Defect Holography and the Fractional Quantum Hall Effect

Charles M. Melby-Thompson Kavli IPMU

Strings and Fields, YITP (26 July, 2014)

work in progress with M. Fujita, R. Meyer, S. Sugimoto

Defect Holography

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Strings and Fields, YITP (26 July, 2014)

work in progress with M. Fujita, R. Meyer, S. Sugimoto **Today's focus:**

Holography of domain walls in Yang-Mills–Chern-Simons theory.

level

Yang-Mills-Chern-Simons? (YMCS)

3D:
$$S = -\frac{1}{4g^2} \int_{M_3} d^3 x \operatorname{Tr}(F^{\mu\nu}F_{\mu\nu}) + \frac{k}{4\pi} \int_{M_3} \omega_3(A)$$

 $\omega_3(A) = \operatorname{Tr}(A \wedge dA + \frac{2}{3}A \wedge A \wedge A) \quad \Leftarrow \text{ Chern-Simons term}$

CS term only gauge invariant up to a total derivative:

$$\delta_{\Lambda}\omega_3(A) = d\operatorname{Tr}(\Lambda F)$$

Consider domain walls separating phases at different levels => not gauge invariant!

Must be additional degrees of freedom living on the defects to compensate New degrees transform under a $U(|\Delta k|)$ global symmetry.



Holographic Embedding of YMES

N=4 SYM AdS5

Compactify N=4 SYM on a circle, impose anti-periodic boundary conditions on fermions 🖒 SUSY

Geometry: AdS soliton

Massive fermions induce scalar masses

Gauge field confines

=> gapped theory

Dual geometry caps off at a scale determined by the compactification radius

$$ds^{2} = w^{2} dx_{3}^{2} + f(w)w^{2} d\theta^{2} + \frac{R^{2}}{f(w)} \frac{dw^{2}}{w^{2}}$$

cap at $w = w_{0}$ $f(w) = 1 - \frac{w_{0}^{4}}{w^{4}}$



Holographic Embedding of YMES



Wilson line

Computed by attaching a string.

AdS => conformal => no dependence on size (after renormalization)

AdS soliton => Wilson lines larger than the compactification radius hit the bottom

Dominated by area of worldsheet at bottom => area law *confinement

Infrared physics = confined phase of 3D YM

Fujita, Li, Ryu, Takayanagi Holographic Embedding of YMCS

How to get the CS in YM-CS?

Starting with D3 branes, introduce k units of C₀ flux on the circle.

- WZ part $\frac{1}{8\pi^2} \int_{M_4} \text{Tr}(F \wedge F)C_0 = \frac{1}{8\pi^2} \int_{M_4} \omega_3(A) \wedge (k \, d\theta) = \frac{k}{4\pi} \int_{M_3} \omega_3(A)$ Chern-Simons
- => With twisted fermion b.c., dual low-energy effective theory is YMCS, in the confined phase.



String worldsheet can attach to D7 probe branes: string worldsheet probe branes at soliton tip Dominant contribution to on-shell action from Wilson loop on D7 brane => level-rank duality

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Holographic Embedding of Domain Walls

Interested in defects changing the level $k \rightleftharpoons b$ need to change the number of branes



Will mostly consider interval of finite length with level k = 1; k = 0 elsewhere.





Spectrum of operators

Consider only bosonic operators uncharged under SO(6) or SO(2) global symmetries. The brane is extended infinitely in the t, \times directions.

WZ term + F₅ flux 🖒 CS term on brane worldvolume

Level = N, number of D3 branes

Fields (near right-hand boundary; left-hand boundary fields have opposite chirality):

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Fields	Asymptotic form
У	$y_0 - \frac{Rw_*^3}{4w^4} + \cdots$
Α	$a^{(4)}w^4 + \dots + (\text{flat})$
Α	$(a_{-}^{(4)} \text{ terms}) + (\text{flat}) + a_{+}^{(-4)}w^{-4} + \cdots$

Spectrum of operators

Fields	Asymptotic form
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Source	Response	Dimension
У		3
۵	۵	5
٥	a	1



- irrelevant chiral operator
- chiral current => U(k) symmetry

Spectrum of operators

Harvey, Roysten 0804.2854

Source	Response	Dimension
У		3
۵	a	5
۵	a	1

Operator

 $\Phi^{(3)} \sim \bar{q}\gamma_+ F_{-y}q$

 $V^{(5)} \sim \bar{q}\gamma_+ F_{-y}F_{-y}q$

c.f.

 $J_+ \sim \bar{q}\gamma_+ q$

D3-D7 brane intersection

System Properties
Metric:
$$ds^2 = w^2 dx_3^2 + f(w)w^2 d\theta^2 + \frac{R^2}{f(w)}\frac{dw^2}{w^2}$$

Embedding: $y(w) = \int_{w_*}^w \frac{Rw^4 dw}{\sqrt{(w^6 - w_*^6)(w^4 - w_0^4)}}$
Free energy: $F = 2R\left(\int_{w_*}^{\Lambda} \frac{w^6 dw}{\sqrt{(w^6 - w_*^6)(w^4 - w_0^4)}} - \frac{1}{2}\Lambda^2\right)$

For *L* large, *F* grows linearly with length.

Behavior due to the bulk geometry ending; if the effective background were AdS_4 , there would be no *L* dependence.

Interpretation as order parameter for confinement?

System Properties

Edge mode [right-hand side chiralities]

Conserved current sector given by boundary values of flat connections: $A = d\varphi$ Source: $A_+^{(\text{flat})} =$ external gauge field \mathcal{A}_+ VeV: $A_-^{(\text{flat})} =$ current j_- Flatness condition: $\partial_+ A_- = \partial_- A_+$ When source is turned off, flatness $\Rightarrow \partial_+ j_- = 0$ so current is chiral.

Anomaly [right-hand side chiralities]

c.f.: Jensen 1012.4831 Yee, Zahed 1103.6286

c.f.: Yee, Zahed 1103.6286

Flatness condition: $\partial_+ A_- = \partial_- A_+ \implies \partial_\mu j^\mu = -\partial_+ j_- \propto \partial_- A_+ = E_{\text{external}}$

anomaly in 2 dimensions

Consistent with expectation from brane construction.

System Properties

Irrelevant vector operator:

 $A_{-}|_{\text{right}} \sim a_{-}w^{4} + \cdots$ at the right-hand boundary leads to $A_{-}|_{\text{left}} \sim c a_{-}w^{-4} + \cdots$ at the left boundary, and the opposite for A_{+}

Thus, $V_+|_{\text{right}} \sim A_+|_{\text{left}}$ and $V_-|_{\text{left}} \sim A_-|_{\text{right}}$; the field varied to obtain the VeV does not determine the VeV. $\langle V_-|_{\text{left}} V_+|_{\text{right}} \rangle \sim e^{-2\pi L/\ell}$ with *l* the circle compactification length. Implies mediation by field with mass on the KK scale.

Meson spectrum, response to bulk fields, general correlation functions, multiples types of phase transitions, ...

Surprisingly rich.

Fractional Quantum Hall Effect

Quantum Hall Effect:

Apply a strong magnetic field normal to a conducting film Fujita, Li, Ryu, Takayanagi

0901.0924

When an electric field is applied, a perpendicular current flows: $\mathbf{E} \perp \mathbf{J}$ For strong magnetic fields, this conductivity becomes quantized.

Effective field theory description by Chern-Simons action.

Gapped, exhibits edge currents, relies crucially on interactions (unlike IQHE)

Response to external EM field (encoded in *B* **field) reproduces response in edge currents (probably)**

Some peculiar subtleties in holographic renormalization...

Examples of Hall behavior in holographic systems may shed light on FQHE.

Summary

Holography for level-changing defects in YM-CS

Few degrees of freedom

Surprisingly rich structure:

anomalies, affine Lie symmetries, unusual correlation properties, relationship with confinement, (large N) symmetry breaking, phase transitions...

Relationship to FQH

