Ultrahigh-energy string collision and rotating string production

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- collaboration with

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string scattering - two view points.

- Gravity — BH — **[Giddings-Gross -Maharana]**
- QCD (hadron) — gluon scatt, Pomeron etc. — **[Alday-Maldacena]** **[Brower-Polchinski -Strassler-Tan]**

We calculate a production cross-section of a rotating string.

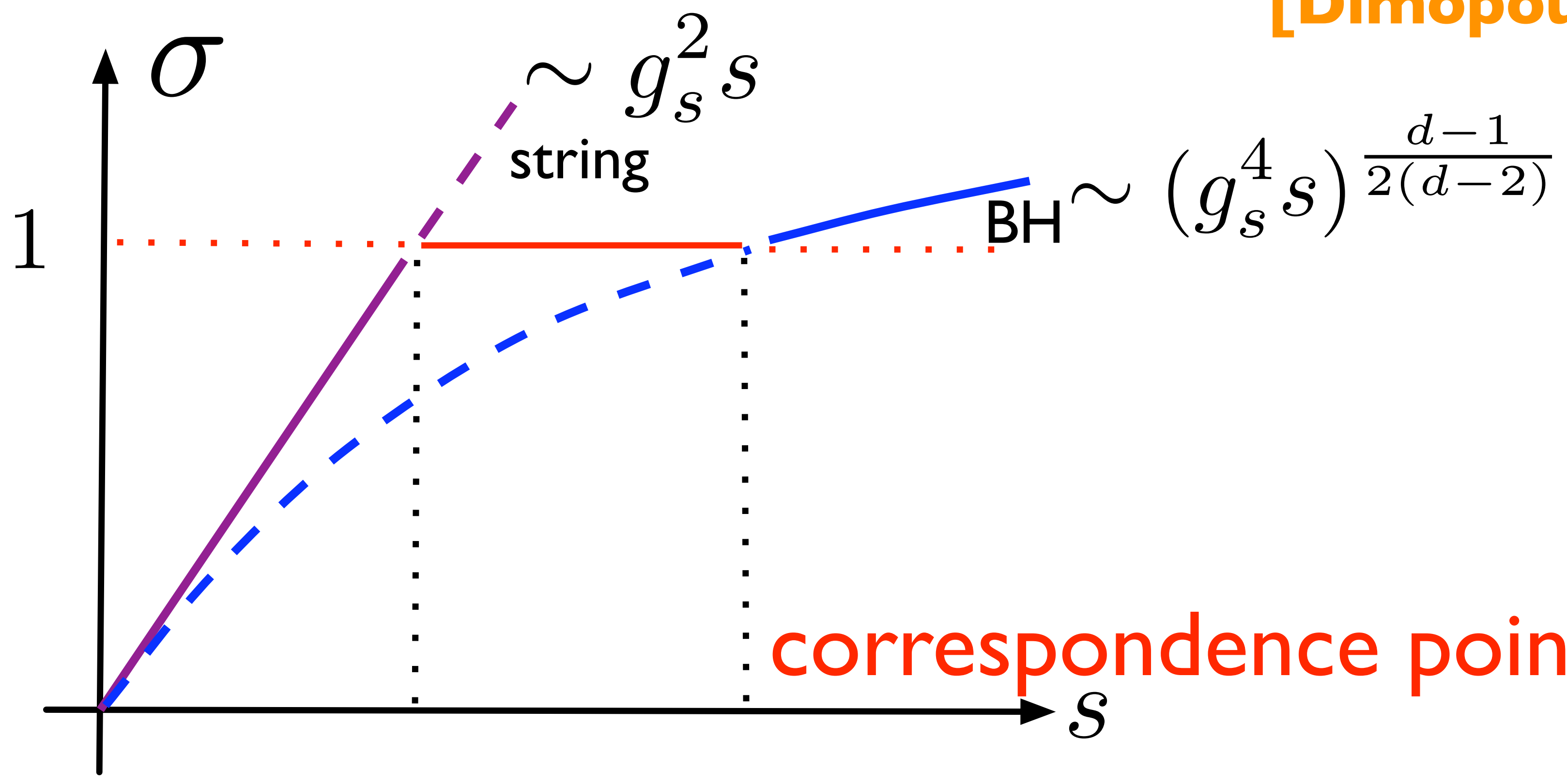
is comparable to

Kerr BH

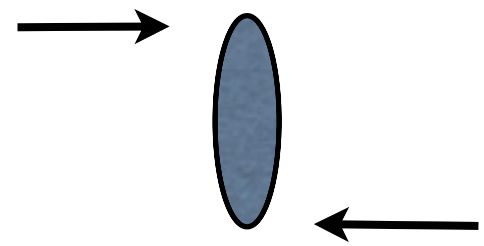
hadron form factor

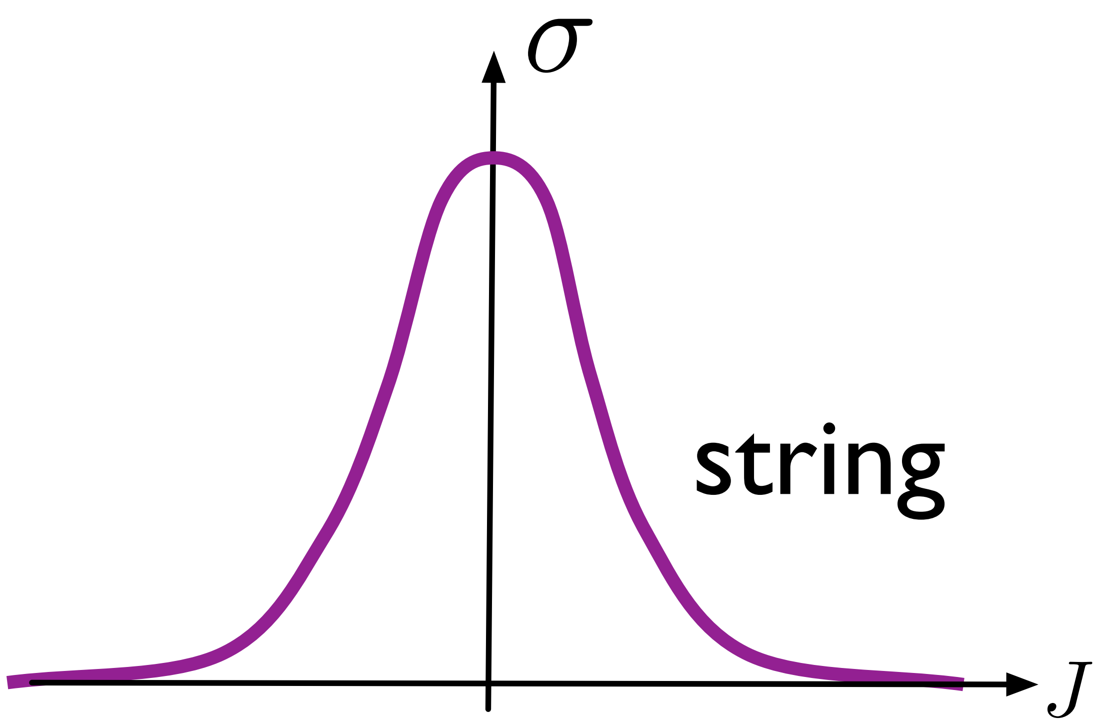
However at tree level

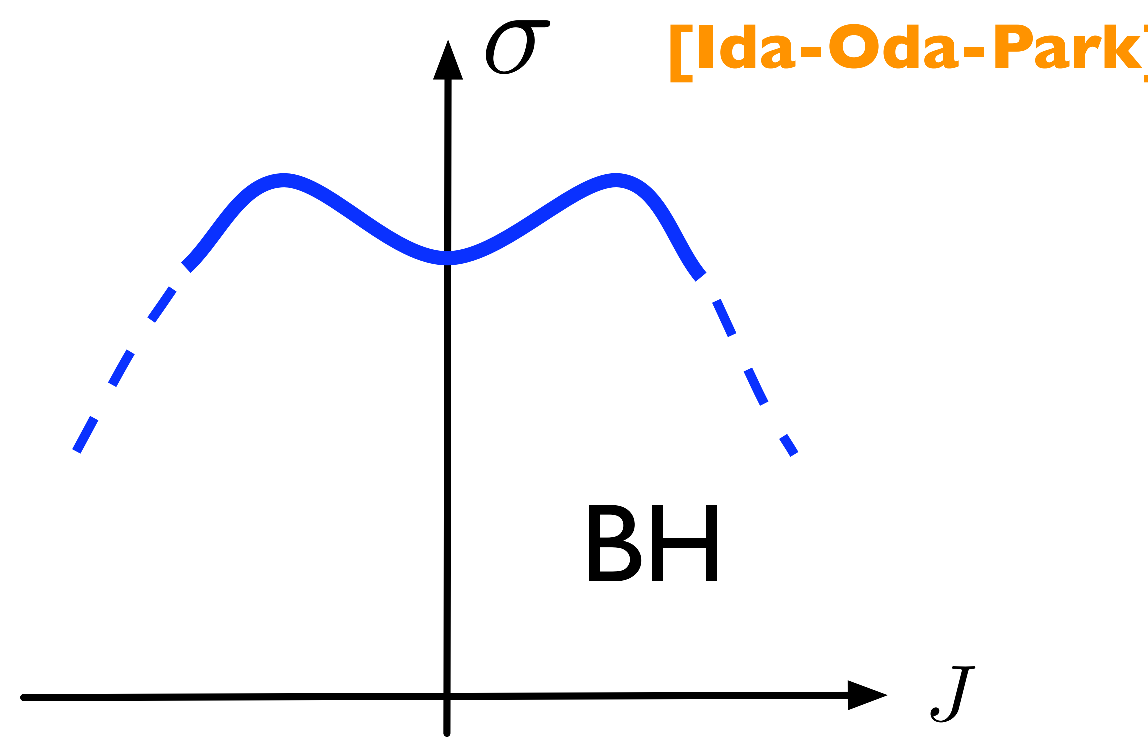
[Dimopoulos-Emparan]



rotating case







String amp. to Pomeron

Regge amp. $A(s,t) \sim s^{\alpha_0+\alpha' t}$ **at** $s \gg |t| \sim 0$

$|t| \sim p_\perp^2$ $$\int \frac{d^{D-2}p_\perp}{(2\pi)^{D-2}} e^{ip_\perp\cdot x_\perp} A(s,t) \sim \frac{s^{\alpha_0}}{(4\pi\alpha' \ln s)^{\frac{D-2}{2}}} e^{-\frac{x_\perp^2}{4\alpha' \ln s}}$$

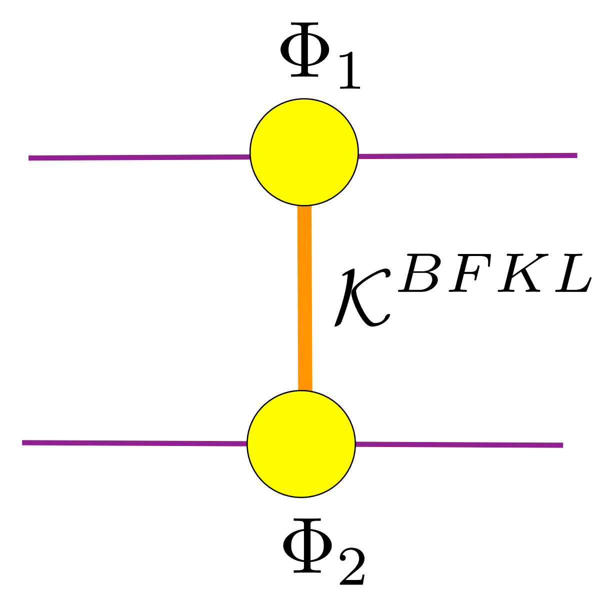
in view of AdS/CFT, this corresponds to the BFKL kernel.

$$\mathcal{K}(r,r',s) = \frac{s^{j_0}}{\sqrt{4\pi\mathcal{D}\ln s}} e^{-\frac{\ln^2(r/r')}{4\mathcal{D}\ln s}}$$

[Brower-Polchinski-Strassler-Tan]

$$\mathcal{K}^{BFKL}(p_\perp,p'_\perp,s) = \frac{s^{j_0}}{\sqrt{4\pi\mathcal{D}\ln s}} e^{-\frac{\ln^2(p_\perp/p'_\perp)}{4\mathcal{D}\ln s}}$$

$$A(s,t) = s\int \frac{d^2k_1}{(k_1-q)^2}\int \frac{d^2k_2}{k_2^2}\Phi_1(k,q)\mathcal{K}^{BFKL}(k_1,k_2,s,t)\Phi_2(k_2,q)$$



our amplitude might have some implication to Pomeron physics.

考える過程

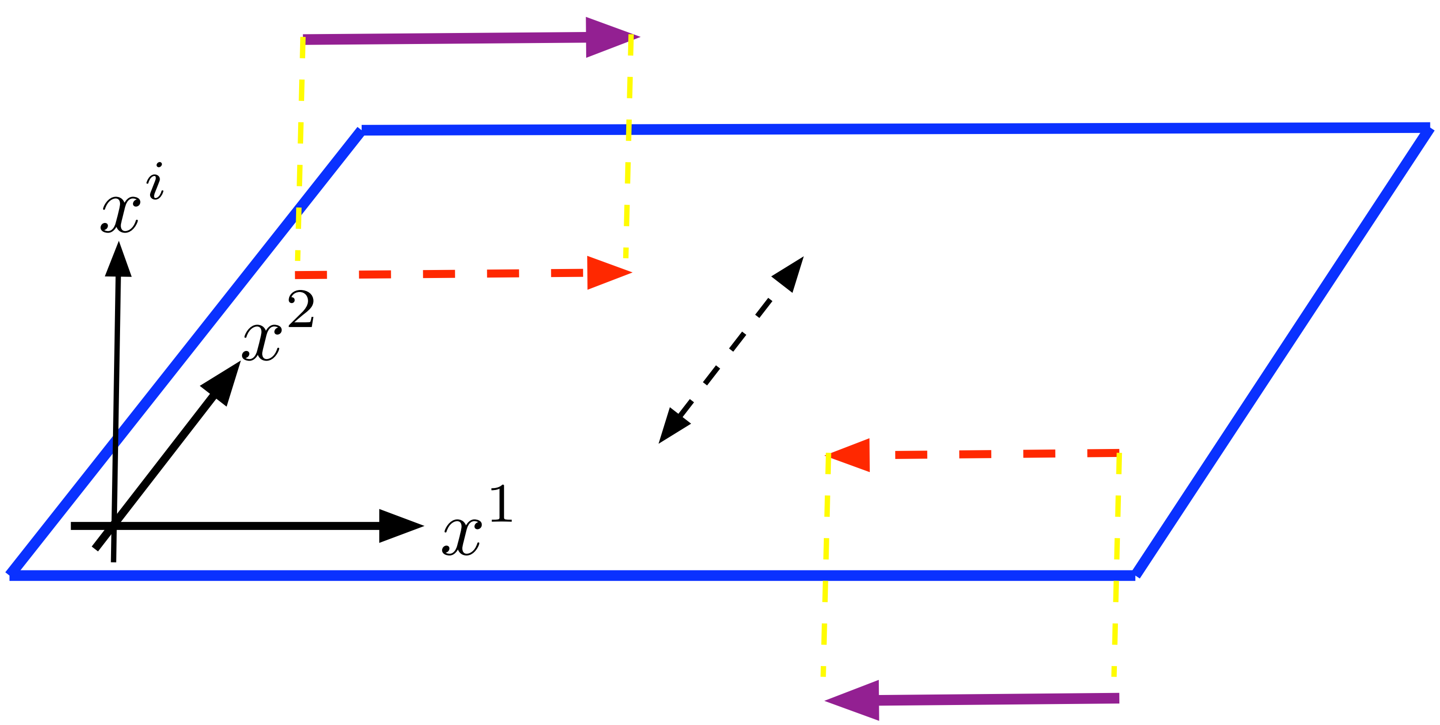
１方向にだけ運動量を持つ、二つの状態（スピン０）の散乱

重心エネルギー　$P^2 = (p+k)^2 = s$

ゼロ以外の固有値を持ちうる角運動量演算子　（初期運動量の方向を１方向にしたので
これ以外の角運動量成分はゼロ）

$$J_{1i}$$

これら i=2~d までの演算子はすべて可換でないので、J_{12} を対角にする基底をとる



calculation

（1 方向にだけ運動量を持つ）二つの tachyons の散乱を考える $V(k) =: e^{ik_\mu X^\mu}:$

終状態が J_{12} 以外にどのような角運動量を持っているのかは観測しない

振幅ではなく、確率そのものを考える。

$$Prob(V(p), V(k) \to \Phi_{N,J}) = \sum_{\Phi|(N,J)} |\langle \Phi | V(k) | p \rangle|^2$$

状態 N への射影演算子 $\hat{P}_N$ 　角運動量 J への射影演算子 $\hat{Q}_J$ をつかって

$$\begin{aligned} &Prob(V(p), V(k) \to \Phi_{N,J}) \\ &= \sum_{i,j=all} \langle i | V(-k) \hat{Q}_J \hat{P}_N | j \rangle \langle j | V(k) \hat{P}_0 | i \rangle \\ &= \mathrm{tr}[V(-k,1) \hat{Q}_J \hat{P}_N V(k,1) \hat{P}_0] \end{aligned}$$

具体的に射影演算子は

$$\hat{P}_N = \oint \frac{dz}{2\pi i z} z^{\hat{N}-N} \qquad \hat{N} = \sum_{n\mu} \alpha_{-n\mu}\alpha_n^\mu \qquad [\alpha_n^\mu, \alpha_m^\nu] = n\delta_{n,m}\delta^{\mu\nu}$$

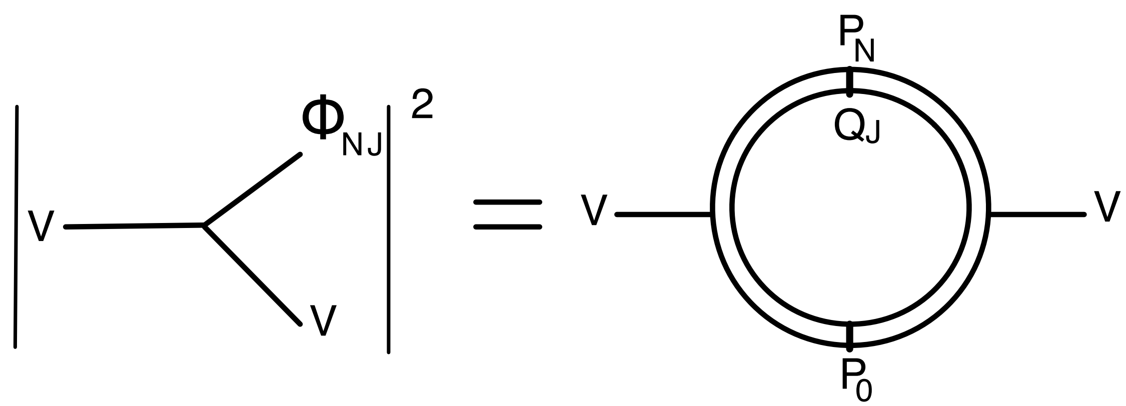
$$\hat{Q}_J = \oint \frac{dz}{2\pi i z} z^{\hat{J}-J} \qquad \hat{J} = -i\sum_n \frac{1}{n}(\alpha_{-n}^1\alpha_n^2 - \alpha_{-n}^2\alpha_n^1)$$

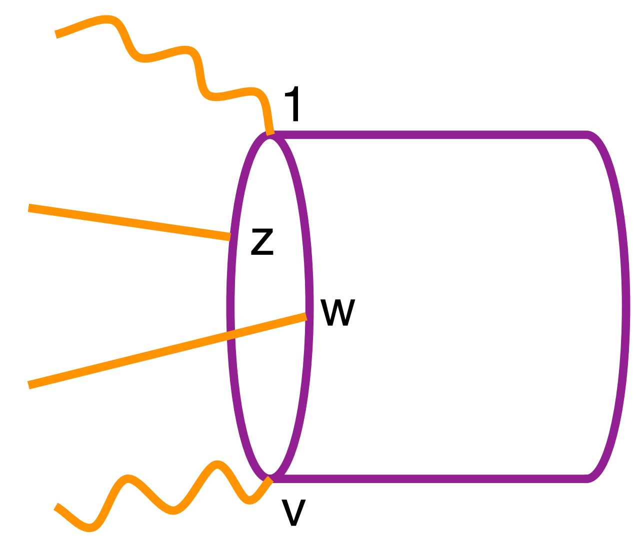
$$z^{\hat{N}} V(k,1) z^{-\hat{N}} = V(k,z)$$ 等によって確率は

$$= \oint \frac{dv}{2\pi i v} v^{-s} \oint \frac{dz}{2\pi i z} z^{-J} \oint \frac{dw}{2\pi i w} \mathrm{tr}[V(-k,1) z^{\hat{J}} V(k,v) w^{\hat{N}}]$$

終状態のon-shell条件

$$N = P^2 = (p+k)^2 = s$$





以下の計算

トレース部分の計算の後、contour 積分を順番に行う。

トレース部分 $\mathrm{tr}[V(-k,1)z^{\hat{J}}V(k,v)w^{\hat{N}}]$

$$\hat{N}=\sum_n \alpha_{-n}\cdot\alpha_n$$

$$\hat{J}=-i\sum_n \frac{1}{n}(\alpha^1_{-n}\alpha^2_n-\alpha^2_{-n}\alpha^1_n)$$

$$[\alpha^\mu_n,\alpha^\nu_m]=n\delta_{n,m}\delta^{\mu\nu}$$

$$\mathrm{tr}\,(A)=\int\frac{d^2u}{\pi}e^{-|u|^2}\langle u|A|u\rangle \qquad |u\rangle=\exp\left(\sum_{n=1}^{\infty}\alpha^\mu_{-n}\frac{u^\mu_n}{\sqrt{n}}\right)|0\rangle$$

$$V(k,v)=:e^{ik_\mu X^\mu}:=\exp\left(\sum_{n=0}^{\infty}k\cdot\alpha_{-n}\frac{v^n}{n}\right)\exp\left(-\sum_{n=0}^{\infty}k\cdot\alpha_n\frac{v^{-n}}{n}\right)$$

$$z^{\hat{J}}=\exp\left(-i\ln z\sum_n\frac{1}{n}(\alpha^1_{-n}\alpha^2_n-\alpha^2_{-n}\alpha^1_n)\right) \qquad w^{\hat{N}}=\exp\left(\ln w\sum_n\alpha_{-n}\cdot\alpha_n\right)$$

トレース部分の答え

$$\mathrm{tr}[V(-k,1)z^{\hat{J}}V(k,v)w^{\hat{N}}] \qquad c = \frac{1}{2}(z+z^{-1})$$

$$= \prod_{n=1} \frac{1}{1-2cw^n+w^{2n}} \exp\left[(k_1^2+k_2^2)\frac{(c-w^n)v^n+(1-cw^n)(w/v)^n-2w^n(c-w^n)}{n(1-2cw^n+w^{2n})}\right]$$

$$\times \prod_{\mu=0,3,\ldots,d} \frac{1}{1-w^n} \exp\left[k_\mu k^\mu \frac{v^n+(w/v)^n-2w^n}{n(1-w^n)}\right] \cdot (1-w^n)^2$$

$$k_\mu = (k_0, k_1, 0, \cdots, 0) \qquad k_0^2 = k_1^2 - 2 = s/4$$ on-shell condition

$$= \prod_{n=1} \frac{(1-w^n)^{3-d}}{1-2cw^n+w^{2n}} \exp\left[(\frac{s}{4}+2)\frac{(c-w^n)v^n+(1-cw^n)(w/v)^n-2w^n(c-w^n)}{n(1-2cw^n+w^{2n})} - \frac{s}{4}\frac{v^n+(w/v)^n-2w^n}{n(1-w^n)}\right]$$

Once again, the trace part is

$$\mathrm{tr}[V(-k,1)z^{\hat{J}}V(k,v)w^{\hat{N}}]$$

$$= \prod_{n=1} \frac{(1-w^n)^{3-d}}{1-2cw^n+w^{2n}} \exp\left[(\frac{s}{4}+2)\frac{(c-w^n)v^n+(1-cw^n)(w/v)^n-2w^n(c-w^n)}{n(1-2cw^n+w^{2n})} \right.$$

$$\left. -\frac{s}{4}\frac{v^n+(w/v)^n-2w^n}{n(1-w^n)} \right]$$

w integration picks up constant terms w.r.t w

$$\oint \frac{dw}{2\pi i w}\mathrm{tr}[V(-k,1)z^{\hat{J}}V(k,v)w^{\hat{N}}]$$

$$= \exp\left[(s/4-(s/4+2)c)\ln(1-v)\right]$$

z integration gives the modified Bessel fn.

$$c = \frac{1}{2}(z+z^{-1})$$

$$\oint \frac{dz}{2\pi i z} z^{-J} \exp\left[(s/4-(s/4+2)c)\ln(1-v)\right]$$

$$= (1-v)^{s/4} I_J(-(s/4+2)\ln(1-v))$$

残りは v 積分.

$$P(s,J) = \oint \frac{dv}{2\pi i v} v^{-s}(1-v)^{s/4} I_J(-(s/4+2)\ln(1-v))$$

So far exact for $\forall(s,J)$.

We may evaluate the integral numerically for given (s,J).

厳密さは多少犠牲にして、large s での近似式を求める。

変形 Bessel 関数の積分表示を使えば v 積分は実行できて

$$P(s,J) = \frac{s'^{J-1}}{\sqrt{\pi}\Gamma(J+1/2)2^J} \int_0^{2s'} dx Q_J\left(1-\frac{x}{s'}\right)\left(-\frac{\partial}{\partial x}\right)^J \frac{\Gamma(s+2-x)}{\Gamma(s+1)\Gamma(2-x)}$$

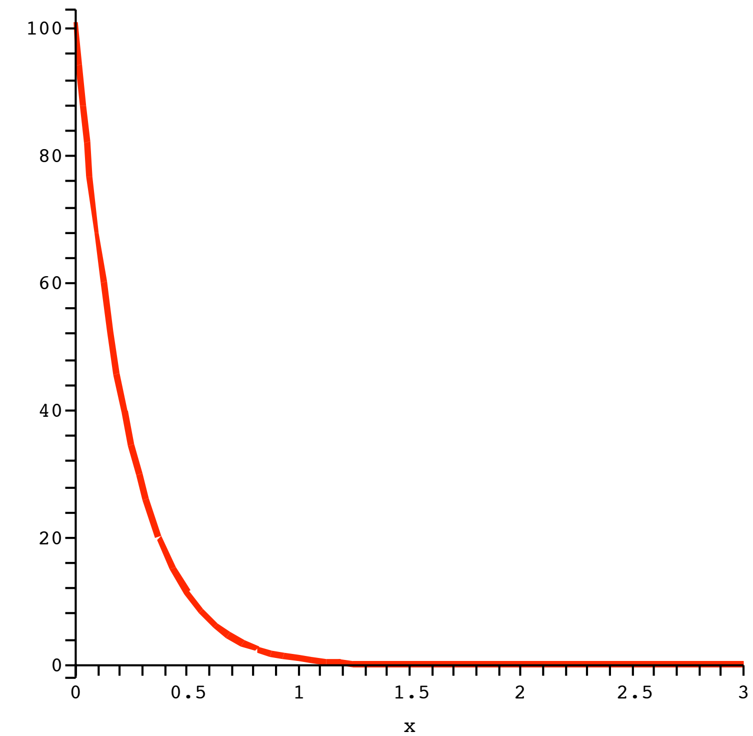
$$Q_J(x) = (1-x^2)^{J-1/2} \qquad s' = s/4+2$$

$$P(s,J)=\frac{s'^{J-1}}{\sqrt{\pi}\Gamma(J+1/2)2^J}\int_0^{2s'}dxQ_J\left(1-\frac{x}{s'}\right)\left(-\frac{\partial}{\partial x}\right)^J\frac{\Gamma(s+2-x)}{\Gamma(s+1)\Gamma(2-x)}$$

Stirling formula によって

cf. Regge limit

$$\frac{\Gamma(s+2-x)}{\Gamma(s+1)\Gamma(2-x)}\sim\frac{s^{1-x}}{\Gamma(2-x)}\sim s^{1-x}$$



$$P(s,J)\sim\frac{s^{1-s'}(s'\ln s/2)^J}{\sqrt{\pi}\Gamma(J+1/2)}\int_{-1}^{1}dzQ_J(z)\,s^{s'z}$$

$$=s^{1-s'}I_J(s'\ln s)$$

cross-section is $\sigma(s,J)=Prob(s,J)/s$

Finally we obtain

$$\sigma(s,J) \sim e^{-z} I_J(z) \qquad z = \frac{s}{4}\ln s$$

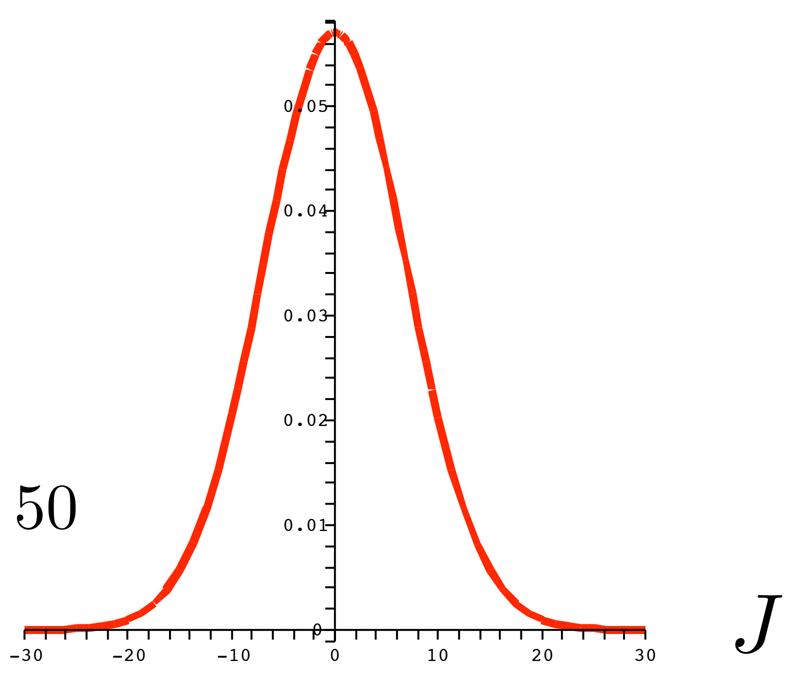
In order to see the behavior in J, we perform a saddle point evaluation for the modified Bessel function :

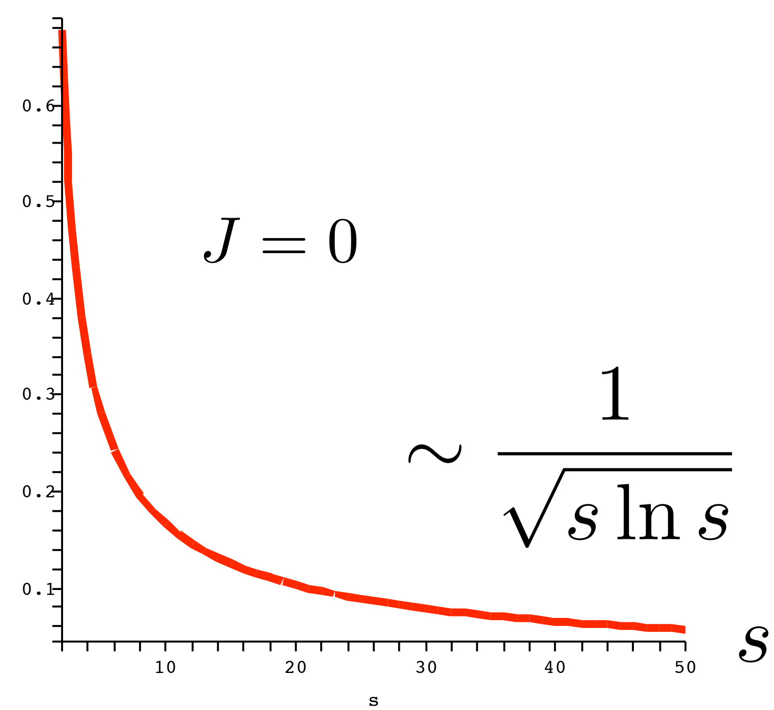
$$I_J(z) = \frac{1}{\pi}\int_0^{\pi} d\theta e^{z\cos\theta}\cos(J\theta)$$

$$\longrightarrow \quad \sigma(s,J) \sim \frac{1}{\sqrt{2\pi z\sqrt{1+\frac{J^2}{z^2}}}} \exp\left[-z + z\sqrt{1+\frac{J^2}{z^2}} - J\ln\left(\frac{J}{z}+\sqrt{1+\frac{J^2}{z^2}}\right)\right]$$

$$|z| \gg 1$$

$s = 50$



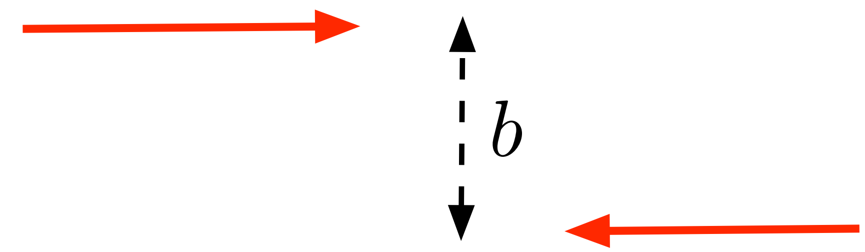


leading order in J/z

$$\sigma(s,J) \sim \frac{1-\frac{J^2}{4z^2}}{\sqrt{2\pi z}} e^{-\frac{J^2}{2z}} \qquad z = \frac{s}{4}\ln s$$

Recall

$$J = b\sqrt{s}/2$$



In the impact parameter space

$$\sigma(s, J = b\sqrt{s}/2) \sim \frac{1}{\sqrt{\pi s \ln s/2}} \left(1 - \frac{b^2}{s\ln^2 s}\right) e^{-\frac{2b^2}{\ln s}}$$

string form factor : $b \sim \sqrt{\alpha' \ln s}$

consistent with the Regge argument

however, $J_{total} \neq J_{12}$ さらにBFKL ? ⟶ Future work

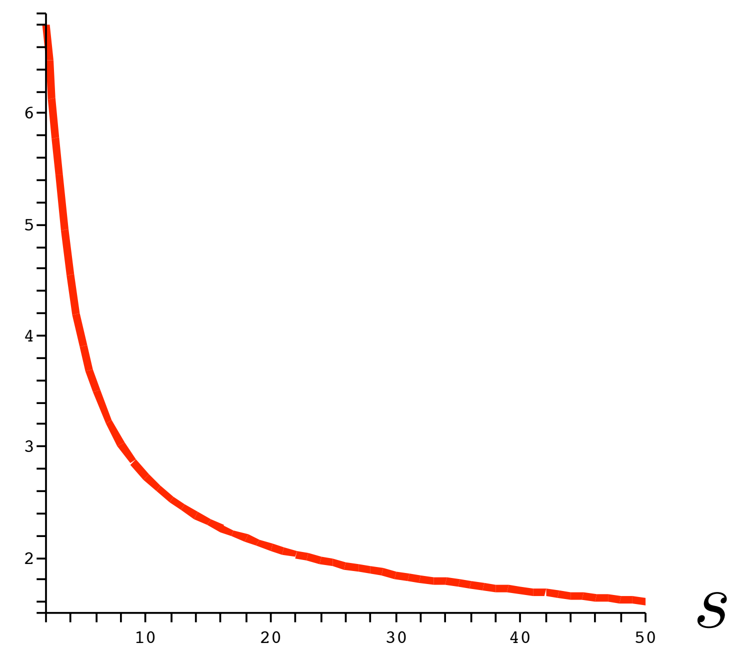
さらに

we may obtain a similar formular for the partial wave expansion of the Veneziano amplitude :

$$A_L \sim \frac{\pi^{3/2} s e^{-s \ln s/2}}{(s \ln s/4)^{1/2}} I_{L+1/2}(s \ln s/2)$$

cross-section for L=0 (head on head)

$$\sigma(s, L=0) = \frac{2\pi}{\ln s}\left(1 - e^{-s \ln s}\right)$$



$\exists$ relation $A_L(s) \leftrightarrow P(s, J)$?

$\sigma(s, L, J)$?

summary

We calculate the production cross-section
of a rotating string and obtain a simple formula.

which is different from the one in Kerr BH

We expect our amplitude has some applications
to the hadron physics such as Pomeron etc.