Wall-Crossing of D4/D2/D0 on the Conifold

(arXiv: 1007.2731 [hep-th])

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(In collaboration with Satoshi Yamaguchi)

Introduction

The d=4, N=2 string thoery have a special class of quantum states called **1/2 BPS states**, whose "degeneracy" or index is **piecewise constant** in the moduli space.

$$\Omega(Q;t) = -\frac{1}{2}\mathrm{Tr}[(-1)^F F^2]$$

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moduli space

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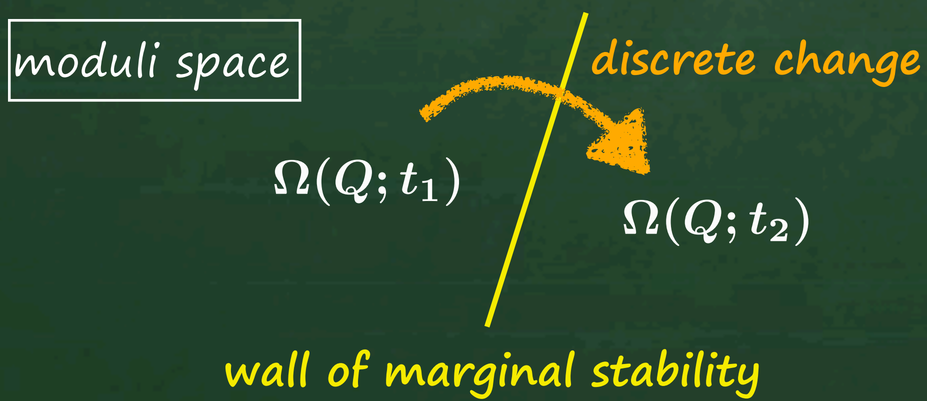

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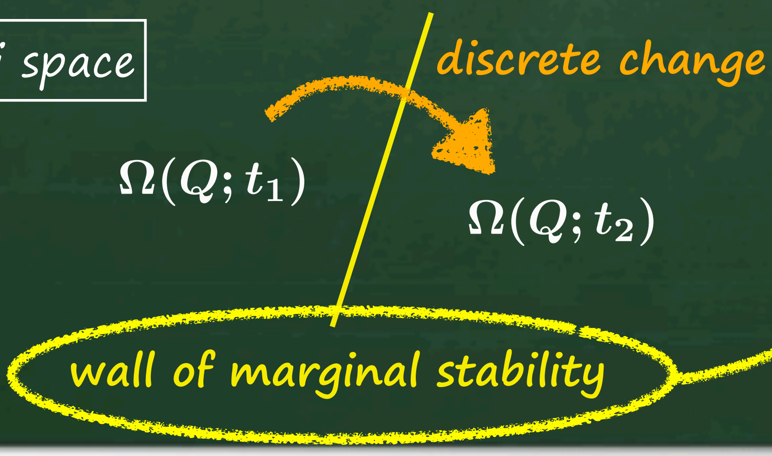

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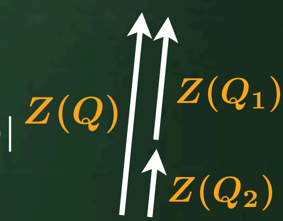

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$$\arg[Z(Q)] = \arg[Z(Q_1)]$$
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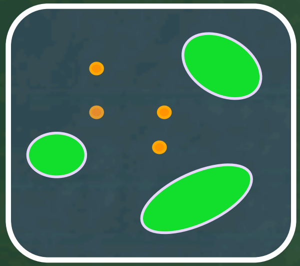
<u>Branes vs Black holes</u>

wrapped D-branes in CY3



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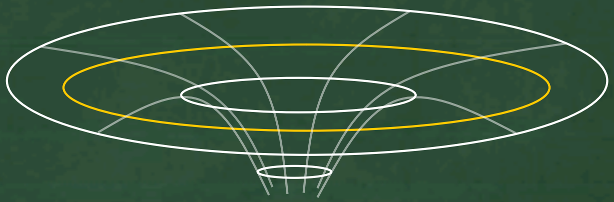
BPS black holes in 4dim
(single- or multi-centered)



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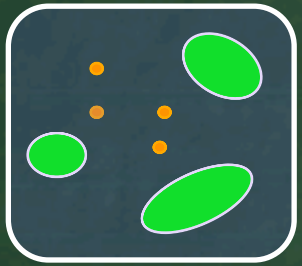

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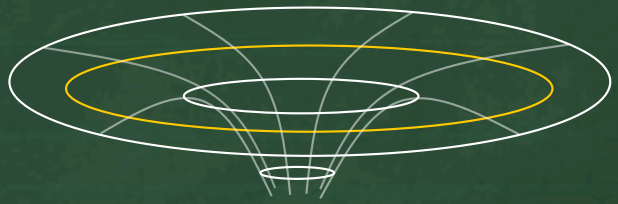

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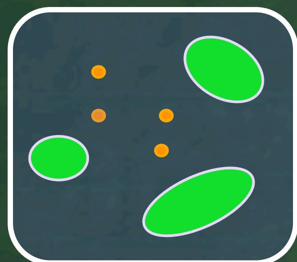

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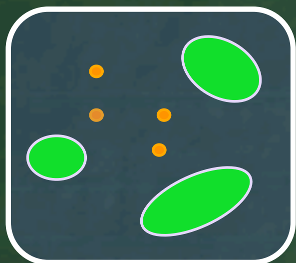

vacuum moduli = Calabi-Yau moduli

KS-formula

Recently, Kontsevich and Soibelman have proposed a wall-crossing formula that tells us **how the degeneracy changes** at the walls of marginal stability.

discrete change

$\Omega(Q;t_1)$ $\Omega(Q;t_2)$

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The main topic of this talk

Type IIA on Calabi-Yau

We study the wall-crossing of **one** non-compact **D4**-brane with **arbitrary numbers of D2/D0** on the **resolved conifold**.

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The vacuum moduli are the **Kahler moduli of the conifold.**

We evaluate the **partition function** of D4/D2/D0 in various chambers in the moduli space by using the **Kontsevich-Soibelman formula (KS-formula).**

Resolved conifold

Definition

$$\mathbb{C}^4 \supset \mathcal{M}_y := \{(z_1, z_2, z_3, z_4)\,;\ \ |z_1|^2 + |z_2|^2 - |z_3|^2 - |z_4|^2 = y\}$$

U(1)-action : $(z_1, z_2, z_3, z_4) \longrightarrow (e^{i\theta} Z_1, e^{i\theta} z_2, e^{-i\theta} z_3, e^{-i\theta} z_4)$

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$\begin{cases} \text{compact 2-cycle} \times 1 \\ \text{compact 4-cycle} \times 0 \end{cases}$

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Two limits $y \to \pm\infty$ correspond to large 2-cycle limits.

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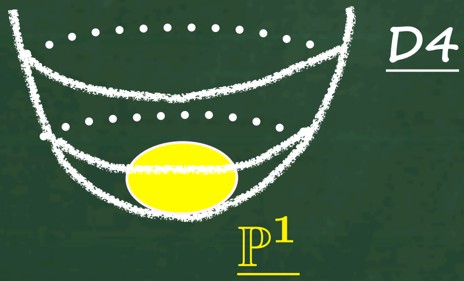

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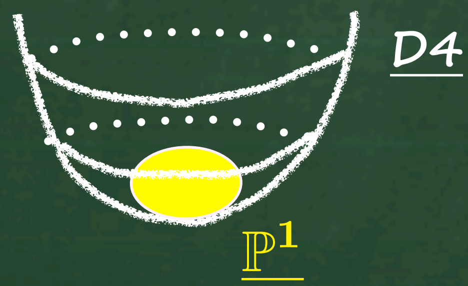

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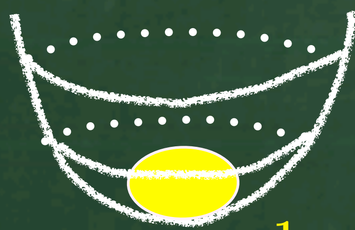

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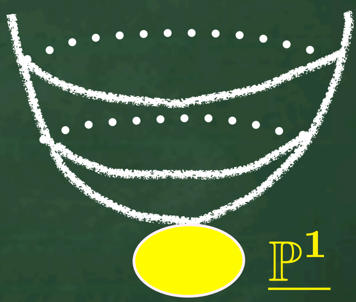

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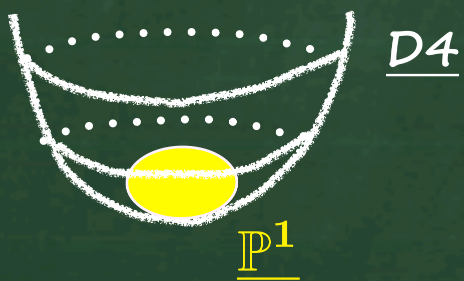

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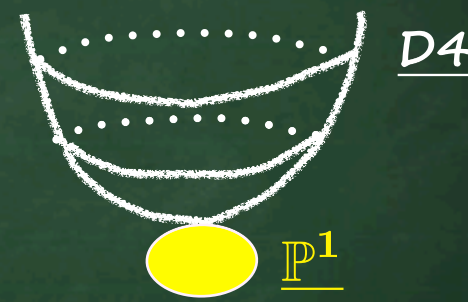

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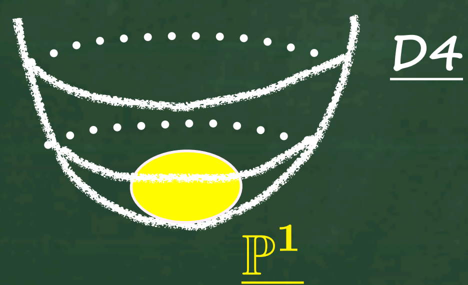

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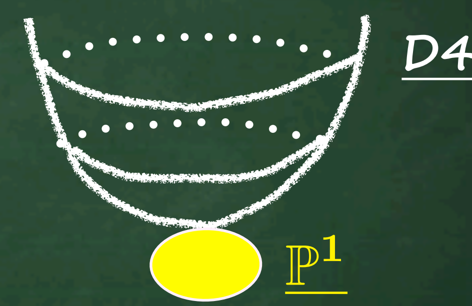

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The point $y = 0$ is geometrically singular but the spectrum has no singularity if we tune the <u>B-field</u> for the compact 2-cycle.

Walls of marginal stability

Kahler moduli

$$t = z\mathcal{P} + \Lambda e^{i\varphi}\mathcal{P}'$$

$(\mathcal{P},\, P' \in H^2(X))$

$z = x + iy$: Kahler parameter for the compact cycle

(x : B-field, y : size)

$\Lambda e^{i\varphi}$: Kahler parameter for other non-compact cycles

In the final result, the local limit $\Lambda \to \infty$ should be taken.

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central charges

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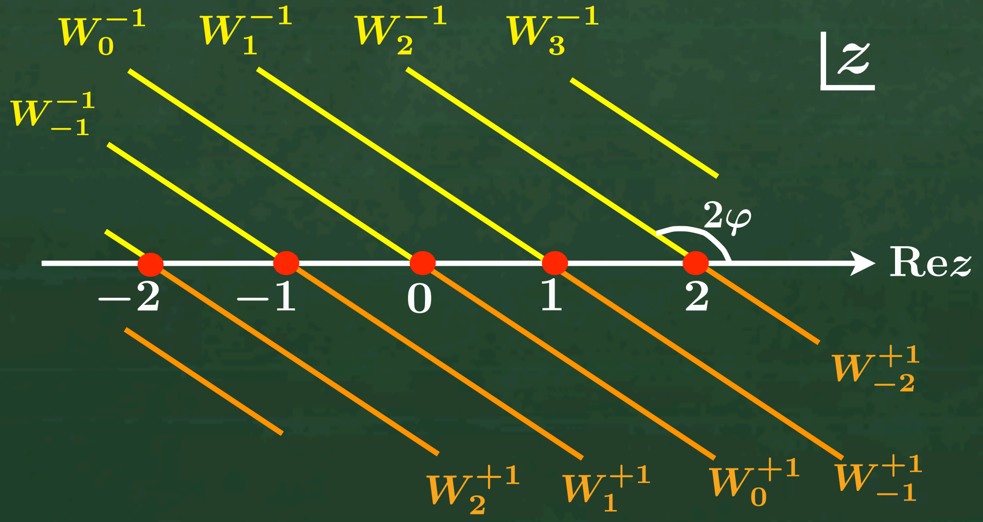
These walls are labeled by $(\pm 1, n)$. So we denote them $W_n^{\pm 1}$.



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We will do this by using **the Kontsevich-Soibelman's wall-crossing formula.**

Kontsevich-Soibelman wall-crossing formula

Consider an infinite dimensional Lie algebra with a commutation relation

$$[e_{Q_1}, e_{Q_2}] = (-1)^{\langle Q_1,Q_2\rangle} \langle Q_1, Q_2\rangle \, e_{Q_1+Q_2}$$

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The product A depends on the moduli t in two ways, the **degeneracy** $\Omega(Q;t)$ and the **order in the product** depend on t .

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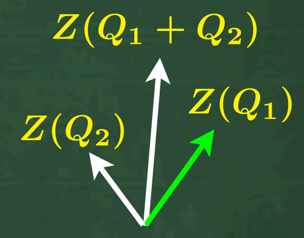

and the following product

$$A = \prod_{Q_{\mathrm{BPS}}}^{\longrightarrow} K_{Q_{\mathrm{BPS}}}^{\Omega(Q_{\mathrm{BPS}};t)} \qquad \left(K_Q = \exp\Big[\sum_{n=1}^{\infty} \frac{1}{n^2} e_{nQ}\Big] \right)$$

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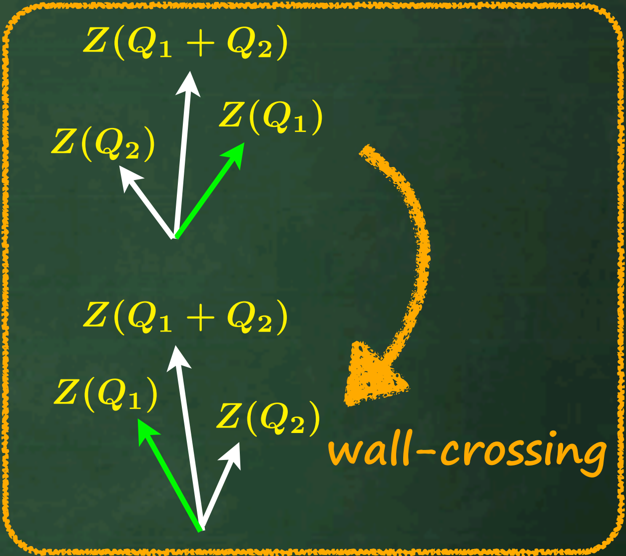

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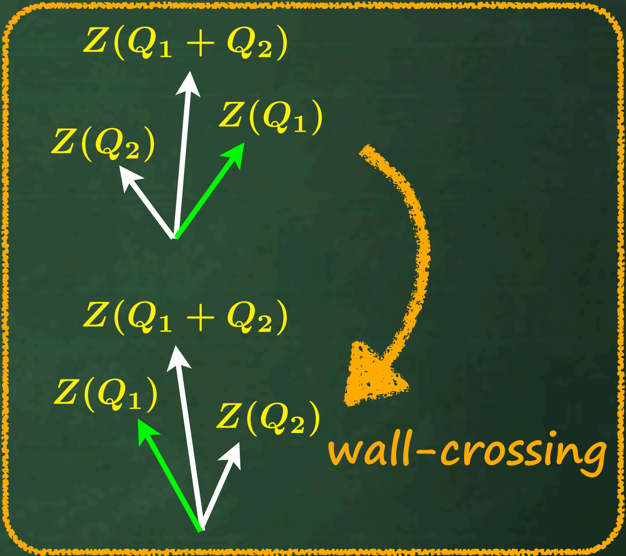

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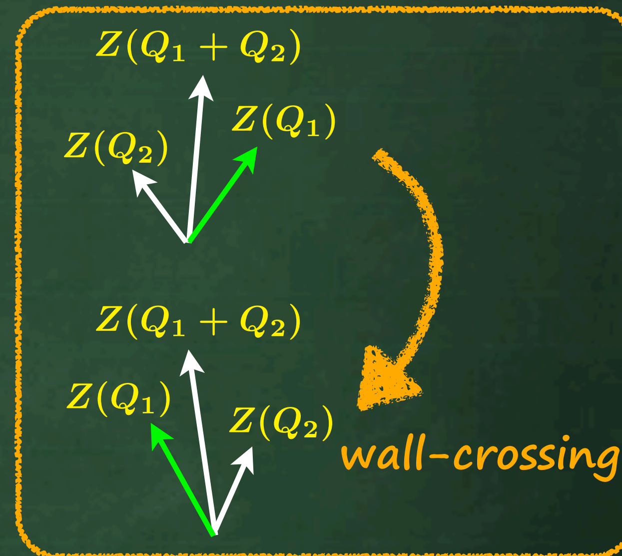

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We can read off the change in $\Omega(Q_{\mathrm{BPS}};t)$ from the invariance of A .

Partition functions

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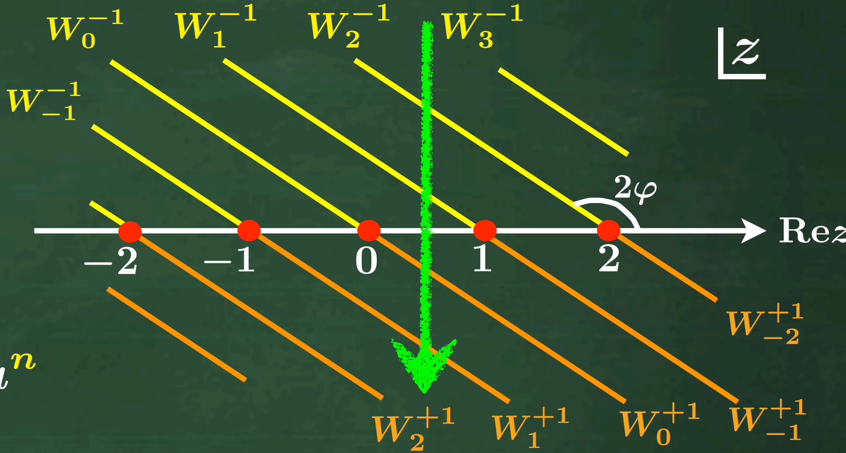

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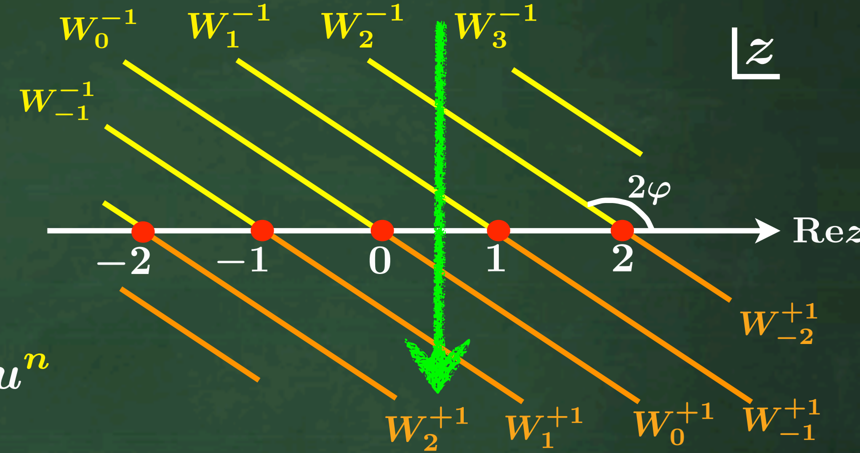

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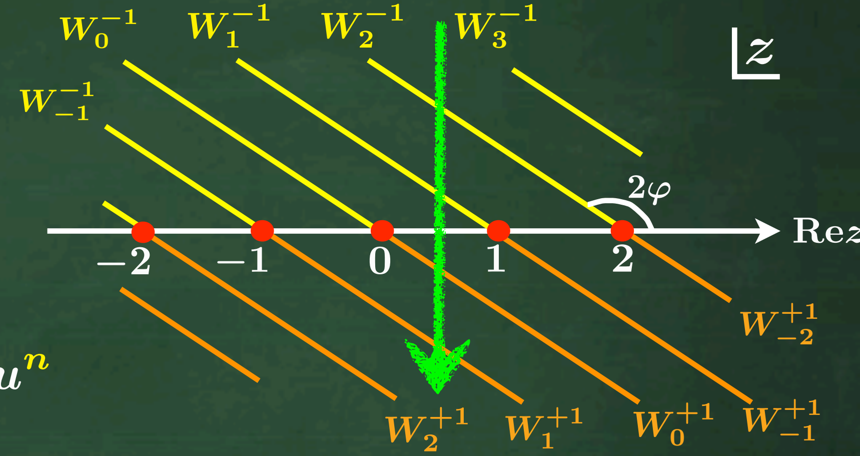

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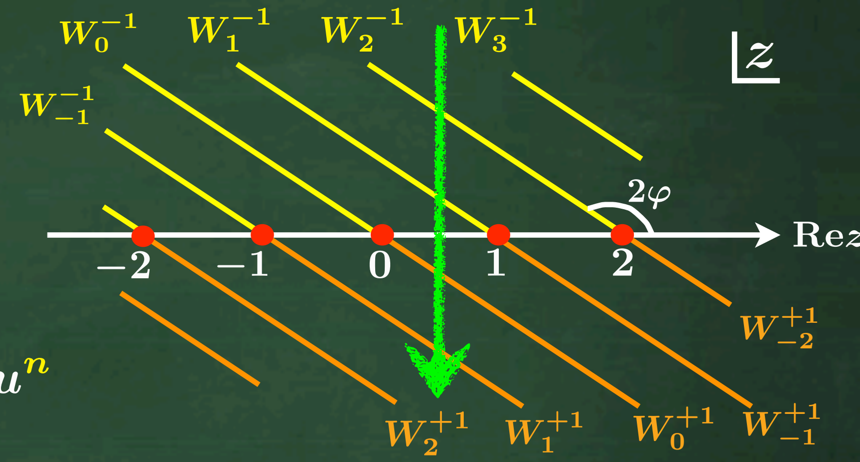

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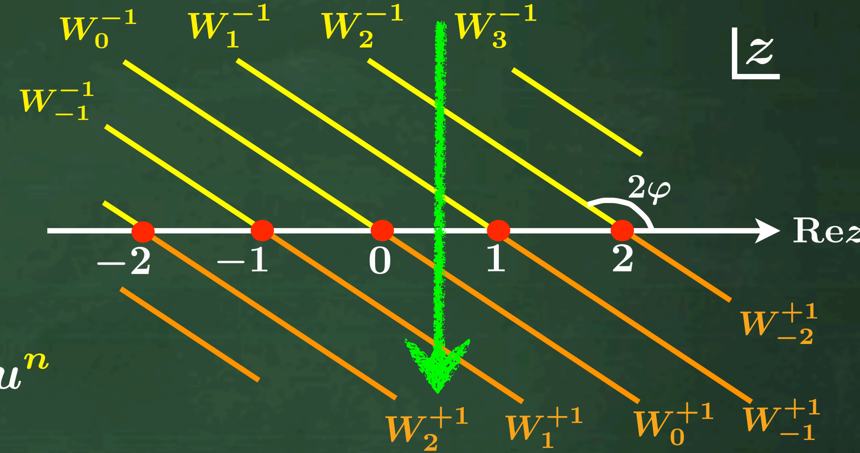

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This is consistent with the fact that $\chi(C_4)$ decreases by one through the flop transition.

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Summary

- We have discussed the wall-crossing phenomena of **D4/D2/D0** bound states on the resolved conifold.
- We considered one non-compact D4-brane and various numbers of D2/D0 on it.
- We identified all **walls of marginal stability**.
- By moving the Kahler moduli, we can consider the **flop transition** of the conifold through which **the topology of the conifold is changed**.
- We evaluate the partition function in all chambers by using the **Kontsevich-Soibelman wall-crossing formula**.
- The result is **completely consistent with the known facts** about the field theory on D4-branes and the flop transition.

That‘s all for my presentation.

Thank you very much.