Tadpole Resummations
in
String Theory

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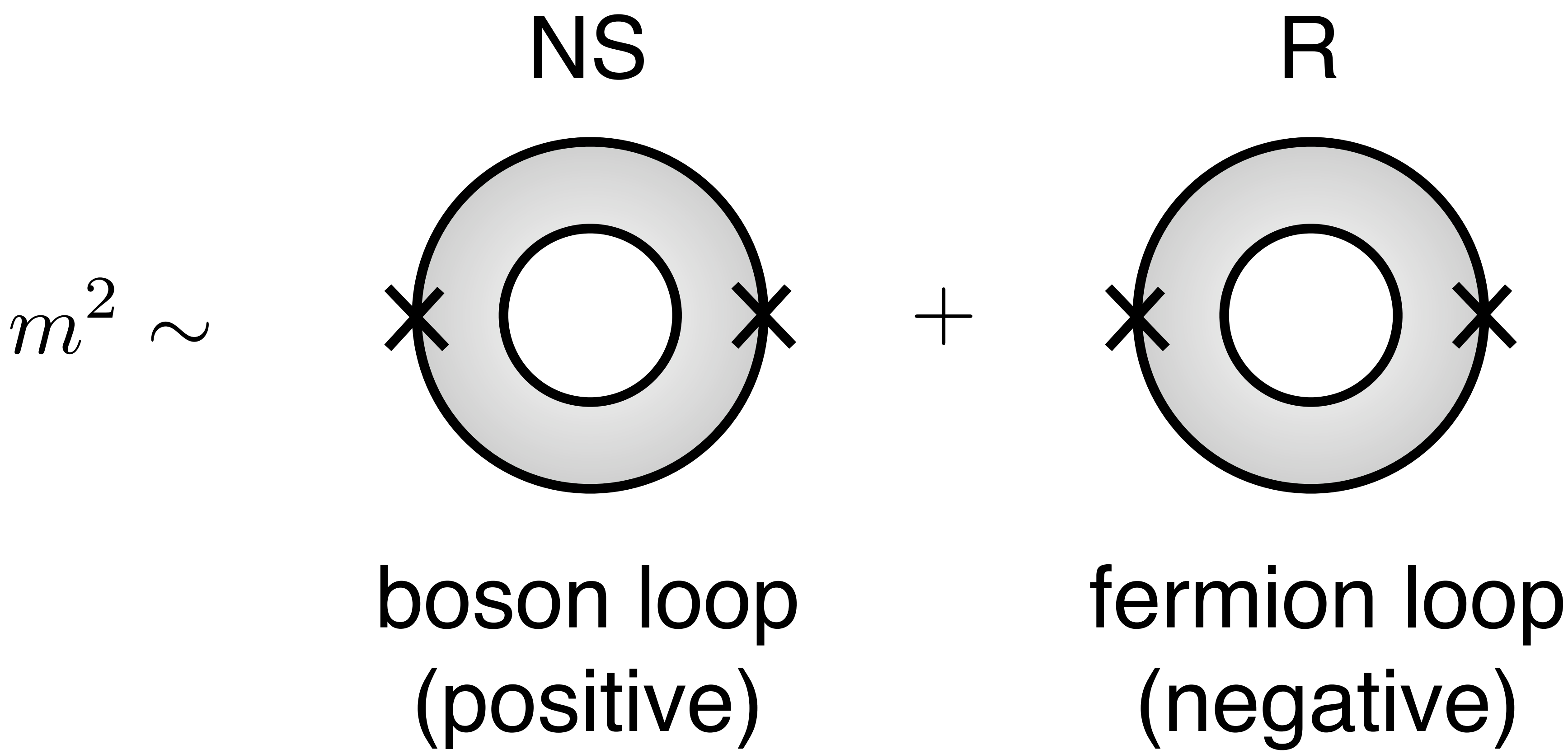
1. Motivation and Introduction

Possibility of low-scale string models without SUSY
(using D-branes at orbifold singularities
with brane SUSY breaking mechanism)

- At tree-level there are many massless scalar fields with non-trivial gauge charges.
- Non-zero one-loop corrections to the masses of those scalar fields are expected because of lack of SUSY.

Some extra scalar fields would become massive and decouple, and some others would obtain negative mass squared and would become Higgs doublet fields in the Standard Model for electroweak symmetry breaking.

Some concrete calculations have shown the possibility of radiative symmetry breaking.



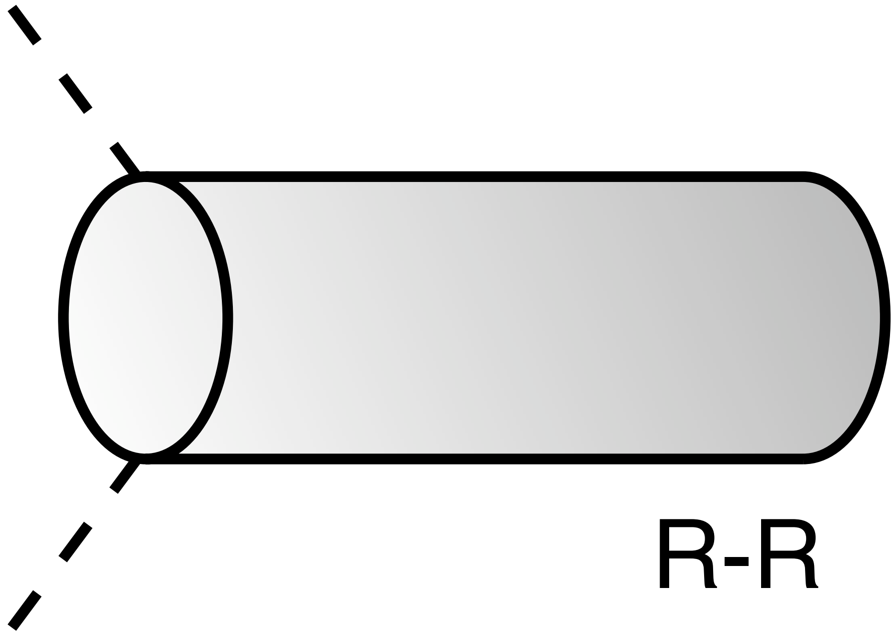
possible to have:

$$m^2 \simeq -\frac{g^2}{16\pi^2}\frac{1}{\alpha'}f(R/\sqrt{\alpha'}) < 0$$

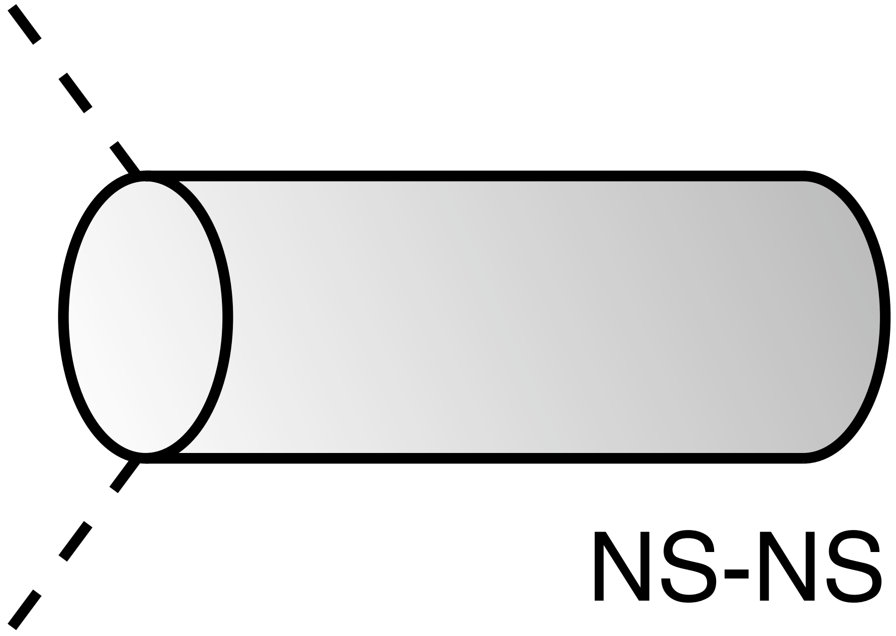
Antoniadis-Benakli-Quiros (2000), N.K. (2006)

“NS-NS tadpole problem”

Two kinds of closed-string tadpole contributions
assuming open-closed string duality



massless R-R tadpoles
should be canceled
for the consistency
of the model.



massless NS-NS tadpoles
are not required to cancel
for consistencies

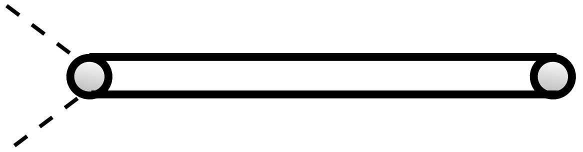
In general string models without SUSY have
NS-NS tadpoles of massless dilaton and graviton.

- The conceptual difficulty (or understanding):

  The background geometry and fields configurations are not the “solution” of String Theory.

  -> This should be cured by Fishler-Susskind mechanism which is very difficult to do unfortunately.

- The actual difficulty:



$$\lim_{p \to 0} \frac{1}{p^2}$$

  Open-string one-loop calculations get infrared divergences.
  Some reasonable technique to evade these divergences?

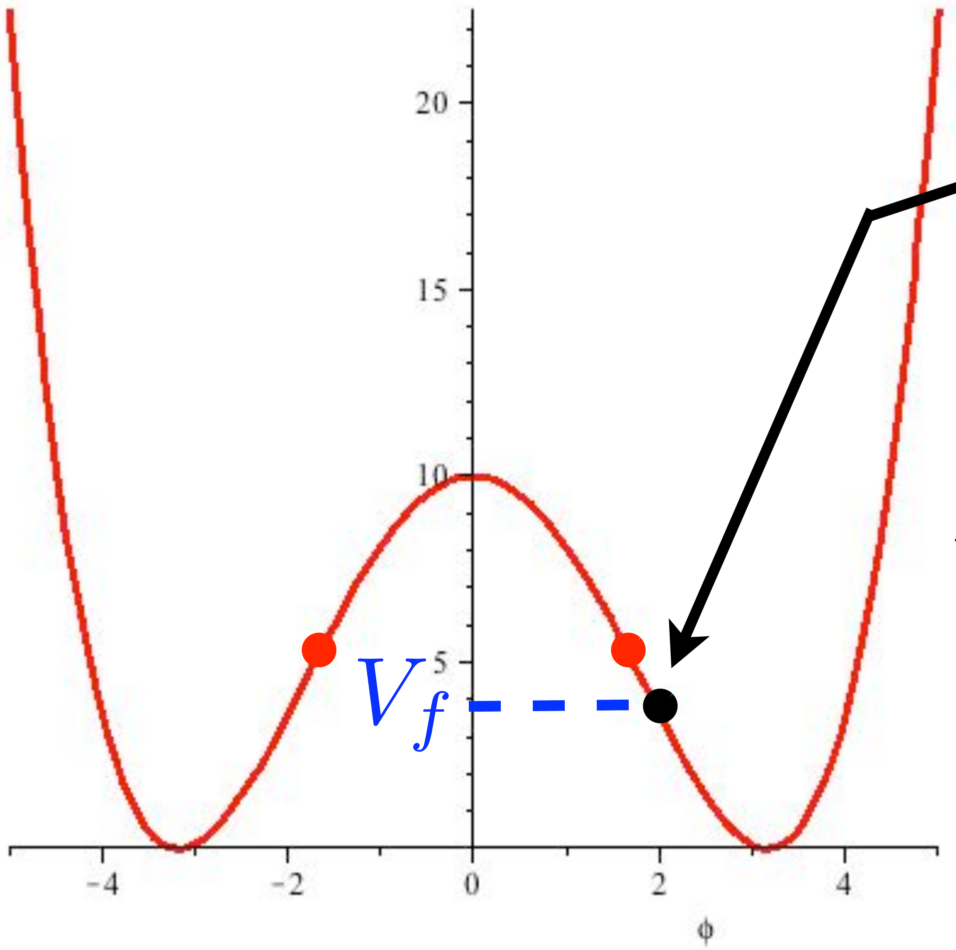
  -> “tadpole resummations”

Tadpole resummations in field theory

Dudas-Nicolosi-Pradisi-Sagnotti (2005)

Possibility to obtain true values of physical quantities in “wrong vacua”

ex.

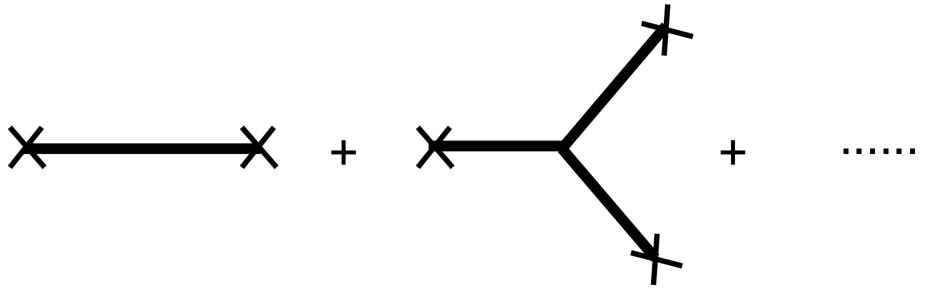


“wrong vacua”

$$V = -4a(v^2 - v_f^2)v_f\phi + \cdots$$

tadpole

$V_{\rm res} =$



$$V_f + V_{\rm res} = 0$$

Is the same is possible in String Theory?

Similar resummations are possible in String Theory using boundary state formalism

The technique gives one positive result:

The vacuum energy of a “Dp-brane” in Bosonic String Theory is canceled by the tree-level contribution from tadpole resummations,

$$\Lambda_p^{\mathrm{cl}} + \Lambda_p^{\mathrm{res}} = 0 \qquad (\Lambda_p^{\mathrm{cl}} = T_p)$$

N.K. (2008)

which is consistent with Sen’s conjecture of “Dp-brane” decay in Bosonic String Theory: “tachyon condensation”.

Some non-trivial checks will be given.

2. Some techniques

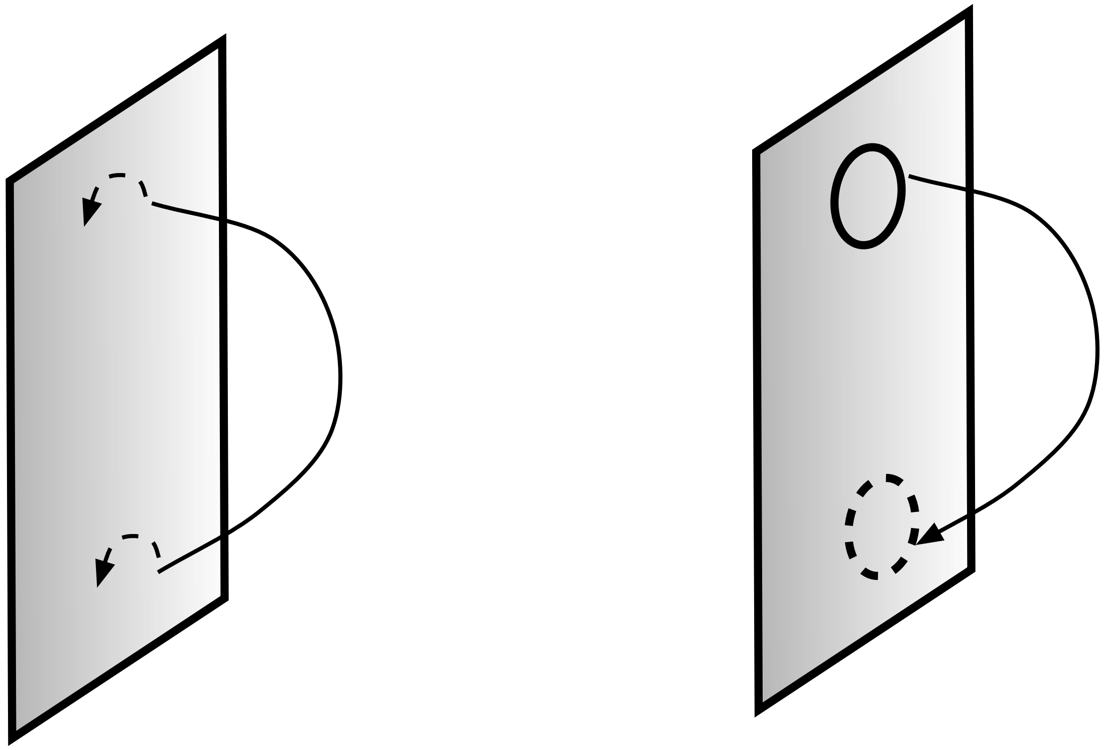
In this talk we mainly consider Bosonic String Theory, although the same is applicable to Superstring Theory.

Boundary State Formalism

the cylinder amplitude

open string ⟷ closed string

one loop — tree



Dp-brane boundary state

$$\frac{1}{2!}\langle B_p|D|B_p\rangle$$

second order of tadpole insertion

propagator operator

The Cylinder Amplitude
-- one-loop correction to the vacuum energy of Dp-brane --

$$A_p = \frac{1}{2!}\langle B_p|D|B_p\rangle = \frac{1}{2!}V_{p+1}N_p^2\Delta_p$$

$$N_p \equiv T_p/2: \ \text{normalization factor of} \, |B_p\rangle$$

$$\Delta_p \equiv \frac{\pi\alpha'}{2}\int_0^\infty ds \int \frac{d^{d_\perp}p}{(2\pi)^{d_\perp}} e^{-\frac{\pi\alpha'}{2}p_\perp^2 s}\frac{1}{(\eta(is))^{24}}$$

“propagator”

$$= \frac{\pi\alpha'}{2}\int_0^\infty ds \frac{1}{(2\pi^2\alpha' s)^{d_\perp/2}}\frac{1}{(\eta(is))^{24}}$$

$$d_\perp \equiv d-(p+1) \ \text{ with } \ d=26$$

Infrared divergence due to massless tadpoles of dilaton, graviton and tachyon.

Tadpole couplings in boundary states

$$A^{\mu\nu} \equiv \langle 0;k|a_1^{\mu}\tilde{a}_1^{\nu}|B_p\rangle = -\frac{T_p}{2}V_{p+1}S^{\mu\nu}$$

$$S^{\mu\nu} \equiv (\eta^{\alpha\beta}, -\delta^{ij})$$

$$\begin{cases} A_{\text{grav}} = A^{\mu\nu}\epsilon^{(h)}_{\mu\nu} = -V_{p+1}T_p\eta^{\alpha\beta}\epsilon^{(h)}_{\alpha\beta}, \\ A_{\text{dil}} = A^{\mu\nu}\epsilon^{(\phi)}_{\mu\nu} = V_{p+1}T_p a. \end{cases}$$

$$\left(a \equiv \frac{d-2p-4}{2\sqrt{d-2}} \right)$$

D-brane effective action in Einstein frame

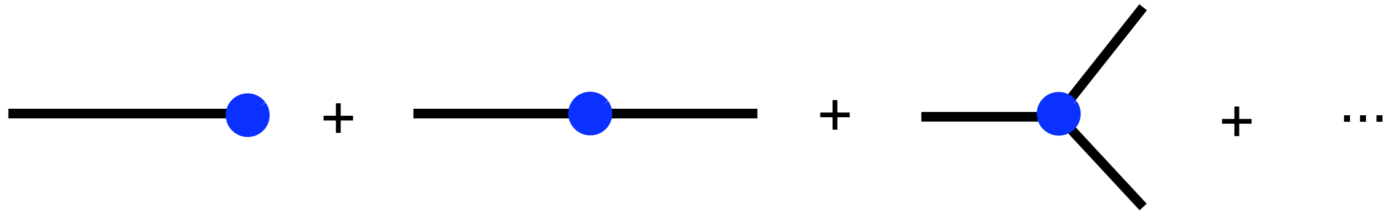
$$S_{Dp} = -T_p \int d^{p+1}\xi e^{-a\phi}\sqrt{-\det g_{\alpha\beta}}$$

ignoring B-field
and gauge field

$$\mathcal{L}_{\text{dil}} = -T_p e^{-a\phi} = -T_p + T_p a\phi - \frac{1}{2!}T^p a^2\phi^2 + \frac{1}{3!}T^p a^3\phi^3 - \cdots$$

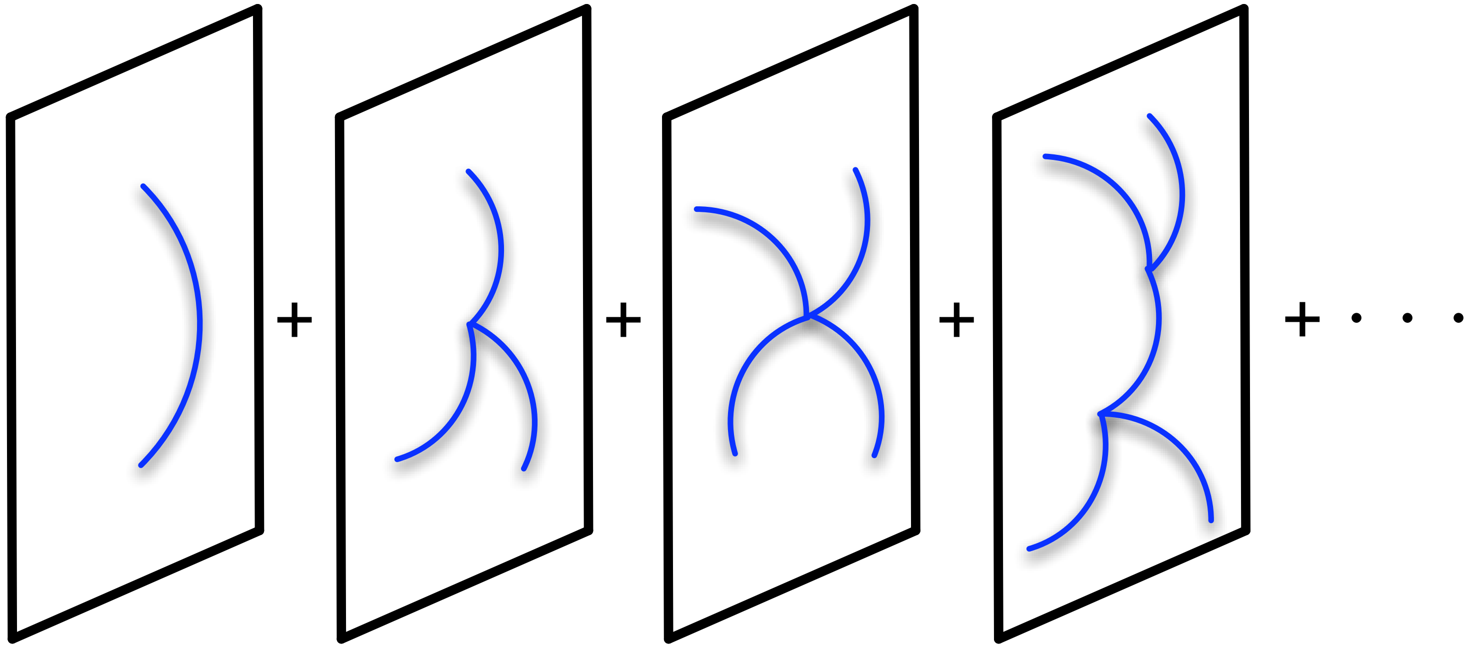
$$\Lambda_p^{\text{cl}} = T_p$$

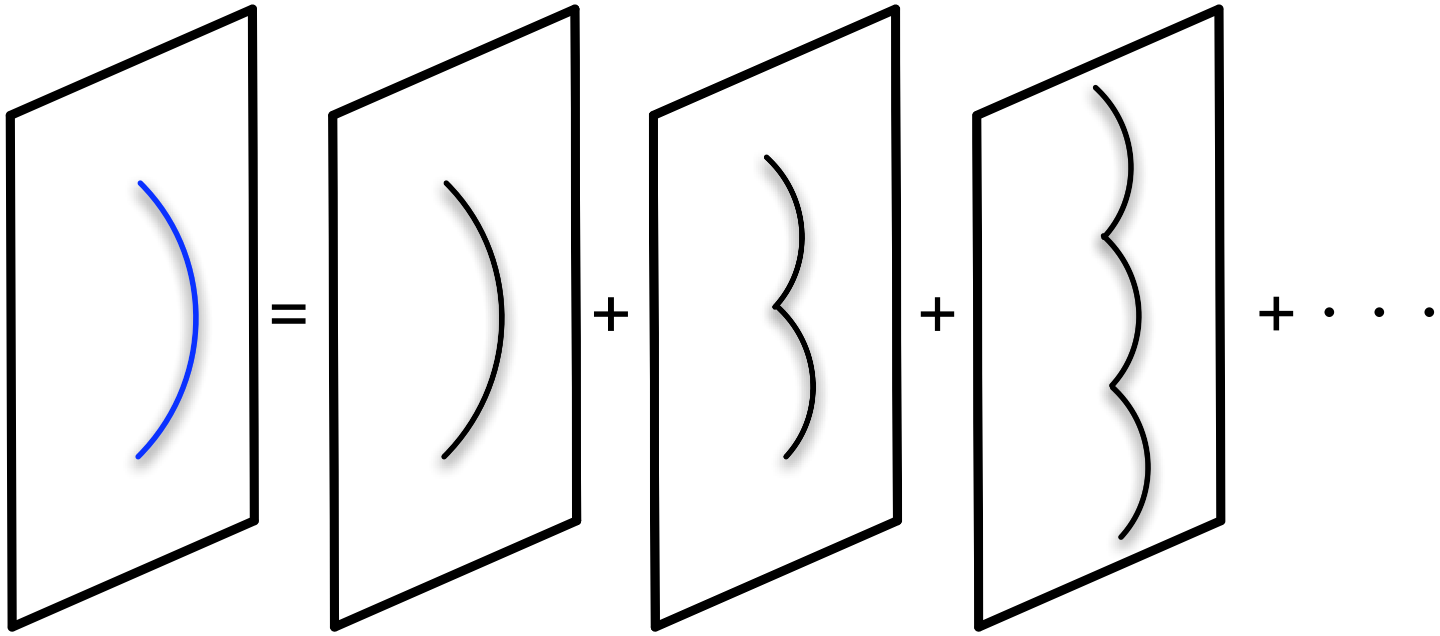
vacuum energy



Strategy for tadpole resummations

$$\Lambda_p = \Lambda_p^{\mathrm{cl}} +$$



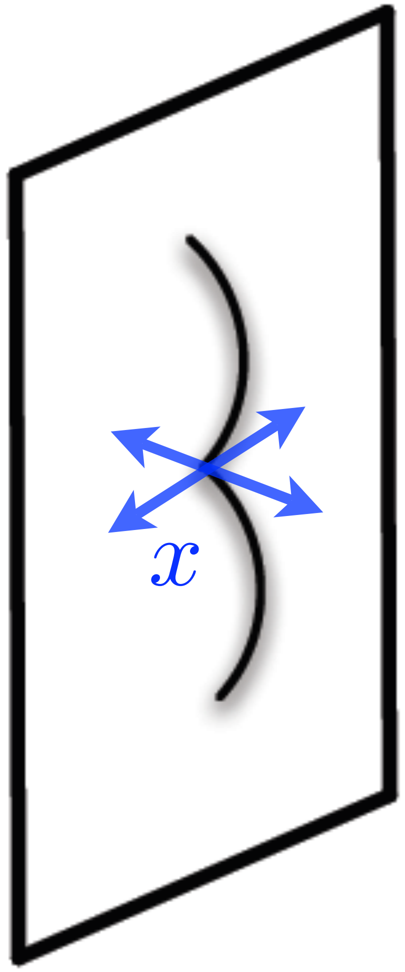


“Closed strings bouncing on a Dp-brane”

Multi-point vertices in boundary state formalism

$$\hat{M} \equiv \int d^d x \delta^{d_\perp}(x) |\tilde{B}_p(x)\rangle (-T_p) \langle \tilde{B}_p(x)|$$

integrating over Dp-brane world volume



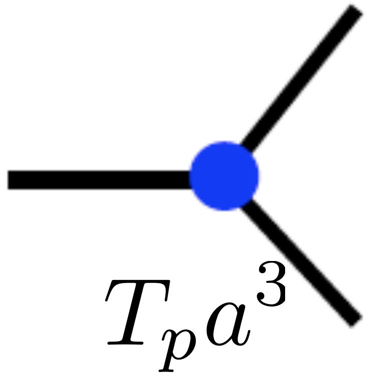
$$|\tilde{B}_p(x)\rangle \equiv \frac{1}{T_p} \delta^{p+1}(\hat{x} - x) |B_p\rangle$$

normalization
for correct coupling

specify one point
on Dp-brane

$$\left(T_p a \times \frac{1}{T_p} \right)^2 \times (-T_p) \times d^d x \delta^{d_\perp}(x) \to -T_p a^2 \times V_{p+1}$$

Similar for three-point vertex and higher



$$\hat{M}^{(3)} \equiv \frac{1}{3!} T_p \left(\frac{N_p}{T_p}\right)^3 \int d^d x \delta^{d_\perp}(x)$$
$$\times |\tilde{B}_p(x)\rangle |\tilde{B}_p(x)\rangle |\tilde{B}_p(x)\rangle$$

Closer look at “propagator”

$$\Delta_p = \frac{\pi\alpha'}{2} \int_0^\infty ds \frac{1}{(2\pi^2\alpha' s)^{d_\perp/2}} \frac{1}{(\eta(is))^{24}}$$
$$\to \frac{\pi\alpha'}{2} \int^\infty ds \frac{1}{(2\pi^2\alpha' s)^{d_\perp/2}} \left\{\boxed{e^{2\pi s}} + \boxed{24} + \mathcal{O}(e^{-2\pi s})\right\}$$

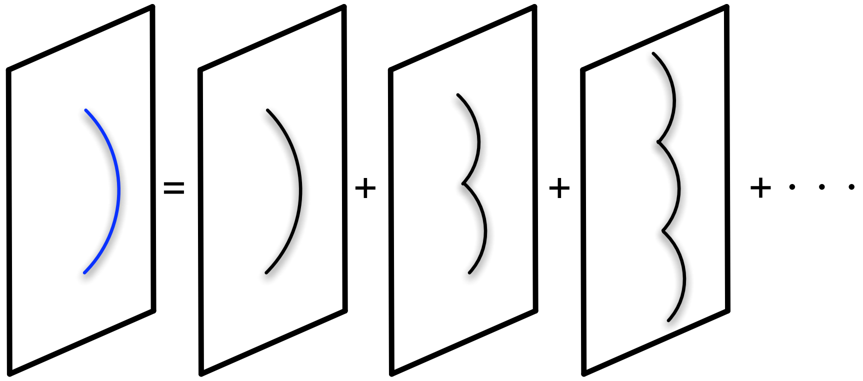
tachyon tadpoles

dilaton/graviton tadpoles

Regularize via an “ultraviolet” cutoff on s

3. Actual calculations on vacuum energies

Full two-point function



"one bounce"

$$\frac{1}{2!}\langle B_p|D\hat{M}D|B_p\rangle = \frac{1}{2!}\int d^dx\,\delta^{d_\perp}(x)\langle B_p|D|\tilde{B}_p(x)\rangle(-T_p)\langle\tilde{B}_p(x)|D|B_p\rangle$$

$$= \frac{1}{2!}V_{p+1}N_p^2\left(\frac{N_p}{T_p}\right)^2(-T_p)(\Delta_p)^2$$

similar to the cylinder

Similar for "two bounces", and more

$$A_p^{(2)} = \frac{1}{2!}\left\{\langle B_p|D|B_p\rangle + \langle B_p|D\hat{M}D|B_p\rangle + \langle B_p|D\hat{M}D\hat{M}D|B_p\rangle + \cdots\right\}$$

geometric series

$$\equiv \frac{1}{2!}\langle B_p|D_M|B_p\rangle$$

$$= \frac{1}{2!}V_{p+1}N_p^2\frac{\Delta_p}{1+T_p(N_p/T_p)^2\Delta_p} \to \frac{1}{2!}V_{p+1}T_p$$

$$\boxed{\Lambda_p^{(2)} = -\frac{1}{2!}T_p}$$

Full three-point function

vertex

# of contractions

$$\frac{1}{3!} = \frac{1}{3!} \times \frac{1}{3!} \times 3!$$

third order of tadpole insertions

$$A_p^{(3)} = \frac{1}{3!} T_p \int d^d x \delta^{d_\perp}(x) \left(\langle B_p | D_M | \tilde{B}_p(x) \rangle \right)^3$$

$$= \frac{1}{3!} V_{p+1} T_p \left(\frac{(N_p^2/T_p)\Delta_p}{1 + T_p (N_p/T_p)^2 \Delta_p} \right)^3 \to \frac{1}{3!} V_{p+1} T_p$$

$$\boxed{\Lambda_p^{(3)} = -\frac{1}{3!} T_p}$$

Full Vacuum Energy of Dp-brane

$$\Lambda_p^{\text{res}} \equiv -\left(A_p^{(2)} + A_p^{(3)} + \cdots \right) / V_{p+1}$$

$$= -T_p \sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \boxed{-T_p}$$

exactly cancels

$$\boxed{\Lambda_p^{\text{cl}} = T_p}$$

Consistent with Sen's conjecture

4. Some Consistency Checks

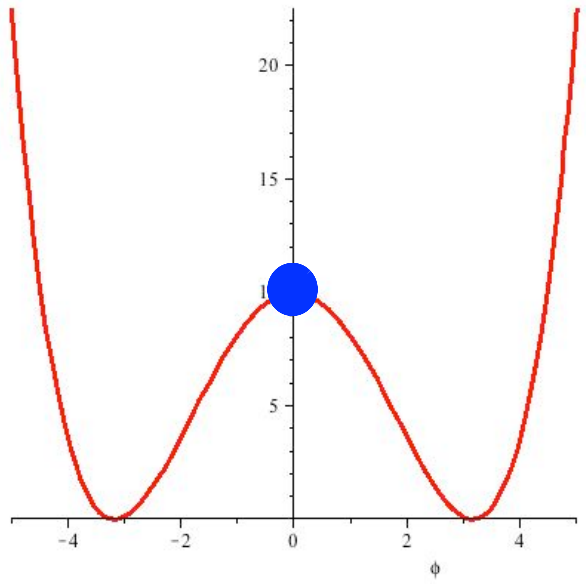
“tadpole resummations” on a D25-brane in SO(8192) theory

8192 D25-branes in unoriented Bosonic String Theory

Douglas-Grinstein (1987), Marcus-Sagnotti (1987), Weinberg (1987)

-> (unstable) “solution” of String Theory

- no tadpoles of dilaton and graviton
- tachyon (with tadpole)



“tadpole resumations” should not give corrections to the vacuum energy, even though tachyon exists.

two kinds of boundary states and three kinds of amplitudes

$$|B_{25}\rangle \quad \text{and} \quad \boxed{|C_{25}\rangle}$$

for orientifold
fixed plane: O25

$$\begin{aligned}
\mathcal{A} &= \frac{1}{2!}\langle B_{25}|D|B_{25}\rangle \\
\mathcal{M} &= \frac{1}{2!}\left(\langle B_{25}|D|C_{25}\rangle + \langle C_{25}|D|B_{25}\rangle\right) \\
\mathcal{K} &= \frac{1}{2!}\langle C_{25}|D|C_{25}\rangle
\end{aligned}$$

Full cylinder amplitude with bouncing on D25 and O25

$$\begin{aligned}
\mathcal{A}^{(2)} &= \frac{1}{2!}V_{26}N_{25}^2 \boxed{\frac{\Delta_{25}}{1+T_{25}(N_{25}/T_{25})^2\Delta_{25}}} \times \boxed{\frac{1}{1-T_{25}(N_{25}/T_{25})^2\Delta_{25}}} \\
&\to 0
\end{aligned}$$

(the same for the
other amplitudes)

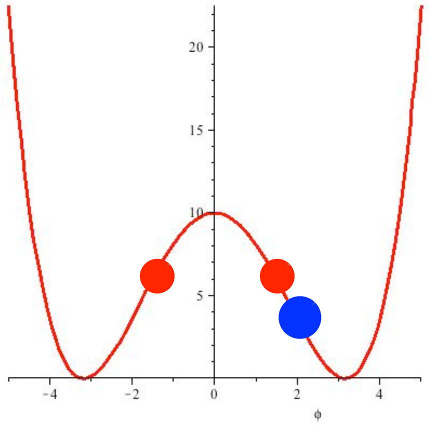
No Correction to Vacuum Energy

(Ignoring tachyon divergence -> $\mathcal{A}^{(2)} + \mathcal{M}^{(2)} + \mathcal{K}^{(2)} = 0$, and the same for multi-point functions. No correction to the vacuum energy.)

“tadpole resummations” on a D9-brane in Sugimoto model

32 D9-branes in a specially unoriented Superstring Theory
-> not a “solution” of String Theory, but stable
(D9-brane is not expected to decay.)

- NS-NS tadpoles (dilaton and graviton)
- no tachyon



“tadpole resummations” should not give the correction to the D9-brane vacuum energy which cancels the classical vacuum energy, even though the system is not a “solution”.

$$\begin{aligned} \mathcal{A}^{(2)} &= \frac{1}{2!} V_{10} N_9^2 \frac{\Delta_{\rm NS}}{1+T_9(N_9/T_9)^2\Delta_{\rm NS}} \times \frac{1}{1+T_9(N_9/T_9)^2\Delta_{\rm NS}} \\ &\to 0 \end{aligned}$$

no correction to D9-brane vacuum energy

Summary

1) The importance of NS-NS tadpole problem in string models without supersymmetry is emphasized.

2) The procedure of “tadpole resummations” is formulated in String Theory.

3) “Tadpole resummations” give a consistent results with the “tachyon condensation” in Bosonic String Theory.

4) Some non-trivial checks are presented.

Possible in String Field Theory?

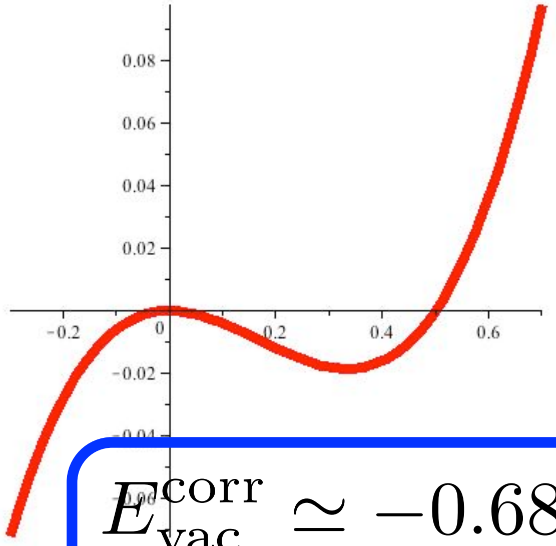
5. Problems and Limitations of this Procedure

On the relation with open-string field theory analysis on “tachyon condensation”

From Taylor-Zwiebach (TASI 2001):

tachyon potential
(tachyon mode only)

$$V(\varphi) = -\frac{1}{2}\varphi^2 + \mu\varphi^3$$

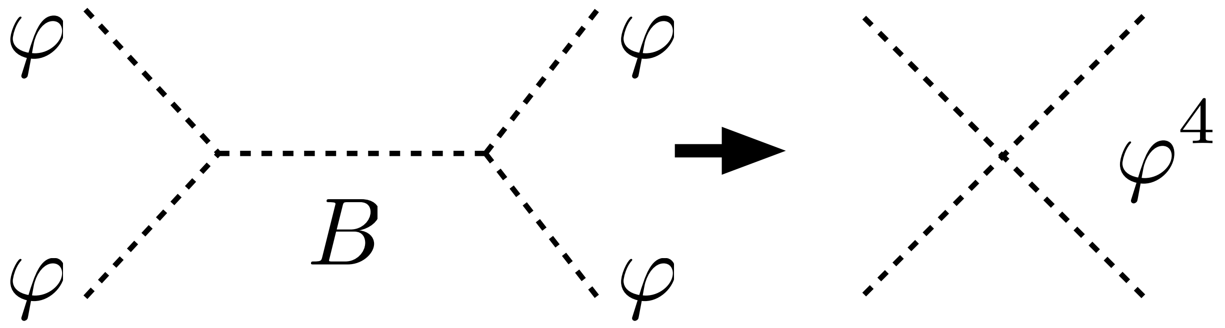


$$\boxed{E_{\rm vac}^{\rm corr} \simeq -0.68T_{25}}$$

a level truncation: $\Psi_s = \varphi|0\rangle + B(\alpha_{-1}\cdot\alpha_{-1})|0\rangle + \beta(b_{-1}c_{-1})|0\rangle + \cdots$

$$\begin{aligned} V &= -\frac{1}{2}\varphi^2 + 26B^2 - \frac{1}{2}\beta^2 \\ &+ \mu\left[\varphi^3 - \frac{130}{9}\varphi^2 B - \frac{11}{9}\varphi^2\beta + \cdots\right] \end{aligned}$$

$$\boxed{E_{\rm vac}^{\rm corr} \simeq -0.95938T_{25}}$$



heavy modes are integrated out

Open string massive modes are important in the analysis of “tachyon condensation” using open-string field theory. Their role is not evident in the procedure of “tadpole resummations”.

One thing we can say:

Open-closed string duality, which is essential in the procedure of “tadpole resummations”, requires infinite number of open string modes. The effects of open string massive modes are implicitly included.

Gravitational back reactions

The procedure of “tadpole resummations” includes “propagations” of closed string in the direction perpendicular to D-branes, assuming flat space-time.

The existence of D-branes should change the background geometry (and background fields).

ex. spontaneous compactifications: Dudas-Mourad (2000)

↓

It may affect the results of “tadpole resummations”, although it is not included in the analysis based on string field theory.