Fatten up identity based solution in string field theory

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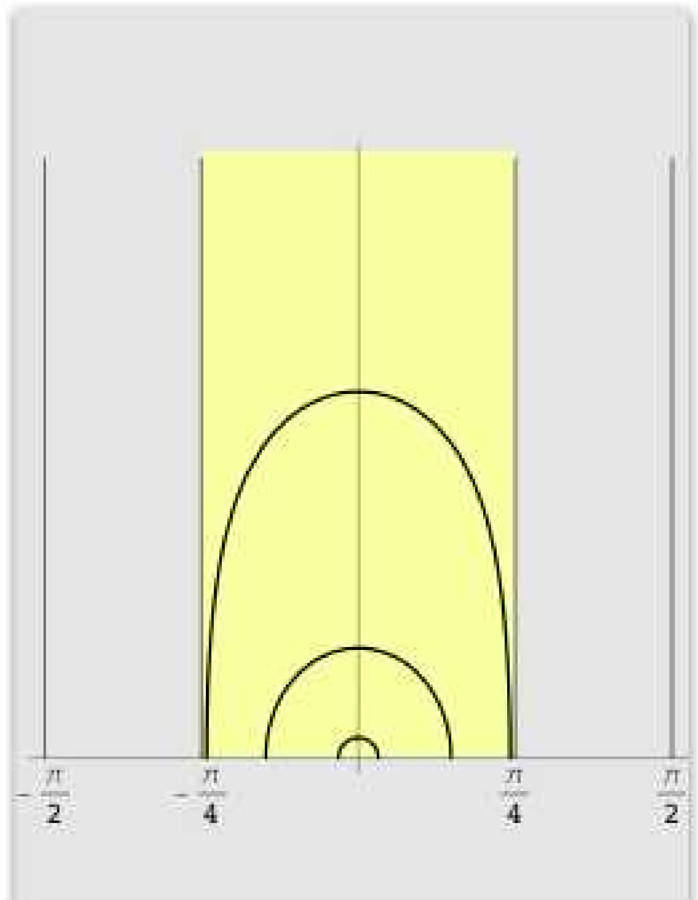
Schnabl's analytic method in SFT

- Powerful method for explicit calculations of open cubic SFT
- Crucial feature: use of special conformal coordinate called "sliver frame"
- Star product is simplified, but not too simplified



Marginal solutions

- Insertion of J(z_1)J(z_2)...J(z_n) in the solution collide each other
- Regular OPE case [Schnabl, KORZ, Kishimoto-Michishita, Okawa, Erler]
- Singular OPE case . . . regularization and counter term needed
  - Up to 3rd order [KROZ]
  - Full order for ∂X [Fuchs-Kroyter-Potting, Fuchs-Kroyter
  - Full order, in general [Kiermaier-Okawa]

Our claim

- New solution of cubic open SFT which corresponds to marginal deformation
- The solution is obtained by "fattening up" Talahashi-Tanimoto (identity based) solution on the zero width cylinder
- Valid for singular OPE case
- Real (not complex)

Solution generation

if $Q_B\phi$ and $\phi * \phi = 0$

$$\Rightarrow \quad \boxed{\Psi = P_\alpha * \frac{1}{1+\phi * A^{(\alpha+\beta)}} * \phi * P_\beta}$$

satisfies EOM $\quad Q_B\Psi + \Psi * \Psi = 0$

[Erler, ORZ, Kishimoto-Michishita]

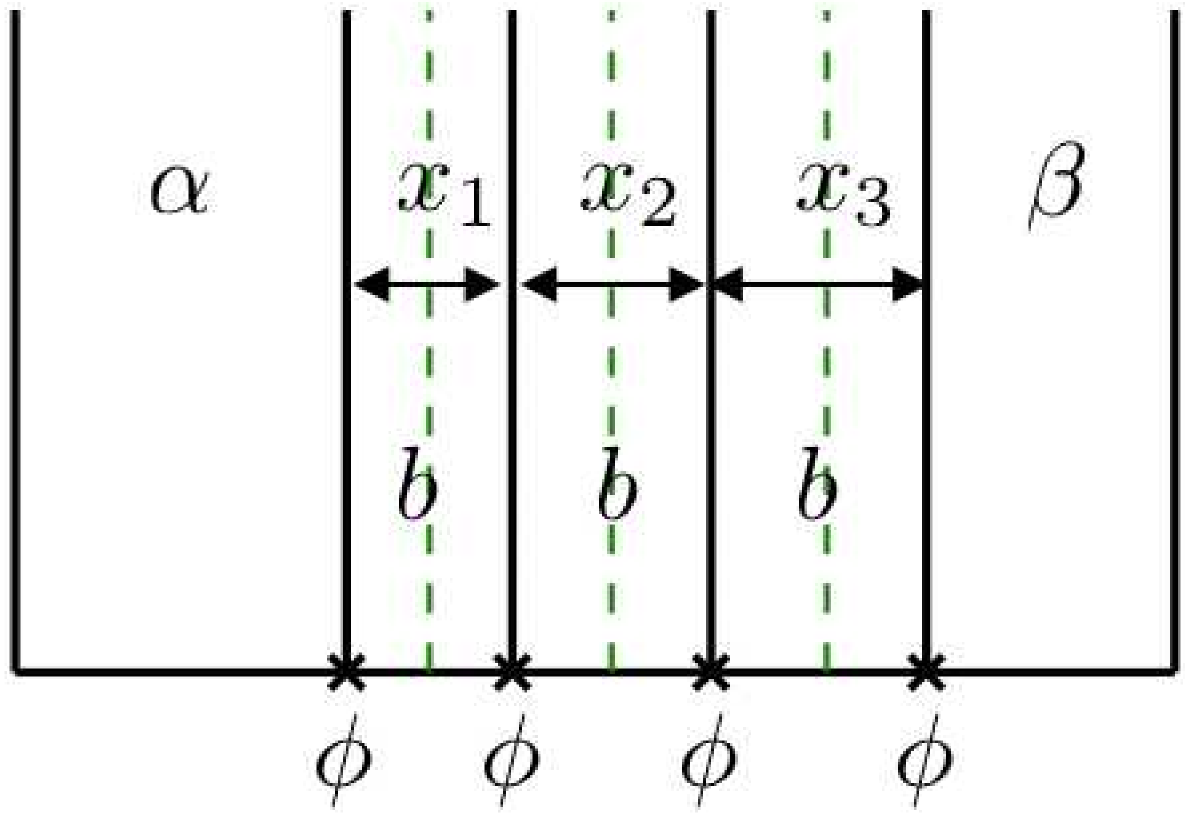
[Kishimoto-S.Z]

ϕ has zero width for known solutions

“fattening up” zero width solution

$$\frac{1}{1+\phi * A} * \phi = \phi - \phi * A * \phi + \phi * A * \phi * A * \phi + \cdots$$

$$\int_0^{\alpha+\beta} \int_0^{\alpha+\beta} \int_0^{\alpha+\beta} dx_1 dx_2 dx_3$$



Takahashi-Tanimoto marginal solution

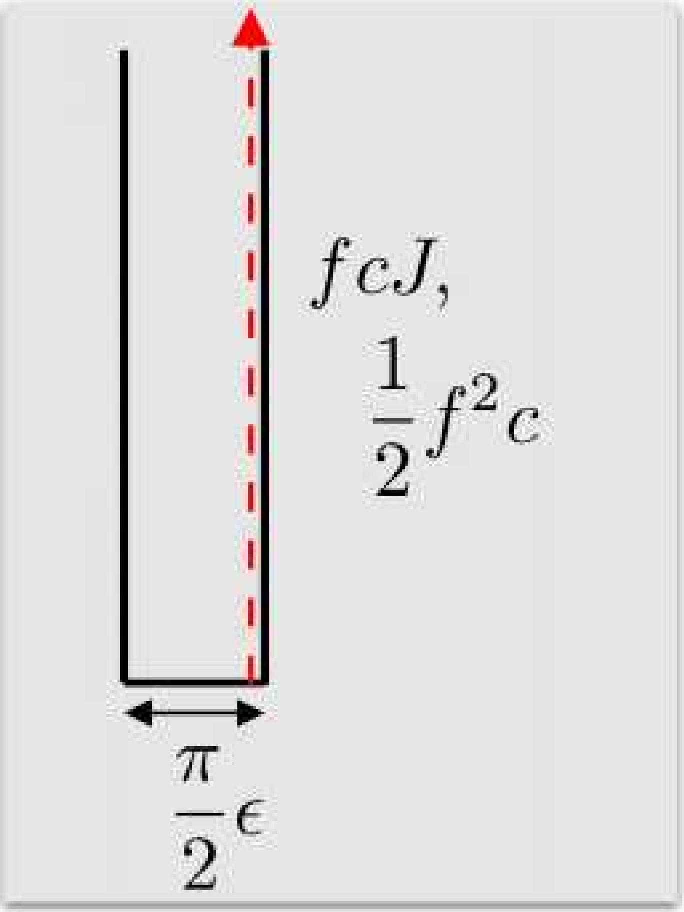
- Marginal solution widh zero width
- Available for singular OPE case
- Before fattening it up, let us study an expression in sliver frame

TT in sliver frame

$$\Psi_m^{\rm TT} = -\lim_{\epsilon\to 0}\widehat{U}_{1+\epsilon}\int_{-\infty}^{\infty}\frac{dt}{2\pi}\left(\lambda_a f(t)c\left(it+\frac{\pi}{4}\epsilon\right)J^a\left(it+\frac{\pi}{4}\epsilon\right)\right.$$
$$\left.+\frac{1}{2}g^{ab}\lambda_a\lambda_b f(t)^2 c\left(it+\frac{\pi}{4}\epsilon\right)\right)|0\rangle$$

$$f(t) = \frac{4}{\cosh^2(2t)}$$

$$J^a(z)J^b(w) \sim \frac{-g^{ab}}{\sin^2(z-w)} + \frac{f^{ab}{}_c}{\sin(z-w)}J^c(w)$$



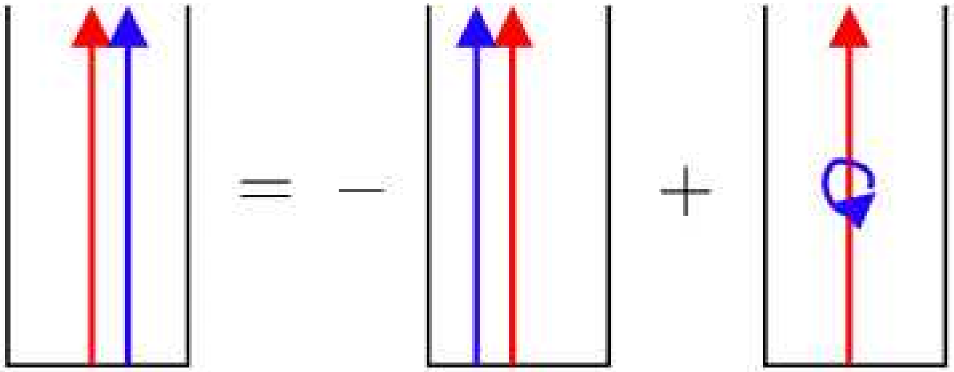
Equation of motion

$$\langle \eta, Q_B \Psi_{\rm m}^{\rm TT} \rangle = -\int_{-\infty}^{\infty} \frac{dt}{2\pi} \frac{1}{2} (f(t))^2 \left\langle \eta\left(\frac{\pi}{4}\right) c\partial c(it) \right\rangle_{C_{\frac{\pi}{2}}}$$

$$\langle \eta, \Psi_{\rm m}^{\rm TT} * \Psi_{\rm m}^{\rm TT} \rangle = \frac{1}{2} \int_{\gamma_1} \frac{dz_1}{2\pi i} \oint_{z_2} \frac{dz_2}{2\pi i} \lambda_a \lambda_b \left\langle \eta\left(\frac{\pi}{4}\right) J^a(z_1) J^b(z_2) c(z_1) c(z_2) \right\rangle_{C_{\frac{\pi}{2}}}$$

$$= +g^{ab} \lambda_a \lambda_b \int_{-\infty}^{\infty} \frac{dt}{2\pi} \frac{1}{2} (f(t))^2 \left\langle \eta\left(\frac{\pi}{4}\right) c\partial c(it) \right\rangle_{C_{\frac{\pi}{2}}}$$

$$Q_B \Psi_{\rm m}^{\rm TT} + \Psi_{\rm m}^{\rm TT} * \Psi_{\rm m}^{\rm TT} = 0$$



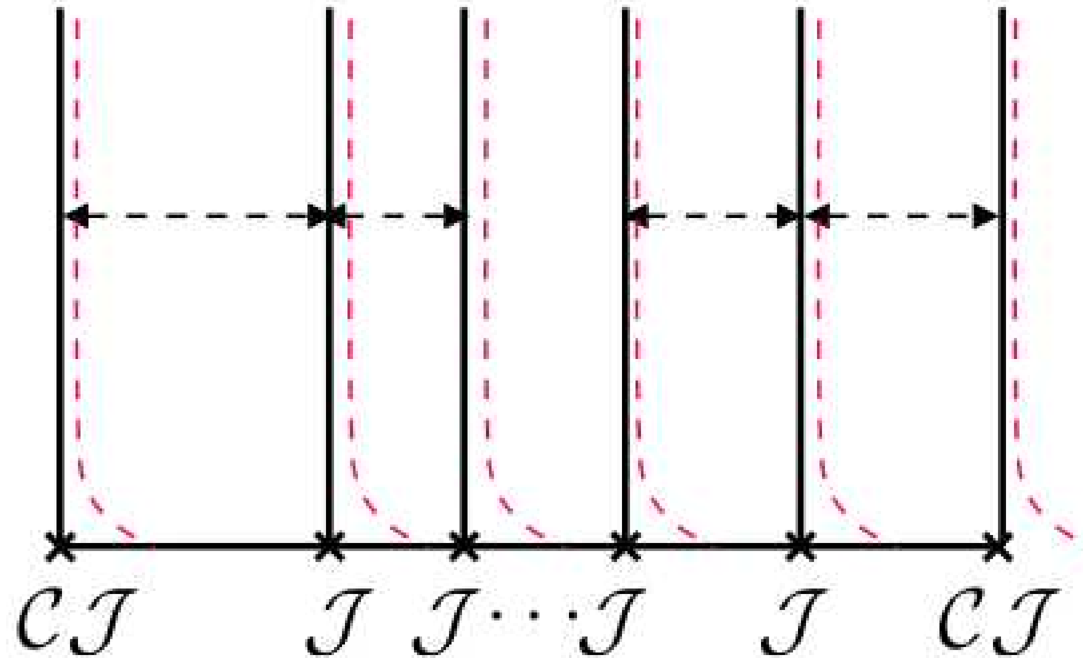
Fatten up TT solution

$$\Psi = P_\alpha * \frac{1}{1 + \Psi_{\rm m}^{\rm TT} * A^{(\alpha+\beta)}} * \Psi_{\rm m}^{\rm TT} * P_\beta$$

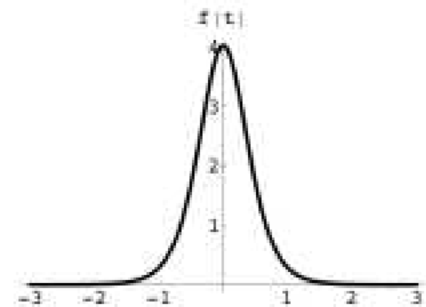
- Formally satisfies the E.O.M.
- Nontrivial issue: “regular” solution?

$$(\Psi_{\rm m}^{\rm TT} * A^{\alpha+\beta})^k * \Psi_{\rm m}^{\rm TT} =$$

$$(\mathcal{B}_0 + \mathcal{B}_0^\dagger)\times$$



“smeared” currents



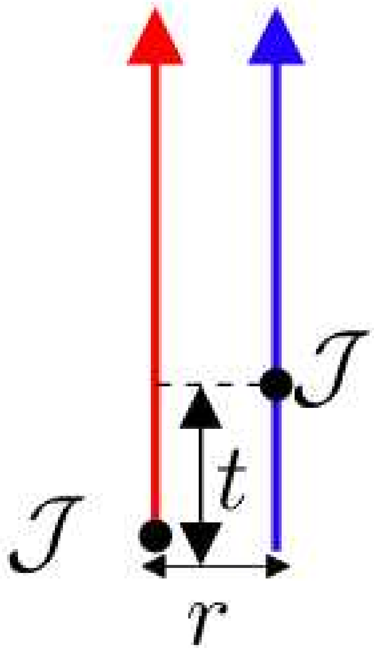
$$\mathcal{J}(x) = -\int_{-\infty}^{\infty} \frac{dt}{2\pi} \left(\lambda_a f(t) J^a(x+it) + \frac{1}{2} g^{ab} \lambda_a \lambda_b f(t)^2 \right)$$

$$\mathcal{CJ}(x) = -\int_{-\infty}^{\infty} \frac{dt}{2\pi} c(x+it) \left(\lambda_a f(t) J^a(x+it) + \frac{1}{2} g^{ab} \lambda_a \lambda_b f(t)^2 \right)$$

$$\mathcal{CJ}(r)\mathcal{CJ}(0), \quad \mathcal{CJ}(r)\mathcal{J}(0), \quad \mathcal{J}(r)\mathcal{J}(0)$$

$$\Rightarrow \qquad I_{p,q}(r) = 2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{dt}{2\pi}\frac{ds}{2\pi}f(s)f(s-t)(is)^p(-it)^q$$

$$\times \frac{1-\cosh(2t)\cos\pi r + i\sinh(2t)\sin\pi r}{(\cosh(2t)-\cos\pi r)^2}$$



contribution from small t
is dominant

finite !

- A possible divergence comes from a region where two smeared currents collide

- We showed that

$$\mathcal{C}\mathcal{J}(x)\mathcal{C}\mathcal{J}(0), \quad \mathcal{C}\mathcal{J}(x)\mathcal{J}(0), \quad \mathcal{J}(x)\mathcal{J}(0)$$

  are finite at x ~ 0

- This ensures that our solution has finite coefficients each order of the coupling λ

- But this is not a whole story

- In our solution,
(λ expansion $\neq$ mode expansion w.r.t. $\mathcal{L}_0$)

- For example, a coefficient of a "width x" contribution

$$\widehat{U}_{1+x}\tilde{c}_1 \left|0\right\rangle$$

includes sum of all (c J J J J...J) contractions so includes infinite powers of λ

- Evaluation of such coefficients is very complicated. But if evaluated, it would give "effective coupling" $\beta(\lambda)$

- This story looks quite similar to "renormalization of boundary state"

[Callen-Klevbanov-Ludwig-Maldacena ,Kogetsu-Teraguchi]

- Anyway, the validity of our solution should be further explored by
    - Explicit examples of some marginal currents
    - Estimating infinite sum
    - Evaluating classical action (it should vanish)