right) B^{\mu\nu},\ T : \left(H^{\dagger} D_{\mu} H \right) \left(H^{\dagger} D^{\mu} H \right)$$

Naively, we can expect them to be finite

similar to the Higgs mass

These parameters are important physical quantities to test the SM and constrain the physics beyond the SM

(well-known that QCD-like technicolor is excluded)

In this work, we investigate the structure of divergence for 1-loop contributions to S & T parameters in the gauge-Higgs unification

Results:

In 5D case, S & T are both finite
In more than 5D case, both divergent
⇒ Natural from the power counting argument

However, the gauge-Higgs unification predicts

$S - 4\cos\theta_W T$ becomes finite in 6D cases

because S & T are related by the higher dim. gauge inv.

PLAN

- Introduction
- Operator Analysis
- Calculation of T-parameter
- Calculation of S-parameter
- Summary

Operator Analysis

S & T parameters are calculated
as the coefficients of dimension six operators

$$\left(H^{\dagger} W^{3}_{\mu\nu} H\right) B^{\mu\nu} \text{ for } S, \left(H^{\dagger} D_{\mu} H\right)\left(H^{\dagger} D^{\mu} H\right) \text{ for } T$$

in 4D sense

Higher dimensional gauge symmetry can forbid these local operators similar to Higgs mass ???

↓

NO

In gauge-Higgs unification,

S & T parameters are "unified" due to higher dim gauge inv

$$Tr\left[\left(D_L F_{MN}\right)\left(D^L F^{MN}\right)\right] \supset \frac{1}{2}\left(8m^4\right)\left(W_\mu^3\right)^2 + \left(2m^4\right)W_\mu^+ W^{\mu-}$$

$$+2\sqrt{3}m^2 p^2 g_{\mu\nu} W^{3\mu} B^\nu + 2\sqrt{3}m^2\left(p^2 g_{\mu\nu} - p_\mu p_\nu\right)W^{3\mu} B^\nu$$

$$\to S = -\frac{16\pi}{g^2}\cos\theta_W 8m^2, \quad T = -\frac{4\pi}{g^2}8m^2$$

$$\left[S = -\frac{16\pi}{g^2\tan\theta_W}\Pi'_{3Y}, \quad T = -\frac{4\pi}{g^2\sin^2\theta_W}\frac{\Delta M^2}{M_W^2}\left(\sqrt{3}/2 \to \sin\theta_W, m^2 \to M_W^2\right)\right]$$

$S - 4\cos\theta_W T$

becomes finite even in more than 5D

Consider a minimal SU(3) gauge-Higgs model compactified on $M^D \times S^1/Z_2$ with a triplet fermion

Although this model is NOT realistic,

- $\sin^2\theta_W = \frac{3}{4} \left(\exp : \sin^2\theta_W \approx 0.23 \right)$
- $m_t = 0,\ m_b = M_W$
- SU(2) x U(1) → U(1) by <A5> assumed

Enough to investigate the divergence structure for 1-loop contributions to S & T parameters

Lagrangian

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\left(F_{MN}F^{MN}\right) + i\bar{\Psi}\Gamma^M D_M \Psi \qquad \Gamma^M = \left(\gamma^\mu, i\gamma^5\right)$$

$$F_{MN} = \partial_M F_N - \partial_N F_M - ig_{D+1}\left[A_M, A_N\right] \left(M, N = 0,1,2,3,5\right)$$

$$D_M = \partial_M - ig_5 A_M \left(A_M = A_M^a \lambda^a/2 : \lambda^a : \text{Gell-Mann matrices}\right)$$

$$\Psi = \left(\psi_1, \psi_2, \psi_3\right)^T$$

Boundary conditions:

$$A_\mu = \begin{pmatrix} (+,+) & (+,+) & (-,-) \\ (+,+) & (+,+) & (-,-) \\ (-,-) & (-,-) & (+,+) \end{pmatrix}, A_{D+1} = \begin{pmatrix} (-,-) & (-,-) & (+,+) \\ (-,-) & (-,-) & (+,+) \\ (+,+) & (+,+) & (-,-) \end{pmatrix}, \Psi = \begin{pmatrix} \psi_{1L}(+,+) + \psi_{1R}(-,-) \\ \psi_{2L}(+,+) + \psi_{2R}(-,-) \\ \psi_{3L}(-,-) + \psi_{3R}(+,+) \end{pmatrix}$$

SU(3) → SU(2) x U(1)

Lagrangian

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Higgs is identified with 0 mode of A5

(KK modes of A5 are absorbed into KK gauge bosons)

Lagrangian

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\left(F_{MN}F^{MN}\right) + i\bar{\Psi}\Gamma^M D_M \Psi \qquad \Gamma^M = \left(\gamma^\mu, i\gamma^5\right)$$

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Chiral fermions are easily obtained

4D effective Lagrangian in terms of mass eigenbasis

$$\mathcal{L}^{(4D)}_{\text{fermion}} = \sum_{n=1}^{\infty}\left[\left(\bar{\psi}_1^{(n)}, \bar{\tilde{\psi}}_2^{(n)}, \bar{\tilde{\psi}}_3^{(n)}\right)\begin{pmatrix} i\gamma^\mu\partial_\mu - m_n & 0 & 0 \\ 0 & i\gamma^\mu\partial_\mu - (m_n + m) & 0 \\ 0 & 0 & i\gamma^\mu\partial_\mu - (m_n - m)\end{pmatrix}\begin{pmatrix}\psi_1^{(n)} \\ \tilde{\psi}_2^{(n)} \\ \tilde{\psi}_3^{(n)}\end{pmatrix}\right.$$

$$\left. + \frac{g}{2}\left(\bar{\psi}_1^{(n)}, \bar{\tilde{\psi}}_2^{(n)}, \bar{\tilde{\psi}}_3^{(n)}\right)\begin{pmatrix} W_\mu^3 + \frac{B_\mu}{\sqrt{3}} & W_\mu^+ & W_\mu^+ \\ W_\mu^- & -\frac{W_\mu^3}{2} - \frac{B_\mu}{2\sqrt{3}} & -\frac{W_\mu^3}{2} + \frac{B_\mu}{2\sqrt{3}} \\ W_\mu^- & -\frac{W_\mu^3}{2} + \frac{B_\mu}{2\sqrt{3}} & -\frac{W_\mu^3}{2} - \frac{B_\mu}{2\sqrt{3}}\end{pmatrix}\gamma^\mu\begin{pmatrix}\psi_1^{(n)} \\ \tilde{\psi}_2^{(n)} \\ \tilde{\psi}_3^{(n)}\end{pmatrix}\right]$$

$$+ i\bar{t}_L\gamma^\mu\partial_\mu t_L + \bar{b}\left(i\gamma^\mu\partial_\mu - m\right)b + \frac{g}{\sqrt{2}}\left(\bar{t}\gamma^\mu L b W_\mu^+ + \bar{b}\gamma^\mu L t W_\mu^-\right) + \frac{g}{2}\left(\bar{t}\gamma^\mu L t + \bar{b}\gamma^\mu L b\right)W_\mu^3$$

$$+ \frac{\sqrt{3}g}{6}\left(\bar{t}\gamma^\mu L t + \bar{b}\gamma^\mu L b - 2\bar{b}\gamma^\mu R b\right)B_\mu$$

$$\begin{pmatrix}\tilde{\psi}_1^{(n)} \\ \tilde{\psi}_2^{(n)} \\ \tilde{\psi}_3^{(n)}\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\sqrt{2} & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1\end{pmatrix}\begin{pmatrix}\psi_1^{(n)} \\ \psi_2^{(n)} \\ \psi_3^{(n)}\end{pmatrix}$$

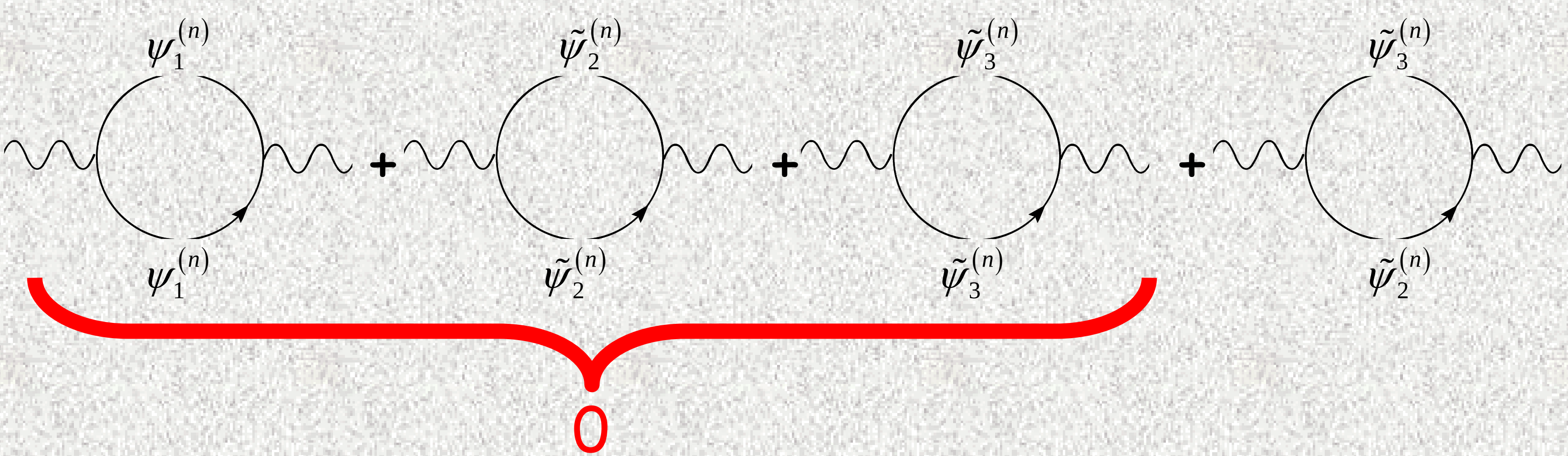
$$L \equiv \frac{1}{2}(1-\gamma_5),\ R \equiv \frac{1}{2}(1+\gamma_5),\ m_n = \frac{n}{R},\ g = \frac{g_5}{\sqrt{2\pi R}},\ m = \frac{1}{2}gv\,(= M_W)$$

Calculation of T-parameter

T-parameter is calculated from the mass squared difference between neutral and charged W-bosons

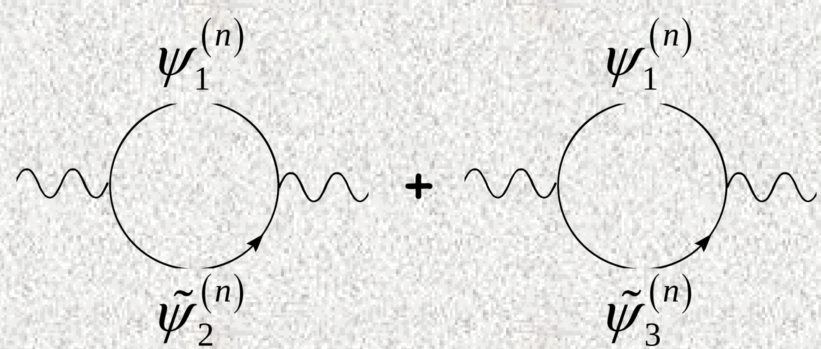
$$\Delta M^2 \equiv \delta M^2_{W^3} - \delta M^2_{W^\pm}$$

$\delta M^2_{W^3} =$



Same as the quantum correction to the photon mass in QED

$\delta M^2_{W^\pm} =$



Mode sum before Momentum integral

$$T = T_{(\mathrm{div})} + T_{(\mathrm{sc})}$$

$$L \equiv 2\pi R, \rho \equiv Lk, \alpha \equiv Lm$$

$$T_{(\mathrm{div})} = -\pi \frac{2^{\frac{3}{2}D-3}}{(4\pi)^{D/2}} \frac{\left(1-2^{3-D}\right)(D-1)}{D(3-D)} \frac{\Gamma\left(\frac{5-D}{2}\right)\Gamma\left(\frac{D-1}{2}\right)^2}{\Gamma\left(\frac{3}{2}\right)\Gamma(D-1)} L^{4-D}\alpha^{D-3}$$

$$T_{(\mathrm{sc})} = \frac{2^{D/2}\pi}{\alpha^2 D} L^{4-D} \int_0^1 dt \int \frac{d^D\rho}{(2\pi)^D} \left[-\frac{D}{\rho}\left(\frac{\sinh\rho}{\cosh\rho-1}-1\right) - \frac{D}{2\rho}\left(\frac{\sinh\rho}{\cosh\rho-\cos\alpha}-1\right) \right.$$

$$-\frac{D}{2}\frac{\left(1+(D-2)\frac{\alpha^2}{\rho^2}\right)}{\sqrt{\rho^2+4t(1-t)\alpha^2}}\left(\frac{\sinh\sqrt{\rho^2+4t(1-t)\alpha^2}}{\cosh\sqrt{\rho^2+4t(1-t)\alpha^2}-\cos\left[(2t-1)\alpha\right]}-1\right)$$

$$\left. +\frac{D}{2}\frac{\left(4+(D-2)\frac{\alpha^2}{\rho^2}\right)}{\sqrt{\rho^2+4t(1-t)\alpha^2}}\left(\frac{\sinh\sqrt{\rho^2+4t(1-t)\alpha^2}}{\cosh\sqrt{\rho^2+4t(1-t)\alpha^2}-\cos(t\alpha)}-1\right) \right]$$

Finite value evaluation in 5D:

Momentum integral before the mode sum

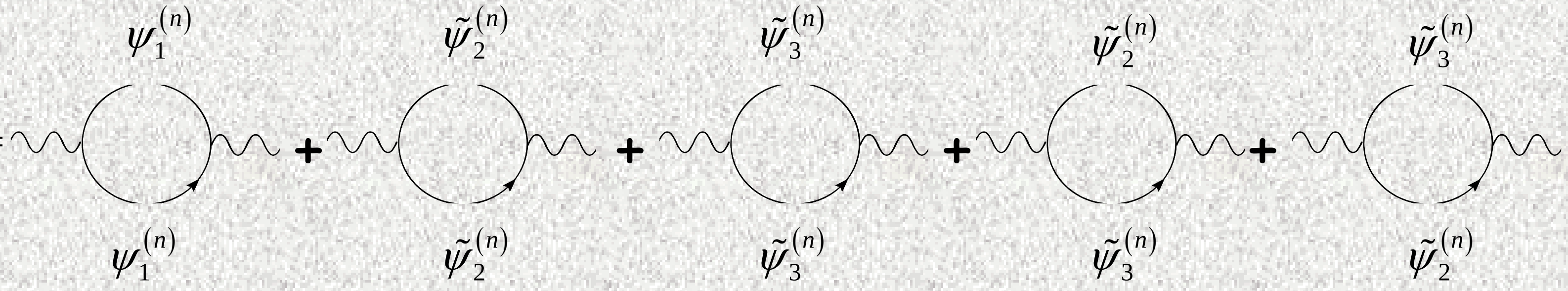
$$T_{(n\neq 0)}(5D) \approx \frac{2}{5\pi m^2}\sum_{n=1}^{\infty}\frac{m^4}{m_n^2} = \frac{\pi}{15}(mR)^2 \qquad \left(m_n^2 \gg m^2\right)$$

- Pole term vanishes because of $\int_0^1 dt\,(1-2t) = 0$
- $\mathcal{O}(m^4) \Leftrightarrow \left(H^\dagger D_\mu H\right)\left(H^\dagger D^\mu H\right)$
- $\mathcal{O}\left(1/m_n^2\right)$ Decoupling nature of KK modes
- $T \to 0$ for $m \to 0$ (Custodial limit)

Calculation of S-parameter

S-parameter is calculated from kinetic mixing of W_μ^3 & B_μ

$$i\Pi'_{3Y}\, p^2 g_{\mu\nu} + \cdots =$$



$$S = S_{(\mathrm{div})} + S_{(\mathrm{sc})}$$

$$S_{(\mathrm{div})} = \frac{9\pi 2^{3D/2-5}}{(4\pi)^{D/2}\,\Gamma(5/2)} \frac{D-1}{D-3} \frac{\Gamma\left(\frac{5-D}{2}\right)\Gamma\left(\frac{D+1}{2}\right)^2}{\Gamma(D+1)} (2\pi R)\, m^{D-3}$$

$$S_{(\mathrm{sc})} = \frac{\pi}{3} 2^{D/2} (D-2) L^{4-D} \int \frac{d^D\rho}{(2\pi)^D} \left[\frac{1}{2\rho^3}\left(\frac{\sinh\rho}{\cosh\rho - \cos\varepsilon} - 1 \right) + \frac{1}{4\rho^3}\left(\frac{\sinh\rho}{\cosh\rho - \cos\alpha} - 1 \right) \right.$$

$$+ \frac{9}{8}\int_0^1 dt \left\{ \left(2t(1-t)-1\right)\frac{1}{\rho^2} + 2t(1-t)\alpha^2 \frac{D-4}{\rho^4} \right\} \times$$

$$\left. \frac{1}{\sqrt{\rho^2 + 4t(1-t)\alpha^2}} \left(\frac{\sinh\sqrt{\rho^2 + 4t(1-t)\alpha^2}}{\cosh\sqrt{\rho^2 + 4t(1-t)\alpha^2} - \cos\left[(2t-1)\alpha\right]} - 1 \right) \right]$$

Finite value of S in 5D is calculated by expanding in terms of m/m_n

$$S(5D) \approx \frac{\pi}{3(2\pi)^2} \sum_{n=1}^{\infty} \frac{46}{5} \left(\frac{m}{m_n} \right)^2 = \frac{23\pi}{180} (mR)^2$$

- $\left[2 + 1 - 18 \int_0^1 dt\, t(1-t) \right] \times (\text{log divergence}) = 0$
- $\mathcal{O}(m^2) \Leftrightarrow \left(H^\dagger W^a_{\mu\nu} \frac{\tau^a}{2} H \right) B^{\mu\nu}$
- $\mathcal{O}(1/m_n^2)$

Decoupling nature of KK mode

In more than 5D

$$T_{(\text{div})} = -\pi \frac{2^{3D/2-3}}{(4\pi)^{D/2}} \frac{\left(1-2^{3-D}\right)(D-1)}{D(3-D)} \frac{\Gamma\left(\frac{5-D}{2}\right)\Gamma\left(\frac{D-1}{2}\right)^2}{\Gamma\left(\frac{3}{2}\right)\Gamma(D-1)} (2\pi R) m^{D-3}$$

$$S_{(\text{div})} = -\frac{9\pi 2^{3D/2-5}}{(4\pi)^{D/2}} \frac{D-1}{3-D} \frac{\Gamma\left(\frac{5-D}{2}\right)\Gamma\left(\frac{D+1}{2}\right)^2}{\Gamma\left(\frac{5}{2}\right)\Gamma(D+1)} (2\pi R) m^{D-3}$$

$$\Rightarrow \quad 6D : S_{(\text{div})} = \frac{3(5-1)}{8\left(1-2^{3-5}\right)} T_{(\text{div})} = 2T_{(\text{div})}$$

Divergence is indeed canceled in S - 2T
[S - 4 $\cos\theta_W$ T → S - 2T (SU(3) model)]

Finite value is calculated by doing the momentum integral before the mode sum and expanding in terms of m/mn

$$S_{(n\neq 0)}(6D) = -\frac{2\sqrt{2}}{5\pi}\sum_{n=1}^{\infty}\left[-\frac{m^2}{m_n} + \frac{3}{14}\frac{m^4}{m_n^3}\right]$$

$$T_{(n\neq 0)}(6D) = -\frac{\sqrt{2}}{5\pi}\sum_{n=1}^{\infty}\left[-\frac{m^2}{m_n} + \frac{1}{12}\frac{m^4}{m_n^3}\right]$$

$$\Rightarrow \quad S_{(n\neq 0)}(6D) - 2T_{(n\neq 0)}(6D) = \frac{11\sqrt{2}}{210\pi} m^4 R^3 \zeta(3)$$

1st term in S and T indicates the log divergence and is canceled in S - 2T

Comments

1: In our model, only one extra spatial dimension is compactified as $M^D \times S^1/Z_2$

Our arguments of finiteness for 6D case is meaningless?? because it is not realistic

However, our argument with respect to UV divergence is not affected by the shape of the compactified space because it is the IR property not UV one

Finiteness of $S - 4\cos\theta_W T$ holds true even in 6D theory compactified on T^2/Z_2, for example although the finite value itself might be changed

2: For higher than 6 dimensions,
the coefficients of the gauge invariant operators with mass dimensions > 6 diverge

$$S : \left(H^\dagger H\right)^n H^\dagger W^3_{\mu\nu} H B^{\mu\nu}$$

$$T : \left(H^\dagger H\right)^n \left(H^\dagger D_\mu H\right)\left(H^\dagger D^\mu H\right) \left(n \geq 1\right)$$

Divergences from these operators
⇒ No prediction in more than 6D

Summary

- We have investigated the divergence structure of one-loop contributions to S & T parameters in the gauge-Higgs unification scenario
- S & T are finite in 5D, but divergent in more than 5D which are natural results from the power counting
- In 6D case, $S - 4\cos\theta_W T$ becomes finite because S & T are related ("unified") ⇒ prediction!!
- Interesting to study these parameters in more realistic gauge-Higgs unification models and obtain the predictions
- g-2 in the gauge-Higgs unification becomes finite in any spacetime dimensions ⇒ Adachi's Talk

Substituting KK mode expansions

$$A_{\mu,5}^{(+,+)}(x,y)=\frac{1}{\sqrt{2\pi R}}\left[A_{\mu,5}^{(0)}(x)+\sqrt{2}\sum_{n=1}^{\infty}A_{\mu,5}^{(n)}\cos\left(\frac{n}{R}y\right)\right],$$

$$A_{\mu,5}^{(-,-)}(x,y)=\frac{1}{\sqrt{\pi R}}\sum_{n=1}^{\infty}A_{\mu,5}^{(n)}\sin\left(\frac{n}{R}y\right),$$

$$\Psi_{1L,2L,3R}^{(+,+)}(x,y)=\frac{1}{\sqrt{2\pi R}}\left[\psi_{1L,2L,3R}^{(0)}(x)+\sqrt{2}\sum_{n=1}^{\infty}\psi_{1L,2L,3R}^{(n)}\cos\left(\frac{n}{R}y\right)\right],$$

$$\Psi_{1R,2R,3L}^{(-,-)}(x,y)=\frac{1}{\sqrt{\pi R}}\sum_{n=1}^{\infty}\psi_{1R,2R,3L}^{(n)}\sin\left(\frac{n}{R}y\right)$$

and integrating out 5th coordinate "y",
making a chiral rotation $\psi_{1,2,3}\to e^{-i\pi\gamma_5/4}\psi_{1,2,3}$ to remove i γ_5,

we obtain 4D effective Lagrangian

$$\mathcal{L}^{(4D)}_{\text{fermion}} = \sum_{n=1}^{\infty} \Bigg[i\left(\bar\psi_1^{(n)}, \bar\psi_2^{(n)}, \bar\psi_3^{(n)}\right) \gamma^\mu \partial_\mu \begin{pmatrix} \psi_1^{(n)} \\ \psi_2^{(n)} \\ \psi_3^{(n)} \end{pmatrix}$$

$$+\frac{g}{2}\left(\bar\psi_1^{(n)}, \bar\psi_2^{(n)}, \bar\psi_3^{(n)}\right) \begin{pmatrix} W_\mu^3 + \frac{B_\mu}{\sqrt{3}} & \sqrt{2}W_\mu^+ & 0 \\ \sqrt{2}W_\mu^- & -W_\mu^3 + \frac{B_\mu}{\sqrt{3}} & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}}B_\mu \end{pmatrix} \gamma^\mu \begin{pmatrix} \psi_1^{(n)} \\ \psi_2^{(n)} \\ \psi_3^{(n)} \end{pmatrix}$$

$$-\left(\bar\psi_1^{(n)}, \bar\psi_2^{(n)}, \bar\psi_3^{(n)}\right) \begin{pmatrix} m_n & 0 & 0 \\ 0 & m_n & -m \\ 0 & -m & m_n \end{pmatrix} \begin{pmatrix} \psi_1^{(n)} \\ \psi_2^{(n)} \\ \psi_3^{(n)} \end{pmatrix} \Bigg]$$

$$+ i\bar t_L \gamma^\mu \partial_\mu t_L + \bar b\left(i\gamma^\mu\partial_\mu - m\right)b + \frac{g}{\sqrt{2}}\left(\bar t\gamma^\mu L b W_\mu^+ + \bar b \gamma^\mu L t W_\mu^-\right) + \frac{g}{2}\left(\bar t \gamma^\mu L t + \bar b \gamma^\mu L b\right) W_\mu^3$$

$$+\frac{\sqrt{3}g}{6}\left(\bar t\gamma^\mu L t + \bar b \gamma^\mu L b - 2\bar b \gamma^\mu R b\right) B_\mu$$

$$L \equiv \frac{1}{2}(1-\gamma_5),\ R \equiv \frac{1}{2}(1+\gamma_5),\ m_n = \frac{n}{R},\ g = \frac{g_5}{\sqrt{2\pi R}},\ m = \frac{1}{2}gv\left(= M_W\right)$$

Mixing occurs between SU(2) doublet component & singlet component

Each of mass eigenvalues has a **periodicity** with respect to m

$$m_n \pm \left(m + \frac{1}{R} \right) = m_{n\pm 1} \pm m$$

Characteristic feature of gauge-Higgs unification

(c.f. $\sqrt{m_n^2 + m^2}$ for UED)

4: Perturbativity

Comparing n-loop graph to (n+1)-loop graph in (D+1) dimensions, we find the ratio (Λ :cutoff scale)

$$\frac{\text{(n+1)-loop}}{\text{n-loop}} = \frac{g_{D+1}^2 \Lambda^{D-3}}{(4\pi)^{\frac{D+1}{2}} \Gamma\left(\frac{D+1}{2}\right)} = \frac{g_D^2 (2\pi R) \Lambda^{D-3}}{(4\pi)^{\frac{D+1}{2}} \Gamma\left(\frac{D+1}{2}\right)}$$

For the perturbation to make sense,

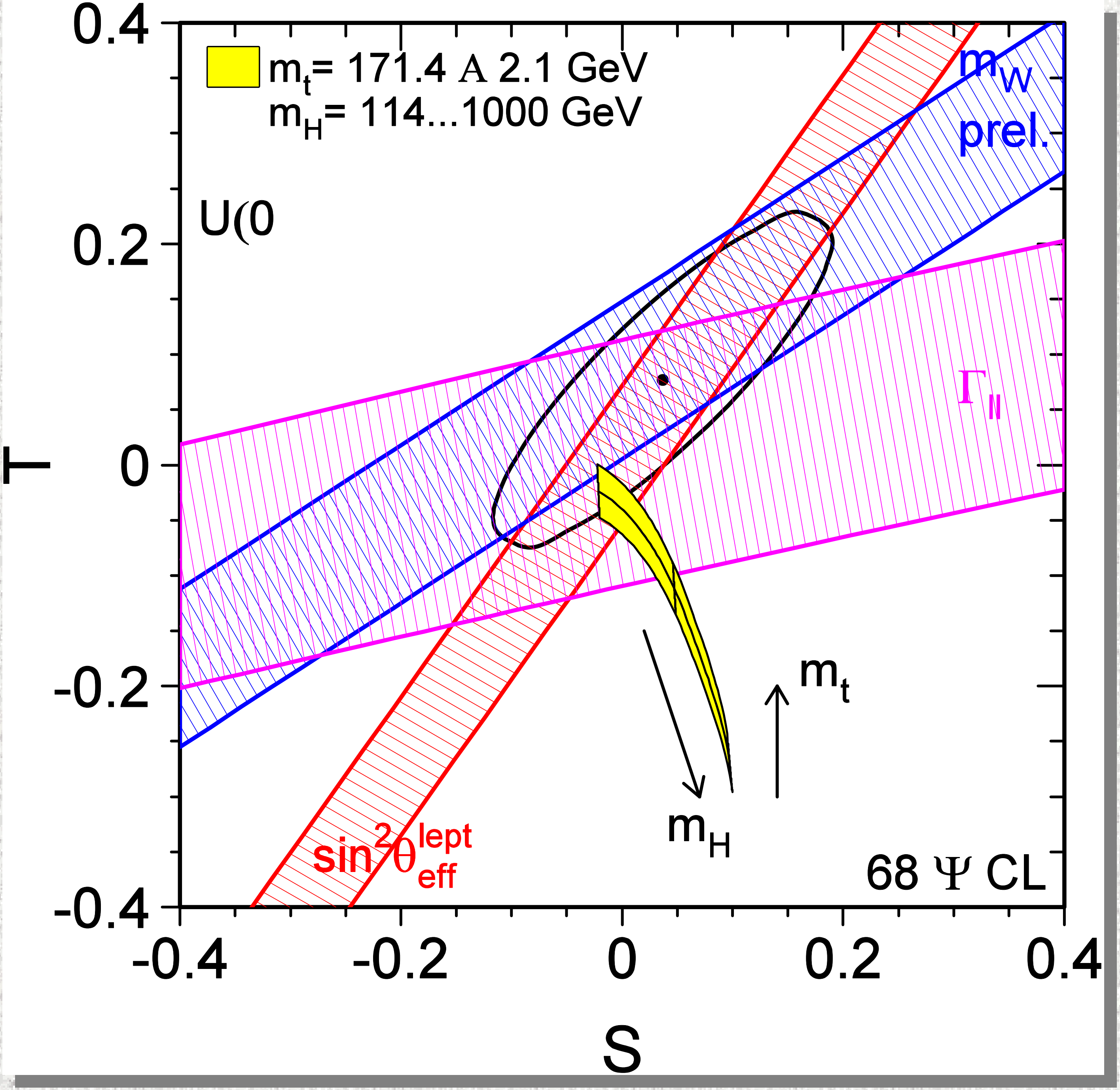
the above ratio must be less than one

For example, D=4 case,

$$\frac{g_4^2 (2\pi R) \Lambda}{(4\pi)^{5/2} \Gamma(5/2)} < 1 \Rightarrow R\Lambda < 12\pi^2 \Rightarrow \Lambda < \mathcal{O}(10^2) \times \frac{1}{R}$$

Cutoff scale Λ cannot be largely separated

from the compactification scale



mt= 171.4 A 2.1 GeV
mH= 114...1000 GeV
U(0
mW
prel.
ΓII
mt
mH
sin2θefflept
68 Ψ CL
0.4
0.2
0
-0.2
-0.4
-0.4
-0.2
0
0.2
0.4
T
S

Combining the neutral and the charged W boson contributions, we obtain KK mode contributions to T-parameter as

$$T_{(n\neq 0)} = -\frac{2^{D/2}}{(4\pi)^{D/2-1} M_W^2} \sum_{n=1}^{\infty} \int_0^1 dt\, \Gamma\left(2-\frac{D}{2}\right) \left[\frac{m^2 + (1-2t) m_n m}{\left[m_n^2 + m^2 + 2(1-2t) m_n m\right]^{2-D/2}} - \left\{ \frac{-m_n m + t\left(2 m_n m + m^2\right)}{\left[m_n^2 + t\left(2 m_n m + m^2\right)\right]^{2-D/2}} + (m \to -m) \right\} \right]$$

This quantity vanishes in the limit $m \to 0$, which corresponds to the custodial symmetry limit in our model

2: For higher than 6D, the coefficients of the gauge inv. operators with mass dimensions > 6 diverge

$$S : \left(H^\dagger H\right)^n H^\dagger W^3_{\mu\nu} H B^{\mu\nu}$$

$$T : \left(H^\dagger H\right)^n \left(H^\dagger D_\mu H\right)\left(H^\dagger D^\mu H\right) \ (n \geq 1)$$

Divergences from these operators

⇒ No prediction in more than 6D

3: Brane localized terms spoil our results?? ⇒ No problem!

$$S : \left(H^\dagger H\right)^n H^\dagger W^3_{\mu\nu} H B^{\mu\nu} \to A_5^{2n} A_5 W^3_{\mu\nu} A_5 B^{\mu\nu}$$

$$T : \left(H^\dagger H\right)^n \left(H^\dagger D_\mu H\right)\left(H^\dagger D^\mu H\right) \to A_5^{2n} \left(A_5 D_\mu A_5\right)\left(A_5 D^\mu A_5\right)$$

@branes forbidden by the shift symmetry

$$A_5 \to A_5 + \partial_5 \alpha(x,y)$$

which is still operative @branes