# *Chiral Magnetic Effect*

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# Strong $\theta$ Angle, Strong $\mathcal{CP}$ Problem and Heavy-Ion Collisions

# $\mathcal{P}$ and $\mathcal{CP}$ Violation in the YM Theory

## Gauge Actions

- **$\mathcal{P}$- and $\mathcal{CP}$- even ($\mathcal{T}$-even) terms**

$$F_{\mu\nu}F^{\mu\nu} = 2F_{0i}F^{0i} + F_{ij}F^{ij}$$

**Even** w.r.t. spatial and temporal indices

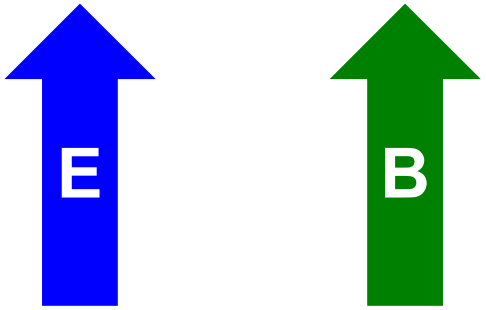
- **$\mathcal{P}$- and $\mathcal{CP}$- odd ($\mathcal{T}$-odd) terms**

$$F_{\mu\nu}\tilde{F}^{\mu\nu} = 2F_{01}F^{23} + 2F_{02}F^{31} + 2F_{03}F^{12}$$

**Odd** w.r.t. spatial and temporal indices

Parallel $\boldsymbol{E}$ and $\boldsymbol{B}$

$$F_{\mu\nu}\tilde{F}^{\mu\nu} = 2\boldsymbol{E}\cdot\boldsymbol{B}$$




vector axial vector

# *Terminology*

## Topological Charge (Pontryagin Index)

$$Q=\frac{1}{32\pi^2}\int d^4x\, F^a_{\mu\nu}\tilde{F}^a_{\mu\nu}$$

$$\tilde{F}^a_{\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^a_{\alpha\beta}$$

$$\frac{1}{32\pi^2}F^a_{\mu\nu}\tilde{F}^a_{\mu\nu}=\partial_\mu K_\mu$$

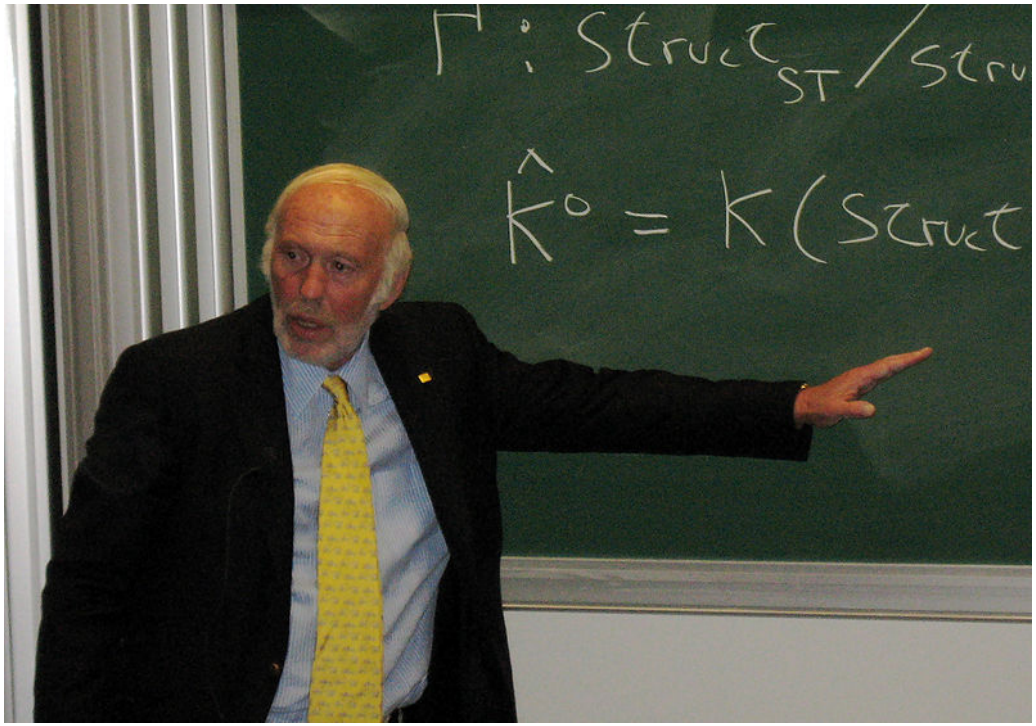
$$K_\mu=\frac{1}{16\pi^2}\epsilon_{\mu\alpha\beta\gamma}\left(A^a_\alpha\partial_\beta A^a_\gamma+\frac{1}{3}\epsilon^{abc}A^a_\alpha A^b_\beta A^c_\gamma\right)$$

# *Terminology*

## Chern-Simons Number

$$\nu = \int d^3x\, K_0 = \frac{1}{16\pi^2}\int d^3x\, \epsilon^{ijk}\left(A_i^a \partial_j A_k^a + \frac{1}{3}\epsilon^{abc} A_i^a A_j^b A_k^c\right)$$

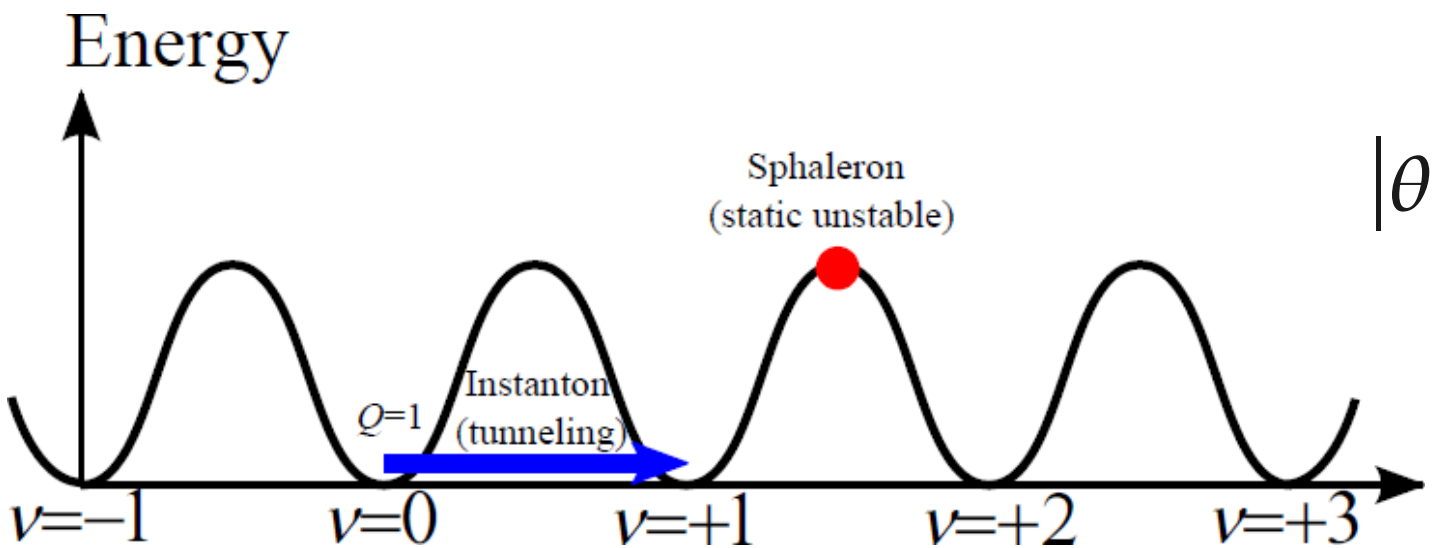
$$Q = \int d^4x\,(\partial_0 K_0 - \partial_i K_i) = \int dt \frac{d}{dt}\int d^3x\, K_0 = \nu(t=+\infty) - \nu(t=-\infty)$$



# $\theta$ -Vacuum and Strong CP Problem

## Topological Structure and $\theta$ -Vacuum

**Manton**
**Faddeev**
**Jackiw-Rebbi**




$$|\theta\rangle = \sum_{\nu} e^{i\theta\nu}|\nu\rangle$$

(Bloch state)

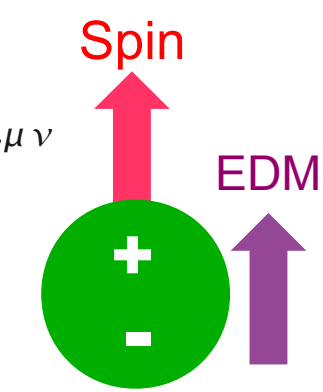
## Strong CP Problem

$$\langle\theta|\theta\rangle \rightarrow \quad S_{\mathrm{QCD}} = -\frac{1}{2g^2}\mathrm{tr}\, F_{\mu\nu}F^{\mu\nu} + \theta\frac{1}{16\pi^2}\mathrm{tr}\, F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$|d_n| \sim (e\, m_q / m_N^2)\theta$$

$$|\theta| < 0.7\times 10^{-11}$$

**No CP breaking** (Why?)



# *Finite-θ Hadronic World*

## $\theta$ can be eliminated by $U(1)_A$ rotation

One solution to the strong CP problem is the presence of massless quarks (almost excluded...)

## Effect of strong $\theta$-angle to hadron physics

$$U \rightarrow e^{i\theta} U$$

Scalar meson

$$\sigma \sim \bar{\psi}\psi \rightarrow \bar{\psi} e^{i\gamma_5\theta}\psi \sim \sigma\cos(\theta) + \eta_0 \sin(\theta)$$

Pseudo-scalar meson

$$\eta_0 \sim \bar{\psi} i\gamma_5 \psi \rightarrow \bar{\psi} e^{i\gamma_5\theta} i\gamma_5 \psi \sim \eta_0 \cos(\theta) + \sigma \sin(\theta)$$

**$\eta_0$ condensates in addition to the chiral $\sigma$ condensate**

# *Possibility for Finite η Condensate*

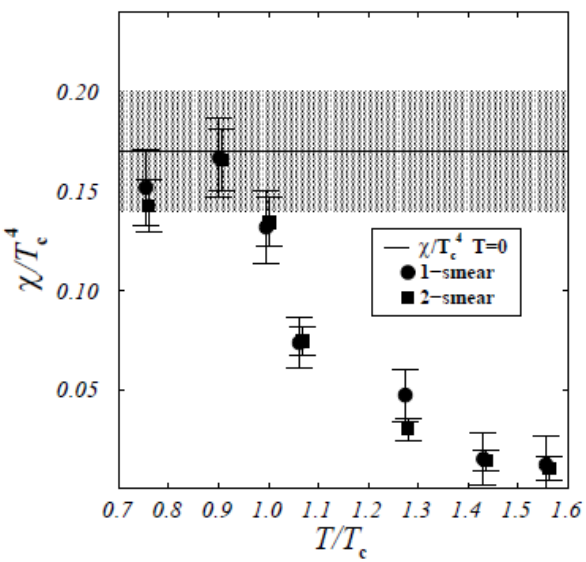
## If $U(1)_A$ symmetry is NOT broken

$\sigma$ and $\eta$ are degenerate ($\eta$ may have a chance as much as the $\sigma$ condensate develops)

## $U(1)_A$ is broken but can be "effectively" restored

$U(1)_A$ breaking effective interaction is induced by the topological susceptibility

**Veneziano-Di Vecchia**




**Susceptibility drops off at high temperature $T \sim T_c$**

$$n(T) = \left(\frac{8\pi^2}{g^2}\rho^{-5} e^{-8\pi^2/g^2(\rho)}\right) \exp\left[-\pi^2\rho^2 T^2\left(\frac{2N_c + N_f}{3}\right)\right]$$

**Gross-Pisarski-Yaffe**

**Lattice Simulation**
**Alles et al (1996)**

# *Relativistic Heavy-Ion Collisions*


**Heavy-Ion** (nucleus)
Au, Pb, Cu, ...

$\sqrt{s_{NN}} = 200\,\mathrm{GeV}, 62\,\mathrm{GeV}, \ldots$

**Quark-Gluon Plasma**

**Baym**
**Shuryak**

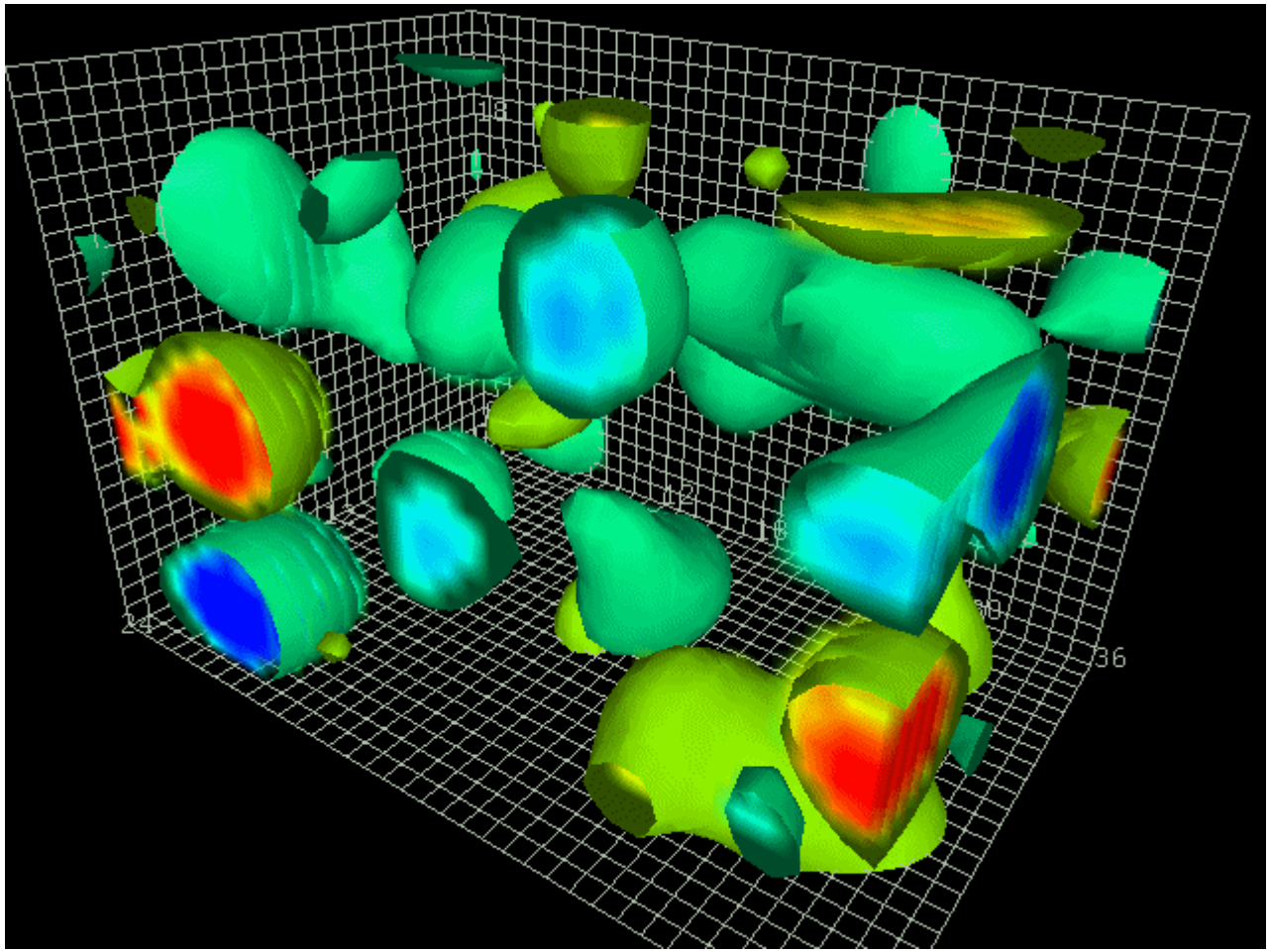
Direct photon measurement
(not from $\pi^0$, $\eta'$ etc)
$\rightarrow$ Initial $T \sim 4\times10^{12}\,\mathrm{K} \sim \mathrm{GeV}$

c.f. $T_c \sim \Lambda_{\mathrm{QCD}}$ $\quad \rho \sim 0.3\,\mathrm{fm}$

# Topological Contents in the QCD Vacuum and the Real-time Fluctuations

# *Lattice Simulation*

## Topological Charge Distribution at $T$ =0



This is not a function of “Real-Time” but of the simulation step.

**Derek's Visual QCD**

# *Is the high-T QCD Vacuum Topologically Trivial?*

## Yes … in terms of Instantons (Euclidean)

Instantons are exponentially suppressed at high $T$.

$$n(T) = \left( \frac{8\pi^2}{g^2} \rho^{-5} e^{-8\pi^2/g^2(\rho)} \right) \exp\left[ -\pi^2 \rho^2 T^2 \left( \frac{2N_c + N_f}{3} \right) \right]$$

## No … in terms of Sphalerons (Minkowskian)

Sphalerons are parametrically enhanced at high $T$.

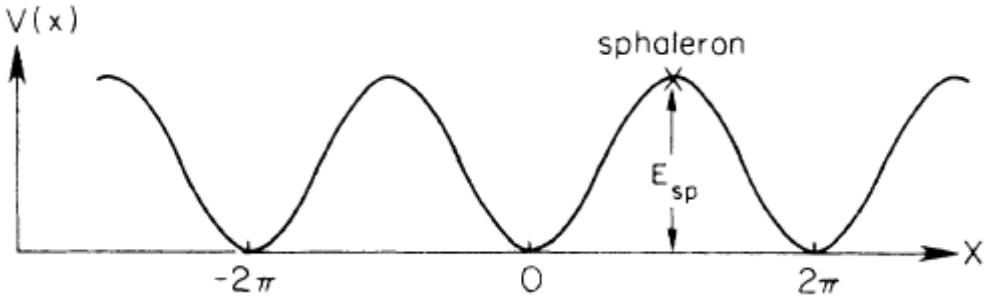
$$\Gamma \sim \alpha_s^5 T^4$$

**QCD sphalerons are abundant in hot and dense matter created in the relativistic heavy-ion collisions**

**Arnold-McLerran (1987)**

# *Topological Rate in Real- and Imaginary-Time*

## Pendulum (Arnold-McLerran)




Chern-Simons number $x(t)/2\pi$

Topological charge $n=\frac{1}{2\pi}\int_0^\beta d\tau\,\dot{x}$

Finite-$T$ Euclidean Action $S_E=\int_0^\beta d\tau\left(\frac{1}{2}\dot{x}^2+i\frac{\theta}{2\pi}\dot{x}\right)$

Topological Susceptibility (Diffusion Rate) $A(t)=\left\langle T\left(\frac{x(t)-x(0)}{2\pi}\right)^2\right\rangle$

Real-time (classical approx.) $A(t)\simeq\frac{t^2}{4\pi^2}\bar{v}^2=\frac{t^2}{4\pi^2\beta}$

Imaginary-time $A(-i\beta)=\langle n^2\rangle\simeq 2\exp\left(-2\pi^2/\beta\right)\cos\theta$

# *Analytical Continuation*

## Diffusion Rate at High *T*

$$A(t)=\left\langle T\left(\frac{\delta x(t)-\delta x(0)}{2\pi}\right)^2\right\rangle +O(e^{2\pi^2/\beta})$$

$$=\frac{1}{4\pi^2 m}\left(\frac{\exp(-im|t|)-1}{\exp(-\beta m)-1}-\frac{\exp(im|t|)-1}{\exp(\beta m)-1}\right) +O(e^{2\pi^2/\beta})$$

$$\to\frac{1}{4\pi^2}\left(\frac{t^2}{\beta}+it\right) +O(e^{2\pi^2/\beta})$$

$$A(t=-i\beta)\to O(e^{2\pi^2/\beta})$$

**Instantons (Euclidean windings) are suppressed at high *T* but communications in real time are not and dominated by the contribution from the zero-winding sector.**

# *Topological Diffusion Rate*

$$\Gamma = \frac{1}{2} \lim_{t \to \infty} \lim_{V \to \infty} \int d^4 x \langle q(x) q(0) + q(0) q(x) \rangle$$

$$\langle Q^2 \rangle = 2 \Gamma V t$$

Random Walk at Finite $T$

In the strong-coupling AdS/CFT by Son and Starinets (hep-th/0205051)

$$\Gamma = \frac{(g_{\mathrm{YM}}^2 N)^2}{256 \pi^3} T^4$$

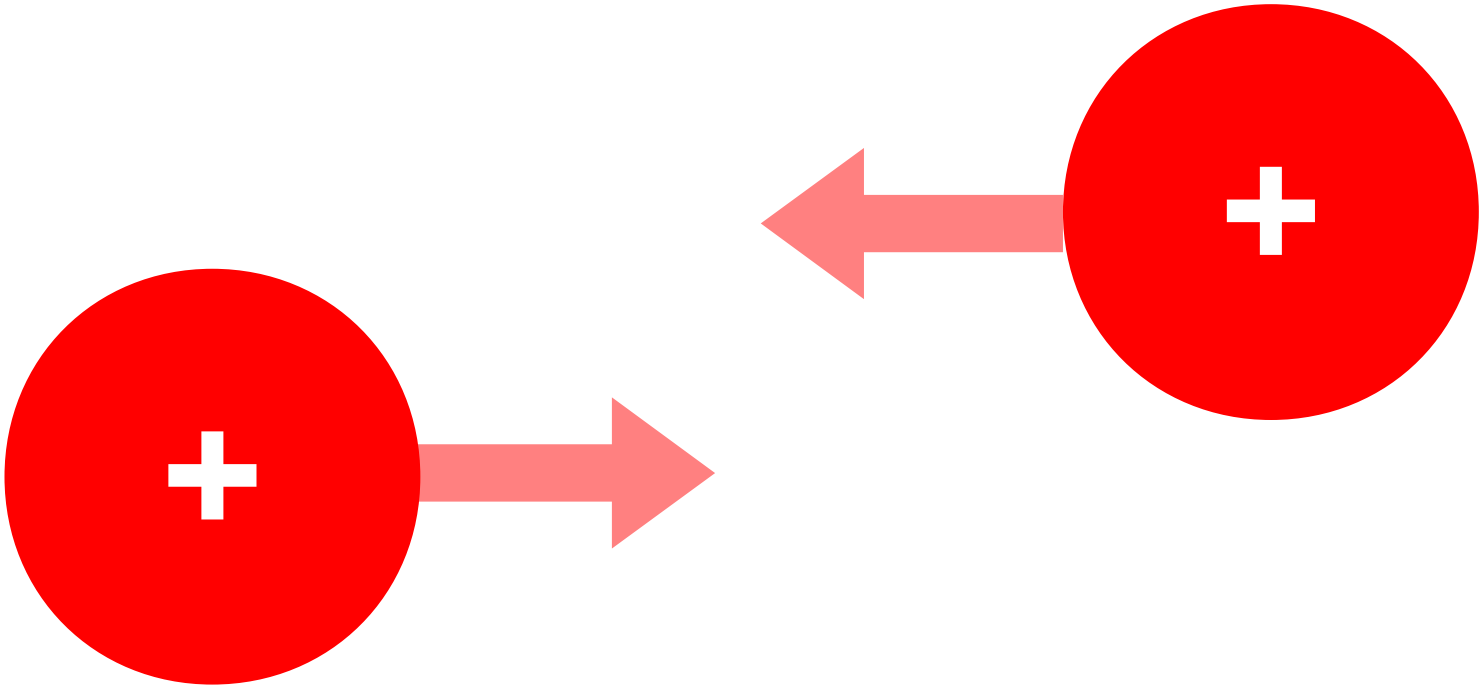
In the weak-coupling perturbation by Arnold, Son, Yaffe, Bodeker, Moore, etc

$$\Gamma = \mathrm{const} \cdot (g_{\mathrm{YM}}^2 N)^5 \ln \frac{1}{g_{\mathrm{YM}}^2 N} T^4$$

# Connection to the Heavy-Ion Collisions
# How to detect the topological effects?

# *Non-Central Collision*

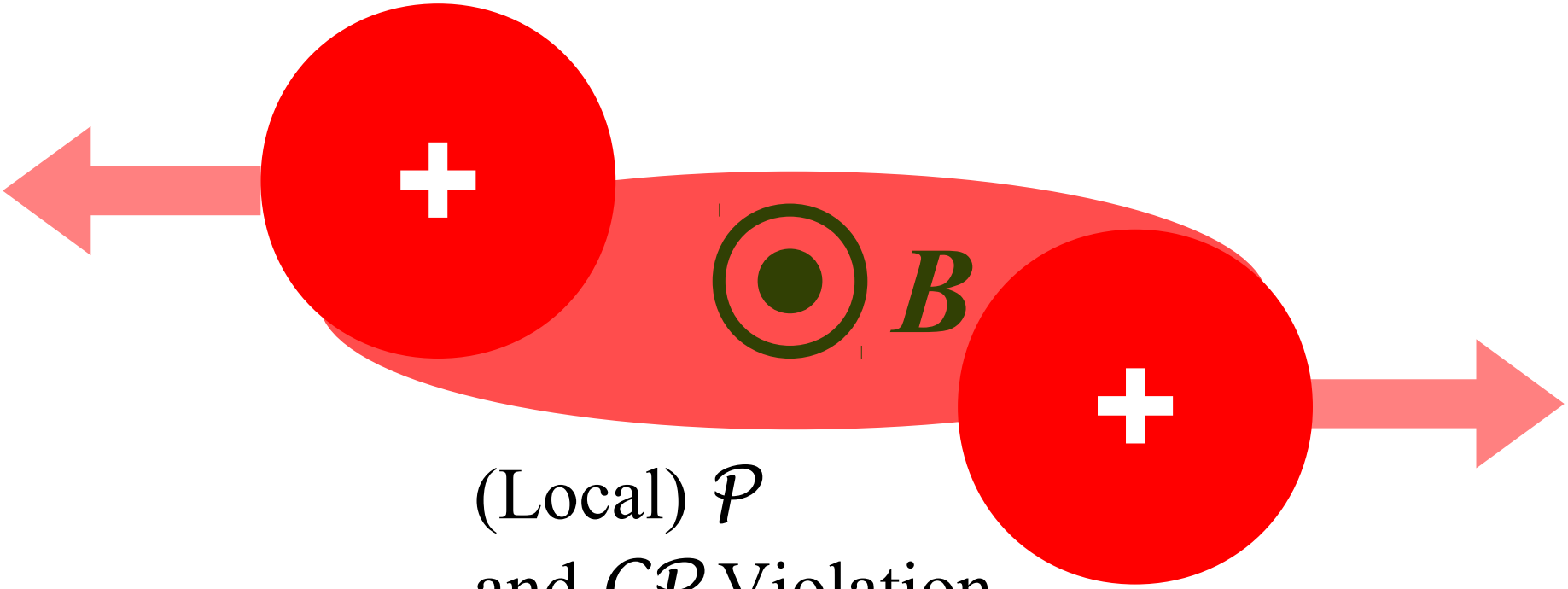
**Before Collision (seen from above)**



**Centrality is determined by $N_{part}$**

# *Non-Central Collision*

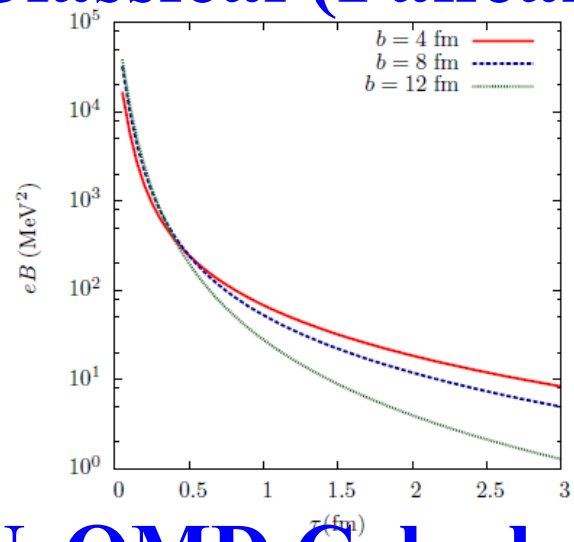
**After Collision**

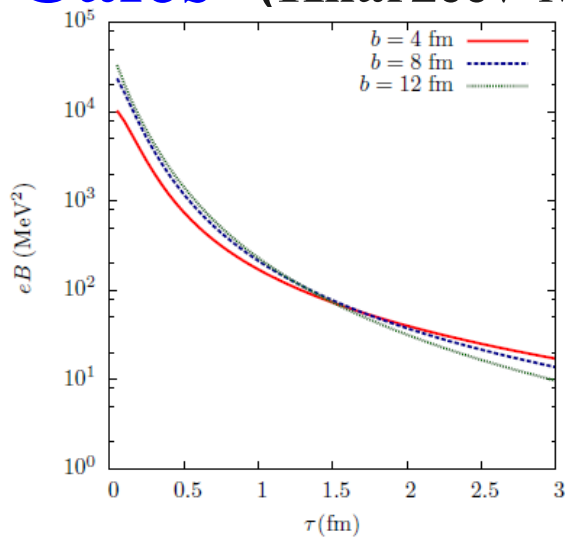



(Local) $\mathcal{P}$
and $\mathcal{CP}$ Violation

# *Estimated Magnetic Fields*

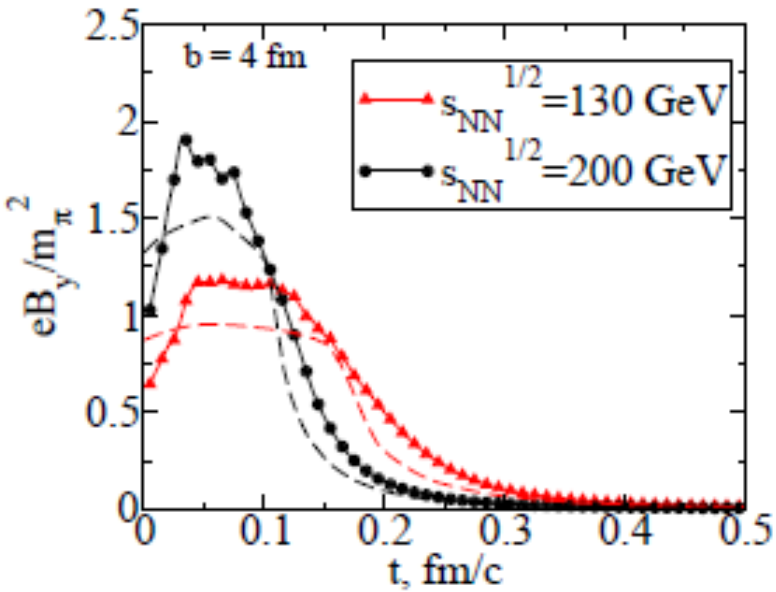
## Classical (Pancake) Calcs (Kharzeev-McLerran-Warringa)

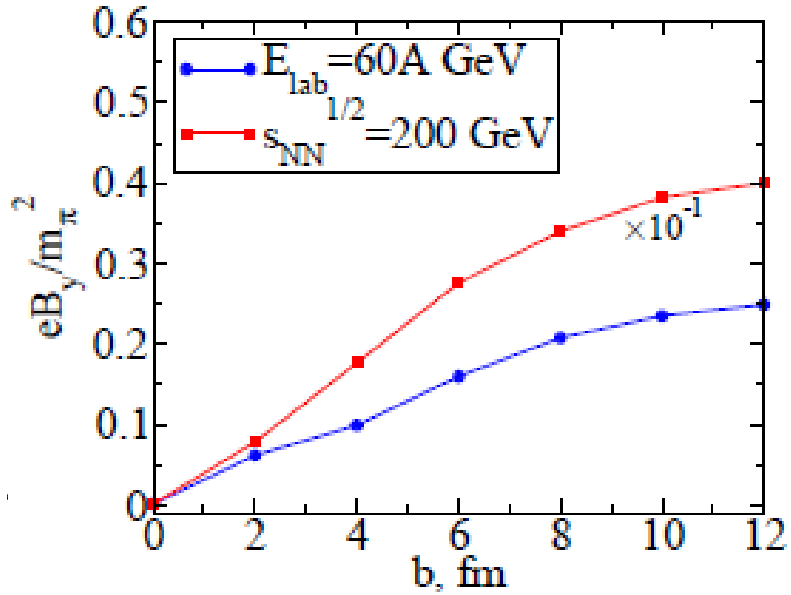



$$eB = 1\,[\mathrm{MeV}^2]$$
$$\rightarrow B \simeq 1.7\times 10^{14}\,[\mathrm{Gauss}]$$

## UrQMD Calculations (Skokov-Illarionev-Toneev)

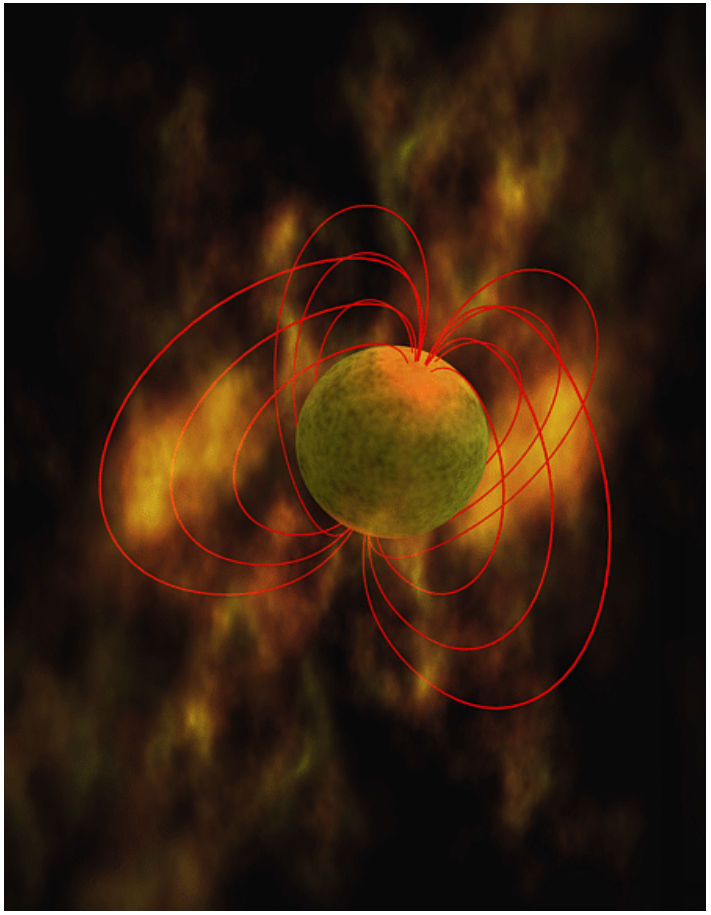


# *How Big?*

$$eB \sim m_\pi^2 \rightarrow 10^{18}\ \text{Gauss}$$

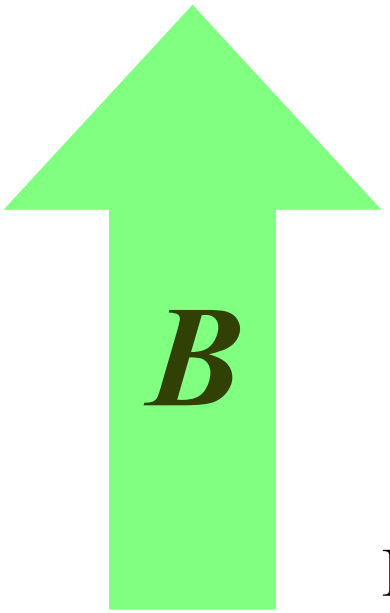
$$10^3 \sim 10^6 \times$$

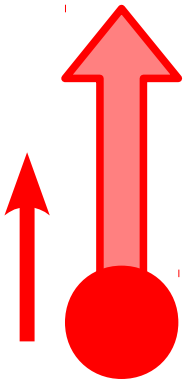


Neutron Star
(Magnetar)

# *Chiral Magnetic Effect*

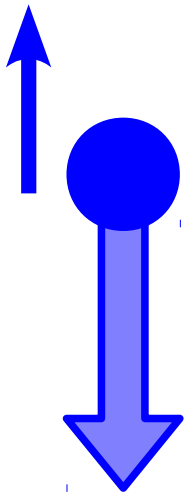
## Classical Picture



**Right-handed Quark**
= momentum parallel to spin

**Left-handed Quark**
= momentum parallel to spin



$$\boldsymbol{J} \neq 0 \quad \text{if} \quad N_5 = N_R - N_L \neq 0$$

**Kharzeev-McLerran-Warringa (2007)**

# *Anomaly Relations*

## Induced $N_5$ by Topological Effects

$$\frac{dN_5}{dt} = -\frac{g^2 N_f}{8\pi^2}\int d^3x \ \mathrm{tr}\, F_{\mu\nu}\tilde{F}^{\mu\nu}$$

QCD Anomaly Relation

Introduce $\mu_5$ to describe induced $N_5$

## Induced $J$ by the presence of $N_5$ and $B$

$$\boldsymbol{j} = \frac{e^2\mu_5}{2\pi^2}\boldsymbol{B}$$

QED Anomaly Relation

$$\left(\boldsymbol{j} = \sum_{i=\mathrm{flavor}} \frac{q_i^2\mu_5}{2\pi^2}\boldsymbol{B} \quad \text{in QCD}\right)$$

**Metlitski-Zhitnitsky (2005)**
**Fukushima-Kharzeev-Warringa (2008)**

# *Derivation (naïve calculation)*

## Thermodynamic Potential (*UV divergent*)

$$\Omega = -V N_c \sum_f \frac{|q_f B|}{2\pi} \sum_{s=\pm} \sum_{n=0}^{\infty} \alpha_{n,s}^f \int \frac{dp_3}{2\pi} \left[ \omega_{n,s} + 2T \ln\left(1 + e^{-\beta \omega_{n,s}}\right) \right]$$

$$\omega_{n,s}^2 = \left( \sqrt{p_3^2 + 2|q_f B| n} + \mathrm{sgn}(p_3) s \mu_5 \right)^2 + m^2$$

$$\alpha_{n,s} = \begin{cases} 1 & n > 0, \\ \delta_{s+} & n = 0, \quad eB > 0 \\ \delta_{s-} & n = 0, \quad eB < 0 \end{cases}$$

## Current (*UV finite*)

$$j_3 = \left. \frac{\partial \Omega}{\partial A_3} \right|_{A_3 = 0} \quad \longleftarrow \quad \frac{\partial}{\partial A_3} = e \frac{\mathrm{d}}{\mathrm{d}p_3}$$

Only surface terms!

$$j_3 = e \frac{|eB|}{4\pi^2} \sum_{s,n} \alpha_{n,s} \left[ \omega_{n,s}(p_3 = \Lambda) - \omega_{n,s}(p_3 = -\Lambda) \right]$$

$$= e \frac{|eB|}{2\pi^2} \sum_{s,n} \alpha_{n,s} s \mu_5 = \frac{e^2 B \mu_5}{2\pi^2}$$

# *Derivation (energy conservation)*

## Energy Conservation (Nielsen-Ninomiya 1983)

**Electric field E → Energy shift (Fermi energy)**

$$p_F^R = eEt \qquad p_F^L = -p_F^R$$

Landau Levels

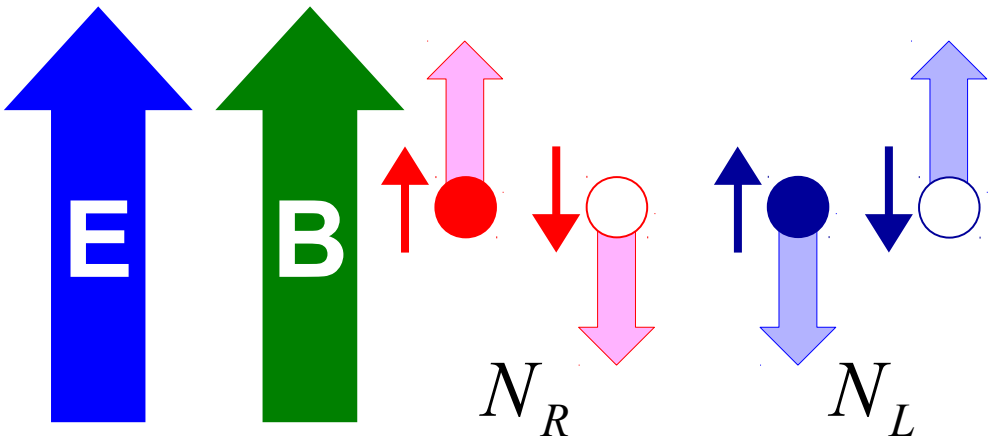
**Density of states** $\mathrm{d}n/\mathrm{d}z = p_F^R/2\pi \qquad \mathrm{d}^2n/\mathrm{d}x\,\mathrm{d}y = eB/2\pi$

$$\frac{p_F^R}{2\pi}\cdot\frac{eB}{2\pi} = \frac{e^2}{4\pi^2}\,\boldsymbol{E}\cdot\boldsymbol{B}\,t \quad \Rightarrow \quad \frac{\mathrm{d}^4N_5}{\mathrm{d}t\,\mathrm{d}^3x} = \frac{e^2}{2\pi^2}\,\boldsymbol{E}\cdot\boldsymbol{B}$$

**Energy cost**

$$\int \mathrm{d}^3x\,\boldsymbol{j}\cdot\boldsymbol{E} = \mu_5\frac{\mathrm{d}N_5}{\mathrm{d}t}$$

$$= \frac{e^2\mu_5}{2\pi^2}\int \mathrm{d}^3x\,\boldsymbol{E}\cdot\boldsymbol{B}$$



# *CME from Inhomogeneous θ*

## Space-time Dependent $\theta$ -angle

$$\gamma\cdot\partial\psi\rightarrow\gamma\cdot\partial\left(e^{i\gamma_5\theta/2N_f}\psi\right)=e^{i\gamma_5\theta/2N_f}\left(\gamma\cdot\partial+i\left(\gamma\cdot\partial\theta/2N_f\right)\gamma_5\right)\psi$$

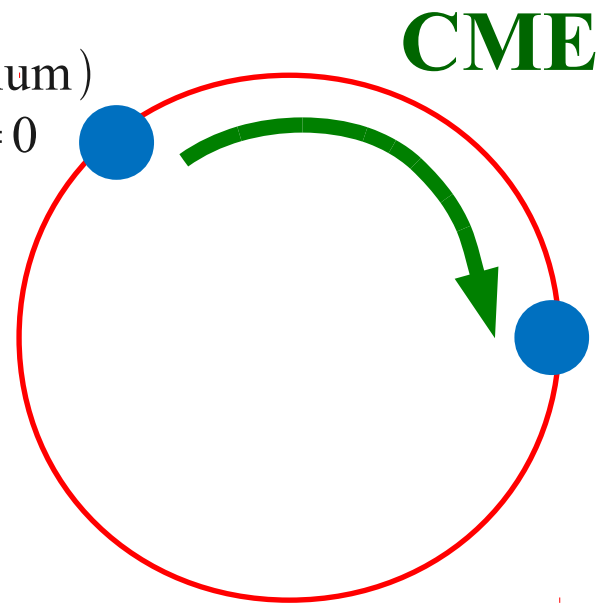
$$\partial_0\theta/2N_f=\mu_5$$

## Schematic Picture

**No CME**

$\theta\neq 0$ (in-medium)
$\langle\sigma\rangle\neq 0 \quad \langle\eta_0\rangle\neq 0$

**Kharzeev, Pisarski, Tytgat, Krasnitz, Venugopalan, Voloshin, ...**




$\theta=0$ (vacuum)
$\langle\sigma\rangle\neq 0 \quad \langle\eta_0\rangle=0$

**No CME**

# *Witten Effect and CME*

## Maxwell-Chern-Simons Theory

$$\vec{\nabla}\times\vec{B}-\frac{\partial\vec{E}}{\partial t} = \vec{j}+c\left(\dot{\theta}\vec{B}-\vec{P}\times\vec{E}\right)$$

$$\vec{\nabla}\cdot\vec{E} = \rho+c\vec{P}\cdot\vec{B}$$

$$\vec{\nabla}\times\vec{E}+\frac{\partial\vec{B}}{\partial t} = 0$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$P_\mu = \partial_\mu\theta$$

Induced Electric Current $j=c\left(\dot{\theta}\vec{B}-\vec{P}\times\vec{E}\right)$

Induced Electric Charge $q=c\vec{P}\cdot\vec{B}=-c\theta g$

**Witten, Wilczek**

# *CME from AdS/QCD Models*

## Chiral Magnetic Current

*Sakai-Sugimoto Model:*
Rebhan et al, JHEP 0905, 084 (2009)
Lifshytz-Lippert, PRD80, 066005 (2009)
*Sakai-Sugimoto Model & Reissner-Nordstrom BH:*
Yee, JHEP 0911, 085 (2009)
*Soft-wall AdS/QCD:*
Gorsky-Kopnin-Zayakin, 1003.2293

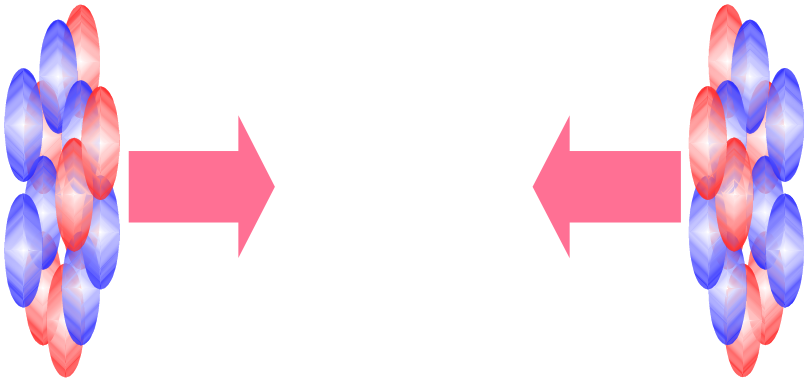
## Confusion and (maybe) a Resolution

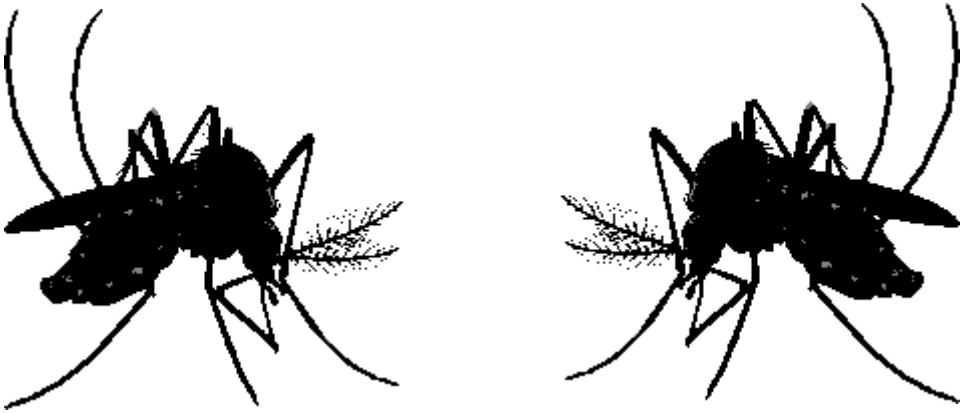
$S_{CS}$ and Bardeen's counter terms change the CME currents?
– Axial gauge fields are not dynamical ones
so the counter terms should not be applied. **Rubakov (2010)**

# Experimental Status

# *Relativistic Heavy-Ion Collisions*

Nucleus (Au) Collision
Energy per nucleon-nucleon collision = 200GeV
$p_0$ =100GeV, $M$ =1GeV
$\rightarrow \gamma \sim 100$

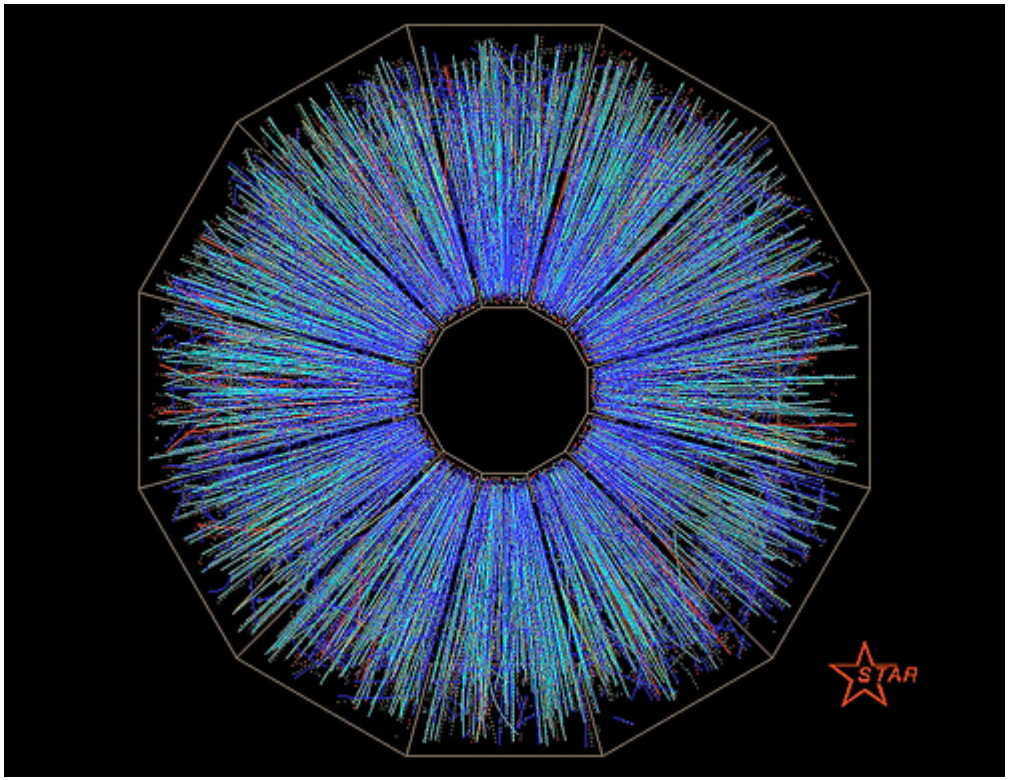




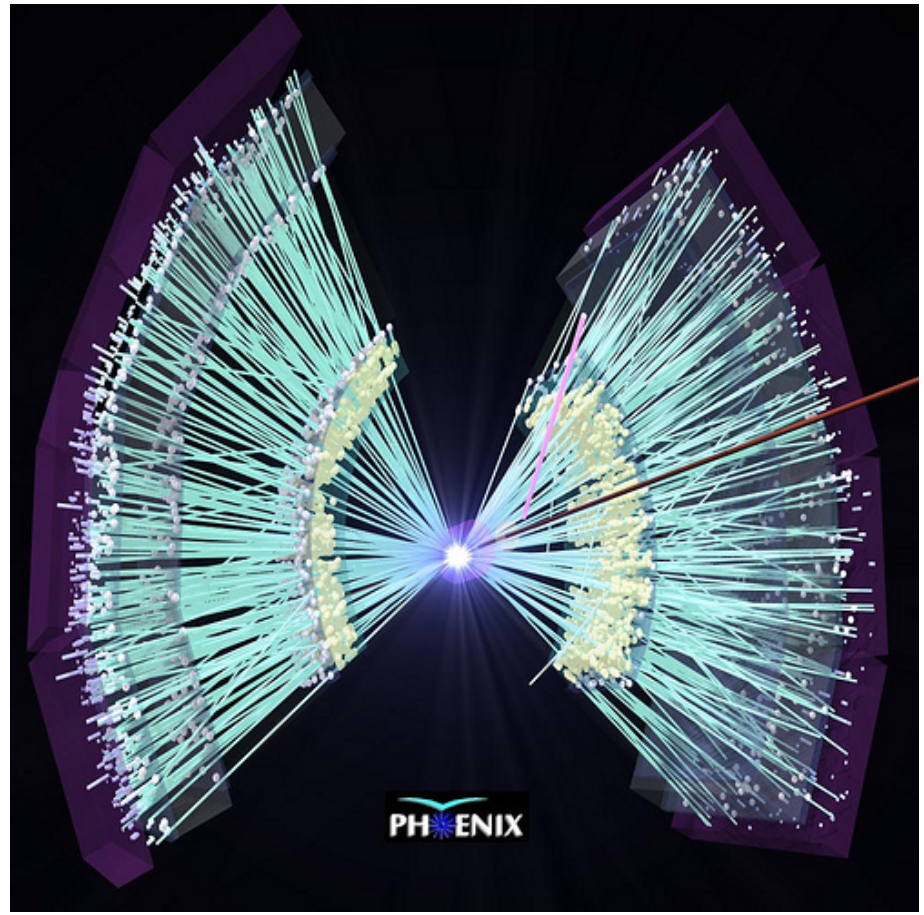
Same as the kinetic energy by flying mosquitoes
$M$ ~3mg, $v$ ~10cm/s

# *Experimental Observation*

## Brookhaven National Laboratory (Gallery)



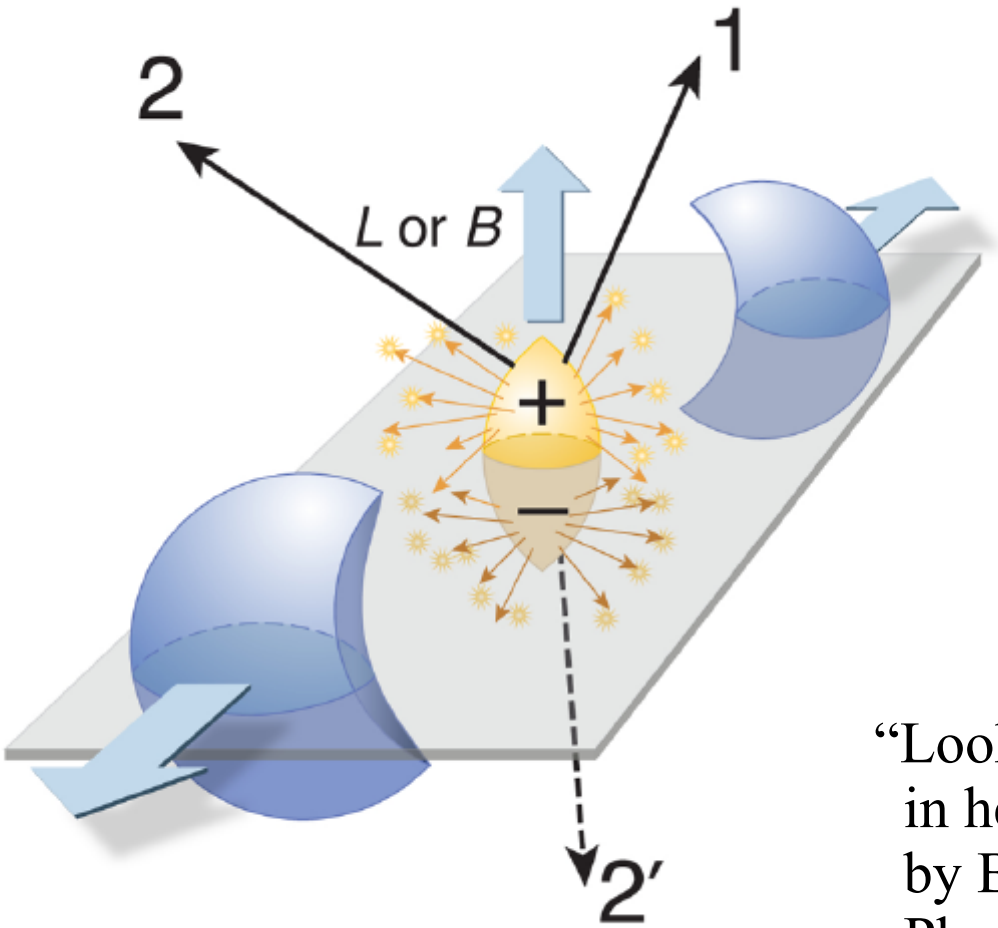

STAR Detector




PHENIX Detector

$\times \sim 100$ M events

# *Charge Separation*

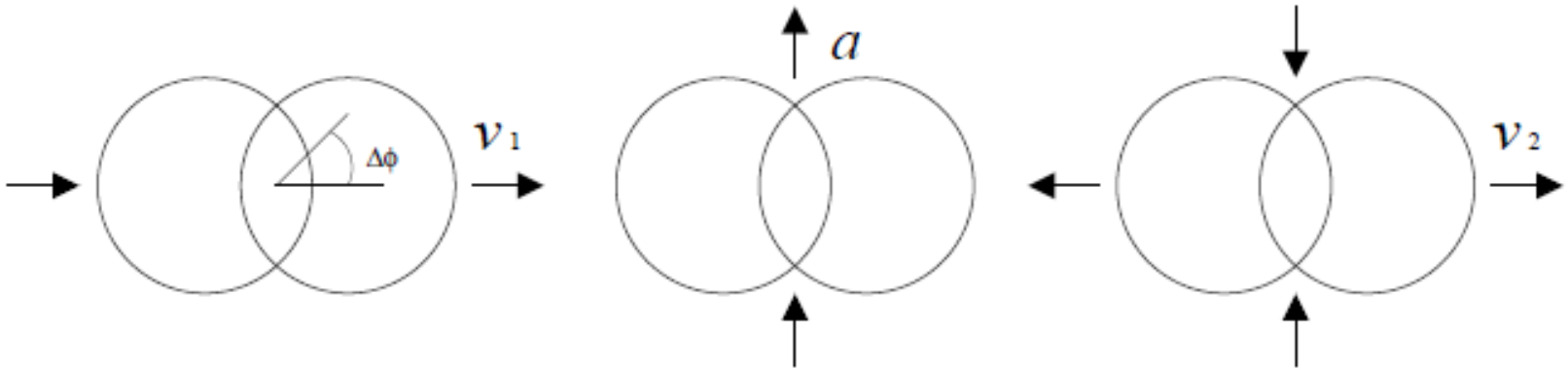



"Looking for parity violation in heavy-ion collisions"
by Berndt Müller
Physics 2, 104 (2009)

# *Observable by Voloshin*

## Measured Multiplicity

$$\frac{\mathrm{d}N_\pm}{\mathrm{d}\phi} \propto 1 + 2v_{1\pm}\cos(\Delta\phi) + 2a_\pm \sin(\Delta\phi) + 2v_{2\pm}\cos(2\Delta\phi) + \cdots$$




$\phi$ : Azimuthal angle

$v_1$ : Directed flow

$v_2$ : Elliptic flow

# *Observable by Voloshin*

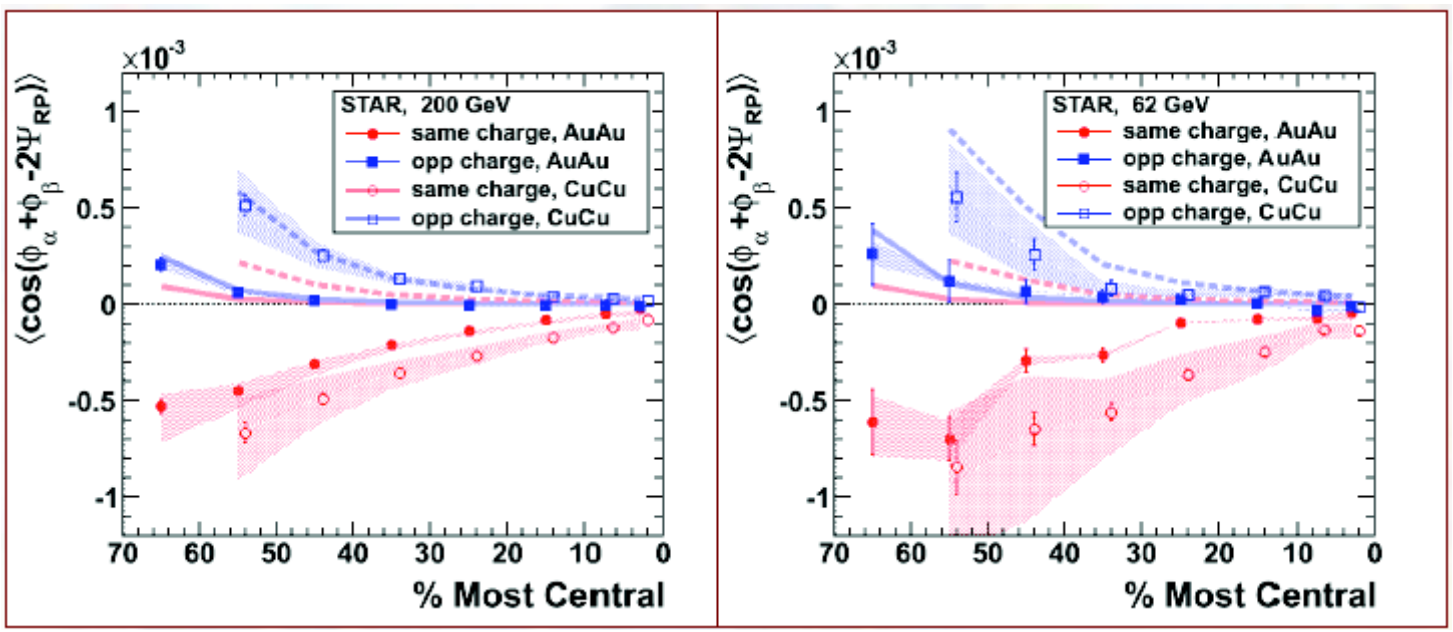
**Observable (fluctuation measurement)**

$$\langle\langle \cos(\Delta\phi_\alpha + \Delta\phi_\beta) \rangle\rangle \equiv \left\langle\left\langle \frac{1}{N_\alpha N_\beta} \sum_{i=1}^{N_\alpha} \sum_{j=1}^{N_\beta} \cos(\Delta\phi_{\alpha,i} + \Delta\phi_{\beta,j}) \right\rangle\right\rangle$$

$$= \langle\langle \cos\Delta\phi_\alpha \cos\Delta\phi_\beta \rangle\rangle - \langle\langle \sin\Delta\phi_\alpha \sin\Delta\phi_\beta \rangle\rangle$$

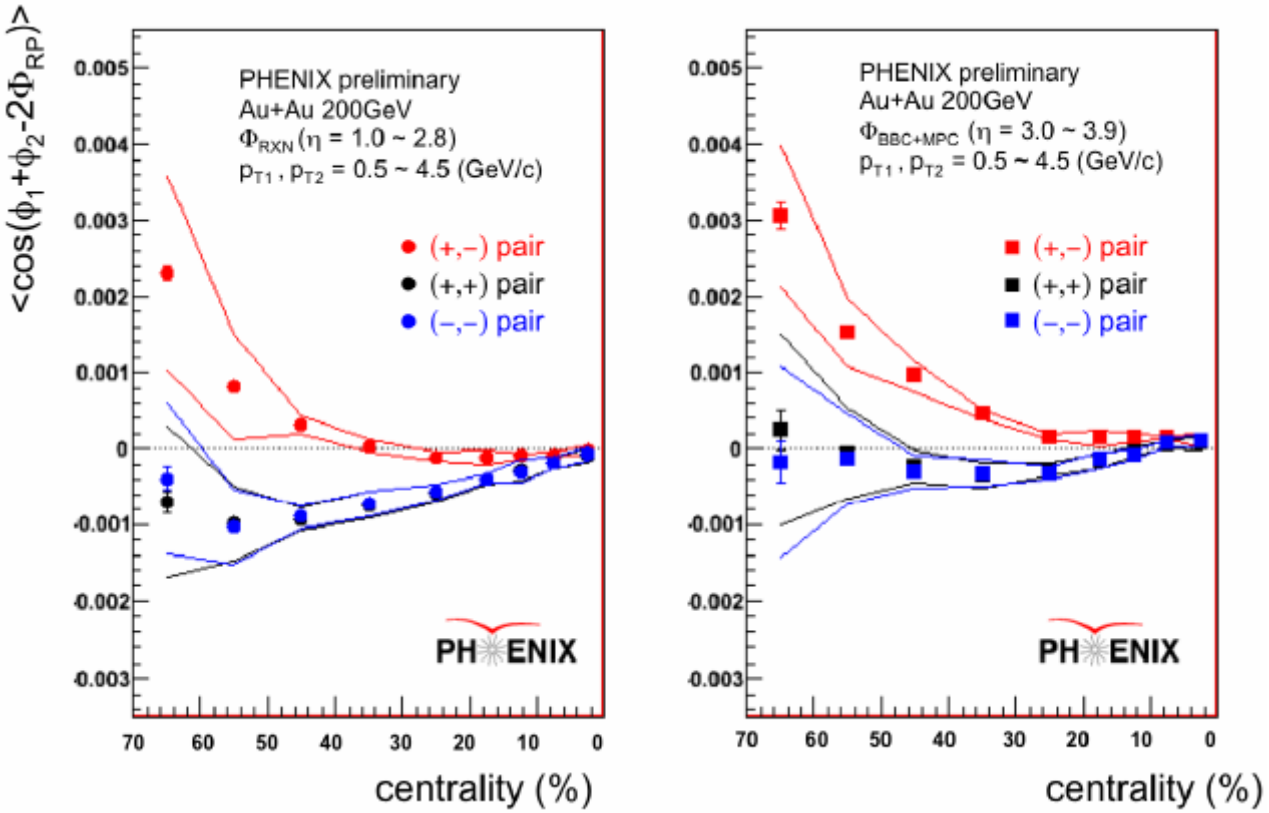
$$= \left( \langle\langle v_{1,\alpha} v_{1,\beta} \rangle\rangle + B^{\text{in}}_{\alpha\beta} \right) - \left( \langle\langle a_\alpha a_\beta \rangle\rangle + B^{\text{out}}_{\alpha\beta} \right).$$

**STAR Results**



# *Confirmation by PHENIX*

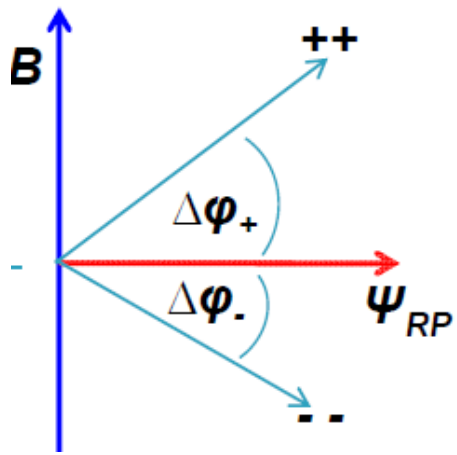
## Good agreement between STAR and PHENIX




**Not conclusive – Backgrounds from Flow, Decay, etc**

# *Multi-Particle Correlation*

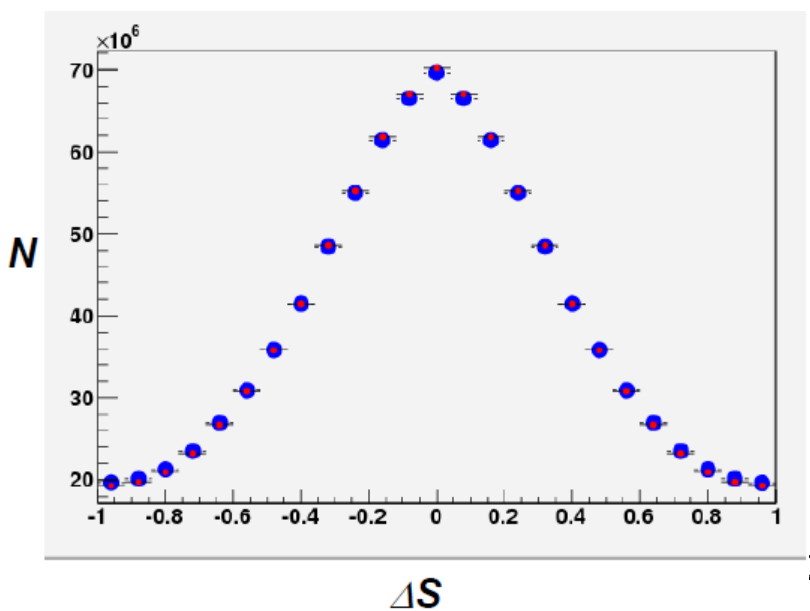
## Two Distribution Functions

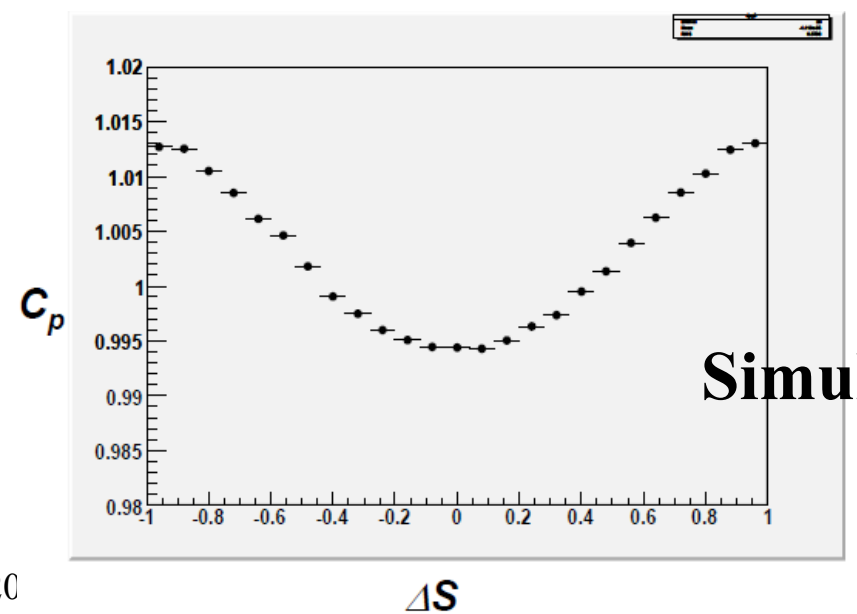



$$\langle S_p^{h+} = \sin\Delta\phi_{p+}\rangle \qquad \langle S_n^{h-}\rangle \qquad \langle S_p^{h\mp}\rangle \qquad \langle S_n^{h\mp}\rangle$$

$$C_p(\Delta S) = \frac{N\left(\langle S_p^{h+}\rangle - \langle S_n^{h-}\rangle\right)}{N\left(\langle S_p^{h\mp}\rangle - \langle S_n^{h\mp}\rangle\right)}$$

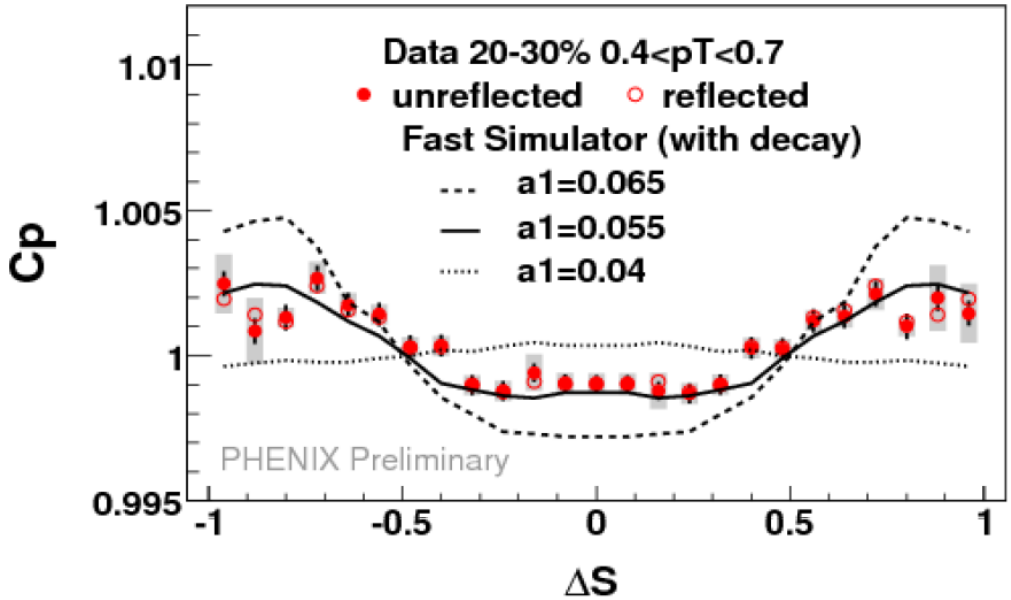
**PHENIX**
**Talk by R.Lacey**

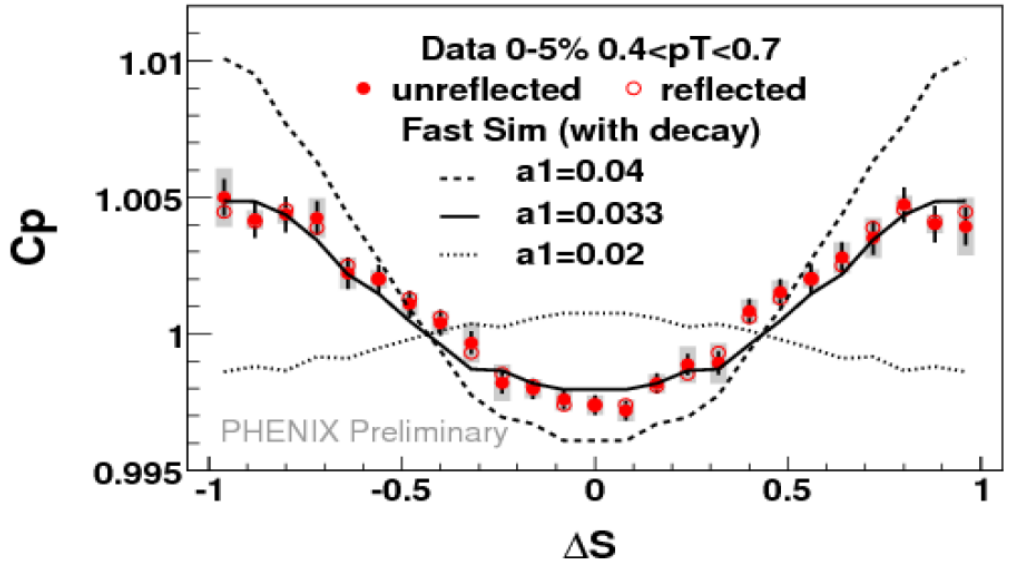



**Simulation**

# *Preliminary Data (talk by R. Lacey)*



# Some Attempts to the Current Fluctuations

# *Charge Asymmetry by Currents*

## Induced Charge

$$\sum_{i=1}^{N_+} \cos\Delta\phi_{+,i} \approx -\sum_{i=1}^{N_-} \cos\Delta\phi_{-,i} \propto J_x$$

$$\sum_{j=1}^{N_+} \sin\Delta\phi_{+,j} \approx -\sum_{i=1}^{N_-} \sin\Delta\phi_{-,i} \propto J_z$$

## Induced Charge Fluctuation

$$\cos(\Delta\phi_\alpha + \Delta\phi_\beta) \propto \frac{\alpha\beta}{N_\alpha N_\beta}\left(J_\perp^2 - J_\parallel^2\right)$$

$$\langle J_\parallel^2\rangle_{\mu_5} = \langle J_\parallel\rangle_{\mu_5}^2 + \chi_{\mu_5}^\parallel\,, \qquad \langle J_\perp^2\rangle_{\mu_5} = \langle J_\perp\rangle_{\mu_5}^2 + \chi_{\mu_5}^\perp$$

# *CME and Non-CME Contributions*

## Electric-current Correlation Function

$$\langle\langle \cos(\Delta\phi_+ + \Delta\phi_+) \rangle\rangle = -\langle\langle \cos(\Delta\phi_+ + \Delta\phi_-) \rangle\rangle$$
$$= \frac{c}{N_\pm^2} \int \mathrm{d}\mu_5\, \mathcal{W}(\mu_5) \left( \langle J_\perp^2 \rangle_{\mu_5} - \langle J_\parallel^2 \rangle_{\mu_5} \right)$$
$$= -\frac{c}{N_\pm^2} \int \mathrm{d}\mu_5\, \mathcal{W}(\mu_5) \left( \langle J_\parallel \rangle_{\mu_5}^2 + \chi_{\mu_5}^\parallel - \chi_{\mu_5}^\perp \right),$$

**Disconnected Part**
**CME (known)**

**Connected Part**
**non-CME background**

# *Susceptibility Computation*

## Expression

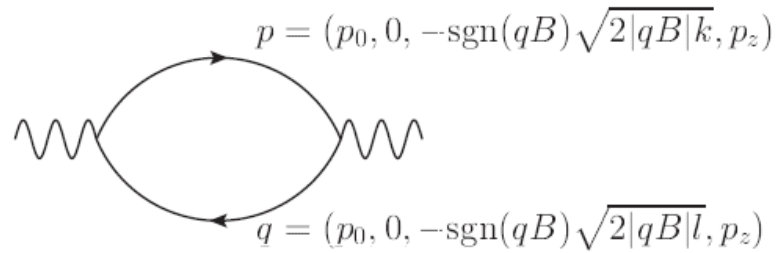
$$\chi_i = \mathrm{i}\, VTN_c \sum_f \frac{q_f^2 |q_f B|}{2\pi} \sum_{k,l} \int \frac{\mathrm{d}p_z}{2\pi} \int^T \frac{\mathrm{d}p_0}{2\pi} \int \mathrm{d}x\, \mathrm{d}y$$

$$\times\, \mathrm{tr}\Big[\gamma^i P_k(x)(\not{\tilde{p}} + \mu_5\gamma^0\gamma^5 - M_f)^{-1}(\Gamma_+(\tilde{p}) + \Gamma_-(\tilde{p}))P_k(y)$$

$$\times\, \gamma^i P_l(y)(\not{\tilde{q}} + \mu_5\gamma^0\gamma^5 - M_f)^{-1}(\Gamma_+(\tilde{q}) + \Gamma_-(\tilde{q}))P_l(x)\Big]$$

$$\Gamma_\pm(p) \equiv \frac{1}{2}(1 \pm \hat{\boldsymbol{p}}\cdot\boldsymbol{\gamma}\gamma^0\gamma^5)$$

$$P_k(x) = \frac{1}{2}\big[f_{k+}(x) + f_{k-}(x)\big] + \frac{\mathrm{i}}{2}\big[f_{k+}(x) - f_{k-}(x)\big]\gamma^1\gamma^2$$




$$f_{k+}(x) = \phi_k(x - p_y/(qB)) \qquad (k = 0, 1, 2, \ldots),$$
$$f_{k-}(x) = \phi_{k-1}(x - p_y/(qB)) \qquad (k = 1, 2, 3, \ldots)$$

$$\phi_k(x) = \sqrt{\frac{1}{2^k k!}}\left(\frac{|qB|}{\pi}\right)^{1/4} \exp\left(-\frac{1}{2}|qB|x^2\right) H_k\left(\sqrt{|qB|}x\right)$$

(Ritus' method)

# *Susceptibility Difference*

## UV-Finite Results

$$\chi_{\mu_5}^{\parallel} - \chi_{\mu_5}^{\perp} = VTN_c \sum_{f,s} \frac{q_f^2 |q_f B|}{4\pi^2} \frac{\Lambda}{\omega_{\Lambda\lambda}} \left(1 + \frac{s\mu_5}{\Lambda}\right) \left[1 - n_F(\omega_{\Lambda\lambda}) - \bar{n}_F(\omega_{\Lambda\lambda})\right]$$

$$\xrightarrow[\Lambda\to\infty]{} VTN_c \sum_f \frac{q_f^2 |q_f B|}{2\pi^2} .$$

Only the *Landau zero-mode* contributes to the final result. The longitudinal and transverse difference is *UV finite* and insensitive to any IR scales.

# *Heuristic Argument*

**Current generation rate**
**= Chirality (Schwinger process) rate**

c.f. Iwazaki
KF-Kharzeev-Warringa

$$\left\langle \frac{\mathrm{d}J_\parallel}{\mathrm{d}x_0} \right\rangle = V N_c \sum_f \frac{q_f^2 |q_f B| E}{2\pi^2} + O(E^2)$$

**Linear response theory**

$$\left\langle \frac{\mathrm{d}J_\parallel}{\mathrm{d}x_0} \right\rangle = -\int \mathrm{d}^3x\, \mathrm{d}^4x' \left\langle \frac{\mathrm{d}j_\parallel(x)}{\mathrm{d}x_0} j_\parallel(x') \right\rangle_{\mathrm{ret}} A_z(x') + O(A_z^2)$$

**Integration by parts in the gauge** $(d/dx_0)A_z = E_z$

$$\left\langle \frac{\mathrm{d}J_\parallel}{\mathrm{d}x_0} \right\rangle = \int \mathrm{d}^3x\, \mathrm{d}^4x' \, \langle j_\parallel(x) j_\parallel(x') \rangle_{\mathrm{ret}} E + O(E^2)$$

# *Heuristic Argument*

**Current generation rate**
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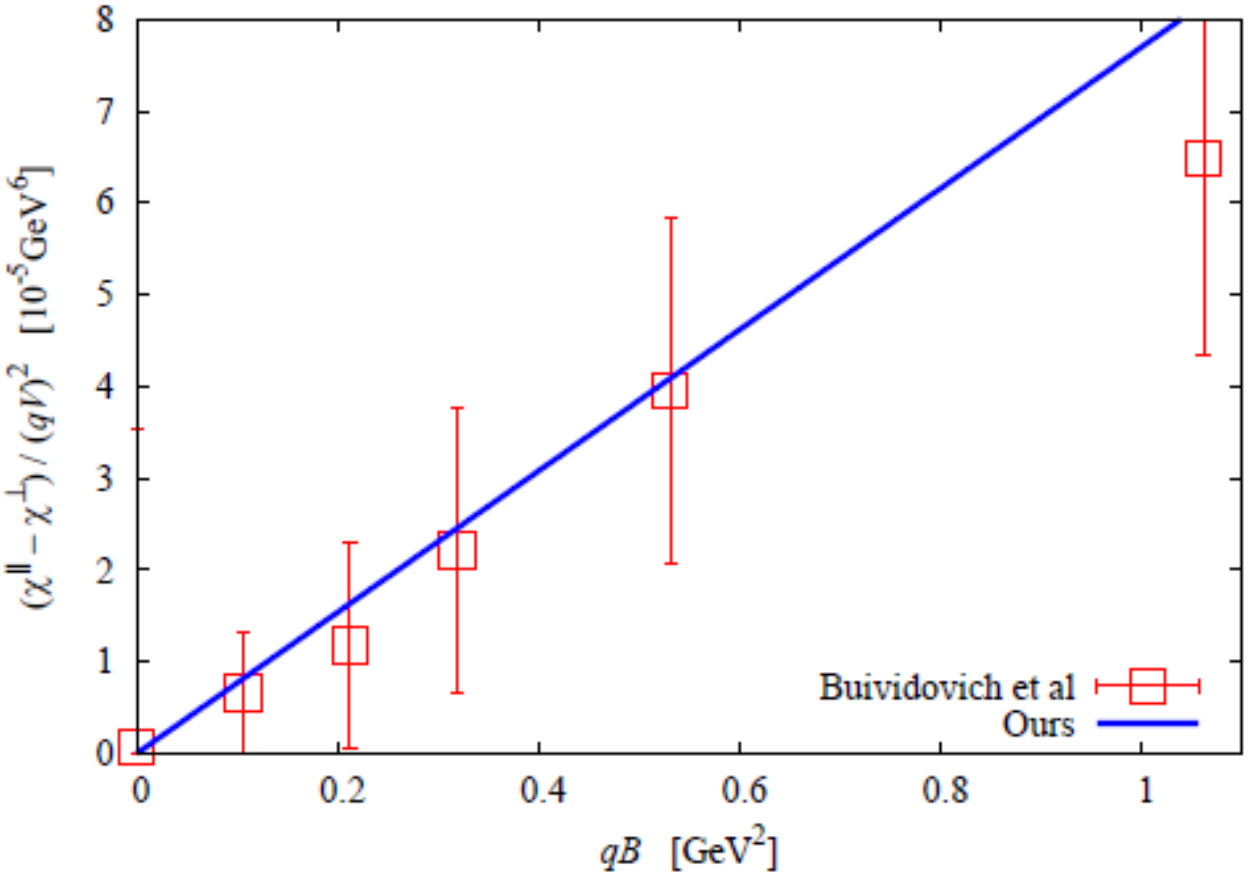
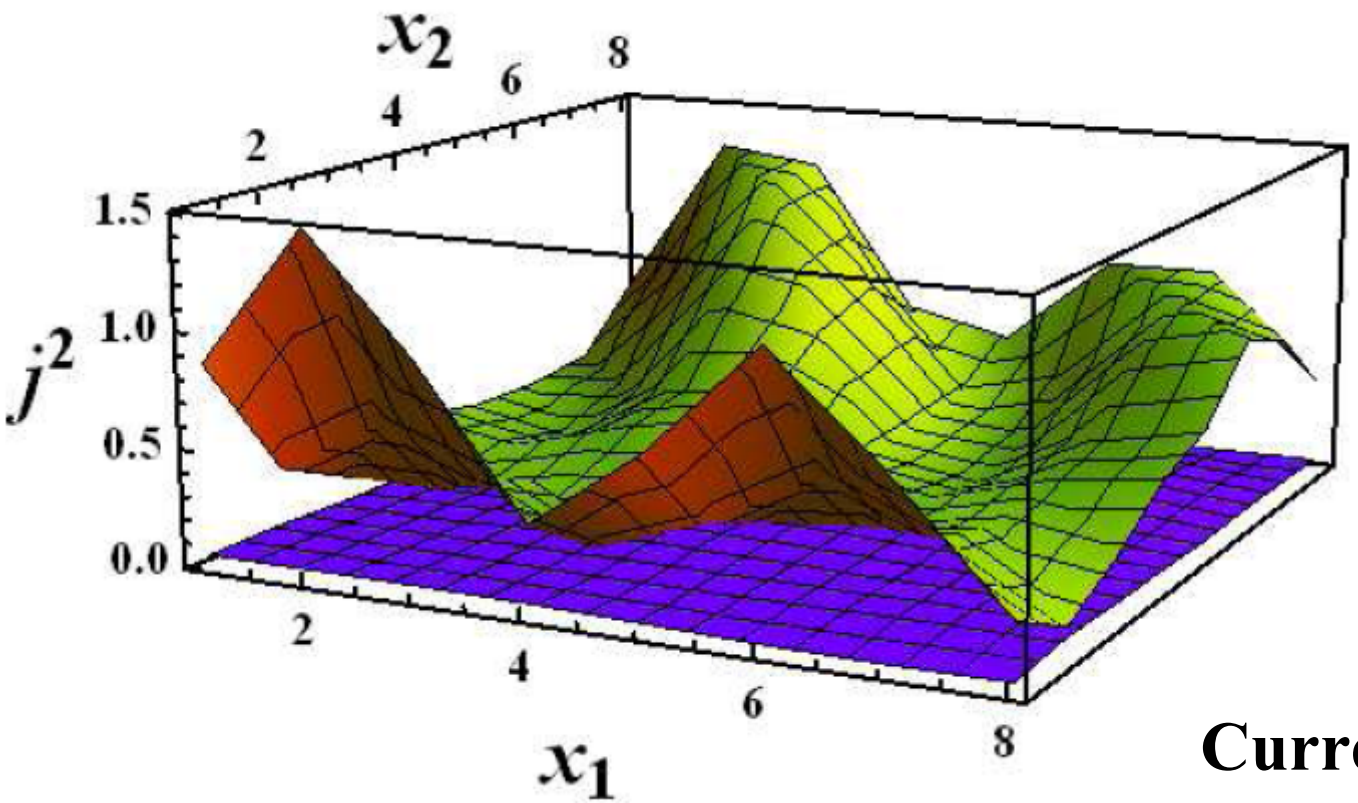
# *Comparison with Lattice QCD*

**Buividovich, Chernodub, Luschevskaya, Polikarpov (2009)**



# *More Attempts from Lattice QCD*

## Polikarpov et al (Instanton + Magnetic Field)

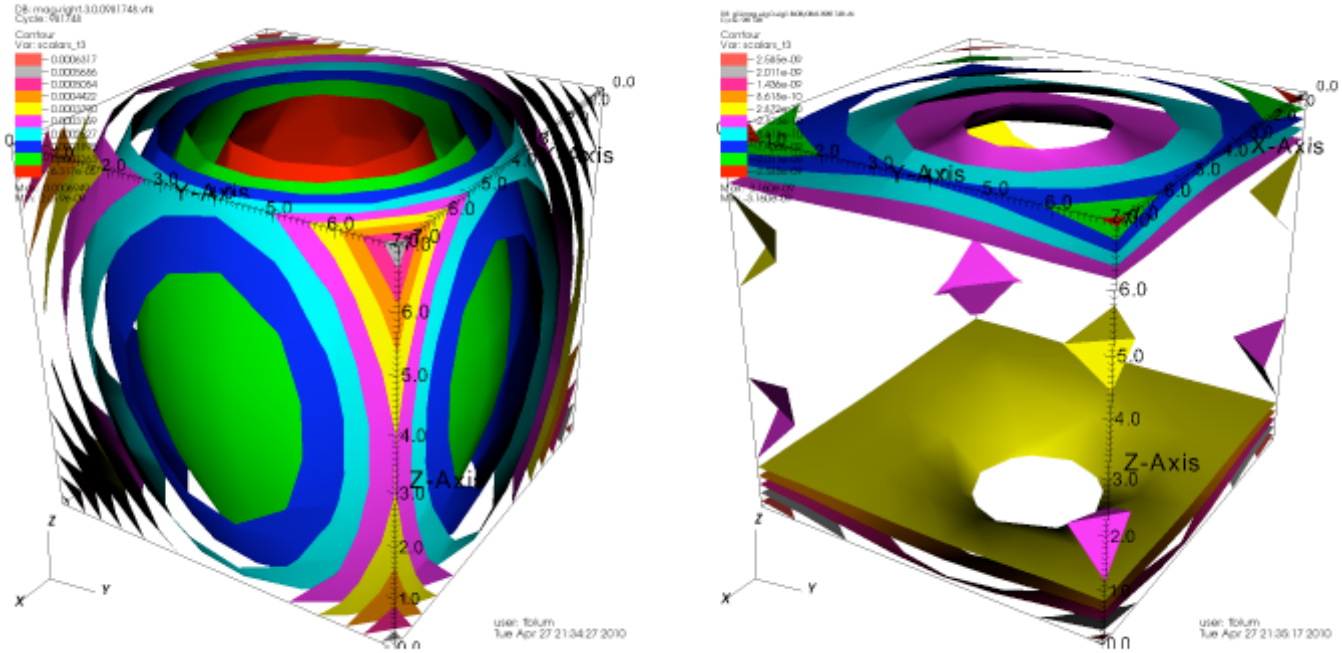



**Current Squared**

**Chiral fermions are crucial ← Overlap fermion**

# *More Attempts from Lattice QCD*

## Blum et al (Instanton + Magnetic Field)




**Charge Separation**

**QCD+QED simulations are ongoing**

# *Instead of Conclusions*

**Please visit: http://quark.phy.bnl.gov/~kharzeev/cpodd/**

## P- and CP-odd Effects in Hot and Dense Matter

RIKEN BNL Research Center Workshop
April 26-30, 2010 at Brookhaven National Laboratory

Home | Registration | Agenda | Contact Us | Event Information

Workshop on
### P- and CP-odd Effects in Hot and Dense Matter

**Registration* Deadline:** March 1, 2010

***Additional Guest Registration** (Deadline: March 1, 2010)
*If you are a Non-U.S. Citizen, you must complete a Guest Registration Form in addition to the Workshop Registration. Due to a required review process, all Non-U.S. Citizens must be registered and approved in the BNL Guest Information System (GIS) before they will be allowed access to the site.* Please complete this as soon as possible to ensure sufficient time for complete approval. *The link to GIS will be provided to you after you have registered for this event.*

**Event Date**
April 26-30, 2010

**Event Location**
Brookhaven National Laboratory
Physics Building, Bldg 510
Large Seminar Room

**Event Coordinator**
Pam Esposito
Bus: 631-344-3097
Fax: 631-344-4067
Email: pesposit@bnl.gov

### Motivation & Plans

Our goal is to initiate a cross-disciplinary discussion of topological effects (such as Chern-Simons number generation) in hot and dense QCD matter, including the implications for cosmology and condensed matter physics. We plan to critically review the current state of the theory, the experiment and the unsolved questions; to formulate new research directions; and to make the predictions for the future program of RHIC as well as for the heavy ion program at the LHC, FAIR and NICA.