

# Discrete Flavor Symmetry in String Model

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# Introduction

- String → Standard Model
    - String theory
      - A candidate which describe quantum gravity and unify four forces
      - Is it possible to realize phenomenological properties of Standard Model ?
    - (Supersymmetric) Standard Model
      - We have to realize **all** properties of Standard model
        - Four-dimensions,
        - N=1 supersymmetry,
        - Standard model group(  $SU(3)*SU(2)*U(1)$  ),
        - Three generations,
        - Quarks, Leptons and Higgs,
        - No exotics,
        - Yukawa hierarchy,**
        - Proton longevity,
        - R-parity,
        - Doublet-triplet splitting,
        - Moduli stabilization,
        - ...
- The key is **non-Abelian discrete symmetry**

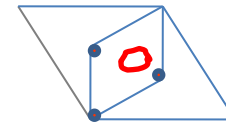
- String  $\rightarrow$  Standard Model ----- String compactification : 10-dim  $\rightarrow$  4-dim

Orbifold compactification, Calabi-Yau, Intersecting D-brane, Magnetized D-brane, F-theory, M-theory, ...

- Orbifold compactification

- SM or several GUT gauge symmetries
- N=1 supersymmetry
- Chiral matter spectrum

Dixon, Harvey, Vafa, Witten '85,'86  
Ibanez, Kim, Nilles, Quevedo '87



- MSSM searches in orbifold vacua :

Embedding higher dimensional GUT into string

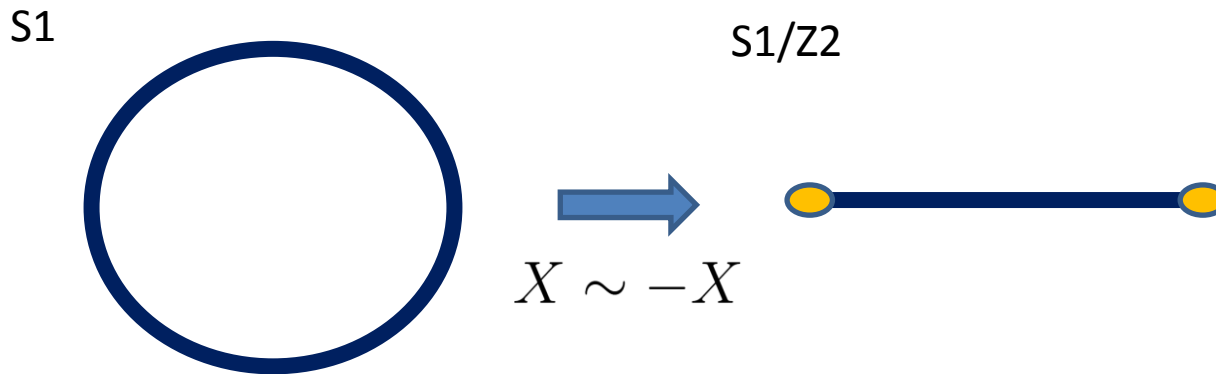
- Three generations,
- Quarks, Leptons and Higgs,
- No exotics,
- Top Yukawa,
- Proton longevity,
- R-parity,
- Doublet-triplet splitting,
- ...

Kobayashi, Raby, Zhang '04  
Buchmuller, Hamaguchi, Lebedev, Ratz '06  
Lebedev, Nilles, Raby, Ramos-Sanchez, Ratz,  
Vaudrevange, Wingerter '07  
Kim, Kyae '07  
.....

# Discrete flavor symmetry in heterotic orbifolds

- Orbifold compactification of heterotic string theory

Kobayashi, Nilles, Plöger, Raby, Ratz '07



- Boundary condition

$$X^i(\sigma + \pi) = (\theta^k X)^i + n_a e_a^i$$

- String coupling selection rule

Conjugacy class :  $(\theta, me)$

$$\prod_j (\theta, m^j e) = (1, (1 - \theta)\Gamma)$$

■ 1 dimensional orbifold :  $S^1/Z_2$

-- String coupling selection rule

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

-- Relabeling fixed points

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

These Abelian discrete symmetries generate the non-Abelian discrete symmetry

$$D_4 = (Z_2 \times Z_2) \rtimes Z_2$$

■ 2 dimensional orbifold : T2/Z3

-- String coupling selection rule

$$\begin{pmatrix} \omega & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^{-1} \end{pmatrix}$$

-- Relabeling fixed points

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

These Abelian discrete symmetries generate the non-Abelian discrete symmetry

$$\Delta(54) = (Z_3 \times Z_3) \rtimes S_3$$

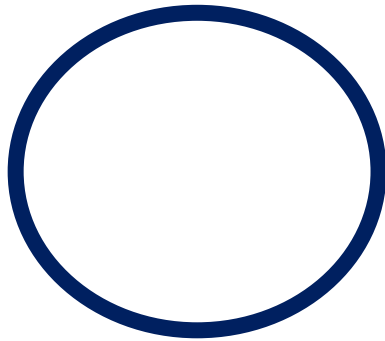
Non-Abelian discrete symmetries have a stringy origin, which are determined by the structure of the extra dimension space

# Gauge origin of discrete flavor symmetry

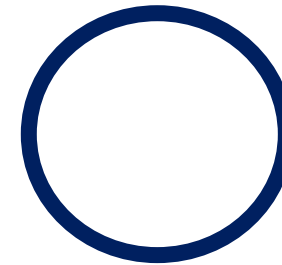
- ◆ symmetry enhance point in moduli space

Beye, Kobayashi, Kuwakino  
arXiv:1406.4660 [hep-th]

$S^1$



$U(1)$  gauge symmetry



$SU(2)$  gauge symmetry

$S^1/\mathbb{Z}_2$



$D_4$  discrete symmetry



$U(1)$  gauge symmetry

◆ 1-dimensional orbifold model at symmetry enhance point

-- Currents

$$H' = i\partial X' = \frac{1}{\sqrt{2}} (E_+ + E_-),$$

$$E'_\pm = e^{\pm i\alpha X'} = \frac{1}{\sqrt{2}} H \mp \frac{1}{2} (E_+ - E_-)$$

$$\left( \begin{array}{l} H = i\partial X \\ E_\pm = e^{\pm i\alpha X} \end{array} \right)$$

-- Massless spectrum

Sector	Field	$U(1)$ charge	$Z_4$ charge
U	$U$	0	0
U	$U_1$	$\alpha$	0
U	$U_2$	$-\alpha$	0
T	$M_1$	$\frac{\alpha}{4}$	$\frac{1}{4}$
T	$M_2$	$-\frac{\alpha}{4}$	$-\frac{1}{4}$



-- This model has symmetry :  $U(1) \rtimes Z_2$

Z2 symmetry can be described by

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

-- Non zero VEV of Kahler moduli field breaks the U(1) symmetry to Z4 Abelian discrete symmetry

$$T = \frac{1}{\sqrt{2}}(U_1 + U_2)$$
$$\langle U_1 \rangle = \langle U_2 \rangle$$

Z4 symmetry can be described by

$$\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$$

-- Symmetry breaking patterns are summarized as

$$SU(2) \xrightarrow{\text{orbifolding}} U(1) \rtimes Z_2 \xrightarrow{\langle T \rangle} D_4$$

◆ 2-dimensional orbifold model at symmetry enhance point

-- Massless spectrum

Sector	Field	$U(1)^2$ charge	$Z_3^2$ charge
U	$U$	$(0, 0)$	$(0, 0)$
U	$U_1$	$-\alpha_1$	$(0, 0)$
U	$U_2$	$-\alpha_2$	$(0, 0)$
U	$U_3$	$\alpha_1 + \alpha_2$	$(0, 0)$
T	$M_1$	$\frac{\alpha_1}{3}$	$(\frac{1}{3}, \frac{1}{3})$
T	$M_2$	$\frac{\alpha_2}{3}$	$(-\frac{1}{3}, 0)$
T	$M_3$	$-\frac{\alpha_1 + \alpha_2}{3}$	$(0, -\frac{1}{3})$

-- This model has symmetry :  $U(1)^2 \times S_3$

Z2 symmetry can be described by

$$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

-- Non zero VEV of Kahler moduli field breaks the  $U(1)^2$  symmetry to  $Z_3 \times Z_3$  Abelian discrete symmetry

$$T = \frac{1}{\sqrt{3}}(U_1 + U_2 + U_3)$$

$$\langle U_1 \rangle = \langle U_2 \rangle = \langle U_3 \rangle$$

$Z_3 \times Z_3$  symmetry can be described by

$$\begin{pmatrix} \omega & 0 & 0 \\ 0 & \omega^{-1} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \omega & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \omega^{-1} \end{pmatrix}$$

-- Symmetry breaking patterns are summarized as

$$SU(3) \xrightarrow{\text{orbifolding}} U(1)^2 \times S_3 \xrightarrow{\langle T \rangle} \Delta(54)$$

# Example model

- ❑ Heterotic asymmetric orbifold model
- ❑  $SU(3) \rightarrow U(1)^2 \times S_3$  by orbifolding
- ❑ Three-generation SUSY SM model

Gauge symmetry :  $SU(3)_C \times SU(2)_L \times U(1)_Y \times SU(2)^2 \times SU(4) \times SU(5) \times U(1)^9$

Untwisted sector : 3 x 11 multiplets

Twisted sector : 1 x 114 multiplets

Three-generation quarks and leptons

Suitable  $U(1)$  hyper charges

Vector-like exotics

- ❑ This model realizes  $\Delta(54)$  discrete flavor symmetry in low energy effective theory
- ❑ This gauge origin mechanism can also be applied to string models in non-geometric background

# Summary and discussion

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- A non-Abelian continuous gauge symmetry can be regarded as the origin of a non-Abelian discrete flavor symmetry in heterotic orbifolds
- This can be understood at symmetry enhance point in moduli space
- This mechanism can be applied to non-geometric string models
  
- Application of this mechanism to field theoretical models
  - Higher-dimensional gauge theory on orbifolds
  - Gauge extension of discrete flavor model
  - New  $Z'$  bosons