0-picture superstring における no ghost 定理

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1. Introduction (picture)

picture とは何か？

conformal ghost b, c $\xrightarrow{\text{super partner}}$ super ghost β, γ

$$[\gamma_r, \beta_s] = \delta_{r+s,0}$$

$$\beta_r \left|0_l\right\rangle = 0 \quad r > -l - 3/2$$

$$\gamma_r \left|0_l\right\rangle = 0 \quad r > l + 1/2$$

$\boxed{l}$: picture 数

superstring field theory : 0 picture $\left|0_0\right\rangle = c_1 \left|0\right\rangle$

Preitschopf, Thorn and Yost, 1990
Arefeva, Medvedev, Zubarev, 1990
Berkovits, 1995

natural picture : -1 picture $\left|0_{-1}\right\rangle = \delta(\gamma_{\frac{1}{2}}) \left|0_0\right\rangle$

1. Introduction (概要)

相関関数は picture 数 **-2** の場合にのみ非零

picture changing operator

$$X(z) = [Q_{\rm B}, \Theta(\beta(z))] \qquad +1$$

$$Y(z) = -c(z)\delta'(\gamma(z)) \qquad -1$$

$$\text{amplitude} \sim \langle XX\ldots\rangle$$

示したいこと

・・・ 0-picture superstring theory における <u>no ghost 定理</u>

cf. Kugo-Ojima (for Yang-Mills)

⇕

ノルムの半正定値性(物理的状態)

$$\langle\phi|\, g\, |\psi\rangle$$

♯pic 0 -2 0

Outline

１. Introduction (picture & 概要)

２. Bosonic string の no ghost 定理

（加藤・小川の証明)

３. (-1)-picture の no ghost 定理

４. 0-picture の no ghost 定理

５. 展望

2. Bosonic string（KO 1）

Kato, Ogawa, 1983

$$p^\mu = (p^+, p^-, 0, \cdots, 0)$$

物理的状態:

$$\boxed{\begin{aligned} Q_\mathrm{B} \left|\mathrm{phys}\right\rangle &= 0 \\ b_0 \left|\mathrm{phys}\right\rangle &= 0 \end{aligned}}$$

$$Q_\mathrm{B} = \int \frac{dz}{2\pi i} \left(cT^\mathrm{m} + bc\partial c + \frac{3}{2}\partial^2 c \right)(z)$$

内積 (ノルム) $\left\langle\psi\right| c_0 \left|\phi\right\rangle$

$$\{ \underset{\text{1次}}{Q_\mathrm{B}}, c \} = \underset{\text{2次}}{c\partial c}$$

← Kugo-Ojima を適用できない

2. Bosonic string（KO 2）

$$\boxed{Q_{\mathrm{B}} = c_0 L_0 + b_0 M + \widetilde{Q}_B}$$

$$L_0 = \alpha' p^2 + \sum_{n=1}^{\infty} \alpha_{-n} \cdot \alpha_n + \sum_{n=1}^{\infty} n(c_{-n} b_n + b_{-n} c_n) \qquad M = -2 \sum_{n=1}^{\infty} n c_{-n} c_n$$

3次

$$\widetilde{Q}_B = \sum_{n \neq 0} c_{-n} L_n^{(\mathrm{m})} - \frac{1}{2} \sum_{nm(n+m)\neq 0} (n-m) \mathop{\circ}\limits^{\circ} c_{-n} c_{-m} b_{n+m} \mathop{\circ}\limits^{\circ}$$

$$Q_{\mathrm{B}} \left|\mathrm{phys}\right\rangle = 0 \quad \Longrightarrow \quad \boxed{\widetilde{Q}_B \left|\mathrm{phys}\right\rangle = 0}$$

$$\boxed{\alpha' \to \frac{1}{\hbar^2}\alpha' \quad p^- \to \hbar^2 p^-, \quad x^+ \to \frac{1}{\hbar^2} x^+}$$

$$\boxed{\widetilde{Q}_B(\hbar) = A + \hbar B + \hbar^2 C}$$

2. Bosonic string（KO 3）

$$|\psi(\hbar)\rangle = \sum_{n=0}^{\infty} \hbar^n |\psi_n\rangle$$

物理的状態 $\widetilde{Q}_B(\hbar)\,|\psi(\hbar)\rangle = 0$

$$\boxed{A\,|\psi_n\rangle + B\,|\psi_{n-1}\rangle + C\,|\psi_{n-2}\rangle = 0} \quad \text{for} \quad n \geq 0$$

0次 **補題**

$$A\,|\psi_0\rangle = 0$$

$$A^2 = 0 \qquad A = \sum_{n=1}^{\infty} (c_{-n} p^+ \alpha_n^- + p^+ \alpha_{-n}^- c_n)$$

bilinear

Kugo-Ojima

$$\boxed{|\psi_0\rangle = P_T\,|\psi_0\rangle + A\,|\phi\rangle}$$

横波モードのみからなる状態への射影

2. Bosonic string（KO 4）

フルオーダー

$$\widetilde{Q}_B(\hbar)\,|\phi\rangle = 0$$

$$|\phi\rangle = P_{\mathrm{DDF}}(\hbar)\,|\phi\rangle + \widetilde{Q}_B(\hbar)\,|\psi\rangle$$

正定値ノルム　　　　ゼロノルム

DDF state への射影 $P_{\mathrm{DDF}}(\hbar) = \sum_m \frac{1}{m!}\left(\frac{1}{n_1}A^{\dagger}_{n_1,i_1}\cdots\frac{1}{n_m}A^{\dagger}_{n_m,i_m}\,|0\rangle\,\langle 0|\,A_{n_1,i_1}\cdots A_{n_m,i_m}\right)$

$$A^i_n(\hbar = 0) = \alpha^i_n$$

証明に必要なもの

補題(0次)　A,B,C にまつわる代数関係

$$P_{\mathrm{DDF}}(\hbar)\widetilde{Q}_B = 0 \qquad P_T = P_{\mathrm{DDF}}(\hbar = 0)$$

$$P_{\mathrm{DDF}}(\hbar)^2 = P_{\mathrm{DDF}}(\hbar)$$

2. Bosonic string (おさらい)

物理的状態: $\begin{cases} Q_{\mathrm{B}} \left|\mathrm{phys}\right\rangle = 0 \\ b_0 \left|\mathrm{phys}\right\rangle = 0 \end{cases}$

内積: $\langle \psi | \, c_0 \, | \phi \rangle$

$$\begin{tikzcd} Q_{\mathrm{B}} \arrow[r, "c_0 \quad b_0"] & \widetilde{Q}_B \arrow[r, "\hbar"] & A \arrow[l, bend left] & \boxed{\text{bilinear}} \end{tikzcd}$$

補題(0次)　A,B,C にまつわる代数関係

$$P_{\mathrm{DDF}}(\hbar)\widetilde{Q}_B = 0 \qquad P_T = P_{\mathrm{DDF}}(\hbar = 0)$$

$$P_{\mathrm{DDF}}(\hbar)^2 = P_{\mathrm{DDF}}(\hbar)$$

3. -1 picture

Ito,Morozumi,Nojiri,Uehara,1986
Ohta,1986

証明の流れ：bosonic と同じ

物理的状態： $\begin{cases} Q_{\mathrm{B}} \left|\mathrm{phys}\right\rangle = 0 \\ b_0 \left|\mathrm{phys}\right\rangle = 0 \end{cases}$

内積： $\left\langle\psi\right| c_0 \left|\phi\right\rangle$

$$\begin{tikzcd} Q_{\mathrm{B}} \arrow[r, "c_0 \quad b_0"] & \widetilde{Q}_B \arrow[r, "\hbar"] & A \ \boxed{\text{bilinear}} \arrow[l, bend left] \end{tikzcd}$$

補題(0次)　A,B,C にまつわる代数関係

$$P_{\mathrm{DDF}}(\hbar)\widetilde{Q}_B = 0 \qquad P_T = P_{\mathrm{DDF}}(\hbar = 0)$$

$$P_{\mathrm{DDF}}(\hbar)^2 = P_{\mathrm{DDF}}(\hbar)$$

4. 0-picture

流れ

$$Q_{\mathrm{B}} \xrightarrow{c_0 \quad b_0} \tilde{Q}_B \xrightarrow{\hbar} A \quad \text{bilinear}$$

真空の違い

自明

補題(A のコホモロジー)　A,B,C にまつわる代数関係

$$P_{\mathrm{DDF}}(\hbar)\tilde{Q}_B = 0 \qquad P_T = P_{\mathrm{DDF}}(\hbar = 0)$$

$$P_{\mathrm{DDF}}(\hbar)^2 = P_{\mathrm{DDF}}(\hbar)$$

内積　$\langle \phi | \, g \, | \psi \rangle$

♯pic　0　-2　0

4. 0-picture (真空)

-1picture $\widetilde{Q}_B\left|0_{-1}\right\rangle = 0$ $\{A^i_n, \widetilde{Q}_B\} = 0$

0 次

$$A\left|0_0\right\rangle = \gamma_{\frac{1}{2}} p^+ \psi^-_{-\frac{1}{2}} \left|0_0\right\rangle \neq 0$$

$$\left|\mathrm{vac}_0(\hbar)\right\rangle = \sum_{n\geq 0} \hbar^n \left|\mathrm{vac}_0^{(\mathrm{n})}\right\rangle$$

$$\widetilde{Q}_B(\hbar)\left|\mathrm{vac}(\hbar)\right\rangle = 0$$

$$\boxed{\begin{aligned}\left|\mathrm{vac}_0(\hbar)\right\rangle &= \left(\sqrt{2\alpha'}p^+\psi^-_{-\frac{1}{2}} - \hbar\gamma_{\frac{1}{2}} b_{-1} + \hbar^2\sqrt{2\alpha'}p^2\psi^+_{-\frac{1}{2}}\right)\left|0_0\right\rangle = X_{-\frac{1}{2}}\left|0_{-1}\right\rangle \\ &= X(0)\left|0_{-1}\right\rangle\end{aligned}}$$

$$X_{-\frac{1}{2}} = [Q_B, \Theta(\beta_{-\frac{1}{2}})]$$

4. 0-picture (内積)

$$\langle \phi | \, g \, | \psi \rangle$$

♯pic	0	-2	0

$$g(\hbar) = \sum_{n=0}^{\infty} \hbar^n g^{(n)}$$

$g(\hbar)$ の満たすべき条件:

条件1:

$$[\widetilde{Q}_B, g(\hbar)] = 0$$

条件2:

$$\langle \mathrm{vac}_0(\hbar) | \, g(\hbar) \, | \mathrm{vac}_0(\hbar) \rangle = 1$$

$$\boxed{g(\hbar) = c_0 \Psi^+(\hbar)_{-\frac{1}{2}} \delta(\Gamma(\hbar)_{-\frac{1}{2}}) \delta(\Gamma(\hbar)_{\frac{1}{2}}) \Psi^+(\hbar)_{\frac{1}{2}}}$$

$$\Psi^+(\hbar)_{\frac{1}{2}} = \left(\frac{1}{p^+}\right)^{\frac{1}{2}} \oint \frac{dz}{2\pi i} \psi^+(z) \left(z i\partial X^+(z)\right)^{\frac{1}{2}}$$

$$\Gamma(\hbar)_{\frac{1}{2}} = \left(\frac{1}{p^+}\right)^{\frac{3}{2}} \oint \frac{dz}{2\pi i} \Big(\frac{1}{z}\gamma(z) \left(z i\partial X^+(z)\right)^{\frac{3}{2}} - \frac{1}{2z} c(z)\psi^+(z) \left(z i\partial X^+(z)\right)^{\frac{1}{2}} - \frac{z}{2}\gamma(z)\psi^+\partial\psi^+(z) \left(z i\partial X^+(z)\right)^{\frac{1}{2}} \Big)$$

$$\Psi^+(\hbar)_{-\frac{1}{2}} = \Psi^+(\hbar)^{\dagger}_{\frac{1}{2}}, \quad \Gamma(\hbar)_{-\frac{1}{2}} = \Gamma(\hbar)^{\dagger}_{\frac{1}{2}}$$

4. 0-picture (no ghost 定理 1)

補題(0次)

bilinear

$$A\left|\psi_0\right\rangle = 0$$

Kugo-Ojima

$$\left|\psi_0\right\rangle = P_T\left|\psi_0\right\rangle + A\left|\phi\right\rangle$$

$$P_T = \sum_m \frac{1}{m!}\left(\frac{1}{n_1}\alpha^\dagger_{n_1,i_1}\cdots\frac{1}{r_1}\psi^\dagger_{r_1,j_1}\cdots\boxed{\left|\mathrm{vac}_0^{(0)}\right\rangle\left\langle\mathrm{vac}_0^{(0)}\right| g^{(0)}}\alpha_{n_1,i_1}\cdots\psi_{r_1,j_1}\cdots\right)$$

横波モードのみからなる状態への射影

4. 0-picture (no ghost 定理 2)

補題(Aのコホモロジー)　A,B,C の代数関係

$$P_{\mathrm{DDF}}(\hbar)\widetilde{Q}_B = 0 \qquad P_T = P_{\mathrm{DDF}}(\hbar = 0)$$

$$P_{\mathrm{DDF}}(\hbar)^2 = P_{\mathrm{DDF}}(\hbar)$$

$$|\mathrm{DDF}_0(\hbar)\rangle = (A^\dagger_{n,i} \cdots B^\dagger_{r,i} \cdots)\,|\mathrm{vac}_0(\hbar)\rangle = X(0)\,|\mathrm{DDF}_{-1}\rangle$$

DDF 状態への射影演算子

$$P_{\mathrm{DDF}}(\hbar) = \sum_m \frac{1}{m!}\left(\frac{1}{n_1}A^\dagger_{n_1,i_1}\cdots\frac{1}{r_1}B^\dagger_{r_1,j_1}\cdots\boxed{|\mathrm{vac}_0(\hbar)\rangle\langle\mathrm{vac}_0(\hbar)|\,g(\hbar)}A_{n_1,i_1}\cdots B_{r_1,j_1}\cdots\right)$$

$$P_T = \sum_m \frac{1}{m!}\left(\frac{1}{n_1}\alpha^\dagger_{n_1,i_1}\cdots\frac{1}{r_1}\psi^\dagger_{r_1,j_1}\cdots\boxed{\left|\mathrm{vac}_0^{(0)}\right\rangle\left\langle\mathrm{vac}_0^{(0)}\right|\,g^{(0}}\alpha_{n_1,i_1}\cdots\psi_{r_1,j_1}\cdots\right)$$

4．0-picture (no ghost 定理 3)

$$\widetilde{Q}_B(\hbar)\,|\phi\rangle = 0$$

$$\Downarrow$$

$$|\phi\rangle = P_{\mathrm{DDF}}(\hbar)\,|\phi\rangle + \widetilde{Q}_B(\hbar)\,|\psi\rangle$$

正定値ノルム　　　　ゼロノルム

$$\langle \mathrm{DDF}_0(\hbar)|\, g(\hbar)\, |\mathrm{DDF}_0(\hbar)\rangle = \langle \mathrm{DDF}_{-1}(\hbar)|\, c_0\, |\mathrm{DDF}_{-1}(\hbar)\rangle \boxed{> 0}$$

4. 0-picture (おさらい)

1. 0-picture の真空, 内積を定義

$$|\mathrm{vac}_0(\hbar)\rangle = X(0)\,|0_{-1}\rangle$$

$$g(\hbar) = c_0\Psi^+(\hbar)_{-\frac{1}{2}}\delta(\Gamma(\hbar)_{-\frac{1}{2}})\delta(\Gamma(\hbar)_{\frac{1}{2}})\Psi^+(\hbar)_{\frac{1}{2}}$$

2. 0-picture の no ghost 定理の証明

$$|\phi\rangle = P_{\mathrm{DDF}}(\hbar)\,|\phi\rangle + \widetilde{Q}_B(\hbar)\,|\psi\rangle$$

$$|\mathrm{DDF}_0\rangle = X(0)\,|\mathrm{DDF}_{-1}\rangle$$

5. 展望

new picture changing operator

$$\tilde{Y} = \frac{\psi^+}{i\partial X^+}\delta\left(\Gamma\right)$$ non covariant

$$\Gamma = \left\{Q_B, \frac{\psi^+}{i\partial X^+}\right\} = \gamma + \gamma\frac{\psi^+\partial\psi^+}{(i\partial X^+)^2} + c\partial\left(\frac{\psi^+}{i\partial X^+}\right) - \partial c\frac{\psi^+}{i\partial X^+}$$

$$\lim_{z\to w}\tilde{Y}(z)\tilde{Y}(w) = \partial\left(\frac{\psi^+}{i\partial X^+}\right)\delta(\partial\Gamma)\delta(\Gamma)\frac{\psi^+}{i\partial X^+}$$

n 乗が有限になる ⟶ SFT の構成に使える？