Topological odd-parity superconductor

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Outline

a. What is topological superconductor? (8pages 8min)

b. Bulk-Edge Correspondence (3pages 4min)

c. Topological odd-parity superconductors (7page 7min)

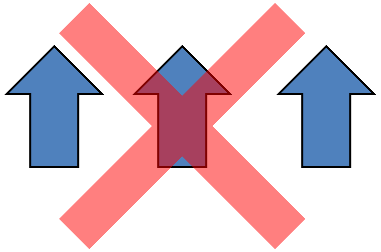
What is topological superconductor ?

- **Topological SC = superconductor with topological order**

What is topological order ?

order which **can not be described by spontaneous symmetry breaking**

$$\xcancel{\langle \phi \rangle \neq 0}$$



Any local order parameter cannot characterize this order

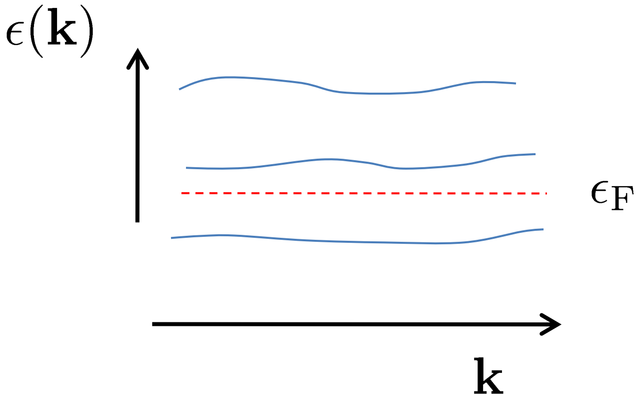
How to characterize topological orders ?

$$\xcancel{\langle \phi \rangle \neq 0}$$

➡ **Topological number of the ground state**

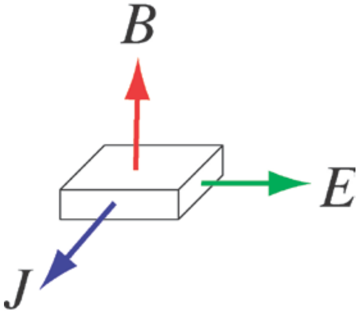
Topological number of the ground state

ex.) Integer Quantum Hall states



N empty bands

M filled bands



$$\sigma_{xy} \equiv \frac{J}{E} = \frac{e^2}{h}\nu$$

"gauge field " $$\mathcal{A}_i(\boldsymbol{k}) = \sum_{n \in \text{filled}} \langle u_n(\boldsymbol{k})|\frac{\partial}{\partial k_i}|u_n(\boldsymbol{k})\rangle$$

Thouless-Kohmoto-Nightingale-den Nijs # (=1st Chern #)

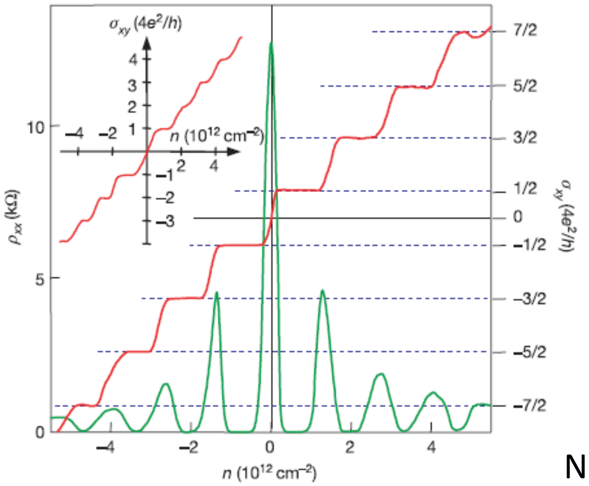
$$\nu = \frac{1}{2\pi i}\int_{\text{BZ}} dk_x dk_y \mathcal{F}_{xy}(\boldsymbol{k})$$

TKNN # explains

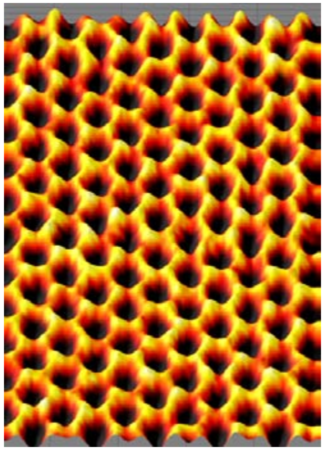
- Quantization of the Hall conductance (or CS terem)

$$\sigma_{xy} \equiv \frac{J}{E} = \frac{e^2}{h}\nu$$

$$\Delta\mathcal{L}_{\mathrm{CS}} = \frac{\nu}{4\pi}\epsilon^{\mu\nu\tau}A_\mu\partial_\nu A_\tau$$



Noveselov et al (05)



graphene

Three importance developments in topological orders

1. New classes of topological orders

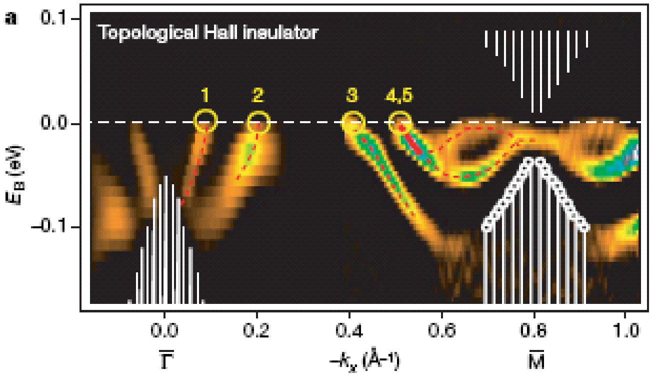
2. Exotic excitations

3. Wide range of application

1. New classes of topological orders

Topological insulators

Kane-Mele (05), Bernevig-Zhang (05),
Moore-Balents(07), Roy(07), Fu-Kane(07)



3dim topological insulator

- $Bi_{0.9}Sb_{0.1}$ （Hsieh et al., Nature (2008)）
- BiSe

ARPES → surface states

Topological superconductors

Qi-Hughes-Raghu-Zhang (08) ,Roy (08),
Schnyder-Ryu-Furusaki-Ludwig (08),
MS (08), MS-Fujimoto (08),
Kou-Wen (09)

2. Exotic excitations

- For superconductors, the Majorana condition is imposed naturally.

quasiparticle in Nambu rep.   quasiparticle   anti-quasiparticle

$$\Psi(x) = \begin{pmatrix} \psi_\uparrow(x) \\ \psi_\downarrow(x) \\ \psi_\uparrow^\dagger(x) \\ \psi_\downarrow^\dagger(x) \end{pmatrix} \quad \Rightarrow \quad \boxed{\Psi(x) = \mathcal{C}\Psi^*(x), \quad \mathcal{C} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}$$

Majorana condition

For topological superconductors, there exists gapless boundary state with linear dispersions

→ Majorana fermions

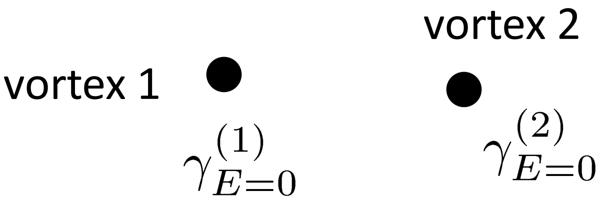
In particular, for a zero mode (E=0) in a vortex,

$$\boxed{\gamma_{E=0}^{\dagger} = \gamma_{E=0}}$$

creation = annihilation

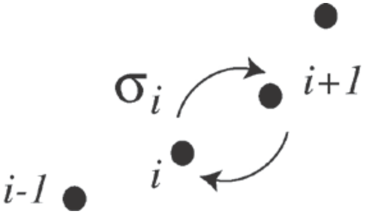
・・contradiction (for a single Majorana zero mode in a vortex)

Fortunately, we always have a pair of the vortices, so it is possible to obtain a well-defined creation op .



$$\boxed{\gamma^{\dagger} = \gamma_{E=0}^{(1)} + i\gamma_{E=0}^{(2)} \quad \{\gamma^{\dagger}, \gamma\} = 1}$$

This non-local definition of creation op. gives rise to non-abelian anyon statistics of the vorticies.



$$\sigma_i \sigma_{i+1} \neq \sigma_{i+1} \sigma_i$$

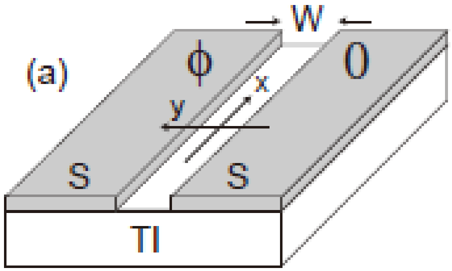
Read-Green (00)
Ivanov (01)
MS-Fujimoto (08)
MS-Takahashi-Fujimoto (09)
MS (09), MS-Takahashi-Fujimoto(10)

3. Wide range of application

1) Non-Abelian statistics of Axion strings MS (03)

Interface between topological insulator and superconductor

Fu-Kane (08)



2) Topological color superconductor

Y. Nishida, Phys. Rev. D81, 074004 (2010)

Outline

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Bulk-edge correspondence

“ Nambu-Goldstone theorem”   “ index theorem”

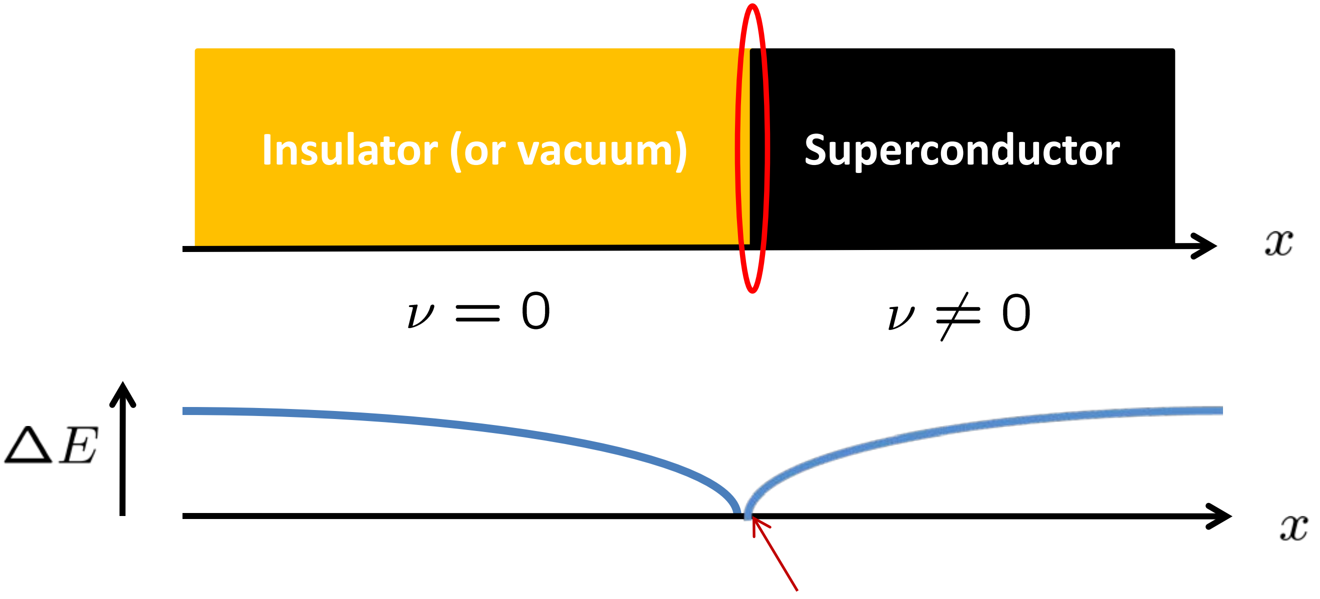
For topological insulators/superconductors, there exist gapless states localized on the boundary.

A change of the topological number = gap closing

When the gap of the system closes, $|u_n(\mathbf{k})\rangle$ $n \in$ filled is ill-defined.

➡ **A discontinuous jump of the topological number**

Therefore,

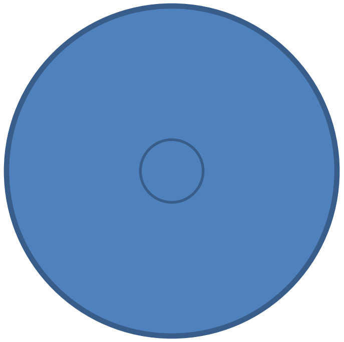


Gapless edge state

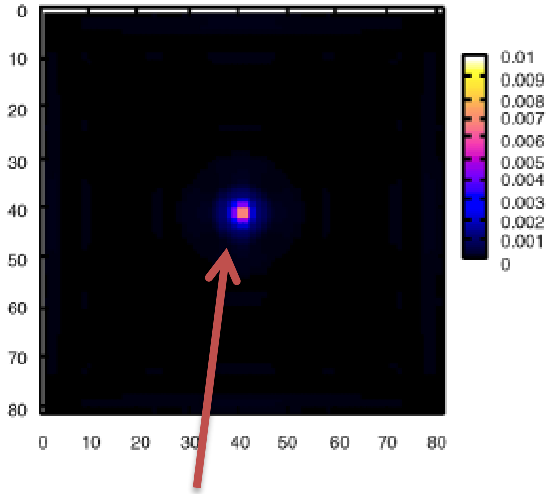
Non-trivial topological number also implies the zero mode in a vortex

◆ vortex = hole in bulk superconductors

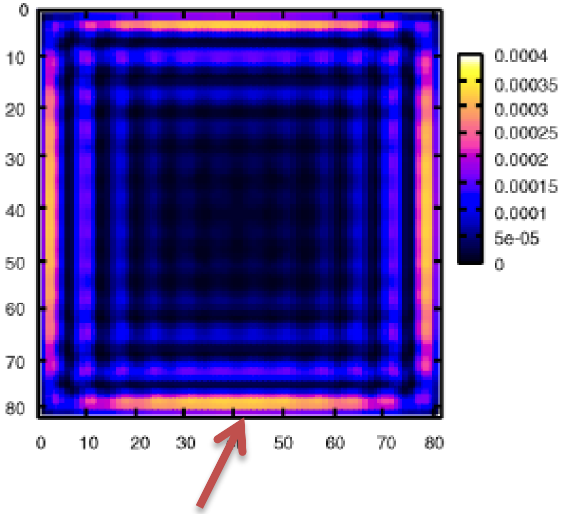
◆ zero mode = gapless state on the edge of hole



Sol. of BdG eq. with a vortex



zero mode in a vortex



zero mode on an edge

MS, Fujimoto PRB (09)

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Topological odd-parity superconductors

MS(08), (09)
Fu-Berg, arXiv:0912.3294

Superconductors

- Spin-singlet (even parity) SC

$$\Delta(\boldsymbol{k})=\Delta(-\boldsymbol{k})$$

- Spin-triplet (odd-parity) SC

$$\Delta(\boldsymbol{k})=-\Delta(-\boldsymbol{k})$$

- Non-centrosymmetric SC
(~ SC without parity symmetry)

Spin-singlet+ spin-triplet

- ● Non-Abelian anyon is possible
- ● The obtained results can be extended to the non-centrosymmetric SC

MS-Fujimoto (09) (10)

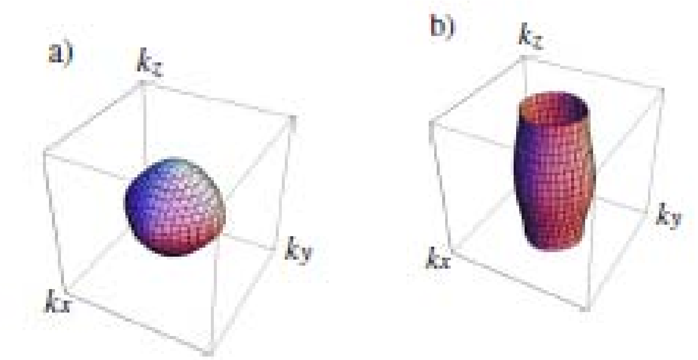
Main result

The topology of the Fermi surface characterizes topological properties of odd-parity (spin-triplet) superconductors

Fermi surface

Euler's character

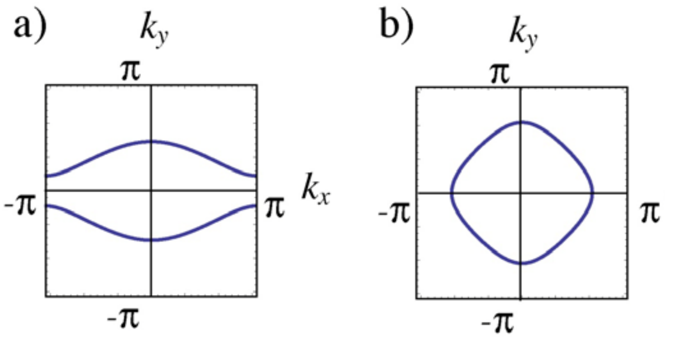
$\chi(S_{\mathrm{F}})$



$\chi(S_{\mathrm{F}}) = 2$ $\chi(S_{\mathrm{F}}) = 0$

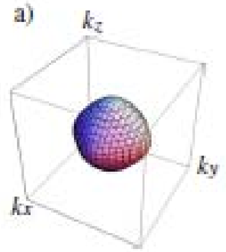
# of connected component

$p_0(S_{\mathrm{F}})$



$p_0(S_{\mathrm{F}}) = 2$ $p_0(S_{\mathrm{F}}) = 1$

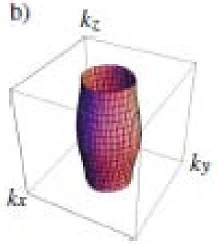
1) For time-reversal invariant odd-parity superconductor



$$(-1)^{\chi(S_{\rm F})/2} = -1$$

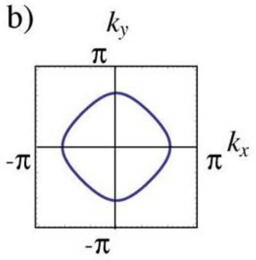
Topological SC

- Gapless boundary state
- pair of zero modes in a vortex



$$(-1)^{\chi(S_{\rm F})/2} = 1$$

2) For time-reversal breaking odd-parity superconductor

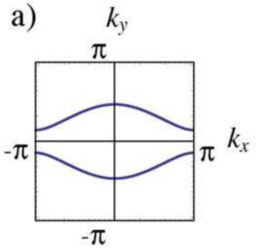


$$(-1)^{p_0(S_{\rm F})} = -1$$

Non-Abelian anyon

ogical SC

- Gap boundary state
- **single** zero m de in a vortex



$$(-1)^{p_0(S_{\rm F})} = 1$$

~~non-Abelian anyon~~

Our idea (outline)

◆ Use special symmetry of Hamiltonian to calculate the topological #

$$\Delta(\boldsymbol{k}) = -\Delta(-\boldsymbol{k})$$

Parity ✗

Parity + U(1) gauge rotation ☺

⇒ Special symmetry of the Hamiltonian

$$\Pi\mathcal{H}(\boldsymbol{k})\Pi^{-1} = \mathcal{H}(-\boldsymbol{k}), \quad \Pi = \begin{pmatrix} P & 0 \\ 0 & -P \end{pmatrix}$$

P : Parity

For parity invariant momenta $\boldsymbol{k} = \Gamma_i$

$$[\Pi, \mathcal{H}(\Gamma_i)] = 0,$$

Occupid states are eigen states with $\Pi = \pm 1$

① To change the eigen value of Π for an occupied state , the energy gap must be closed.

⇒ Eigen value of Π is related to the bulk topological #

② The eigen value of Π is determined by the sign of the electron dispersion at $\boldsymbol{k} = \Gamma_i$

⇒ Electron dispersion characterizes the bulk topological #

Ex.) odd-parity color superconductor

Y. Nishida, Phys. Rev. D81, 074004 (2010)

$$\mathcal{H}_{\text{CSC}} = \int d^3x \Big[\psi^\dagger_{a,f}(-\boldsymbol{\alpha}\cdot\boldsymbol{\partial}+\beta m-\mu)\delta_{ab}\delta_{fg}\psi_{b,g} \\ +\frac{1}{2}\psi^\dagger_{a,f}\Delta_{ab,fg}(\boldsymbol{x})C\gamma^5\psi^*_{b,g} \\ +\frac{1}{2}\psi^T_{a,f}\Delta^\dagger_{ab,fg}(\boldsymbol{x})C\gamma^5\psi_{b,g}\Big]$$

$$\Delta_{ab,fg}(\boldsymbol{x}) = \sum_{i=1,2,3}\Delta_i\epsilon_{iab}\epsilon_{ifg}$$

$$\begin{cases} \Delta_1 = \Delta_2 = \Delta_3 \neq 0 & \text{color-flavor-locked phase} \\ \Delta_1 = \Delta_2 \neq 0, \quad \Delta_3 = 0 & \text{two flavor pairing phase} \end{cases}$$

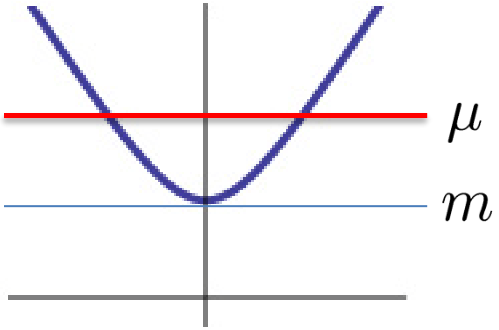
For odd-parity pairing, the BdG Hamiltonian is

$$\mathcal{H}(\boldsymbol{k}) = \begin{pmatrix} \boldsymbol{\alpha}\cdot\boldsymbol{k} + \beta m - \mu & \gamma^5\Delta_0 \\ \gamma^5\Delta_0 & -\boldsymbol{\alpha}\cdot\boldsymbol{k} - \beta m + \mu \end{pmatrix}$$

The BdG Hamiltonian is invariant under parity + U(1) rotation

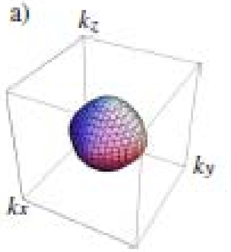
$$\Pi\mathcal{H}(\boldsymbol{k})\Pi^{-1} = \mathcal{H}(-\boldsymbol{k}), \quad \Pi = \begin{pmatrix} \gamma^0 & 0 \\ 0 & -\gamma^0 \end{pmatrix}$$

(A) $\mu^2 + \Delta_0^2 > m^2$



Fermi surface with

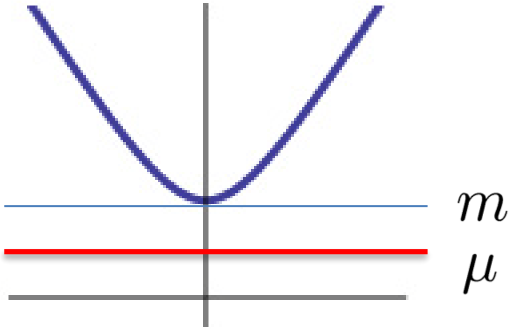
$$(-1)^{\chi(S_{\mathrm{F}})/2} = -1$$



Topological SC

- Gapless boundary state
- Zero modes in a vortex

(B) $\mu^2 + \Delta_0^2 < m^2$



No Fermi surface

Non-topological SC

c.f.) Y. Nishida, Phys. Rev. D81, 074004 (2010)

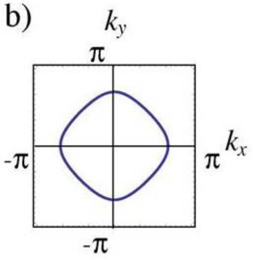
Summary

- We examined topological properties of odd-parity superconductors

- The Fermi surface topology characterizes the topological properties of odd-parity superconductors

- Simple criteria for topological superconductors, in particular that for a non-Abelian topological phase , are provided in terms of the Fermi surface structures.

$$(-1)^{p_0(S_{\rm F})} = -1$$



Reference

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● MS and S.Fujimoto, “Topological Phases of Noncentrosymmetric Superconductors: Edge States, Majorana Fermions, and the Non-Abelian Statistics“, Phys. Rev. B79, 094504 (2009).

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● Anomalous Andreev bound states in noncentrosymmetric superconductors, by Y. Tanaka, Y. Mizuno, T. Yokoyama, K. Yada, and MS, arXiv: 1006.3544, to apppear in PRL

● Non-Abelian Topological Phases in Spin-Singlet Superconductors, by MS, Y. Takahashi, S.Fujimoto, arXiv:1006.4487.

Collaborators

- Satoshi Fujimoto, Dep. Of Phys. Kyoto University

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