

# Melting Spectral Functions of the Vector Mesons in a Holographic QCD Model

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Based on MF, K. Fukushima, T. Misumi, and M. Murata,  
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And MF, T. Kikuchi, K. Fukushima, T. Misumi, M. Murata,  
Phys.Rev.D81:065024 (2010)

# Introduction

- Strongly correlated quark gluon plasma (QGP) has interesting non-perturbative properties.

Example (1) The gauge/gravity correspondence  $\rightarrow$  the shear viscosity/entropy ratio  $\eta/s = 1/(4\pi)$  for D=4 N=4 SYM, while It is difficult to reproduce this result from gauge theory side.

(2) Heavy mesons like J/ $\psi$  survive in the deconfinement phase and melt as temperature increases.

T. Umeda, et. al. '02, M. Asakawa and T. Hatsuda '03

The screening of color forces weaken  $\bar{c}c$  pair binding, manifesting J/ $\psi$  suppression

- We want to compute the finite temperature spectral functions of heavy mesons by using the Soft wall AdS/QCD model.

Non-perturbative corrections!

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# Gauge theory side (large $N_c$ QCD)

- Large  $N_c$  and strongly coupled  $SU(N_c)$  QCD  
 $(g_{YM}^2 N_c \gg 1)$
- Matter field: (u,d,s) transforming under  $U(3)$  and **heavy charm quark  $c$  ( $M_c \sim 1.3$  GeV)**
- The spontaneous breaking of chiral symmetry:  $\langle \bar{c}c \rangle \neq 0$

▪ (C-)Meson field:		$J^{pc}$		$J^{pc}$
scalar	$\bar{c}c,$	$0^{++}$	pseudo-scalar	$\bar{c}\gamma_5 c,$ $0^{-+}$
vector	$\bar{c}\gamma_\mu c,$	$1^{--}$	axial-vector	$\bar{c}\gamma_5\gamma_\mu c,$ $1^{++}$

# Quantum number and Mass of c-mesons

$c\bar{c}$	$IG(J^{PC})$
• $\eta_c(1S)$	$0^+(0^-+)$
• $J/\psi(1S)$	$0^-(1^{--})$
• $\chi_{c0}(1P)$	$0^+(0^{++})$
• $\chi_{c1}(1P)$	$0^+(1^{++})$
• $h_c(1P)$	$?^?(1^{+-})$
• $\chi_{c2}(1P)$	$0^+(2^{++})$
• $\eta_c(2S)$	$0^+(0^-+)$
• $\psi(2S)$	$0^-(1^{--})$
• $\psi(3770)$	$0^-(1^{--})$
• $X(3872)$	$0^?(?^{?+})$
$\chi_{c2}(2P)$	$0^+(2^{++})$
$X(3940)$	$?^?(?^{??})$
• $X(3945)$	$?^?(?^{?+})$
• $\psi(4040)$	$0^-(1^{--})$
• $\psi(4160)$	$0^-(1^{--})$
$X(4160)$	$?^?(?^{??})$
• $X(4260)$	$?^?(1^{--})$
$X(4360)$	$?^?(1^{--})$
• $\psi(4415)$	$0^-(1^{--})$
$X(4660)$	$?^?(1^{--})$

- Mass of mesons  
 $M(\eta_c(1S))=3.0$  [GeV],  
 $M(J/\psi(1S))=3.1$  [GeV],  
 $M(\chi_{c0}(1P)) =3.4$  [GeV],  
 $M(\chi_{c1}(1P)) =3.5$  [GeV],  
 $M(\eta_c(2S))=3.6$  [GeV],  
 $M(\psi(2S))=3.7$  [GeV].

# Regge trajectory of c-mesons?

- Regge trajectory of light mesons!

$$m_n^2 \sim 4cn$$

Spectrum of  $\rho$  meson ( $u\bar{d}$ ) :

$\rho(770[\text{MeV}])$ ,  $\rho(1450)$ ,  $\rho(1700)$ ...

- The spectrum of heavy mesons doesn't obey Regge trajectory in general.
- For charmonium, it is known that mesons also obey Regge trajectory up to the first and second level.

S. Gershtein, A. Likhoded and A. Luchinsky ``06

See also H. Grigoryan, P. Hohler, and M. Stephanov

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# Thermal Field Theory

- The retarded Green's function and **spectral function** of a conserved current

$$C_{xx}(x-y) = -i\theta(x^0 - y^0)\langle [J_x(x), J_x(y)] \rangle$$

$$C_{xx}(x-y) = \int \frac{d^4k}{(2\pi)^4} e^{ik(x-y)} C_{xx}(k)$$

$$\rho(k) = -\text{Im} C_{xx}(k)$$

- Conservation law means  $C_{\mu\nu}(k) = P_{\mu\nu} \Pi(k^2)$ ,  $P_{\mu\nu} = \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}$

$$\Rightarrow C_{xx}(k) = \Pi(k^2) \text{ for } k_\mu = (-\omega, 0, 0, q)$$

Transverse component is chosen.

# Dual gravity side: Soft-wall model

Soft-wall model includes smooth dilaton cut off

$\phi = cz^2$  (z is the radial direction of  $AdS_5$ )

Action: 
$$S = \int dx^5 e^{-\Phi} \sqrt{-g} \left[ \text{Tr}[-|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2} (g^{MN} g^{PQ} F_{MP}^{\mathcal{L}} F_{NQ}^{\mathcal{L}} + g^{MN} g^{PQ} F_{MP}^{\mathcal{R}} F_{NQ}^{\mathcal{R}})] \right]$$
  
with  $M = 0, 1, 2, 3, 4$

A. Karch, E. Katz, D. T. Son, and M. A. Stephanov ``06

In zero temperature, the soft-wall model reproduces

Regge trajectory of light mesons!

$$m_n^2 \sim 4cn$$

Spectrum of  $\rho$  meson  $\rho(770[\text{MeV}])$ ,  $\rho(1450)$ ,  $\rho(1700)$ ...

can be compared with the above result and then  $c = 0.151 \text{ GeV}^2$



# Flavor dependent soft wall model

$$S = \int d^5x \sqrt{-g} \operatorname{tr} (e^{-c\rho z^2} \mathcal{L}_{\text{light}} + e^{-c_{J/\psi} z^2} \mathcal{L}_{\text{heavy}}),$$

- $L_{\text{heavy}}$  and  $L_{\text{light}}$  take almost same structure:

$$= \operatorname{tr} \left[ -|DX|^2 + \frac{3}{L^2}|X|^2 - \frac{1}{4g_5^2} (g^{MN} g^{PQ} F_{L,MP} F_{L,NQ} + g^{MN} g^{PQ} F_{R,MP} F_{R,NQ}) \right]$$

- Fields in  $L_{\text{heavy}}$  and  $L_{\text{light}}$  belong to U(3) and U(1), **decoupling each other.**
- The lowest excitation of heavy mesons  $\rightarrow c_{J/\psi} = 2.43 \text{ GeV}^2$   
In our presentation, we consider only the heavy meson.

# Soft-wall model at finite temperature $T$

**Metric:** AdS Black Hole  $g_{MN}dx^M dx^N = \frac{L^2}{z^2}[-f(z)dt^2 + d\mathbf{x}^2 + \frac{1}{f(z)}dz^2]$

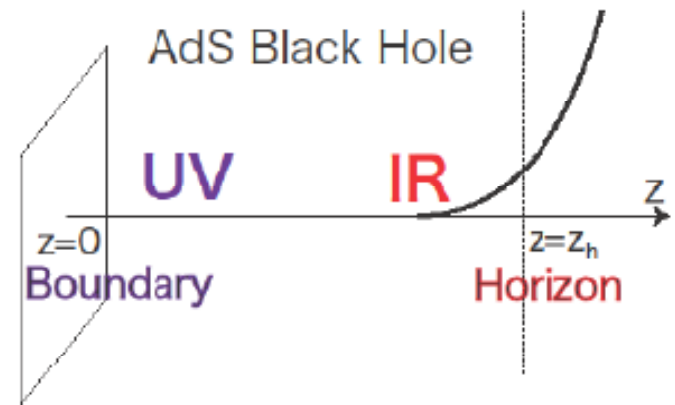
with  $f(z) = 1 - z^4/z_h^4$   $z_h$ :Horizon

Hawking temperature  $T_H = \frac{1}{\pi z_h}$

→ Temperature of the medium

◆ Action at the quadratic order

$$S = \int dx^5 e^{-\Phi} \sqrt{-g} \text{Tr} \left[ -\frac{1}{2g_5^2} (F^V F^V + F^A F^A) - (2X_0 \partial \pi + [V, X_0] + \{A, X_0\})(2X_0 \partial \pi - [V, X_0] + \{A, X_0\}) \right]$$



→ Linearized EOM

Axial:  $\partial_N [e^{-\Phi} \sqrt{-g} g^{MP} g^{NQ} F_{PQ}^A] - e^{-\Phi} \sqrt{-g} g^{MN} \{X_0, \{A_N, X_0\}\} = 0$

Vector:  $\partial_N [e^{-\Phi} \sqrt{-g} g^{MP} g^{NQ} F_{PQ}^V] + \underline{e^{-\Phi} \sqrt{-g} g^{MN} [X_0, [V_N, X_0]]} = 0$



This term vanishes for U(1) case.

- Note that AdS BH is unstable at low temperature and the Hawking-Page phase transition takes place at

$$T_c = 0.492 \sqrt{c_\rho} = 0.19 \text{ GeV} \text{ C. Herzog, ``07}$$

- Note that the transition temperature is determined only by  $\rho$  meson mass  $c_\rho$ .  
In this presentation, we consider the region  $T > T_c$ .

- Chiral symmetry breaking parameter is obtained from the scalar  $X$

Scalar EOM: 
$$X_0''(z) + \left( -2cz + \frac{f-4}{zf} \right) X_0'(z) + \frac{3}{z^2 f} X_0(z) = 0,$$

$$L^{3/2} X_0(z) \sim \frac{1}{2} (M_q z + \Sigma z^3), \quad \text{at } z \sim 0$$

$M_q$ : Mass of c-quark 1.3 GeV  $\rightarrow \Sigma$ : Chiral condensate  $(-3.1 \text{ GeV})^3$

# In-falling solution and Spectral function of the vector meson

- Infalling solution: linear combination of two solutions

$$V(z) = \Phi_2(z) + B(\omega, q)\Phi_1(z) \xrightarrow{z \rightarrow z_h} (1 - z/z_h)^{-i\omega z_h/4}$$

Boundary solutions  $\Phi_2 \approx 1, \quad \Phi_1 \approx z^2 \quad z \rightarrow 0$

- Retarded Green's function:

$$D^R(\omega, \mathbf{q}) = \lim_{\epsilon \rightarrow 0} \frac{\delta^2}{\delta V_0(p)\delta V_0(p)} \exp(-S_{5d}[\bar{V}(p, \epsilon)])|_{V_0=0} : \text{GKP-W relation}$$

$$= \underset{\substack{\uparrow \\ \text{Factor}}}{-C} \lim_{\xi \rightarrow 0} \left( \frac{1}{\xi} V(\xi)^* \partial_\xi V(\xi) \right) = -2C \left[ B(\omega, q) - \frac{\omega^2 - q^2}{2} \ln \left( \frac{e^{\gamma_E}}{2} \sqrt{|\omega^2 - q^2|} \epsilon \right) \right],$$

with  $\bar{V}(p, z) = V(p, z)V_0(p)$

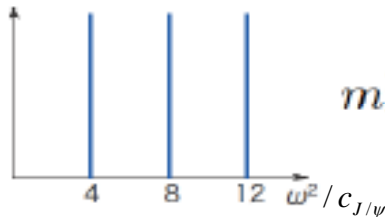
- Spectral function: the imaginary part of  $B(\omega, q)$

$$\rho(\omega, q) = -\frac{1}{\pi} \text{Im} D^R(\omega, q) = \frac{2C}{\pi} \text{Im} B(\omega, q).$$

# ◆ The Numerical Results

## 1. Spectral functions for $q = 0$

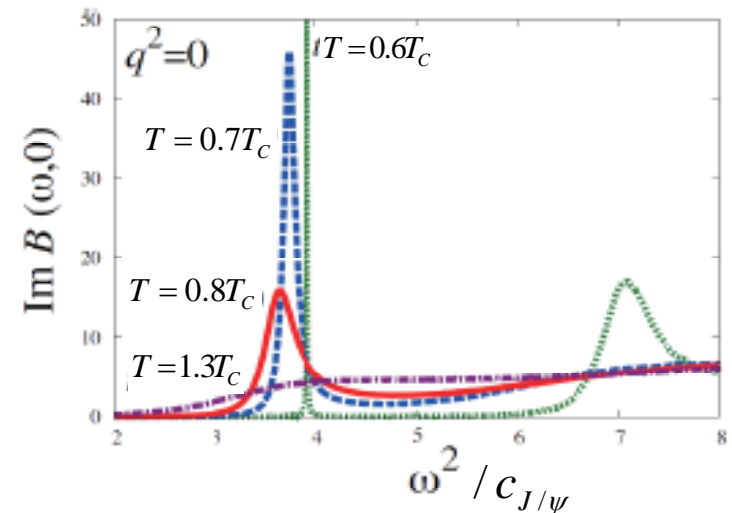
At low temperature, sharp peaks stand in accord with  $t=0$  spectrum.



$$m_n^2 = 4n, \quad (n = 1, 2, 3\dots)$$

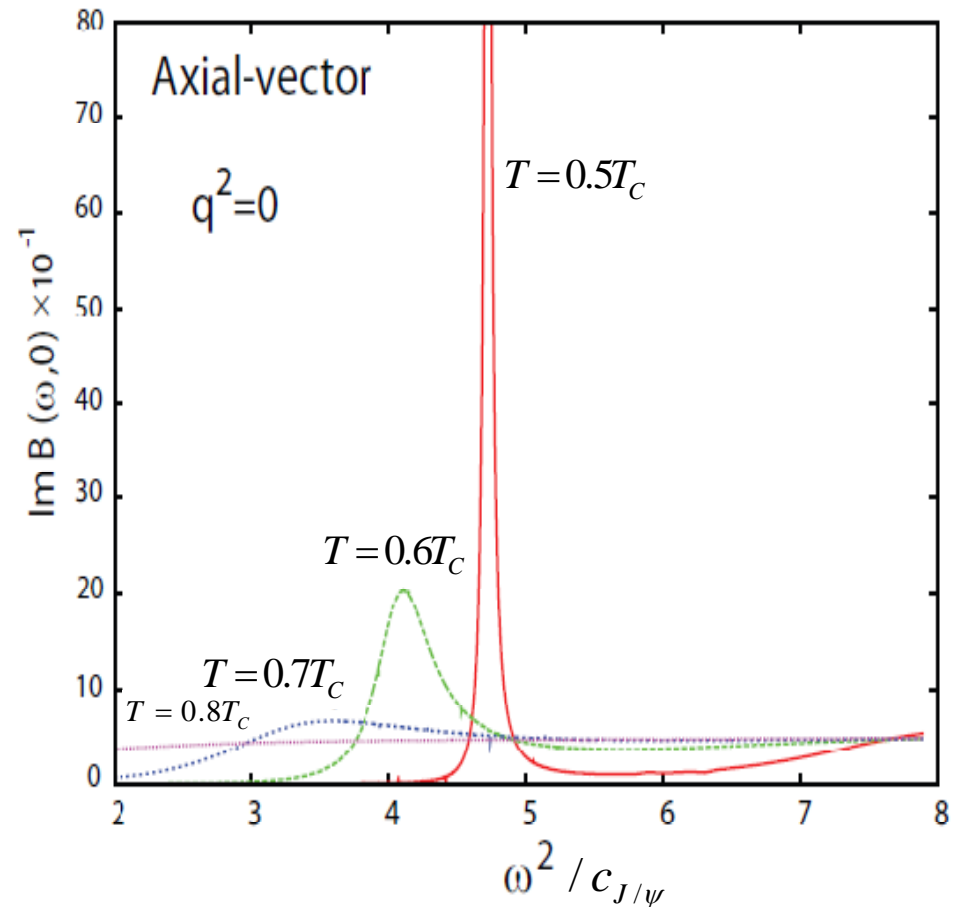
## ➤ Properties

1. The lowest-lying state melts gradually as  $T$  increases.
2. The peak moves to a smaller mass with increasing  $T$ .
3. The excited states melt much earlier and shift more.

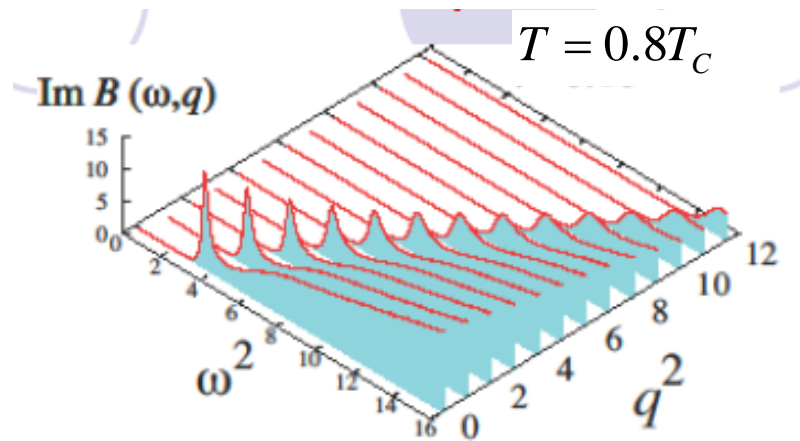


# Spectral function of axial vector mesons

- $\chi_{c1}$  doesn't survive above  $T_c$ .
- Note that chiral condensate  $\Sigma = -(3.1[\text{GeV}])^3$  is not much bigger than the mass  $\rightarrow \Sigma$  doesn't contribute to the spectra function much.
- Zero temperature Mass of  $X_{c1}$  3.4 [GeV] is consistent with experimental result 3.51 [GeV].



# Finite momentum of vector mesons



- The peaks move to a larger mass with increasing  $q$ .
- The Vector Mesons are suppressed under the hot wind.  
(This is consistent with the previous study for N=4 SYM.)

H. Liu, et. al. ``07

# Summary and Discussions

- $J/\psi$  survives above the deconfinement temperature ( $T \sim 1.2T_c$ ), while  $\chi_{c1}$  melts at  $T_c$ .
- The spectral functions of scalar and pseudoscalar can also be computed and are melt at the deconfinement temperature.