Inflaton versus Curvaton in Higher-Dimensional Gauge Theories

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Content

outline

- Problem

  fine tuning of inflaton and curvaton potential

- Approach to the problem

  6D gauge theory ⇒ $M^4 \times T^2$ (toy model)

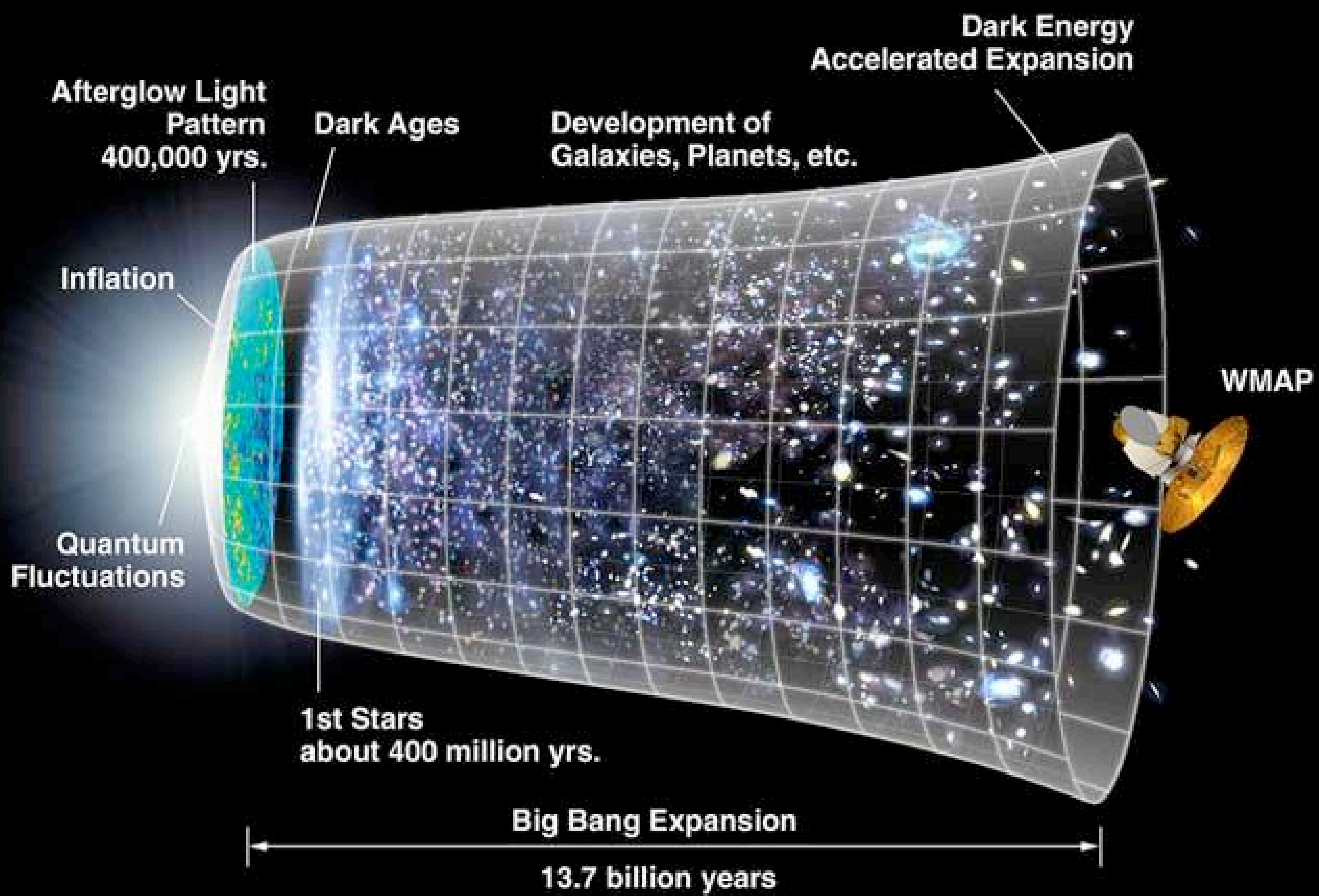
  $A_5^{(0,0)} = \phi\,(\text{inflaton})$ $A_6^{(0,0)} = \sigma\,(\text{curvaton})$

- We find

  It is possible to build a cosmological inflation model

  Curvaton is responsible for only non Gaussian perturbation

1.Introduction



inflation: a period of (quasi)exponential expansion of the early Universe

$$a(t) \sim e^{Ht}$$

$a(t)$: scale factor $H \equiv \dot{a}/a$: Hubble parameter

- The model of cosmological inflation

i. The mechanism of inflation

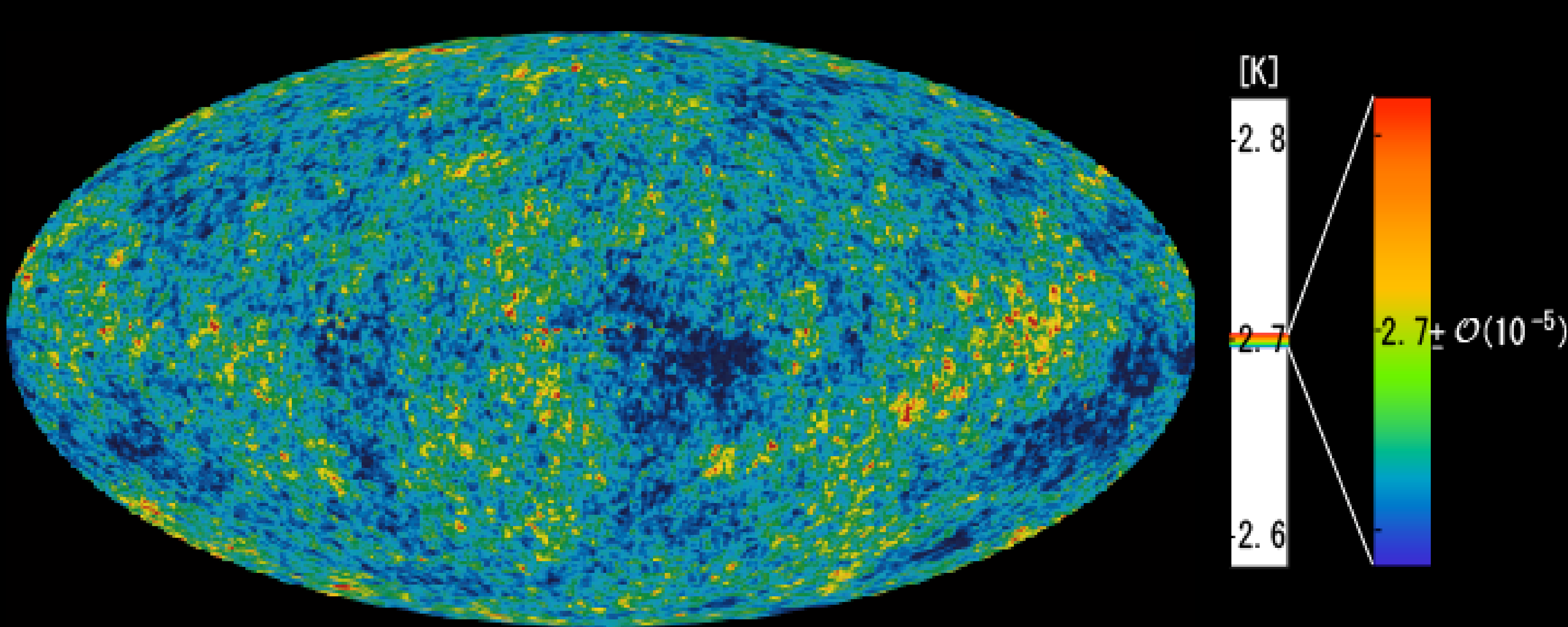
the proper initial condition for the Big-Bang model

~~horizon, flatness, monopole.. problems~~

ii. Generating the curvature perturbation ζ

the origin of the large scale structure and anisotropies in CMB(Cosmic Microwave Background) $\frac{\delta T}{T} \sim 10^{-5}$

anisotropies in CMB



- 宇宙が約39万歳のときの写真。（宇宙の晴れ上がり、decoupling）
- 全体としてほぼ同じ温度である。（現在約2.7K）（ ⟶ 地平線問題 ）

⟶ インフレーションがあれば解決。

- 約数十分角から数度のスケールで見られる約10^{-5} K程度の非等方性

⟶ これを再現するようなmodelのみに制限される。

- The mechanism of inflation

Friedmann eq. $$H^2 = \frac{1}{3M_P^2}\rho$$ ρ : energy density

Inflation occurs if

$$\rho_{\mathrm{univ}} \sim \rho_{\mathrm{vac}} = \mathrm{const.}$$

It can be easily realized by introducing a scalar field, inflaton.

$$\mathcal{L}(\phi)/\sqrt{-g} = \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) \qquad \rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi)$$

e.o.m $$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

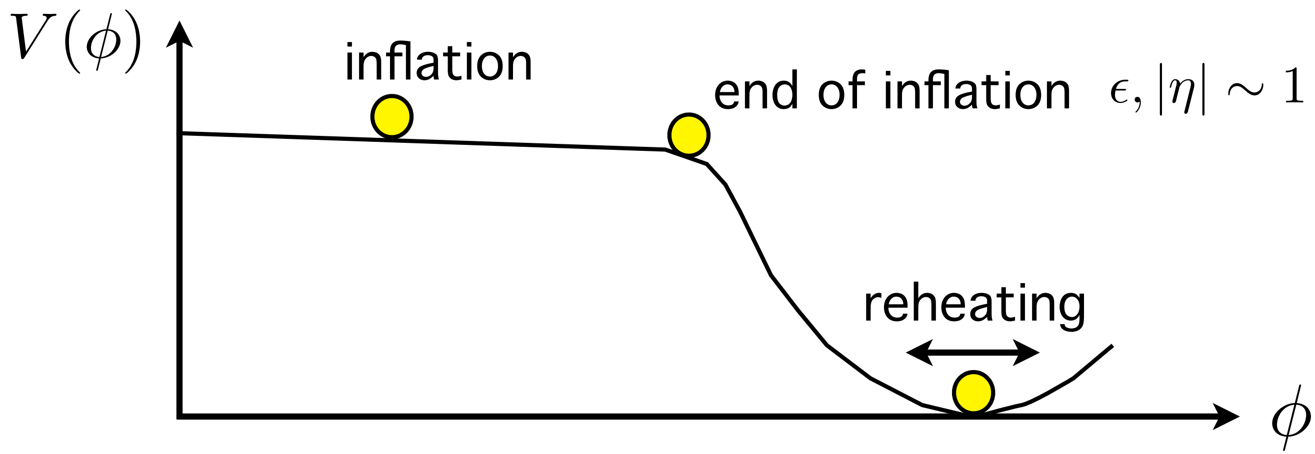
・slow-roll inflation

slow-roll parameters $\epsilon \equiv \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2$ $\eta \equiv M_P^2 \frac{V''}{V}$

slow-roll conditions $\epsilon, |\eta| \ll 1$

⇒ inflaton potential $V(\phi)$ ($\sim \rho_\phi$) flat

⇒ We get $a(t) \sim e^{Ht}$



- Generating the curvature perturbation

1. Inflaton ϕ

   It can be explained both of inflation and the curvature perturbation by only an inflation.

   ⇒ a few severe restrictions on the inflaton potential

2. Curvaton σ (decay) [Lyth et al.(2002)]

   ・ It was introduced to explain the curvature perturbation.
   ※ + non Gaussian perturbation

   ・ Inflaton becomes free from the restriction from the curvature perturbation.

   ・ a large n_s (~ 0.98-0.99)

3. Inflaton and curvaton

- Gaussian (distribution) perturbation

$$\langle \zeta_{\vec{k}} \zeta_{\vec{k}'} \rangle = (2\pi)^3 \delta^3_{\vec{k}+\vec{k}'} P_\zeta(k)$$

$\zeta_{\vec{k}}$: Fourier component of $\zeta(\vec{x})$

two point correlator determines all higher correlators
odd correlators=0
(disconnected)

- non Gaussian perturbation (<< Gaussian)

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \ldots \rangle_{\rm c} \neq 0$$

This gives further restrictions on inflation models.

(PLANCK satellite)

For the (power spectrum of the) curvature perturbation

$\mathcal{P}_\zeta \simeq 10^{-9}$ (WMAP) ($\mathcal{P}_\zeta = (k^3/2\pi^2)P_\zeta$)

- inflaton

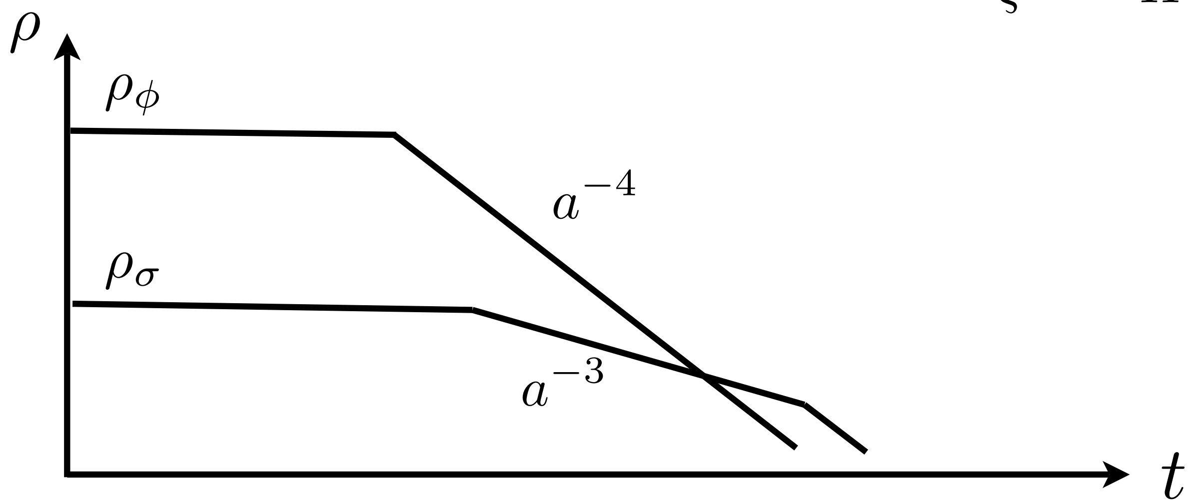
The curvature perturbation is generated during inflation due to the fluctuation of inflaton field, $\delta\phi$.

$$\mathcal{P}_{\rm inf} \simeq V/M_P^4\epsilon \simeq 10^{-9} \qquad \zeta = -H\frac{\delta\phi}{\dot{\phi}}$$

- curvaton

The curvature perturbation is generated after inflation due to the oscillation of curvaton field.

$$\zeta = -H\frac{\delta\rho}{\dot{\rho}} \sim \frac{\delta\rho}{\rho}$$



$\epsilon, \eta \ll 1, \mathcal{P}_\zeta \simeq 10^{-9}$ $\Rightarrow$ severe restrictions to the coupling constants of $V(\phi)(V(\sigma))$

$\Rightarrow$ fine tuning problem in inflation

- It is difficult to build a 4D inflation model which contains quantum effects.
- We consider higher dimensional gauge theory for the model of inflaton and curvaton.
- In this theory, after compactification, a flat(and small) potential arises through the radiative corrections.

A possible solution for the fine-tuning problem

$\Rightarrow$ applications gauge-Higgs [Hatanaka et al.(1998)]

$$A_y^{(0)} = \mathrm{Higgs}\ h$$

extranatural inflation [Arkani-Hamed et al.(2003)]

$$A_y^{(0)} = \mathrm{inflaton}\ \phi$$

Higgs-inflaton [Inami et al.(2009)]

$$A_y^{(0)} = \phi = h$$

- There is an application of the higher dimensional gauge theory to only curvaton. [Dimopoulos et al.(2003)]

- New point

We use the potential for both of inflaton and curvaton derived from the higher dimensional gauge theory.

(We don't write the potential by hand.)

ϕ, σ :

extra components of higher dimensional gauge field, $A_y^{(0)}$

The potential obtained as a function of the Wilson line,

$V = V(e^{i\theta})$ ⟸ gauge invariant, shift symmetry, finite

2. Toy Model and one-loop effective potential

- 6D SU(2) gauge theory $\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\, F_{MN}F^{MN} - i\bar{\psi}\gamma^M D_M \psi.$

$(M, N = 0, \ldots, 5, 6)$

- Compactification → $M^4 \times T^2$

$$A_M(x^\mu, y_5, y_6) = \frac{1}{\sqrt{L_5 L_6}} \sum_{n,m=-\infty}^{\infty} A_M^{(n,m)}(x^\mu) e^{i(ny_5/R_5 + my_6/R_6)}.$$

R_5, R_6 : compactification radii $\quad L_{5,6} = 2\pi R_{5,6}$

- We assume $A_5^{(0,0)} = \phi\,(\text{inflaton}) \qquad A_6^{(0,0)} = \sigma\,(\text{curvaton})$

To evaluate the effective potential we allow $A_5^{(0,0)}$ and $A_6^{(0,0)}$ to have VEVs of the form

$$\langle A_5^{(0,0)} \rangle = \frac{1}{gL_5}\begin{pmatrix} \theta & 0 \\ 0 & -\theta \end{pmatrix}, \quad \langle A_6^{(0,0)} \rangle = \frac{1}{gL_6}\begin{pmatrix} \varphi & 0 \\ 0 & -\varphi \end{pmatrix}. \quad g : \text{coupling}$$

θ and φ (constants) are given by the Wilson line phases $g\int_0^{L_a} dy_a \langle A_a^{(0,0)} \rangle (a = 5, 6)$

※ We assume that quantum gravity effects are negligible.

⇒ Compactification radii, $R_{5,6}$ are stable.

- one-loop effective potential

$$V_{\rm eff}(\theta,\varphi) = -\frac{R_5R_6}{\pi^7}\left[\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\frac{4}{(k^2R_5^2+l^2R_6^2)^3}\left((1+\cos(2k\theta))(1+\cos(2l\varphi))-2\cos(k\theta)\cos(l\varphi)\right)\right.$$
$$\left.+\sum_{l=1}^{\infty}\frac{1}{(l^6R_6^6)}(1+\cos(2l\varphi)-2\cos(l\varphi))+\sum_{k=1}^{\infty}\frac{1}{(k^6R_5^6)}(1+\cos(2k\theta)-2\cos(k\theta))\right]$$
$$+\,{\rm const.}$$

- Inflaton and curvaton are defined as the fluctuations around a minimum.

$$\phi \equiv f_5(\theta(x^\mu)-\pi), \quad \sigma \equiv f_6(\varphi(x^\mu)-\pi) \qquad f_{5,6}=\frac{1}{gL_{5,6}}$$

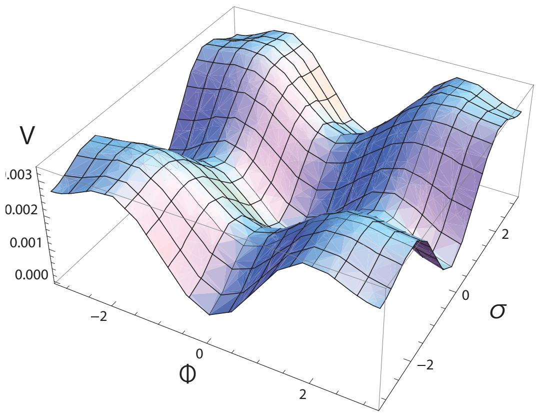
- The leading terms (k=1 and l=1) is a good approximation to $V_{\rm eff}$.

$$V(\phi,\sigma) = -\frac{R_5R_6}{\pi^7}\left[\frac{2}{(R_5^2+R_6^2)^3}\left((\cos(2\frac{\phi}{f_5})-1)(\cos(2\frac{\sigma}{f_6})-1)-2(\cos(\frac{\phi}{f_5})-1)(\cos(\frac{\sigma}{f_6})-1)\right)\right.$$
$$\left.+\left(\frac{1}{R_6^6}+\frac{8}{(R_5^2+R_6^2)^3}\right)(\cos(2\frac{\sigma}{f_6})-1)+\left(\frac{2}{R_6^6}-\frac{8}{(R_5^2+R_6^2)^3}\right)(\cos(\frac{\sigma}{f_6})-1)+(\sigma,f_6\leftrightarrow\phi,f_5)\right].$$

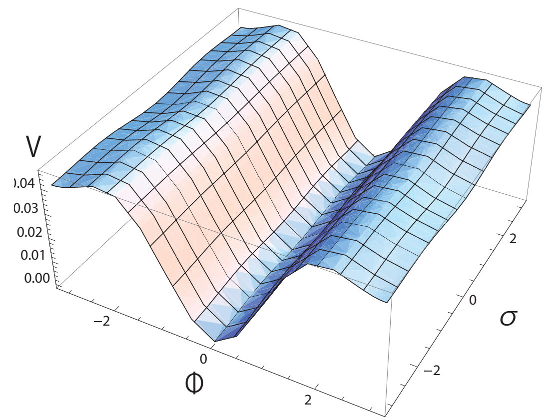
- V ~ interaction terms between the inflaton and curvaton

  +self-interaction terms of the curvaton+inflaton...

- The effective potential with $R_5 = r_R R_6$ for two values of $r_R \equiv R_5/R_6$.



$r_R = 1$



$r_R = 0.5$

- The contribution of the inflaton to the energy density of the Universe becomes dominant as the ratio r_R decreases.

3. Constraints for the curvaton model

- We consider the two alternative situations for $\mathcal{P}_\zeta$

$$\text{(I)}\ \mathcal{P}_\zeta \simeq \mathcal{P}_{\rm cur},\ \ \mathcal{P}_{\rm inf} \ll \mathcal{P}_{\rm cur}$$

$$\text{(II)}\ \mathcal{P}_\zeta = \mathcal{P}_{\rm cur} + \mathcal{P}_{\rm inf}$$

・parameters

$$g, R_5, R_6$$

- Constraints [Lyth et al.(2002),Bartoro et al.(2002),Ichikawa et al.(2008)...]

<u>curvaton scenario</u>

・ Slow-roll inflation

slow-roll condition

$$\epsilon = \frac{1}{2}M_P^2\left(\frac{V_\phi}{V(\phi,\sigma)}\right)^2 \ll 1, \quad |\eta_{\phi\phi}| = M_P^2\left|\frac{V_{\phi\phi}}{V(\phi,\sigma)}\right| \ll 1$$

e-foldings

$$N \equiv \int_{t_*}^{t_f} H dt \simeq 50 - 60$$

・ The curvaton energy density should negligibly during the inflation period.

$$R_5 < R_6$$

⇒ curvaton does not contribute to inflation

- The light field condition $\left|\frac{m_{\sigma *}}{H_*}\right| \ll 1$ $\quad *$: horizon exit

  The curvaton does not diluted away during inflation.

- The curvaton perturbation is Gaussian. $\sigma_*^2 \gg \delta\sigma^2 = \left(\frac{H_*}{2\pi}\right)^2$

for generating the curvature perturbation

$$(\mathrm{I})\quad \mathcal{P}_{\rm cur}=\frac{4}{9}\Omega^2\left(\frac{g'(\sigma_*)}{g(\sigma_*)}\right)^2\left(\frac{H_*}{2\pi}\right)^2=2.45\times10^{-9},\quad \mathcal{P}_{\rm inf}=\frac{1}{24\pi^2M_P^4}\frac{V(\phi_*,\sigma_*)}{\epsilon_*}\ll10^{-9}$$

$$\Omega\equiv\left(\frac{\rho_\sigma}{\rho_{\rm rad}}\right)_{\rm dec}=\left(\frac{\rho_\sigma}{\rho_{\rm rad}}\right)_{\rm osc}\left(\frac{m_\sigma}{\Gamma_\sigma}\right)^{1/2},\quad g(\sigma_*)=2\frac{\rho_\sigma|_{\rm osc}}{m_\sigma^2}\quad \Gamma_\sigma=\frac{g^2}{4\pi}m_\sigma$$

- spectral index $n_s\equiv1-2\epsilon_*+2\eta_{\sigma\sigma*}=0.96,\quad \eta_{\sigma\sigma}\equiv M_P^2\frac{V_{\sigma\sigma}(\phi,\sigma)}{V(\phi,\sigma)}$

  scale dependence of the perturbation

$$(\mathrm{II})\quad \mathcal{P}_\zeta=\mathcal{P}_{\rm cur}+\mathcal{P}_{\rm inf}=2.45\times10^{-9}$$

$$n_s=1-2\epsilon_*-\frac{4\epsilon_*-2\eta_*}{1+\frac{8}{9}\epsilon_*M_P^2\Omega^2\left(\frac{g'(\sigma_*)}{g(\sigma_*)}\right)^2}=0.96$$

- non-Gausianity parameter $f_{NL}<100$

  the size of non-Gaussianity　　　　only an inflaton ⇒ $f_{NL}\sim\mathcal{O}(10^{-2})$

- tensor to scalar ratio $r=\frac{\mathcal{P}_h}{\mathcal{P}_\zeta}\lesssim0.2,\quad \mathcal{P}_h=\frac{8}{M_P^2}\left(\frac{H_*}{2\pi}\right)^2$

  The contribution of gravitational wave to the curvature perturbation is small.

4. Result

- parameters of our model $R_5, r_R \equiv R_5/R_6, f_5 = 1/(2\pi g R_5), \phi_*, \sigma_*$
- We used mathematica for numerical study.
- slow-roll inflation ⇒ $f_5 \gtrsim 10 M_P$

(I) $\mathcal{P}_{\rm cur} = 2.45 \times 10^{-9}$

⇒ spectral index $n_s \sim 0.98 - 0.99$ WMAP data $n_s = 0.960 \pm 0.013$

The curvaton dominance does not holds,
unless we allow an artificially larger error to n_s.

(II) $\mathcal{P}_{\rm cur} + \mathcal{P}_{\rm inf} = 2.45 \times 10^{-9}$

$\underline{f_5 = 10 M_P}$ $\quad N = 50 - 60 \Rightarrow \phi_* \simeq 13 M_P$

- All of the constraints are satisfied.

σ_*[GeV]	r_R	g	R_5 [GeV^{-1}]	R_6 [GeV^{-1}]	f_{NL}
9.1×10^{17}	2.7×10^{-2}	2.8×10^{-4}	2.3×10^{-17}	1.2×10^{-16}	0.7
2.6×10^{15}	0.20	3.8×10^{-4}	1.7×10^{-17}	8.3×10^{-17}	0.5
1.1×10^{15}	4.3×10^{-5}	4.7×10^{-5}	1.4×10^{-16}	6.9×10^{-14}	3.0
4.6×10^{14}	2.6×10^{-5}	4.4×10^{-5}	1.5×10^{-16}	7.6×10^{-14}	0.05
3.4×10^{14}	4.1×10^{-5}	4.7×10^{-5}	1.4×10^{-16}	7.0×10^{-14}	3.0

$g \simeq 3.8 \times 10^{-4} - 4.4 \times 10^{-5}$,

$R_5 \simeq (1.5 \times 10^{-16} - 1.7 \times 10^{-17})\,\mathrm{GeV}^{-1}$, $R_6 \simeq (7.6 \times 10^{-14} - 8.3 \times 10^{-17})\,\mathrm{GeV}^{-1}$

$\sigma_* \simeq (3.4 \times 10^{14} - 9.1 \times 10^{17})\,\mathrm{GeV}$

- f_{NL} is at most $\mathcal{O}(1)$
- $\dfrac{\mathcal{P}_{\rm cur}}{\mathcal{P}_{\rm inf}} \simeq 0.04$ $\quad \Rightarrow$ The curvaton contribution to the curvature perturbation is not sizable.

$\underline{f_5 = 100M_P}$ $N = 50 - 60$ ⇒ $\phi_* \simeq 14M_P$

- There is no parameter region satisfying all of the constraints simultaneously.

  especially $\sigma_*^2 \gg \delta\sigma^2$, $f_{NL} < 100$ and $r = \frac{\mathcal{P}_h}{\mathcal{P}_\zeta} \lesssim 0.2,$

- We have the same result for $f_5 > 100M_P$

- It turned out that the curvaton model is realized only when both the inflaton and the curvaton contribute to the curvature perturbation.

- The contribution of the curvaton to the curvature perturbation is very small compared with that of the inflaton, $\mathcal{P}_{\rm cur}/\mathcal{P}_{\rm inf} \simeq 0.04$.

- The curvaton is responsible for only generating the non-Gaussian perturbation.

  f_{NL} will soon be measured to the accuracy of $\delta f_{NL} \sim 1$

  $f_{NL} \simeq 3$ has a good chance of detection.