Light-cone gauge NSR strings in noncritical dimensions

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Based on
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- JHEP 10 (2009) 035, arXiv:0906.3577
- JHEP 12 (2009) 010, arXiv:0909.4675
- JHEP 01 (2010) 119, arXiv:0911.3704
- arXiv:0912.4811

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§1. Introduction

§2. $X^{\pm}$ CFT for bosonic case

§3. Supersymmetrization (worldsheet sense)

§4. Summary and outlook

Why light-cone gauge string field theory (SFT)?

Why SFT?

- Great success of field theory for point particles.

  Expect the same things for the string case

- has been playing roles in the study of Sen's conjectures

Why light-cone (LC) gauge?

- simple and so convenient to define the theory.

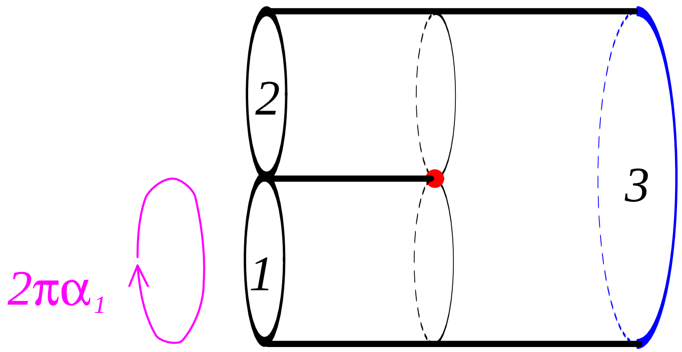
closed superstring field theory ????,
while there are some attempts in open string case

Under these circumstances, it should be important to reconsider the LC gauge closed superSFT.

Light-cone gauge string field theory

$$S = \int dt \left[\frac{1}{2} \langle R(1,2)|\Phi\rangle_1 \left(\alpha_2 i \frac{\partial}{\partial t} - \left(L_0^{\mathrm{LC}(2)} + \tilde{L}_0^{\mathrm{LC}(2)} - \frac{d-2}{8} \right) \right) |\Phi\rangle_2 \right.$$
$$\left. + \frac{2g}{3} \langle V_3(1,2,3)|\Phi\rangle_1 |\Phi\rangle_2 |\Phi\rangle_3 \right]$$

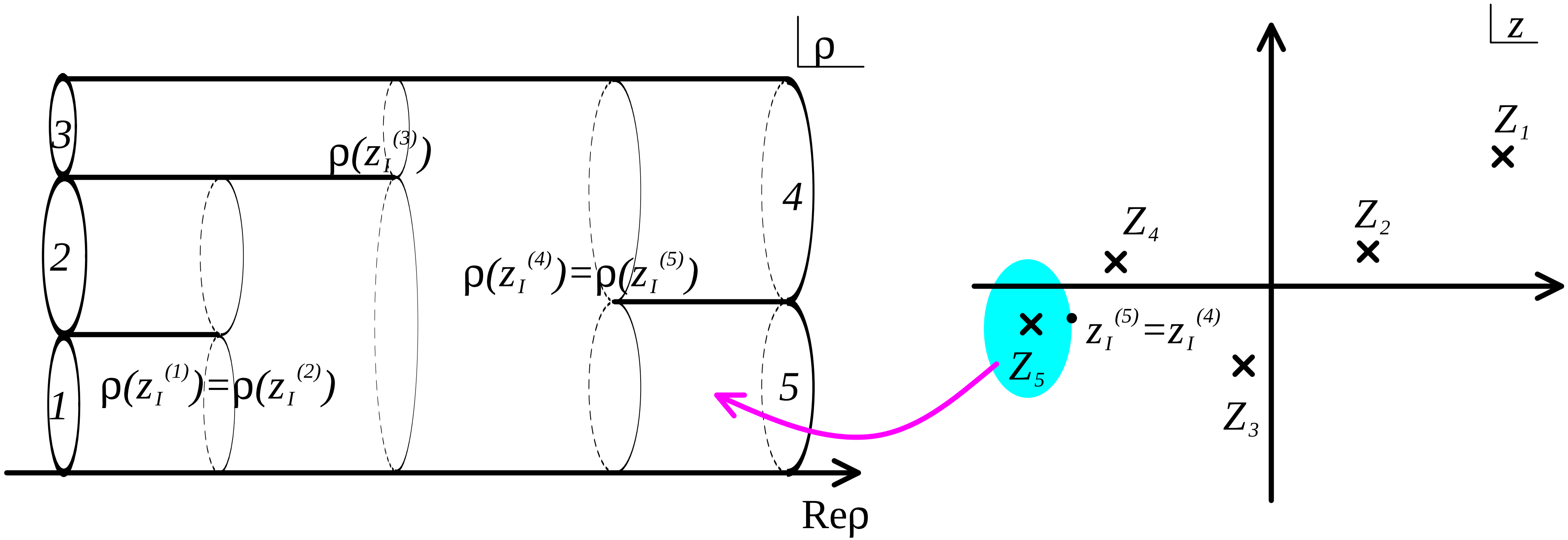
$+$



- $\alpha_r = 2p_r^+$: string-length parameter
- $\Phi\left[t, \alpha, X^i(\sigma); \psi^i(\sigma), \tilde{\psi}^i(\sigma)\right]$: string field
- $T_F^{\mathrm{LC}} \tilde{T}_F^{\mathrm{LC}} |\partial^2 \rho|^{-\frac{3}{2}}$ must be inserted at interaction point
  $\Leftarrow$ Lorentz invariance for $d = 10$ Mandelstam ('74), S.-J. Sin ('89) ...

N-string light-cone diagram and Mandelstam mapping

$$\rho(z) = \sum_{r=1}^{N} \alpha_r \ln(z - Z_r) \ , \quad \alpha_r = 2p_r^+$$



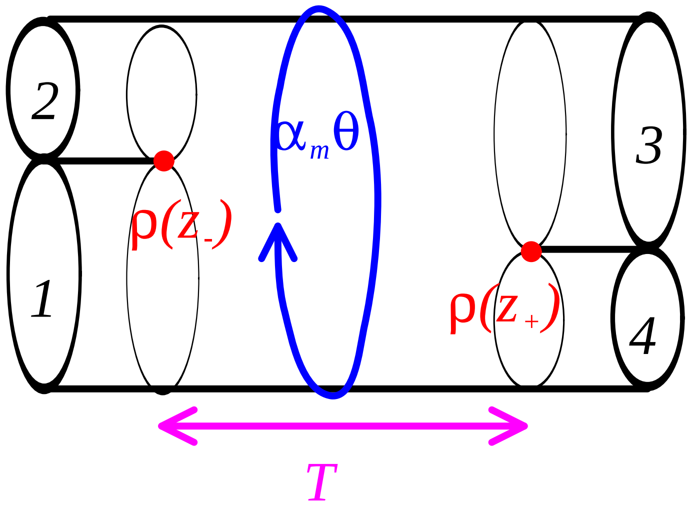
- $N - 2$ interaction points z_I $\quad \longleftarrow \quad$ $\partial\rho(z_I) = 0$

  among them we denote the interaction point for the rth string by $z_I^{(r)}$
- worldsheet metric: $\quad ds^2 = d\rho d\bar{\rho} = e^{\phi} dz d\bar{z} \ , \quad \phi = \ln(\partial\rho\bar{\partial}\bar{\rho})$

Motivation: Divergences caused by colliding T_F^{LC}

e.g. 4pt amplitudes

$$\mathcal{A}_4 = \int d\mathcal{T} d\bar{\mathcal{T}}$$



- $\mathcal{T} = \rho(z_+) - \rho(z_-) = T + i\alpha_m\theta$
- At $\mathcal{T} = 0 \Leftrightarrow z_+ - z_- = 0$, unwanted divergence

$$T_F^{\mathrm{LC}}(z_+) T_F^{\mathrm{LC}}(z_-) \sim \frac{\frac{3}{2}(d-2)}{(z_+ - z_-)^3}$$

Some regularization is necessary even at tree level

What about dimensional regularization in SFT?

▶ Scheme we propose

(1) Formulate LC gauge SFT in $d \neq 10$ ($d \neq 26$ for bosonic case)

(2) take d to be a large negative value

(3) analytic continuation $d \to 10$ ($d \to 26$) in the end

- IR $\cdots$ on-shell condition : $M^2 \equiv 2p^+p^- - p^ip^i = \mathcal{N} + \tilde{\mathcal{N}} - \dfrac{d-2}{8}$

  $\to$ for largely negative d, ground state is non-tachyonic

- Divergences caused by colliding $T_F(z_I)$'s are indeed regularized

  ($\to$ next slide)

▶ Lorentz invariance?

Shift the spacetime dimensions d $\Rightarrow$ central charge shifted

We give up the Lorentz invariance.

N-point tree level amplitudes for (NS,NS) strings

can be expressed as a correlation function of the worldsheet theory (on z-plane)

$$\mathcal{A} \sim \int \prod_{\mathcal{I}=1}^{N-3} d^2\mathcal{T}_{\mathcal{I}} \left\langle \prod_{I=1}^{N-2} \left[|\partial\rho(z_I)|^{-\frac{3}{2}} T_F^{\mathrm{LC}}(z_I) \tilde{T}_F^{\mathrm{LC}}(\bar{z}_I) \right] \prod_{r=1}^{N} V_r^{\mathrm{LC}} \right\rangle_{\mathrm{LC}} e^{-\frac{d-2}{16}\Gamma[\phi]}$$

$$[dX^i d\psi^i]_\phi = [dX^i d\psi^i]_0 \, e^{-\frac{d-2}{16}\Gamma[\phi]} \,, \qquad \Gamma[\phi] = -\frac{1}{\pi}\int d^2z \, \partial\phi\bar{\partial}\phi\text{: Liouville action}$$

$$\boxed{e^{-\Gamma[\phi]} \propto \left| \sum_{s=1}^{N} \alpha_s Z_s \right|^4 \prod_{r=1}^{N} \left[|\alpha_r|^{-2} e^{-2\left(\frac{\mathrm{Re}\,\rho(z_I^{(r)})}{\alpha_r} - \sum_{s\neq r} \frac{\alpha_s}{\alpha_r} \ln|Z_r - Z_s| \right)} \right] \prod_I \left| \partial^2\rho(z_I) \right|^{-1}}$$

Mandelstam ('86)

$$\prod_I |\partial^2\rho(z_I)| = \frac{|\sum_s \alpha_s Z_s|^{2N-2}}{\prod_r |\alpha_r|} \frac{\prod_{I>J} |z_I - z_J|^2}{\prod_{r>s} |Z_r - Z_s|} \implies \boxed{e^{-\frac{d-2}{16}\Gamma[\phi]} \sim |z_I - z_J|^{-\frac{d-2}{8}}}$$

By taking d to be largely negative, the amplitudes are vanishing as $z_I \to z_J$.

$\Rightarrow$ The divergences caused by colliding T_F^{LC} are regularized.

Gauge invariance of the dimensional regularization?

Difficult at the second quantized level.

$$\boxed{\begin{array}{c}\text{gauge invariant SFT}\\ \text{for } d \neq 10 \text{ or } d \neq 26\end{array}}\ (??)$$

$$\Downarrow \text{ gauge fixing } (??)$$

$$\boxed{\begin{array}{c}\text{light-cone gauge SFT}\\ \text{for } d \neq 10 \text{ or } d \neq 26\end{array}}$$

This is not achieved even for $d = 26$.

What we expect is at the first quantized level for $d \neq 26$ or $d \neq 10$

the amplitudes of the LC gauge SFT $\rightarrow$ BRST invariant form

$\boxed{\text{worldsheet theory in conformal gauge } (\hat{c} \text{ or } c = 0)}$

$\boxed{\text{gauge invariant SFT for } d \neq 10 \text{ or } d \neq 26}$ (??)

gauge fixing $\Downarrow \Uparrow$ $\otimes$ $\boxed{X^{\pm} \text{ CFT } (\hat{c} = 12 - d \text{ or } c = 28 - d)}$ $\otimes$ $\boxed{\text{ghosts}}$

$\boxed{\text{worldsheet theory in light-cone gauge } (\hat{c} \text{ or } c = d - 2)}$ $\Longleftrightarrow$ $\boxed{\text{light-cone gauge SFT for } d \neq 10 \text{ or } d \neq 26}$

(Proved for critical dimensions (D'Hoker-Giddings, Aoki-D'Hoker-Phong))

What we have done

- Formulate $X^{\pm}$ CFT

  interacting CFT for $X^{\pm}$, $(\psi^{\pm}, \tilde{\psi}^{\pm})$ with $c = 28 - d$ $(\hat{c} = 12 - d)$

  combined with free transverse CFT and ghost CFT

  $\Rightarrow$ worldsheet CFT with $c = 0$ & nilpotent Q_{B}

- Rewrite tree-level amplitudes (for the (NS,NS) strings) of the light-cone gauge SFT for $d \neq 26$ $(d \neq 10)$ into a BRST invariant form

  $\Rightarrow$ dimensional regularization preserves

  BRST on the worldsheet $\leftarrow$ basics of gauge symmetry of SFT

- Using this CFT, we show that the tree-level amplitudes of the LC gauge SFT coincide with the results in the 1st quantized formulation in the limit $d \to 26$ $(d \to 10)$.

Plan of the talk

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§2. $X^{\pm}$ CFT for bosonic case

§3. Supersymmetrization (worldsheet sense)

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Action

$$S_\pm = -\int d^2z\,(\partial X^+\bar\partial X^- + \bar\partial X^+\partial X^-) + \frac{d-26}{24}\Gamma[\hat\phi]$$

$$\Gamma[\hat\phi] = -\frac{1}{\pi}\int d^2z\,\partial\hat\phi\bar\partial\hat\phi\,, \qquad \hat\phi \equiv \ln\left(-4\partial X^+\bar\partial X^+\right)$$

- energy-momentum tensor

$$\boxed{T_{X^\pm} = \partial X^+\partial X^- - \frac{d-26}{12}\{X^+\,,\,z\}}$$

$$\{X^+\,,\,z\} \equiv \frac{\partial^3 X^+}{\partial X^+} - \frac{3}{2}\left(\frac{\partial^2 X^+}{\partial X^+}\right)^2 \qquad \text{(Schwarzian)}$$

- OPE

$X^+(z,\bar z)X^+(z',\bar z') \sim$ regular , $\quad X^+(z,\bar z)X^-(z',\bar z') \sim \ln|z-z'|^2$

$X^-X^- \leftarrow$ complicated, to be evaluated in the following

Expectation value of X^+

$S_\pm$ contains $\frac{1}{\partial X^+}$

$\Rightarrow$ ∂X^+ should have a non-zero expectation value

$\Rightarrow$ always with insertion $\prod_{r=1}^N e^{-ip_r^+ X^-}(Z_r, \bar{Z}_r)$ $\quad (\sum_{r=1}^N p_r^+ = 0)$

Consider the correlation function of the form

$$\left\langle F[X^+, X^-] \prod_{r=1}^N e^{-ip_r^+ X^-}(Z_r, \bar{Z}_r) \right\rangle_\pm$$

$$\equiv \int [dX^\pm] e^{-S_\pm} F[X^+, X^-] \prod_{r=1}^N e^{-ip_r^+ X^-}(Z_r, \bar{Z}_r)$$

X^+ indeed has an expectation value ($\rightarrow$ next slide)

For $F[X^+]$ (independent of X^-)

By using $-2\pi \sum_{r=1}^{N} p_r^+ \delta^2(z - Z_r) = \partial\bar{\partial}\left(\rho(z) + \bar{\rho}(\bar{z})\right)$,

$$\left\langle F[X^+] \prod_{r=1}^{N} e^{-ip_r^+ X^-}(Z_r, \bar{Z}_r) \right\rangle_{\pm}$$

$$= \int [dX^{\pm}] e^{\frac{1}{\pi}\int d^2z\, X^- \partial\bar{\partial}\left(X^+ + \frac{i}{2}(\rho(z)+\bar{\rho}(\bar{z}))\right)} F[X^+]\, e^{-\frac{d-26}{24}\Gamma[\ln(-4\partial X^+ \bar{\partial} X^+)]}$$

$$\sim \int [dX^+]\, \delta\left(X^+ + \frac{i}{2}(\rho + \bar{\rho})\right) F[X^+]\, e^{-\frac{d-26}{24}\Gamma[\ln(-4\partial X^+ \bar{\partial} X^+)]}$$

$$= F\left[-\frac{i}{2}(\rho + \bar{\rho})\right] e^{-\frac{d-26}{24}\Gamma[\ln(\partial\rho\bar{\partial}\bar{\rho})]}$$

$\Rightarrow$ X^+ has an expectation value

$$X^+(z, \bar{z}) \sim -\frac{i}{2}(\rho(z)+\bar{\rho}(\bar{z})) \quad (\leftarrow \text{Lorentzian light-cone time})$$

$\sim$ light-cone gauge

$F[-\frac{i}{2}(\rho+\bar\rho)]e^{-\frac{d-26}{24}\Gamma[\ln(\partial\rho\bar\partial\bar\rho)]}$ as a generating functional of correlation functions involving X^-

$$
\begin{aligned}
&\left\langle F[X^+]X^-(Z_N,\bar Z_N)\prod_{r=1}^{N-1}e^{-ip_r^+X^-}(Z_r,\bar Z_r)\right\rangle_{\pm} \\
&\sim i\partial_{p_N^+}\left\langle F[X^+]\prod_{r=1}^{N}e^{-ip_r^+X^-}(Z_r,\bar Z_r)\right\rangle_{\pm}\Bigg|_{p_N^+=0} \\
&= i\partial_{p_N^+}\left(F\left[-\frac{i}{2}(\rho+\bar\rho)\right]e^{-\frac{d-26}{24}\Gamma[\ln(\partial\rho\bar\partial\bar\rho)]}\right)\Bigg|_{p_N^+=0}
\end{aligned}
$$

- We can go on $\rightarrow$ correlation functions with more than one X^-

Rather than starting from the action $S_{X^\pm}$, we define the theory based on

$$
\left\langle F[X^+]\prod_r e^{-ip_r^+X^-}(Z_r,\bar Z_r)\right\rangle_{\pm}\sim F\left[-\frac{i}{2}(\rho+\bar\rho)\right]e^{-\frac{d-26}{24}\Gamma[\phi]}
$$

One point function

$$\left\langle \partial X^-(z) \prod_{r=1}^{N} e^{-ip_r^+ X^-}(Z_r, \bar{Z}_r) \right\rangle_\pm$$

- poles not only at $z = Z_r$, but also at $z = z_I$ where no operator insertions

$e^{-ip_r^+ X^-}(Z_r, \bar{Z}_r)$: operator making a hole with length $2\pi\alpha_r$

$\Rightarrow$ non-local effects

$$\left\langle T_{X^\pm}(z) \prod_{r=1}^{N} e^{-ip_r^+ X^-}(Z_r, \bar{Z}_r) \right\rangle_\pm$$

- $T_{X^\pm}(z)$ is regular where no operators are inserted, particularly at $z = z_I$.

$\Longrightarrow$ This is needed to assure the conservation of $Q_{\rm B}$ at $z = z_I$.

Two point function $\langle \partial X^-(z) \partial X^-(z') \rangle$

From the two point function of X^-, one can read off the OPE

$$\partial X^-(z)\partial X^-(z') \sim -\frac{d-26}{12}\partial_z \partial_{z'} \left[\frac{1}{(z-z')^2} \frac{1}{\partial X^+(z)\partial X^+(z')} \right].$$

- $$\boxed{T_{X^\pm}(z) T_{X^\pm}(z') \sim \frac{\frac{1}{2}(28-d)}{(z-z')^4} + \frac{2}{(z-z')^2} T_{X^\pm}(z') + \frac{1}{z-z'} \partial T_{X^\pm}(z')}$$

$\Rightarrow$ Virasoro with $c = 28 - d$ $\quad$ ($\leftarrow$ What is desired!)

CFT for $X^\pm$		free X^i		bc ghosts
$c_\pm = 28 - d$	$\otimes$	$c_{\text{LC}} = d - 2$	$\otimes$	$c_{\text{gh}} = -26$

$c_{\text{total}} = 0 \Rightarrow$ nilpotent BRST charge Q_{B}

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Supersymmetrization via superspace formalism

- superspace

$$z \longrightarrow \mathbf{z} = (z, \theta)$$

$$\partial_z \longrightarrow D = \partial_\theta + \theta \partial_z$$

$$z_1 - z_2 \longrightarrow \mathbf{z}_1 - \mathbf{z}_2 \equiv z_1 - z_2 - \theta_1 \theta_2$$

- supersheet (Berkovits, Aoki-D'Hoker-Phong)

$$\rho(z) = \sum_{r=1}^{N} \alpha_r \ln(z - Z_r) \longrightarrow \boldsymbol{\rho}(\mathbf{z}) = (\rho(\mathbf{z}), \xi(\mathbf{z}))$$

$$\left(\rho(\mathbf{z}) \equiv \sum_{r=1}^{N} \alpha_r \ln(\mathbf{z} - \mathbf{Z}_r) \ , \quad \xi(\mathbf{z}) \equiv \frac{D\rho}{(\partial\rho)^{\frac{1}{2}}}(\mathbf{z}) \right)$$

- superfield

$$X^{\pm}(z, \bar{z}) \longrightarrow \mathcal{X}^{\pm}(\mathbf{z}, \bar{\mathbf{z}}) = X^{\pm} + i\theta\psi^{\pm} + i\bar{\theta}\tilde{\psi}^{\pm}$$

$X^{\pm}$ CFT for superstring case

$$S_{\pm} = -\frac{1}{2\pi}\int d^2\mathbf{z}\ \left(\bar{D}\mathcal{X}^{+}D\mathcal{X}^{-} + \bar{D}\mathcal{X}^{-}D\mathcal{X}^{+}\right) + \frac{d-10}{8}\Gamma_{\mathrm{super}}[\Phi]$$

$$\Gamma_{\mathrm{super}}[\Phi] = -\frac{1}{2\pi}\int d^2\mathbf{z}\,\bar{D}\Phi D\Phi$$ super Liouville action

$$\Phi(\mathbf{z},\bar{\mathbf{z}}) \equiv \ln\left|-4\left(\partial\mathcal{X}^{+} - \frac{\partial D\mathcal{X}^{+}D\mathcal{X}^{+}}{\partial\mathcal{X}^{+}}\right)(\mathbf{z})\right|^2$$

- Super energy-momentum tensor $T_{X^{\pm}}(\mathbf{z})$

$$T_{X^{\pm}}(\mathbf{z}) = \frac{1}{2}D\mathcal{X}^{+}\partial\mathcal{X}^{-} + \frac{1}{2}D\mathcal{X}^{-}\partial\mathcal{X}^{+} - \frac{d-10}{4}S\left(\mathbf{z},\boldsymbol{\mathcal{X}}_L^{+}\right)$$

$S\left(\mathbf{z},\boldsymbol{\mathcal{X}}_L^{+}\right)$ super Schwarzian

Correlation function

always with the insertions $e^{-ip_r^+ \mathcal{X}^-}(\mathbf{Z}_r, \bar{\mathbf{Z}}_r)$:

$$\left\langle F\left[\mathcal{X}^+, \mathcal{X}^-\right] \prod_{r=1}^{N} e^{-ip_r^+ \mathcal{X}^-}(\mathbf{Z}_r, \bar{\mathbf{Z}}_r) \right\rangle_{\pm}$$

This can be evaluated in a similar way to the bosonic case

- expectation value $\quad \mathcal{X}^+(\mathbf{z}, \bar{\mathbf{z}}) \sim -\frac{i}{2}\left(\rho(\mathbf{z}) + \bar{\rho}(\bar{\mathbf{z}})\right)$
- $e^{-\frac{d-26}{24}\Gamma} \longrightarrow e^{-\frac{d-10}{8}\Gamma_{\text{super}}}$: partition function on supersheet (Berkovits ('88))

From correlation functions, we find

- $T_{X^\pm}(\mathbf{z}) T_{X^\pm}(\mathbf{z}') \sim$ (OPE of super Virasoro with $\hat{c} = 12 - d$)

  $T_{X^\pm} + T_{\text{LC}} + T_{\text{gh}} \quad \Longrightarrow \quad \hat{c}_{\text{total}} = 10 \quad \Longrightarrow \quad$ **nilpotent BRST charge!!**

- $T_X^\pm(\mathbf{z})$ is regular at $\mathbf{z} = \mathbf{z}_I \quad \Longrightarrow \quad Q_{\text{B}}$ is conserved there.

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Summary

- We have provided yet another way to realize string theories in noncritical dimensions

worldsheet theory
in conformal gauge
($\hat{c}$ or $c = 0$)

gauge fixing $\Downarrow$ $\Uparrow$ $\otimes$ $X^{\pm}$ CFT ($\hat{c} = 12 - d$ or $c = 28 - d$) $\otimes$ ghosts

gauge invariant SFT
for $d \neq 10$ or $d \neq 26$ (??)

worldsheet theory
in light-cone gauge
($\hat{c}$ or $c = d - 2$)

$\Longleftrightarrow$

light-cone gauge SFT
for $d \neq 10$ or $d \neq 26$

- Using the $X^{\pm}$ CFT, we can show our dimensional regularization scheme works without any contact terms (at tree level for the (NS,NS) sector).

Outlook

- Ramond sector

  spin fields? $\leftarrow$ free field realization using $X^{\pm}$, $\psi^{\pm}$ and ghosts (work in progress)

- multi-loop amplitudes

- gauge invariant SFT ($\leftarrow$ $\alpha = p^{+}$ HIKKO-like theory??)

- Green-Schwarz superstring (in semi light-cone gauge)

  modification of the energy-momentum tensor is similar

$$\{X^{+}, z\} = -\frac{1}{2}(\partial \ln X^{+})^2 + \partial^2 \ln X^{+}$$

(Berkovits-Marchioro, Kunitomo-Mizoguchi, Kazama-Yokoi, ...)

- Is our scheme applicable to Witten's cubic open SFT?

etc.....

Appendix: Towards Ramond sector (work in progress)

Immediate difficulty in the Ramond sector

$\Rightarrow$ spin fields

bosonization??? $\longleftarrow$ $X^{\pm}$ CFT is an interacting theory

- We have found free field realization

∘ bosonic case

$$(X^{+}, X^{-}; b, c, \tilde{b}, \tilde{c}) \longmapsto (X^{+}, X'^{-}; b', c', \tilde{b}', \tilde{c}')$$

$$b' \equiv (\partial X^{+})^{\alpha} b \ , \qquad \tilde{b}' \equiv (\bar{\partial} X^{+})^{\alpha} \tilde{b} \ ,$$

$$c' \equiv (\partial X^{+})^{-\alpha} c \ , \qquad \tilde{c}' \equiv (\bar{\partial} X^{+})^{-\alpha} \tilde{c} \ , \qquad \text{with } \alpha(\alpha+3) = \tfrac{d-26}{12} \ ,$$

$$X'^{-} \equiv X^{-} + \left(-\alpha \frac{cb}{\partial X^{+}} - \frac{3}{2}\alpha \frac{\partial^{2} X^{+}}{(\partial X^{+})^{2}} + \text{c.c.} \right)$$

X^{+} and the primed variables satisfy the free OPE's.

∘ superstring case

$$(X^+, X^-, \psi^+, \psi^-; b, c : \beta, \gamma) \quad \longmapsto (X^+, X'^-, \psi^+, \psi'^-; b', c'; \beta', \gamma')$$

and also for the anti-holomorphic sector have found out.

define spin fields by bosonizing the free primed variables

$$\psi^+ = e^{iH'} \ , \qquad \tilde{\psi}'^- = -e^{-iH'} \ ; \qquad \beta' = e^{-\phi'} \partial \xi' \ , \qquad \gamma' = \eta' e^{\phi'}$$

- Correspondence between the correlation functions expressed in terms of the unprimed variables and those in terms of the primed variables has been found out.

(work in progress)

Free field realization itself is interesting.