Branes in the OSP(1|2) WZNW model

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Ref. T. Creutzig&YH, arXiv:1004.1977 [hep-th]

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Why supergroup models are interesting

1. INTRODUCTION

AdS/CFT correspondence

[Maldacena '97]

d+1 dim. superstrings on Anti-de Sitter (AdS) space

$$ds^2 = \frac{dr^2}{r^2} + r^2 dx^\mu dx_\mu$$

$\Longleftrightarrow$

d dim. conformal field theory（CFT）

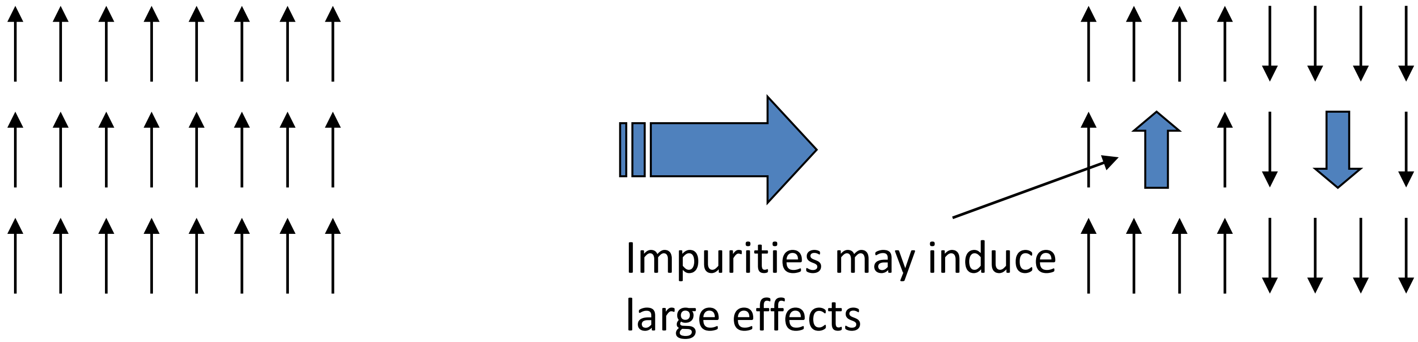
lives at the boundary of AdS space $r \to \infty$

- Difficulty
  - It is quite difficult to go beyond the classical gravity limit of the correspondence
- Superstrings on AdS spaces
  - Problems reduce to solve models of supergroups

$$PSU(1,1|2), \frac{PSU(2,2|4)}{SO(4,1)\times SO(5)}, \cdots$$

Disordered systems

- Impurities
  - Real material, spin glass, quantum Hall effects

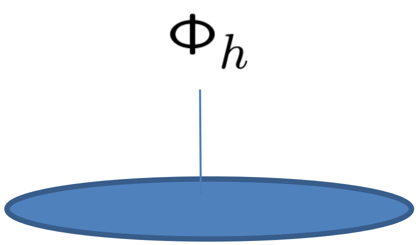


- Methods for random systems
  - Replica method
    - Prepare *n* replica theories and take *n*→0 limit
  - Supersymmetric method
    - Use model with supergroup symmetry

$$OSP(N|N), PSU(N|N), ..$$

Results

- The OSP(1|2) WZNW model
  - We study OSP(1|2) WZNW model as a simple example among supergroup models
  - Models without boundary have been studied by utilizing the relation to *N*=1 super Liouville theory
    [YH-Schomerus '07]
- Branes in the model
  - There are two types of branes
    - Super AdS_2 branes, super spherical branes
  - Compute coupling of closed strings to these branes
  - Quite analogous to branes in AdS_3
    [Lee-Ooguri-Park '01, Ponsot-Schomerus-Teschner '01]



Plan

1. Introduction
2. The OSP(1|2) WZNW model
3. Branes in the OSP(1|2) WZNW model
4. Conclusion

The OSP(1|2) WZNW model and correlations functions

2. THE OSP(1|2) WZNW MODEL

OSP(1|2) WZNW model

- OSP(1|2) Lie superalgebra
  - Generators satisfy (anti-)commutation relations

$$[H, E^{\pm}] = \pm E^{\pm},\ [E^{+}, E^{-}] = -2H,\ [H, F^{\pm}] = \pm\tfrac{1}{2}F^{\pm},$$
$$[E^{\pm}, F^{\mp}] = \pm F^{\pm},\ \{F^{+}, F^{-}\} = -\tfrac{1}{2}H,\ \{F^{\pm}, F^{\pm}\} = \tfrac{1}{2}E^{\pm}$$

- The action of the model
  - Constructed by the standard WZNW action

$$S = \frac{k}{4\pi}\int d^2z \langle g^{-1}\partial g, g^{-1}\bar{\partial} g\rangle + \Gamma_{WZ}$$

  - Two extra Grassmann odd paramters are introduced as

$$g = e^{2\theta F^{+}} e^{\gamma E^{+}} e^{2\phi H} e^{\bar{\gamma} E^{-}} e^{2\bar{\theta} F^{-}}$$

Current algebra & vertex operator

- OSP(1|2) current algebra
  - Global symmetry → Local symmetry

$$h_L(z) \curvearrowright g(z,\bar{z}) \curvearrowleft h_R^{-1}(\bar{z})$$

  - Generators of OSP(1|2) currents are given by fermionic ones $j^{\pm}(z)$ along with bosonic ones $J^{\pm}(z), J^3(z)$
- Vertex operators
  - Vertex operators are defined by

$$J^A(z)\Phi_h(x,\xi|w) \sim -\frac{\mathcal{D}^A\Phi_h(x,\xi|w)}{z-w}$$

  - One extra Grassmann odd paremeter ξ can be interpreted as a superpartner of *x*.

$$\mathcal{D}^{E^+} = \partial_x, \ \mathcal{D}^H = -x\partial_x - \tfrac{1}{2}\xi\partial_\xi - h,$$
$$\mathcal{D}^{E^-} = x^2\partial_x + x\xi\partial_\xi + 2xh,$$
$$\mathcal{D}^{F^+} = \tfrac{1}{2}(\partial_\xi + \xi\partial_x), \ \mathcal{D}^{F^-} = \tfrac{1}{2}x(\partial_\xi + \xi\partial_x) + \xi h$$

Correlation functions

- OSP(1|2) symmetry restricts the form of correlation functions to large extent
  - Three point function

$$\left\langle \prod_{i=1}^{3} \Phi_{h_i}(x_i, \xi_i | z_i) \right\rangle \propto \frac{C(h_i) + \eta\bar{\eta}\tilde{C}(h_i)}{\prod_{i<j} |x_i - x_j + \xi_i\xi_j|^{2h_i+2h_j-2h_k}}$$

  - Fermionic cross ratio
    - Contrary to the bosonic case, the three point function depends on one fermionic cross racio

$$\eta = (x_{12}x_{23}x_{31})^{-\frac{1}{2}}(x_{23}\xi_1 + x_{31}\xi_2 + x_{12}\xi_3 + \tfrac{1}{2}\xi_1\xi_2\xi_3)$$

- Correlation functions are obtained by using
  - OSP(1|2) model $\Longleftrightarrow$ *N*=1 super Liouville [YH-Schomerus '07]
  - c.f. SL(2) model $\Longleftrightarrow$ Liouville theory [Ribault-Teschner '05]

The OSP(1|2) WZNW model with the boundary of world-sheet

3. BRANES IN THE OSP(1|2) WZNW MODEL

Geometry of the branes

- Maximally symmetric branes

$$h_L(z) \curvearrowright g(z,\bar{z}) \curvearrowleft h_R^{-1}(\bar{z})$$

$\Downarrow$ Presence of worldsheet boundary

$$h(t) \curvearrowright g(t) \curvearrowleft \Omega h^{-1}(t)$$

- Conjugacy classes

$$\mathcal{C}^C(g) = \{hg\Omega h^{-1}, h^{\forall} \in OSP(1|2)\}$$

$$\Rightarrow \mathrm{Tr}(g\Omega) = \text{constant}$$

- Geometry of branes

Ω=id.: Super spherical branes Ω=twist: Super AdS_2 branes

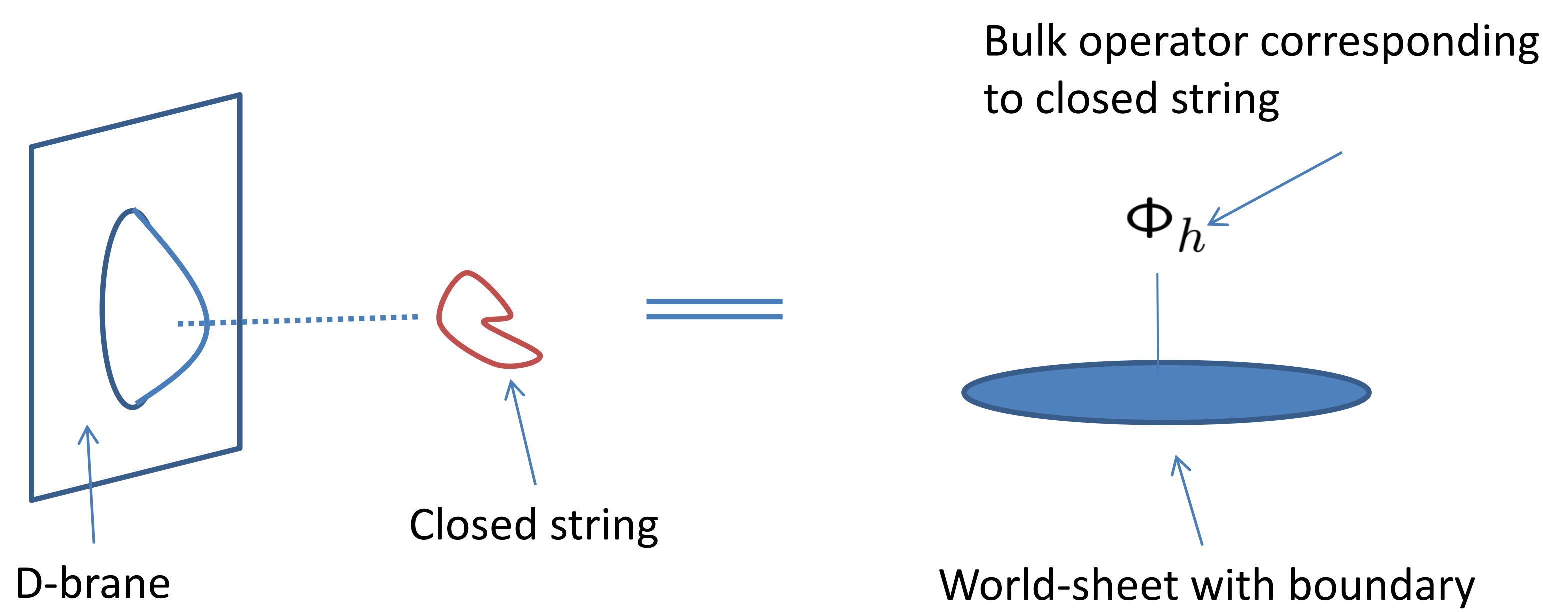
$\Leftrightarrow$

cf. Branes in AdS_3
Ω=id.: Spherical branes
$\Omega=\sigma_1$: AdS_2 branes

[Bachas-Petropoulos '00]

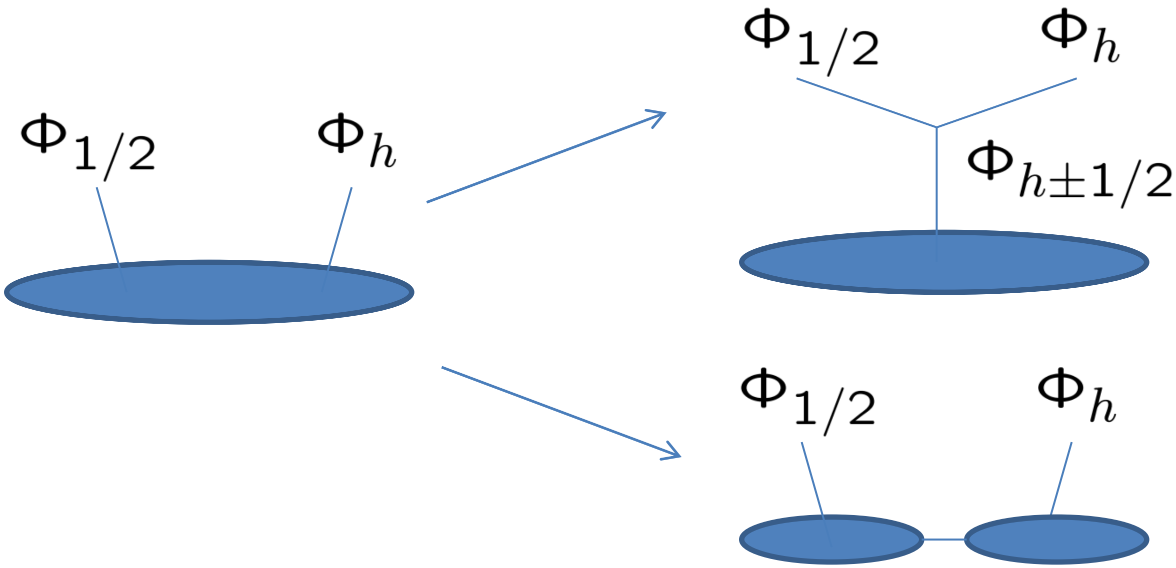
Quantum analysis

- 1 point function of closed string
  - Coupling of closed strings to branes can be read from 1 point function of bulk operator on a disk
    - Presence of some object is found by the coupling to gravity



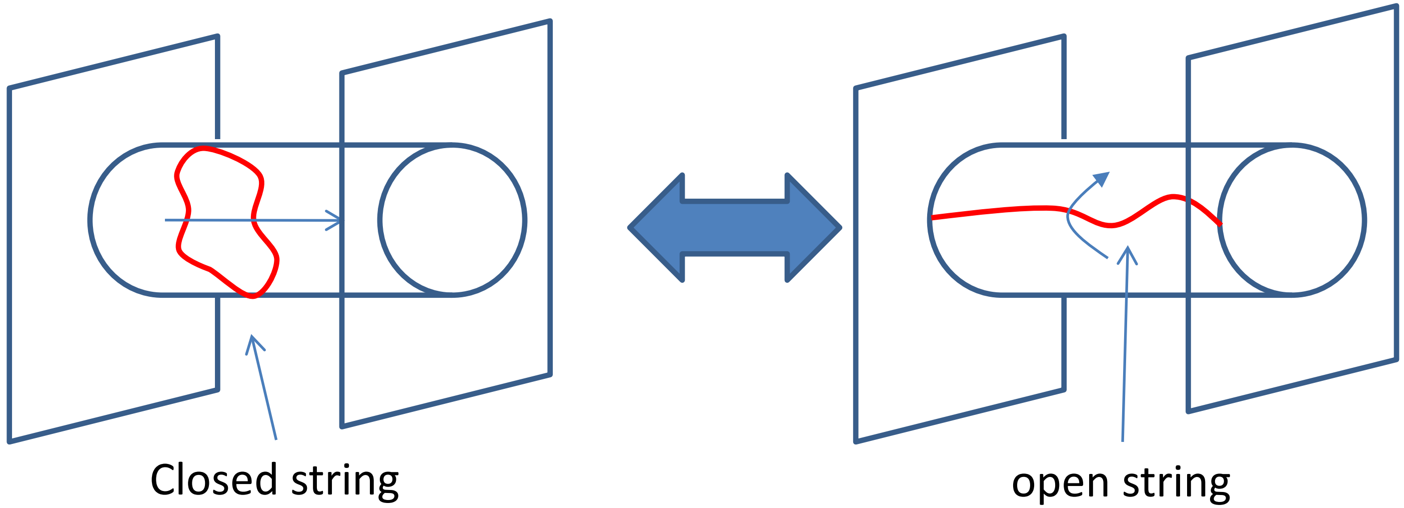
Factorization constraint

- Constraint for the coefficients
  - Two point function on a disk can be factorized in two ways. Comparing the two expressions we obtain a constraint for one point function.



Cardy condition

- Open-closed duality



- Boundary states can be constructed by 1pt function of bulk operator.
- Overlap between boundary states is evaluated by the exchange of closed strings.

- One loop amplitude of open string is computed with density of state $\rho(s)$.
- The density of state is read off from 2pt function of boundary operator.

1 point functions

- 1 point functions of bulk operators on a disk
  - Method 1: Solving the factorization constraint
    - Applicable for both types of branes
    - Difficult to analyze 2pt functions on a disk
  - Method 2: Utilized OSP-super Liouville relation
    - Applicable only for super AdS_2 branes
    - Easy to generalize to the Ramon sector.
- Consistency checks
  - Reproduce classical geometry
  - Open-closed duality
    - Need to include the RR-sector of closed strings

Results

[Creutzig-YH '10]

- 1 point functions $(b^{-2} = 2k - 3, \sigma = \mathrm{sgn}(x + \bar{x}))$
  - Super AdS$_2$ branes

$$\langle \Phi_h(x, \xi | z) \rangle_{\epsilon=+1} \propto \frac{\Gamma(\frac{1}{2} - b^2(2h - \frac{1}{2}))e^{-\sigma(2h - \frac{1}{2})r}}{|x + \bar{x} + \xi\bar{\xi}|^{2h}}$$

$$\langle \Phi_h(x, \xi | z) \rangle_{\epsilon=-1} \propto \sigma \frac{\Gamma(\frac{1}{2} - b^2(2h - \frac{1}{2}))e^{-\sigma(2h - \frac{1}{2})r}}{|x + \bar{x} - \xi\bar{\xi}|^{2h}}$$

Parameters corresponding to positions of branes

  - Super spherical branes

$$\langle \Phi_h(x, \xi | z) \rangle_{\epsilon=+1} \propto \frac{\Gamma(\frac{1}{2} - b^2(2h - \frac{1}{2}))\,\sin(s(2h - \frac{1}{2}))}{|1 + x\bar{x} + \xi\bar{\xi}|^{2h}}$$

$$\langle \Phi_h(x, \xi | z) \rangle_{\epsilon=+1} \propto \frac{\Gamma(\frac{1}{2} - b^2(2h - \frac{1}{2}))\,\cos(s(2h - \frac{1}{2}))}{|1 + x\bar{x} - \xi\bar{\xi}|^{2h}}$$

Summary and future works

4. CONCLUSION

Conclusion

- Branes in the OSP(1|2) WZNW model
  - Two types; super AdS_2 branes, super spherical branes
  - Couplings of closed strings to branes are computed by using factorization constraint and relation to super Liouville theory
  - Cardy condition and classical limit is checked
- Future works
  - Ramond sector should be analyzed more systemativally
  - Generalize to other models, e.g., OSP(2|2) model
  - Mirror symmetry of other cosets, e.g., OSP(1|2)/U(1)
  - Establish SL(N)-Toda relation (related to AGT relation?)
  - Applications to string theory and/or condensed matter physics