Entanglement Entropy of local operator excited states in 2d RCFTs

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based on arXiv:1401.5209 (Phys.Rev.Lett. 112,11602 (2014))with Masahiro Nozaki and Tadashi Takayanagi
and arXiv:1403.0702 (to appear in PRD)with Song He, Kento Watanabe and Tadashi Takayanagi

Motivation

・To study the property of entanglement entropy for excited states . [cf. Calabrese, Cardy 05, 07: Time evolution of excited states called "Quantum Quench".]

➡ In this talk, we consider local operator excited states:

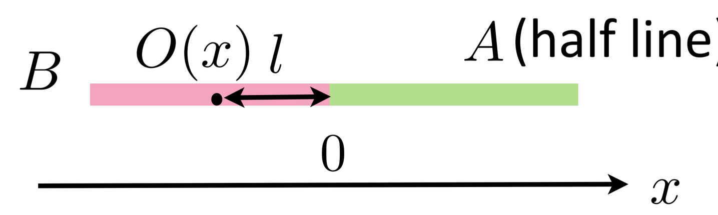
$|O\rangle \equiv O(x)\,|\mathrm{vac}\rangle \quad (t=0)$

・To study the universal property of entanglement entropy in the limit the subsystem is very large.

cf) In the small size limit , there is a property analogous to the first law of thermodynamics: $\Delta S_A[|O\rangle] \propto E_O$

$(\Delta S_A[|O\rangle] = S_A[|O\rangle] - S_A[|\mathrm{vac}\rangle])$

➡ In this talk , we consider the following setup:



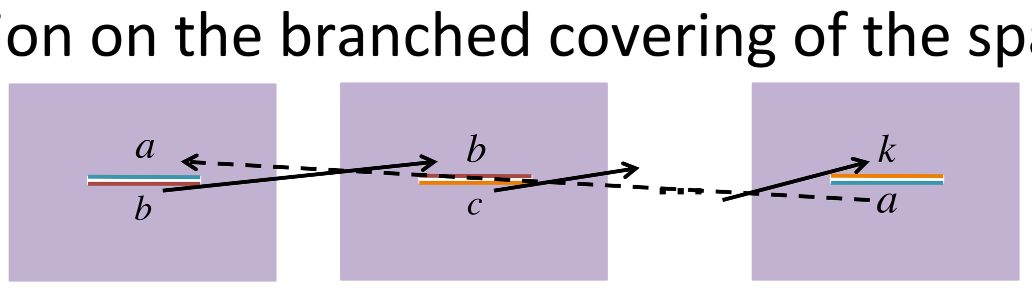
Replica method

To calculate entanglement entropy , first we calculate $\mathrm{Tr}_A\rho_A^n$ instead of $\mathrm{Tr}_A\rho_A\log\rho_A$. Then, we analytically continue n to 1:

$$S_A = -\mathrm{Tr}_A\rho_A\log\rho_A = \frac{1}{1-n}\log\mathrm{Tr}_A\rho_A^n\Big|_{n=1} \text{ :Replica method}$$

ground states

In the path integral formalism, we can represent $\mathrm{Tr}_A\rho_A^n$ in terms of partition function on the branched covering of the spacetime mfd:

$\mathrm{Tr}_A\rho_A^n =$  $= \dfrac{Z_n}{Z_1^n}$

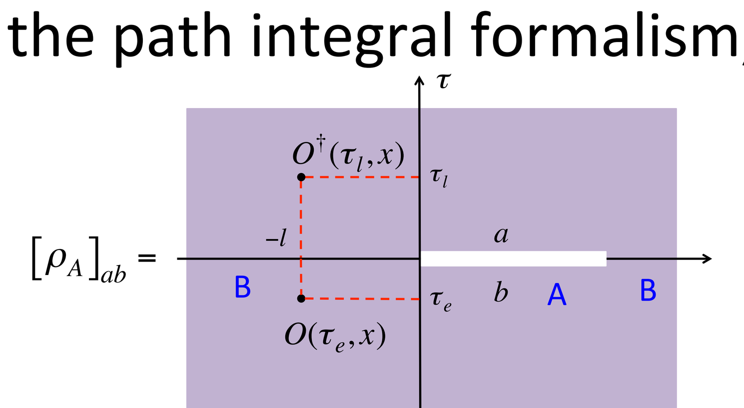
Z_n : partitoin function on $n$sheet covering space Σ_n .

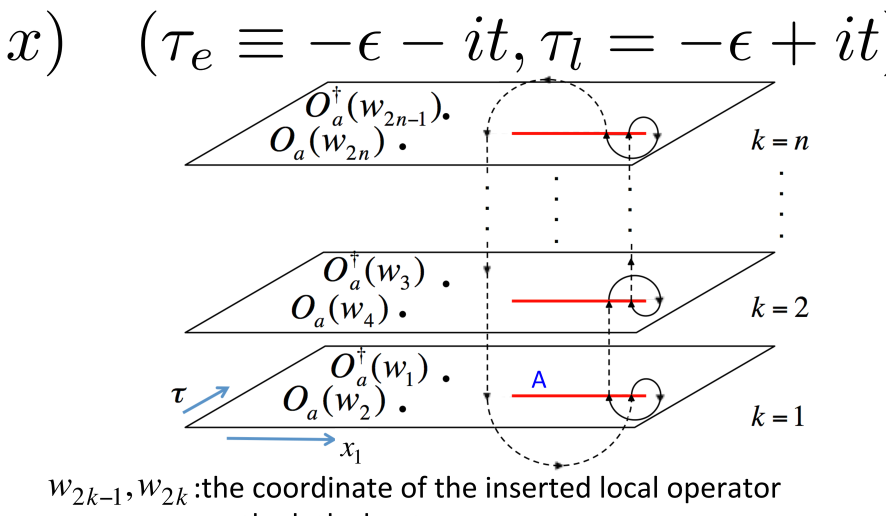
Local operator excited states

In this case, the state takes the following form:

$$\rho_{tot}(t,x) = e^{-iHt}e^{\epsilon H}O(x)\,|0\rangle\langle 0|\,O^\dagger(x)e^{-\epsilon H}e^{iHt}$$
$$= O(\tau_e,x)\,|0\rangle\langle 0|\,O^\dagger(\tau_l,x) \quad (\tau_e \equiv -\epsilon - it, \tau_l = -\epsilon + it)$$

In the path integral formalism,





w_{2k-1}, w_{2k} :the coordinate of the inserted local operator on the k-th sheet .

Finally, we can express the difference between EE of the excited state and the ground state using the correlation function!

$$\Delta\mathrm{Tr}\rho_A^n = \left[\log\langle O^\dagger(w_1)O(w_2)\cdots O^\dagger(w_{2n-1})O(w_{2n})\rangle_{\Sigma_n} - n\log\langle O^\dagger(w_1)O(w_2)\rangle_{\Sigma_1}\right]$$

Results for free scalar field CFTs in 2D

We first consider the free massless scalar field theory:

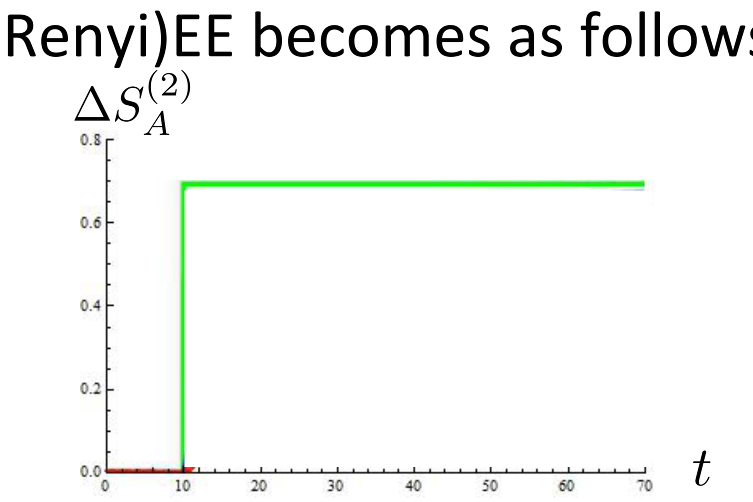
$$S = \frac{1}{2}\int d^2x\,\partial_\mu\phi\partial^\mu\phi$$

Time evolution of (Renyi) Entanglement Entropy

Operator: $O = e^{i\alpha\phi} + e^{-i\alpha\phi}$

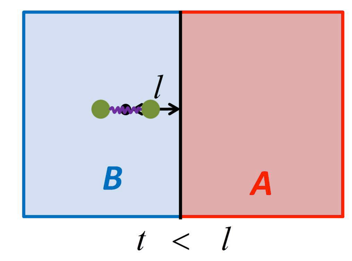
In this case, the time evolution of (Renyi)EE becomes as follows:

$$\Delta S_A^{(n)} = \begin{cases} 0 & (t<l) \\ \log 2 & (t>l) \end{cases}$$

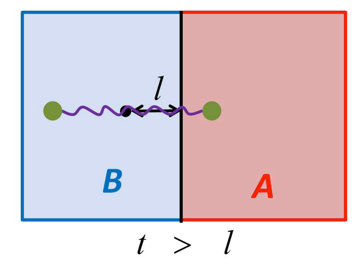


We can this time evolution as follows:
At t = 0 , entangled (quasi) particles are created at x = -l, and they are propagate with the velocity of light.

・If t < l, quasi particles don't reach at entangling surface, so REE doesn't change:



・If t > l , quasi particles pass the entangling surface, so the value of REE increase.



The late time value

In 2d CFT, free boson is decomposed into chiral and anti chiral parts:

$$\phi(x,t) = \phi_L(x-t) + \phi_R(x+t)$$

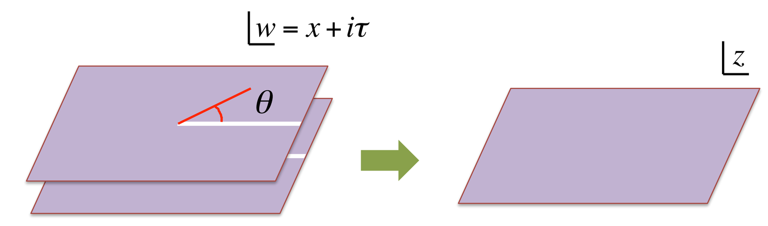
From this ,

$$O(x,t)\,|0\rangle = e^{i\alpha\phi_L}\,|0_L\rangle \otimes e^{i\alpha\phi_R}\,|0_R\rangle + e^{-i\alpha\phi_L}\,|0_L\rangle \otimes e^{-i\alpha\phi_R}\,|0_R\rangle$$
$$\approx |\uparrow\rangle_L \otimes |\uparrow\rangle_R + |\downarrow\rangle_L \otimes |\downarrow\rangle_R \quad ➡ \text{ EPR state!}$$

Result for 2d RCFTs

・n=2 REE

Using conformal mapping



$$\Sigma_2 \to \Sigma_1 : z = \sqrt{w} = \sqrt{re^{i\theta}}(0 \le \theta < 4\pi)$$

We can n = 2 REE in terms of 4-pt function on $\Sigma_1 = \mathbb{C}$:

$$\langle O(w_1,\bar{w}_1)O(w_2,\bar{w}_2)O(w_3,\bar{w}_3)O(w_4,\bar{w}_4)\rangle_{\Sigma_2} = |z_{13}z_{24}|^{-4\Delta_O}\cdot G_O(z,\bar{z})$$

($z = z_{12}z_{34}/z_{13}z_{24}$: cross ratio)

In the late time,

$$\Delta S_A^{(2)} = \log d_O \quad (d_O\text{:quantum dimension})$$

Why?

Because τ_e and τ_l are complex , $\bar{z}$ is not the complex conjugate of z .

$$z = \frac{-(l-t) + \sqrt{(l-t)^2+\varepsilon^2}}{2\sqrt{(l-t)^2+\varepsilon^2}}$$
$$\bar{z} = \frac{-(l+t) + \sqrt{(l+t)^2+\varepsilon^2}}{2\sqrt{(l+t)^2+\varepsilon^2}}$$

The late time value is

$(z,\bar{z}) \simeq (1+\mathcal{O}(\epsilon^2), \mathcal{O}(\epsilon^2)) \to (1,0)$

From this , $G_a(z,\bar{z}) = \sum_b (C_{aa}^b)^2 F_a(b|z)\bar{F}_a(b|\bar{z})$

$$\underset{(z,\bar{z})\to(1,0)}{\simeq} F_{00}[a]\cdot F_a(0|1-z)\bar{F}_a(0|z)$$
$$\simeq F_{00}[a]\cdot(1-z)^{-2\Delta_a}\bar{z}^{-2\Delta_a}$$

where $F_{bc}[a]$ is the fusion matrix defined by

$$F_a(b|1-z) = \sum_c F_{bc}[a]\cdot F_a(c|z) \quad (F_a(b|z)\text{:conformal block})$$

and $F_{00}[a] = 1/d_a$ [Moore-Seiberg 89]

Conclusion

・When the subsystem is very large , the late time value of $\Delta S_A^{(n)}$ becomes finite.

・$\Delta S_A^{(n)}$ is the contribution to EE from the local operator,so (R)EE can detect the degrees of freedom of local operator.

cf) EE for ground states can degrees of freedom of theory (for example central charge)

Future problem

・Holographic viewpoint
・other CFTs (for example D1-D5 orbifold)
・Relation to the topological EE in gapped systems

14年8月9日土曜日