Model for non-perturbative string landscape and appearance of M theory

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Ref)
CIY2 ’10, in progress
CIY ’10, “Fractional-superstring amplitudes, multi-cut matrix models and non-critical M theory,” **Nucl.Phys.B838:75-118,2010 [arXiv:1003:1626]**
CISY ’09, “macroscopic loop amplitudes in the multi-cut two-matrix models,” **Nucl.Phys.B828:536-580,2010 [arXiv:0909.1197]**
H.I ’09, “fractional supersymmetric Liouville theory and the multi-cut matrix models,” **Nucl.Phys.B819:351-374,2009 [arXiv:0902.1676]**

我々が知りたいこと:

摂動論的弦理論が、
非摂動論的弦理論の中で果たす役割は？

弦理論の真空のランドスケープが
非摂動論的どう見えるか？

Non-critical strings、行列模型の枠内で
これらを理解したい。

今回は特に、
これらを考察するための幾つかのモデルを提唱したい。

Non-critical stringにおけるモジュライ空間とは？

（A little different from the usual）

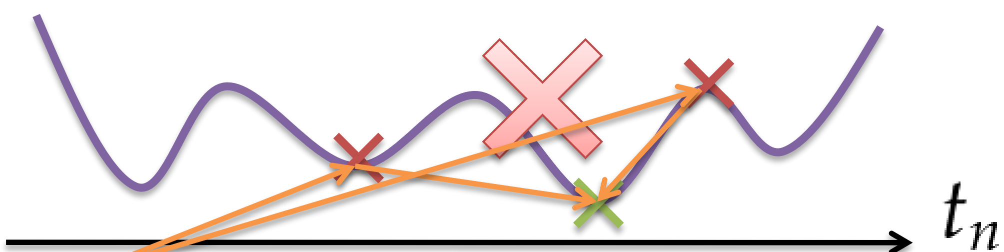
世界面の作用:

$$S = \int d^2z \left(\partial X \bar{\partial} X + \partial\phi\bar{\partial}\phi + \sum_n t_n \cdot \boxed{\mathcal{O}_n(X,\phi)}\right)$$

? ⇒ Chemical Potential of On

On-shell op.(BRST coh.)

演算子 $\mathcal{O}_n$ は、規格化できないモードに対応し、t_n は
モジュライ空間というよりも、super selection パラメータ
[Seiberg-Shenker '92]



それぞれの点で違う行列模型、違うシステム

→ 今回はtn に関する"最小化"は考えない。

Non-critical stringにおけるモジュライ空間とは？

（A little different from the usual）

では、Non-critical string のモジュライ空間とは？

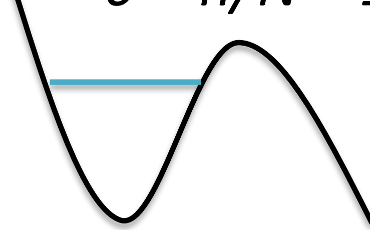
固有値の配置である

固有値

Remember:

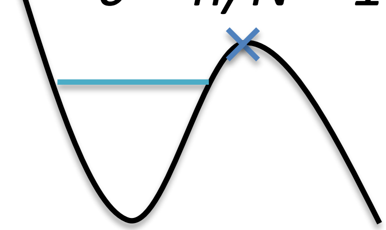
$$\mathcal{Z}=\int dM\, e^{-N trV(M)} \quad \Rightarrow \quad \mathcal{Z}=\int d^N\lambda \prod_{i>j}(\lambda_i-\lambda_j)^2 e^{-N\sum_i V(\lambda_i)}$$

$\vartheta = n/N = 1$



Stable background

$\vartheta = n/N = 1\text{-}1/N$



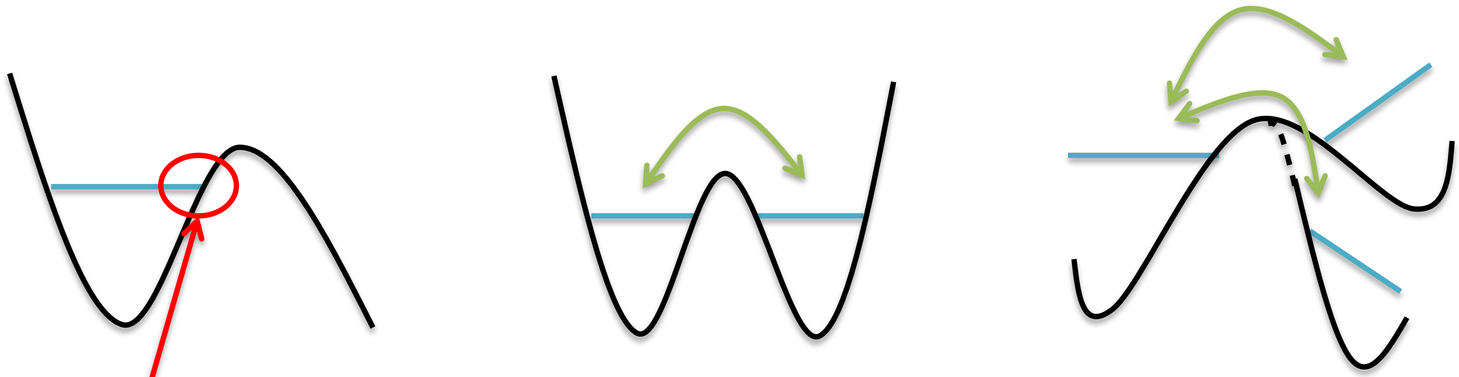
Unstable background
(adding Unstable D-branes)

This system is understood as *openstring Tachyon condensation*

モジュライ空間は、*filling fraction* ϑ で与えられる

Toward the next step??

multi-cut matrix models!



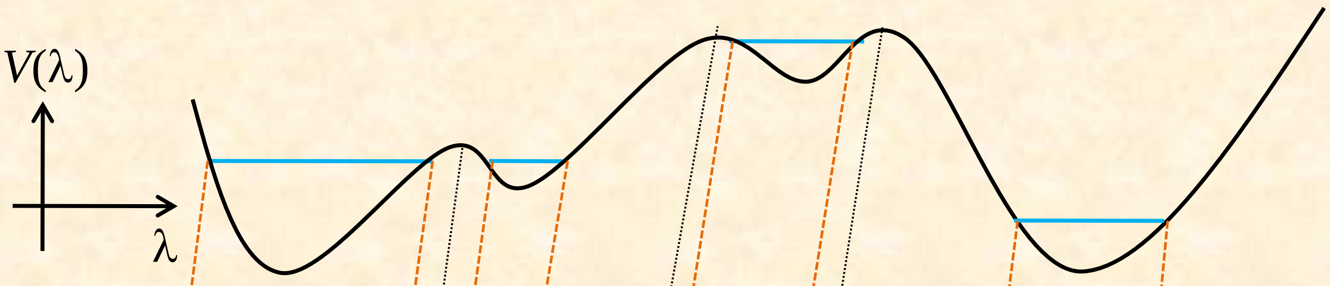
Perturbative string theory

There are more D.O.F to interplay among various perturb. strings

<u>How to define the multi-cut matrix models?</u>

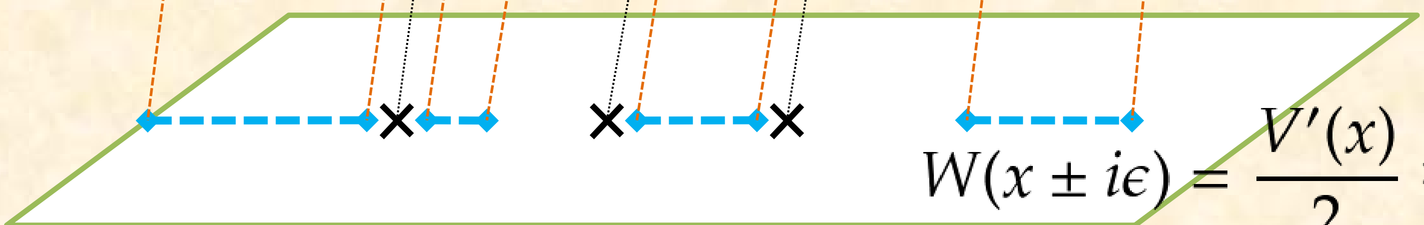
See the spectral curve!

the spectral curve and eiganvalues



We usually introduce *the resolvent*:

$$W(x)=\frac{1}{N}\left\langle tr\frac{1}{x-M}\right\rangle \quad =\int d\lambda\frac{\rho(\lambda)}{z-\lambda}$$

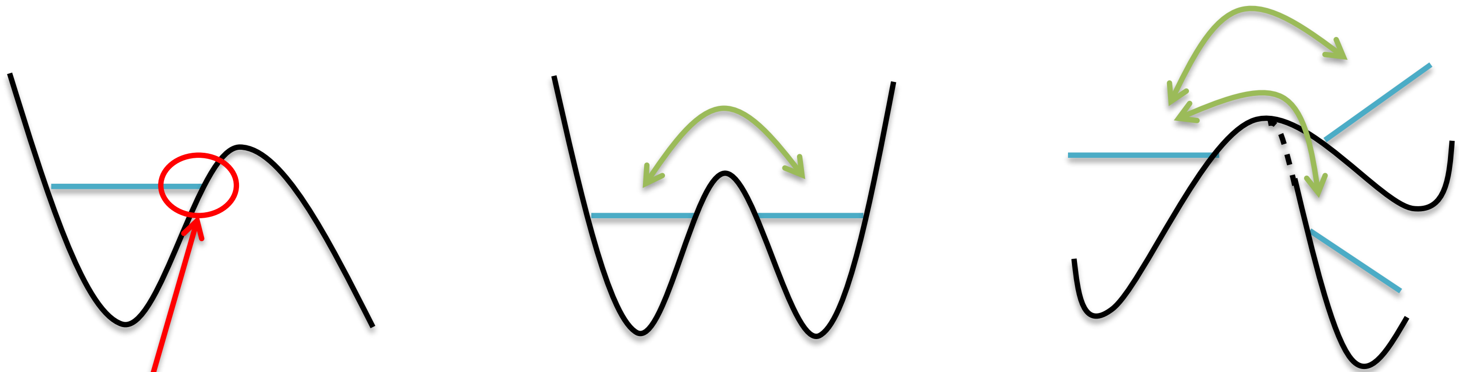


$$W(x\pm i\epsilon)=\frac{V'(x)}{2}\mp\pi i\rho(x)$$

Eigenvalue density

the (algebraic) spectral curve!

Toward the next step?? multi-cut matrix models!



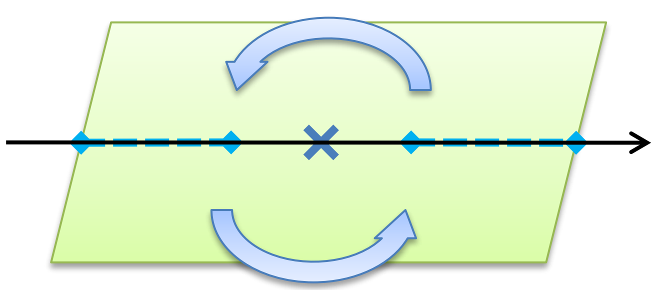
Perturbative string theory

There are more D.O.F to interplay among various perturb. strings

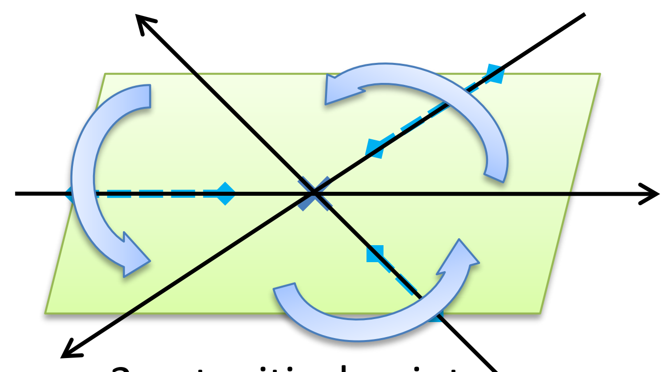
How to define this?

See the spectral curve!

[Crinkovik-Moore '91]

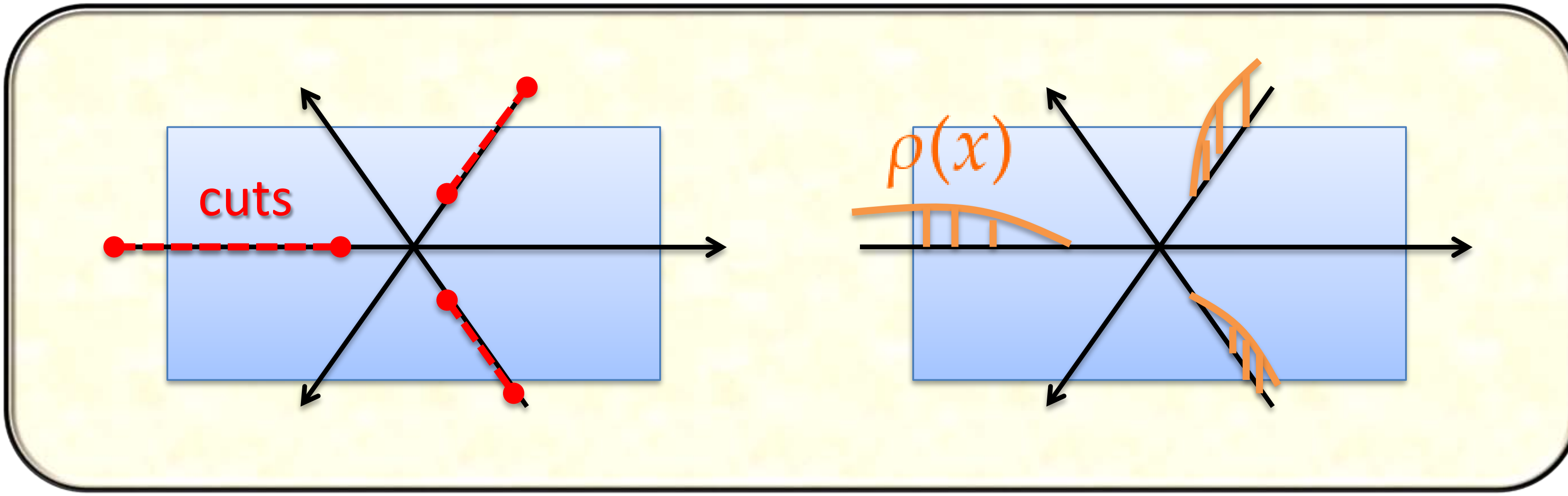


2-cut critical points



3-cut critical points

Intuitively, we expect the following geometry and eigenvalue distribution:



We have obtained ***concrete quantitative*** **(*and analytic*)** amplitudes of these systems [CISY'09, CIY'10]

Actual solutions in the system [CISY'09, CIY'10]

the Z_k symmetric case [CISY'09]: (p,q) critical points with k cuts

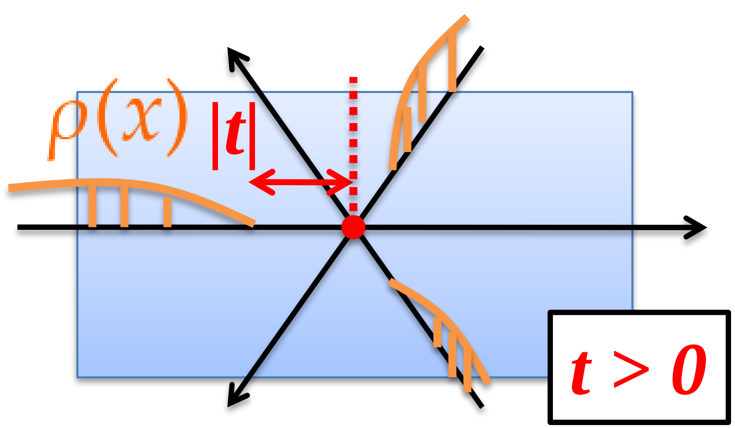
Note: $$W=\mathcal{W}(x)\Leftrightarrow (W(z),x(z))$$

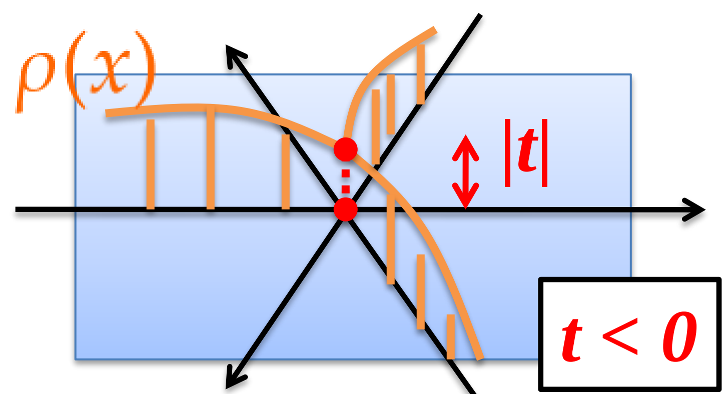
$$W=t^{\frac{q}{2p-1}}P_{q-1}^{\left(\frac{2l-k}{k},-\frac{2l-k}{k}\right)}(z)\sqrt[k]{(z-1)^{k-l}(z+1)^{l}}$$
$$x=t^{\frac{p}{2p-1}}P_{p-1}^{\left(-\frac{2l-k}{k},\frac{2l-k}{k}\right)}(z)\sqrt[k]{(z-1)^{l}(z+1)^{k-l}},$$

(Pn(z) is called Jacobi polynomial)

Fermat Curve

What is t ? e.g.) the 3-cut cases are





the Z_k symmetric case [CISY'09]: (p,q) critical points with k cuts

$$W = t^{\frac{q}{2p-1}} P_{q-1}^{\left(\frac{2l-k}{k},-\frac{2l-k}{k}\right)}(z) \sqrt[k]{(z-1)^{k-l}(z+1)^{l}}$$

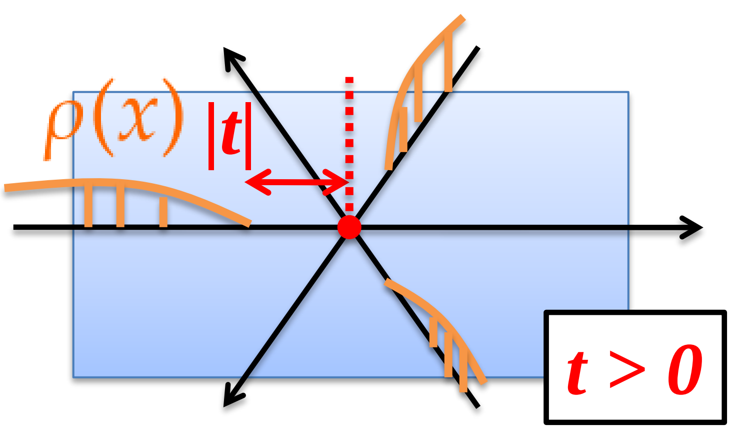
$$x = t^{\frac{p}{2p-1}} P_{p-1}^{\left(-\frac{2l-k}{k},\frac{2l-k}{k}\right)}(z) \sqrt[k]{(z-1)^{l}(z+1)^{k-l}},$$

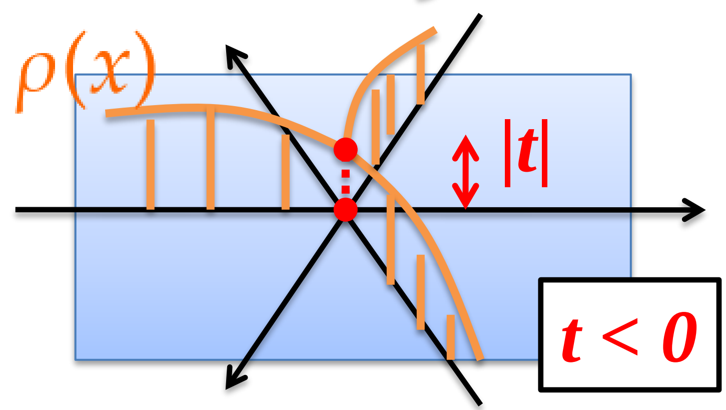
$l = 0,1,\cdots,k-1, \quad \#l \sim k$

label of solutions

$\#l = 2$ is natural because we have two choices

$t > 0$ or $t < 0$





the Z_k symmetric case [CISY'09]: (p,q) critical points with k cuts

$$W = t^{\frac{q}{2p-1}}\, P_{q-1}^{\left(\frac{2l-k}{k},\, -\frac{2l-k}{k}\right)}(z)\, \sqrt[k]{(z-1)^{k-l}(z+1)^{l}}$$

$$x = t^{\frac{p}{2p-1}}\, P_{p-1}^{\left(-\frac{2l-k}{k},\, \frac{2l-k}{k}\right)}(z)\, \sqrt[k]{(z-1)^{l}(z+1)^{k-l}},$$

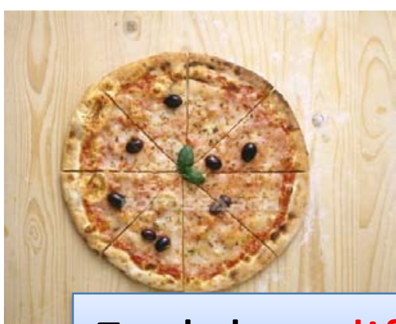
$$l = 0, 1, \cdots, k-1, \quad \# l \sim k$$

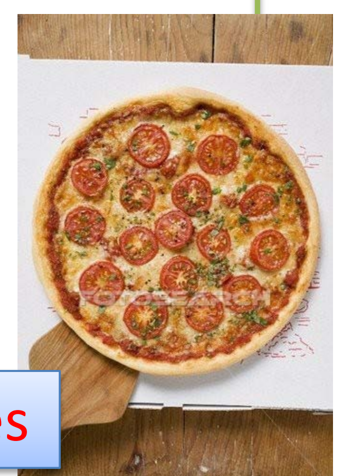
label of solutions

Too many solutions!?

$\# l = 2$ is natural because we have two choices

$$t > 0 \ \text{ or } \ t < 0$$





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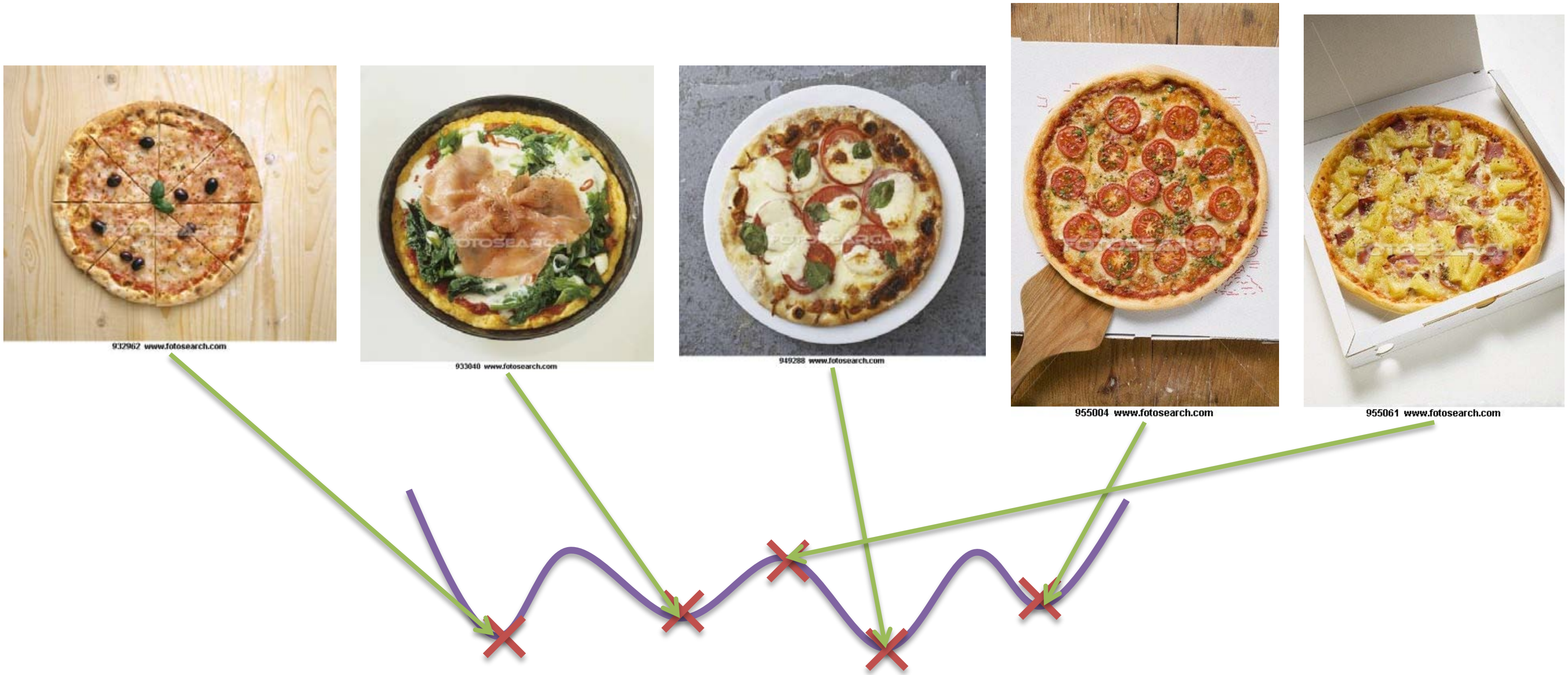
Each has different perturbative amplitudes

This implies that

the string Landscape of multi-cut matrix models is non-trivial



The multi-cut matrix models provide
non-trivial models for non-perturbative string landscape!

the fractional superstring cases [CIY’10]:

簡単な特徴づけ: Two-Matrix Model $$Z=\int dXdYe^{-Ntr\,w(X,Y)}$$

Z_k symmetric critical points:

Z_k symmetry but ~~$X \leftrightarrow Y$ symmetry~~

Fractional superstring critical points:

$X \leftrightarrow Y$ symmetry but ~~Z_k symmetry~~

corresponds to *Fractional superstring theory* [Irie’09]

1-cut

$X(z)$

bosonic

2-cut

$X(z)+\psi(z)$

(WS) supersymmetric

[Takayanagi-Toumbas’03]

[Douglas et.al.’03]

… k-cut …

$X(z)+\psi_{pf_k}(z)$

(WS) k-th fractional supersymmetric

[Irie’08,’09]

the fractional superstring cases [CIY'10]:

(p,q) critical points with k cuts $\quad W=\mathcal{W}(x)\Leftrightarrow(W(\tau),x(\tau))$

cosh solution

$$\begin{cases} W = t^{\frac{q}{2p}}\, ch\left(q\tau+2\pi i\dfrac{j-1}{k}\right) \\ x = t^{\frac{p}{2p}}\, ch\left(p\tau+2\pi i\dfrac{j-1}{k}\right) \end{cases}$$

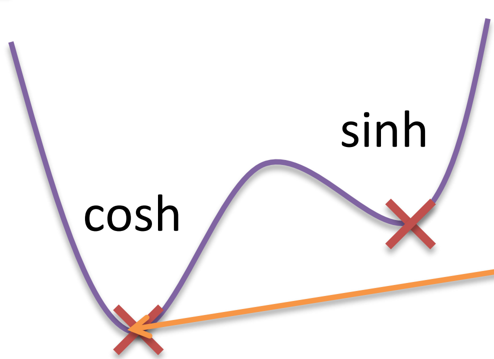
sinh solution

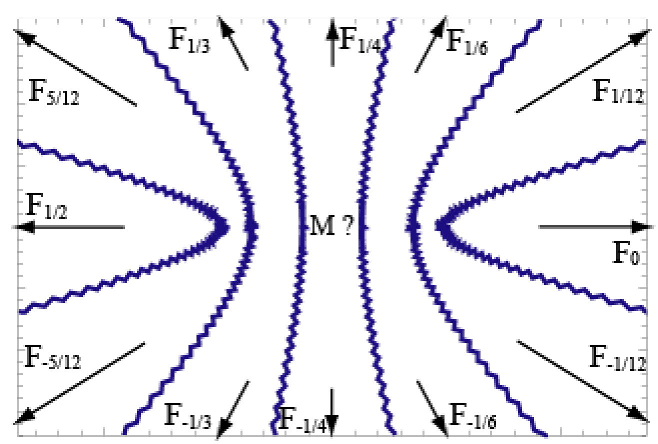
$$\begin{cases} W = t^{\frac{q}{2p}}\, sh\left(q\tau+2\pi i\dfrac{j-1}{k}\right) \\ x = t^{\frac{p}{2p}}\, sh\left(p\tau+2\pi i\dfrac{j-1}{k}\right) \end{cases}$$

There are two solutions:

one solution has k different pieces (j=1,2,…k)

$t>0$





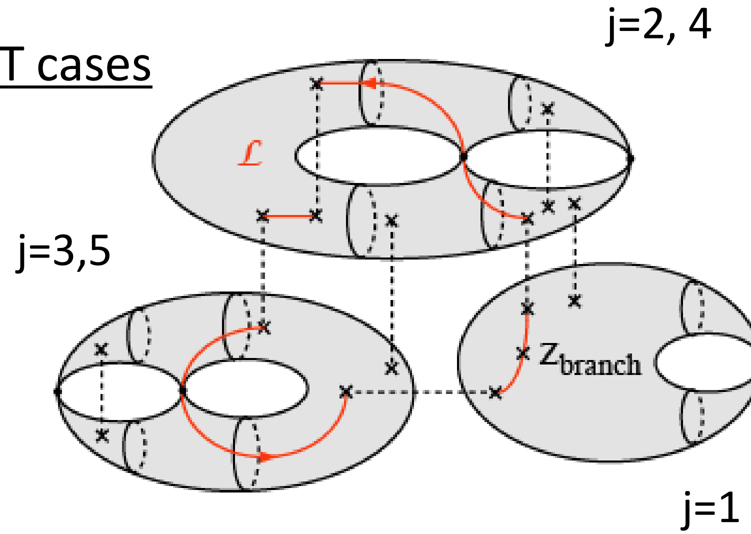
Factorization and Perturbative Isolation [CIY'10]

The algebraic equation of the solution is **reducible**

and is **factorized into** ***irreducible curves***:

usual cases

FSST cases



$$\underbrace{F(x,W)}_{\text{可約}} = \prod_{j=1}^{[\frac{k}{2}]} \underbrace{F_{\nu_j}(x,W)}_{\text{既約}} = 0$$

$$\begin{cases} \nu_j = \dfrac{j-1}{k} \\ \nu_j \sim \nu_j + 1 \end{cases}$$

Factorization and Perturbative Isolation [CIY'10]

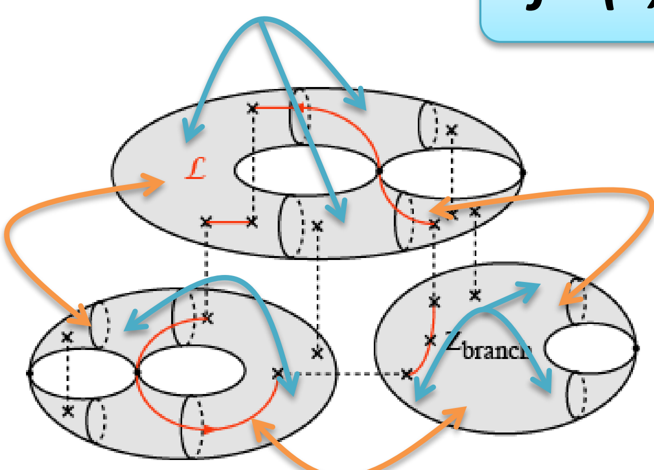
What is the ***physical meaning*** of these factrization?

[Eynard-Orantin '07] **topological recursion can tell us information of All order Perturbative correlators**

The Free-energy and correlators only depend on

- One-point function: F(x,W)=0
- Two-point function B(x,y): the Bergman kernel

If B(x,y)=0 then ***all connected correlators vanish***

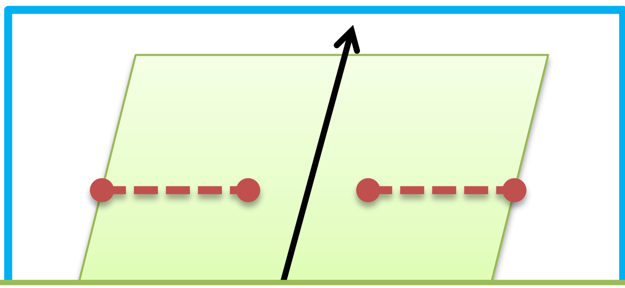


- Perturbative interactions
- Only non-perturbative interactions

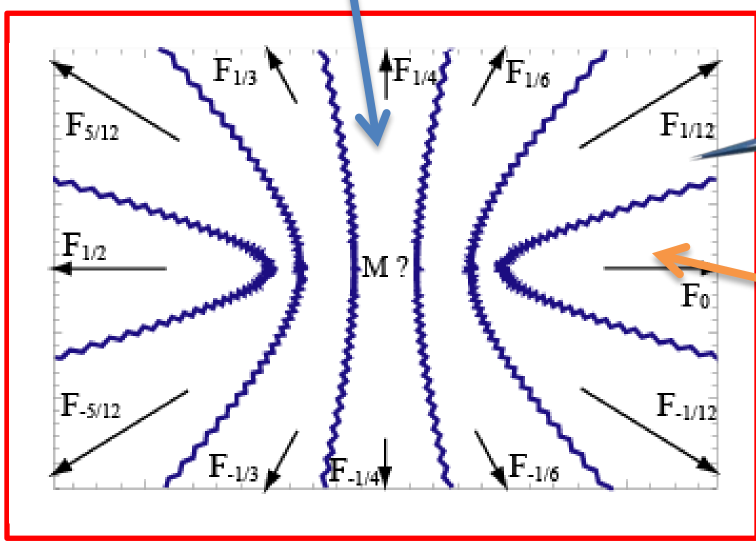
This system has
many ***perturbatively isolated sectors***

Another realization of perturbative strings

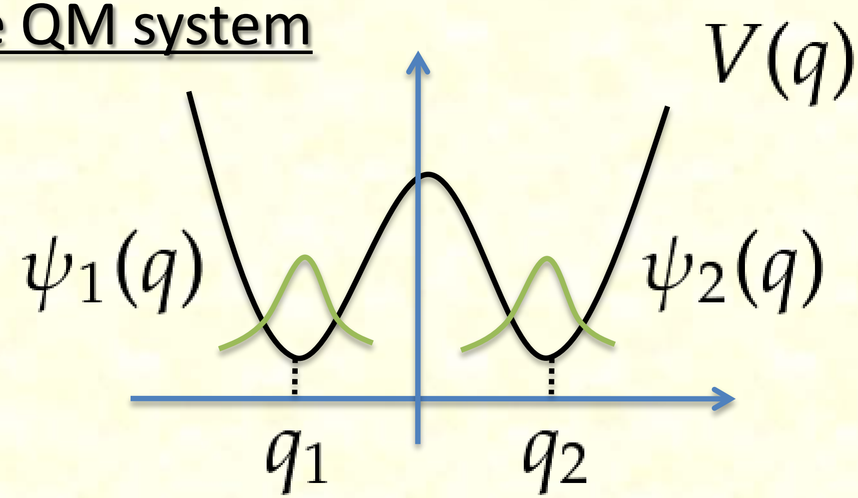
Within perturbation theory, we cannot distinguish *perturbative strings from perturbative isolated sectors!!*



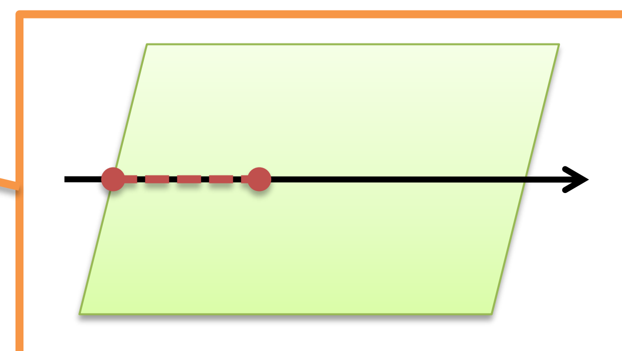
Perturbatively (all order) type 0 Superstring



<u>Like QM system</u>



Perturbatively (all order) Bosonic string



Multi-cut matrix models as non-critical M theory

What is M?

Mother?, Membrane?, Matrix?, Mysterious?, Witten?,
10+1 Dim, Unification of type A spectrum,…

At least except for ***Membrane***, *our system resembles M theory*

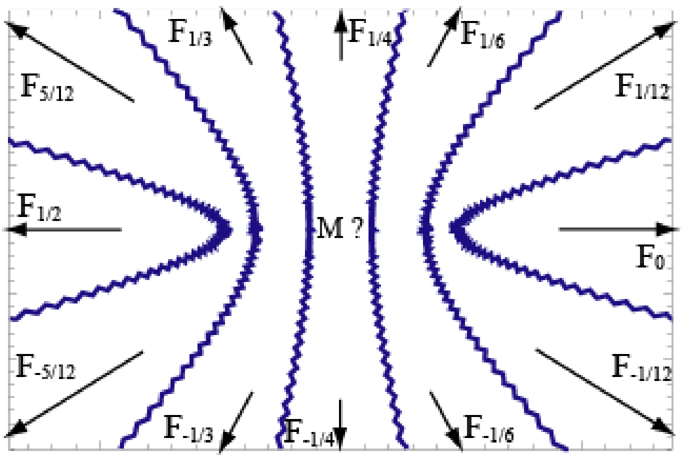
Horava-Keeler's non-critical M theory [Horava-Keeler '05]

In 2005, Horava and Keeler proposed that by adding one angular dimension in 2D string theory: $x \longrightarrow e^{i\theta}x$

One can define the non-critical version of M theory (3D M theory)

Our model itself looks *different from Horava-Keeler's model*, but *the philosophy is the same*.

2+1 Dim,

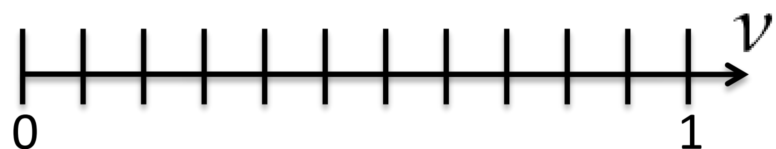


それぞれの $F_{\nu_j}(x, W) = 0$
は、2次元の弦理論である。

それに加え、セクターのラベル

$$\nu_j \sim \nu_j + 1$$

は3次元目（角度方向）に見える。



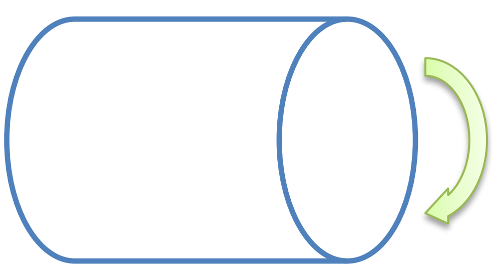
Z_k Charge conjugation は

$$\nu \to \nu + \frac{1}{k}$$

$$\nu = \frac{j-1}{k}, (j = 1, 2, \cdots, k)$$

[Takayanagi-Toumbas’03] [Douglas et.al. ‘03](2-cut) [Fukuma-HI’06] (multi-cut)

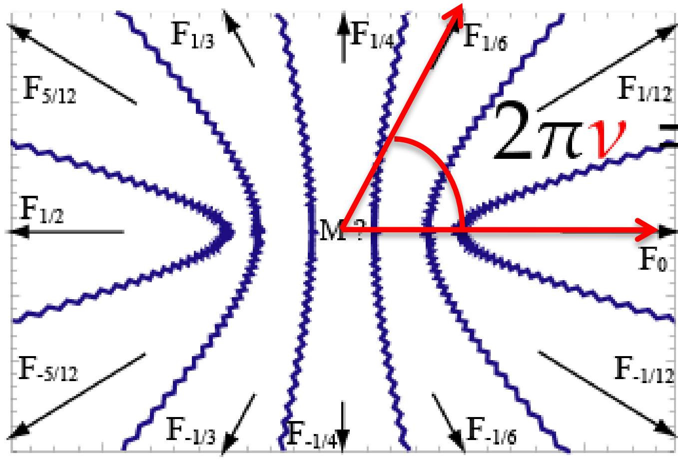
チャージが位置の情報に代わる ➔ Kaluza-Klain reduction!



$$\theta \to \theta + a$$

$$A_\mu(x)$$

Mother theory



$$2\pi\nu = 2\pi\frac{j-1}{k}$$

This 12-cut matrix model (12-Fractional superstring theory) includes all the perturbative strings of

1(=Bosonic)-FSST, 2(=Super)-FSST, 3-FSST,
4-FSST, 6-FSST and 12-FSST

In the same way,

12-FSST C 24-FSST C …… C ∞-FSST

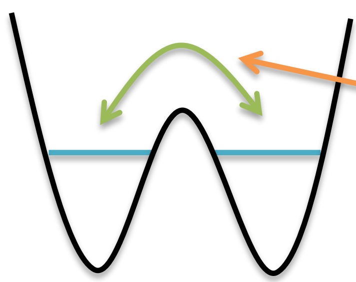
Infinite-cut matrix model (Infinite-Fractional SST)
is the **Mother Theory** of Fractional Superstring Theory

The 3rd direction is non-perturbative

No contradiction with c=1 barrier!

D.O.F is membrane? or else?

Break down of Large N exp.!
[Bonnet-David-Eynard ‘00]



This motion does not have the good Large N exp.

strong coupling dual theory
is described by ***something other than strings***?

We define the strong coupling dual theory
of (p,q) minimal fractional superstring theory
*as **(p,q) minimal non-critical M theory***

Summary

- We proposed a possibility to have various *non-perturbative string landscape models* in the *exactly solvable framework* of 2D string theory.
- We showed various roles which are played by perturbative string theories
- Z_k symmetric critical points
  ➔ Multi-perturbative vacua in string landscape
- FSST critical points
  ➔ Vacuum wave func. itself is non-perturbative
  (superposition of perturb. strings)
- If our non-critical M theory shares various essence of M theory with the critical one, it is very nice!

Note) What is M/String theory limit? [Horava-Keeler '05]

String theory description is good in the *Large* radius limit

$$R_3 = \frac{l_3}{g_s^2}$$

Therefore, non-critical M theory appears in the *Small* radius limit