ホーキング輻射と高階スピンカレント

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1. Introduction

Hawking radiation with the thermal distribution at the Hawking temperature

$$N^{\mp}(\omega) = \frac{1}{e^{\beta\omega} \mp 1}, \qquad T_H = \frac{1}{\beta} = \frac{\kappa}{2\pi} \times \hbar$$

(κ : surface gravity at horizon)

Hawking radiation is a universal phenomenon in a background space-time with an event horizon.

Therefore it is plausible that the Hawking radiation can be derived from

- properties of the horizon,
  null hypersurface, causal structure

- "universal" effective theories near the horizon.

Considering matter fields in black hole backgrounds,

1. Two-dimensional effective theories appear near the horizon.

2. The Planck (Fermi-Dirac) distribution can be reconstructed from quantum anomalies of the two-dimensional effective theory.

   - gravitational anomaly, gauge anomaly
     $\Longrightarrow$ Fluxes of energy, charge and angular momentum from the Hawking radiations

     [Robinson & Wilczek],[Iso, H.U. & Wilczek]

   - anomalous transformations of (higher-spin) currents under conformal transformation
     $\Longrightarrow$ reproduce the Planck (Fermi-Dirac) distribution

     [Iso, Morita & H.U.]

Reissner-Nordström black hole

$$ds^2 = f(r)dt^2 - \frac{1}{f(r)}dr^2 - r^2 d\Omega^2, \qquad A = -\frac{Q}{r}dt,$$

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} = \frac{(r-r_+)(r-r_-)}{r^2} \qquad r_\pm = M \pm \sqrt{M^2 - Q^2}$$

Charged scalar field

$$S = \int d^4x \sqrt{-g}\left[g^{\mu\nu}\left(\partial_\mu + ieA_\mu\right)\phi^*\left(\partial_\nu - ieA_\nu\right)\phi - m^2\phi^*\phi + V(\phi)\right].$$

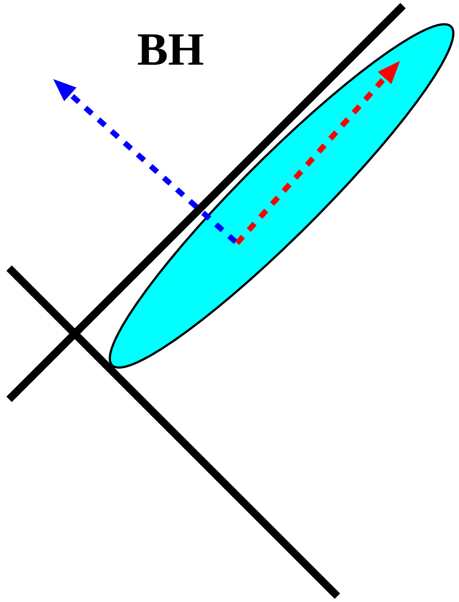
Partial wave decomposition: $\phi = \sum \phi_{lm}(t,r)Y_{lm}(\Omega)$

$$\begin{aligned} S &= \sum_{l,m}\int dtdr_*\ r^2(r_*)\Big[|\left(\partial_t - ieA_t\right)\phi_{lm}|^2 + |\partial_{r_*}\phi_{lm}|^2 \\ &\quad + f(r(r_*))\left(-m^2|\phi_{lm}|^2 + \frac{l(l+1)}{r^2}|\phi_{lm}|^2 + V(\phi_{lm})\right)\Big]. \end{aligned}$$

$$(\ r_* = r + 2M\ln(r/2M - 1)\ )$$

Near horizon, potential term $l(l+1)/r^2$, mass term and interaction terms are suppressed.

$\Longrightarrow$ Each partial wave mode behaves as $d = 2$ massless free field in $(r-t)$ section.



Anomaly method

Outgoing mode = right moving
Ingoing mode = left moving

Ingoing modes near the horizon are irrelevant for the physics in exterior region.

So we first neglect these modes near the horizon.

The two-dimensional effective theory becomes chiral.

$\Longrightarrow$ gauge and gravitational anomalies
= breakdown of gauge and general covariance

But the underlying theory is not anomalous.

Ingoing modes near the horizon cancel the anomalies and do not contribute to the Hawking fluxes.

According to this scenario, we derived the fluxes of energy, charge and angular momentum from Hawking radiation.

$$\langle T^r_t \rangle \Big|_{r\to\infty}, \qquad \langle J^r \rangle \Big|_{r\to\infty}$$

This analysis has been applied to the Hawking radiations of matter fields in

- rotating black holes (BTZ, Kerr-Newman, Myers-Perry black hole)
- black holes with a cosmological constant
- dilatonic black holes
- non-extremal D1-D5 black hole
- black ring

 ....

2. Higher-spin currents and thermal distribution

The flux of energy is a moment of the thermal distribution.

$$\langle T^r_t \rangle \Big|_{r\to\infty} = \frac{1}{48\pi}\kappa^2 \quad \Longleftrightarrow \quad \int_0^\infty \frac{d\omega}{2\pi} \frac{\omega}{e^{\beta\omega} \mp 1}$$

Can the anomaly method reproduce the complete Planck distribution?

Here we show

$$\langle \text{Higher-spin current} \rangle \Big|_{r\to\infty} \quad \Longleftrightarrow \quad \int_0^\infty \frac{d\omega}{2\pi} \frac{\omega^{2n-1}}{e^{\beta\omega} \mp 1}$$

To generalize our method to higher-spin current, we need to know

- conservation laws
- anomalies

} $\cdots$ Ward-Takahashi Identities for higher-spin currents

$\Longrightarrow$ This is difficult. (though this is interesting.)

We bypass this problem by using the conformal field theory technique in the effective field theory near the horizon.

Energy flux from trace anomaly

- General properties of 2-dim CFT (with central charge c)

conformal gauge: $ds^2 = 2e^{\varphi(u,v)}dudv$

The holomorphic quantity $T_{uu}^{(conf)}(u)$ is defined from the conservation equation of the energy-momentum tensor $T_{\mu\nu} = \delta S/\delta g^{\mu\nu}$ and trace anomaly

$$T_{uu}^{(conf)}(u) = T_{uu}(u,v) + \frac{c}{24\pi}\left(\partial_u^2\varphi - \frac{1}{2}(\partial_u\varphi)^2\right)$$

Conformal transformation: $(u,v) \longrightarrow (w(u), y(v))$

$$T_{ww}^{(conf)}(w) = \left(\frac{dw}{du}\right)^{-2}\left(T_{uu}^{(conf)}(u) + \frac{c}{24\pi}\{w,u\}\right)$$

where $\{w,u\}$ is the Schwarzian derivative

$$\{w,u\} = \frac{w'''}{w'} - \frac{3}{2}\left(\frac{w''}{w'}\right)^2$$

We apply these relations to CFT around the horizon of BH.

Kruskal coordinate

Schwarzschild metric: $$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \frac{dr^2}{1 - \frac{2M}{r}} - r^2 d\Omega^2$$

event horizon: $r = 2M$,

light-like coordinates:

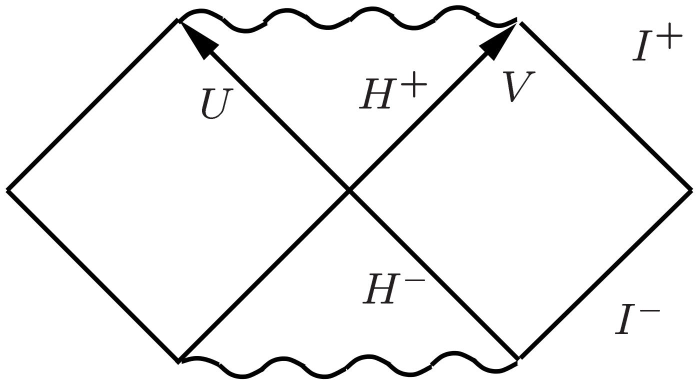
$u = t - r_*$, $v = t + r_*$. ($r_* = r + 2M \ln(r/2M - 1)$)

Kruskal coordinates:

$$U = -4Me^{-u/4M}, \qquad V = 4Me^{v/4M}$$

$$\begin{aligned} ds^2 &= \left(1 - \frac{2M}{r}\right) dudv - r^2 d\Omega^2 \\ &= 2M\frac{e^{-r/2M}}{r} dUdV - r^2 d\Omega^2 \end{aligned}$$

$$UV = -16M^2 \left(\frac{r}{2M} - 1\right) e^{r/2M}$$



Schwarzschild BH

light-like coordinate $u \longrightarrow$ Kruskal coordinate U: $\qquad U = -e^{-\kappa u}$

$$T^{(conf)}_{UU}(U) = \frac{1}{\kappa^2 U^2}\left(T^{(conf)}_{uu}(u) + \frac{c}{24\pi}\{U,u\}\right)$$

Boundary conditions corresponding to the Unruh vacuum

- Regularity condition
  Physical quantities in the Kruskal coordinate system should be regular at the horizon.

$$\Longrightarrow \qquad \langle T^{(conf)}_{UU}\rangle\Big|_{U=0} : \text{finite}$$

- No ingoing flux from the infinity ($r \to \infty$)

$$\Longrightarrow \qquad \langle T^{(conf)}_{vv}\rangle \stackrel{r\to\infty}{\longrightarrow} 0$$

+ (The background geometry is static.)

The asymptotic flux is determined by the value of the Schwarzian derivative:

$$\langle T^r_t\rangle = \langle T_{uu}\rangle - \langle T_{vv}\rangle \quad \stackrel{r\to\infty}{\longrightarrow} \quad -\frac{c}{24\pi}\{U,u\} = \frac{c}{48\pi}\kappa^2$$

$$\Longleftrightarrow \langle T^r_t\rangle = \int_0^\infty \frac{d\omega}{2\pi}\frac{\omega}{e^{\beta\omega}\mp 1} = \begin{cases} \kappa^2/48\pi & \text{for boson, } (c=1) \\ \kappa^2/96\pi & \text{for fermion, } (c=1/2) \end{cases}$$

Generalization to higher-spin current

- In flat $d = 2$ space-time, there are conserved symmetric traceless currents.

  for example, 4-th rank current for a real scalar field:

$$\begin{aligned} J_{\mu\nu\rho\sigma} \;\propto\; & (8\, \partial_\mu\phi\partial_\nu\partial_\rho\partial_\sigma\phi - 12\partial_\mu\partial_\nu\phi\partial_\rho\partial_\sigma\phi - 4\, g_{\mu\nu}\partial^\lambda\phi\partial_\lambda\partial_\rho\partial_\sigma\phi \\ & +8\, g_{\mu\nu}\partial^\lambda\partial_\rho\phi\partial_\lambda\partial_\sigma\phi - g_{\mu\nu}g_{\rho\sigma}\partial^\lambda\partial^\tau\phi\partial_\lambda\partial_\tau\phi) + \text{symm.} \end{aligned}$$

  In general, one can construct even-rank current $J_{\mu_1,\cdots,\mu_{2n}}$ for a real scalar field. (odd-rank current = 0)

- In general curved backgrounds, (conformal gauge: $ds^2 = 2e^{\varphi(u,v)}dudv$)

  $J_{u\cdots u}(u,v) = (\varphi\text{-dependent terms}) + J^{(conf)}_{u\cdots u}(u)$ $\Longrightarrow$ T. Morita's talk

- Similarly to the case of $\langle T\rangle$, $\langle J^{(2n)}\rangle$ is determined by the regularity condition and its transformation property under the conformal transformation.

$$\langle J^{(2n)}\rangle \quad \xrightarrow{r\to\infty} \quad \text{value of generalization of Schwarzian derivative}$$

$$\Longleftrightarrow (2n-1)\text{-th moment of the Planck distribution :} \quad \int_0^\infty \frac{d\omega}{2\pi}\frac{\omega^{2n-1}}{e^{\beta\omega}\mp 1}$$

4-th rank current

holomorphic part of the current

$$J_{uuuu}^{(conf)} = -\frac{2}{5} : \partial_u \phi \partial_u^3 \phi : + \frac{3}{5} : \partial_u^2 \phi \partial_u^2 \phi :$$

First, we consider the transformation property of $: \partial_u \phi \partial_u^3 \phi :$ under $u \longrightarrow w(u)$.

Regularization by the point splitting:

$$: \partial_u \phi \partial_u^3 \phi(u) := \lim_{\epsilon \to 0} \left(\partial_u \phi(u + \epsilon/2) \partial_u^3 \phi(u - \epsilon/2) + \frac{3}{2\pi \epsilon^4} \right)$$

Under $u \to w(u)$: $\quad \partial_u \phi(u) = \partial_u w(u) \partial_w \phi^{(w)}(w(u))$

$$\begin{aligned} : \partial_u \phi \partial_u^3 \phi(u) : \quad = \quad & w' w''' : \partial \phi^{(w)} \partial \phi^{(w)}(w) : + 3(w')^2 w'' : \partial \phi^{(w)} \partial^2 \phi^{(w)}(w) : \\ & + (w')^4 : \partial \phi^{(w)} \partial^3 \phi^{(w)}(w) : - \frac{1}{480\pi} \{w, u\}_{(1,3)} \end{aligned}$$

generalization of the Schwarzian derivative:

$$\{w, u\}_{(1,3)} = 6 \frac{w'''''}{w'} - 20 \left(\frac{w'''}{w'} \right)^2 - 45 \left(\frac{w''}{w'} \right)^4 + 90 \frac{(w'')^2 w'''}{(w')^3} - 30 \frac{w'''' w''}{(w')^2}$$

In the case of BH : $w(u) = U = -e^{-\kappa u}$

$$: \partial_u \phi \partial_u^3 \phi(u) : \;=\; \kappa^4 U^2 : \partial_U \phi^{(U)} \partial_U \phi^{(U)} : + 3\kappa^4 U^3 : \partial_U \phi^{(U)} \partial_U^2 \phi^{(U)} : + \kappa^4 U^4 : \partial_U \phi^{(U)} \partial_U^3 \phi^{(U)} : - \frac{1}{480\pi}\kappa^4$$

Regularity condition at $U = 0$

$$\Longrightarrow \qquad -\langle : \partial_u \phi \partial_u^3 \phi(u) : \rangle = \frac{1}{480\pi}\kappa^4 \qquad \longleftarrow \text{Flux at the infinity}$$

Note: $: \partial_u^2 \phi \partial_u^2 \phi :$ and $: -\partial_u \phi \partial_u^3 \phi(u) :$ give the same contribution to the flux.

Reproduce the 3rd moment of the Planck distribution

$$\langle J^{(conf)}_{uuuu} \rangle = \frac{1}{480\pi}\kappa^4 \quad \Longleftrightarrow \quad \int_0^\infty \frac{d\omega}{2\pi} \frac{\omega^3}{e^{\beta\omega} - 1} = \frac{1}{480\pi}\kappa^4$$

General higher-spin currents

It is sufficient to know the generalized Schwarzian derivatives for : $\partial_u\phi\partial_u^{2n-1}\phi$: in order to derive fluxes for the $2n$-th rank currents.

Conformal transformation of higher-spin currents

Generating function

$$: \partial_u\phi(u)\partial_u\phi(u+a) := \sum_{n=0}^{\infty}\frac{a^n}{n!} : \partial_u\phi(u)\partial_u^{n+1}\phi(u) :$$

Transformation : $u \longrightarrow w(u)$

$$: \partial_u\phi(u)\partial_u\phi(u+a) := \partial_u w(u)\partial_u w(u+a) : \partial_w\phi^{(w)}(w(u))\partial_w\phi^{(w)}(w(u+a)) : + A_b(w,u)$$

$A_b(w,u)$ is a generating function of the Schwarzian derivatives.

$$A_b(w,u) = -\frac{1}{4\pi}\frac{\partial_u w(u)\partial_u w(u+a)}{[w(u)-w(u+a)]^2} + \frac{1}{4\pi a^2}$$

In the case of BH : $w(u) = U = -e^{\kappa u}$

$$:\partial_U \phi^{(U)}(U(u))\partial_U \phi^{(U)}(U(u+a)) := e^{\kappa a}\left(\frac{1}{\kappa U}\right)^2 [:\partial_u \phi_u \partial_u \phi(u+a) : -A_b(U,u)]$$

Regularity condition at the horizon

$$\begin{aligned} \langle :\partial_u \phi(u)\partial_u \phi(u+a):\rangle &= A_b(U,u) \\ &= -\frac{\kappa^2}{16\pi}\frac{1}{\sinh^2\frac{\kappa a}{2}} + \frac{1}{4\pi a^2} = \sum_{n=0}^{\infty}(-1)^n \frac{B_{n+1}\kappa^{2(n+1)}}{8\pi(n+1)}\frac{a^{2n}}{(2n)!} \end{aligned}$$

B_n : Bernoulli number

Hawking flux corresponding to $2n$-th rank current

$$\langle :(-1)^{n-1}\partial_u \phi \partial_u^{2n-1}\phi(u):\rangle = \frac{B_n}{8\pi n}\kappa^{2n} \quad \Longleftrightarrow \quad \int_0^\infty \frac{d\omega}{2\pi}\frac{\omega^{2n-1}}{e^{\beta\omega}-1}$$

Reproduce the $(2n-1)$-th moment of the Planck distribution

Physical meaning of $A_b(U,u)$

$A_b(U,u)$ can be written as

$$A_b(U,u) = \int_0^\infty \frac{d\omega}{2\pi}\frac{\omega}{e^{\beta\omega}-1}\cos(a\omega)$$

This is the temperature dependent part of a finite temperature green function for $\langle T\partial\phi(x)\partial\phi(x+a)\rangle_\beta$

$$\langle T\phi(x)\phi(y)\rangle_\beta = \int \frac{d^2k}{(2\pi)^2}\left(\frac{i}{k^2+i\epsilon} + 2\pi\delta(k^2)\frac{1}{e^{\beta|\omega|}-1}\right)e^{-ik(x-y)}$$

tortoise coordinate $\longrightarrow$ Kruskal coordinate : $U = -e^{-\kappa u}$

$\Longleftrightarrow$ zero temperature $\longrightarrow$ finite temperature with $\beta = 2\pi/\kappa$

Similar analysis can be done for fermionic cases.

Charged black hole

$U(1)$ gauge field : $A_t^{(u)} = -\frac{Q}{r}$ $\Longrightarrow$ $A_U^{(u)}$ diverges at the horizon.

A gauge which is suitable near the horizon

$$A_t^{(U)} = -\frac{Q}{r} + \frac{Q}{r_+} \qquad \Longrightarrow A_U^{(U)} \text{ is regular at the horizon.}$$

$$\left(u, A_t^{(u)}\right) \longleftarrow \text{conformal and gauge transformations} \longrightarrow \left(U, A_t^{(U)}\right)$$

From relations between $\langle : \psi^\dagger \partial_u^n \psi(u) : \rangle_{A^{(u)}}$ and $\langle : \psi^\dagger \partial_U^n \psi(U(u)) : \rangle_{A^{(U)}}$, the moment of the thermal distribution is derived,

$$\langle i^n \psi^\dagger \partial_u^n \psi(u) \rangle_{A^{(u)}} = \int_0^\infty \frac{d\omega}{2\pi} \left[\omega^n N_e(\omega) - (-\omega)^n N_{-e}(\omega)\right],$$

$$N_e(\omega) = \frac{1}{e^{\beta(\omega - \frac{eQ}{r_+})} + 1}.$$

$*$ It is expected that similar analyses can be applied to various black holes.

rotating BH, $\cdots$

3. Summary

- We derived Hawking fluxes of the higher-spin currents by using the CFT technique.

- These fluxes coincide with moments of the Planck (Fermi-Dirac) Distribution.

  $\Longrightarrow$ The Planck distribution can be reconstructed from these quantities.

- Generalizations of the Schwarzian derivative for the higher-spin currents are obtained.

Higher-spin currents for fermion

Generating function

$$:\psi(u)\psi(u+a):=\sum_{n=0}^{\infty}\frac{a^n}{n!}:\psi\partial_u^n\psi(u):$$

Conformal transformation $u \longrightarrow w(u)$: $\psi(u)=(\partial_u w(u))^{1/2}\psi^{(w)}(w(u))$

By a similar analysis to the one in the bosonic case,

$$\begin{aligned}\langle :\psi(u)\psi(u+a):\rangle &= A_f(U,u)\\ &= \frac{i}{2\pi a}\left(\frac{\kappa a}{2}\frac{1}{\sinh\frac{\kappa a}{2}}-1\right)=\sum_{n=0}^{\infty} i^{2n+1}\frac{(1-2^{1-2n})B_n\kappa^{2n}}{4\pi n}\frac{a^{2n-1}}{(2n-1)!}\end{aligned}$$

Hawking flux corresponding to $2n$-th rank current

$$\langle :\frac{i^{2n-1}}{2}\psi\partial_u^{2n-1}\psi(u):\rangle=\frac{(1-2^{1-2n})B_n}{8\pi n}\kappa^{2n} \quad \Longleftrightarrow \int_0^{\infty}\frac{d\omega}{2\pi}\frac{\omega^{2n-1}}{e^{\beta\omega}+1}$$

Reproduce the $(2n-1)$-th moment of the Fermi-Dirac Distribution