Superconformal index on $\mathbb{RP}^2 \times S^1$
&
mirror symmetry

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based on arXiv 1408.xxxx

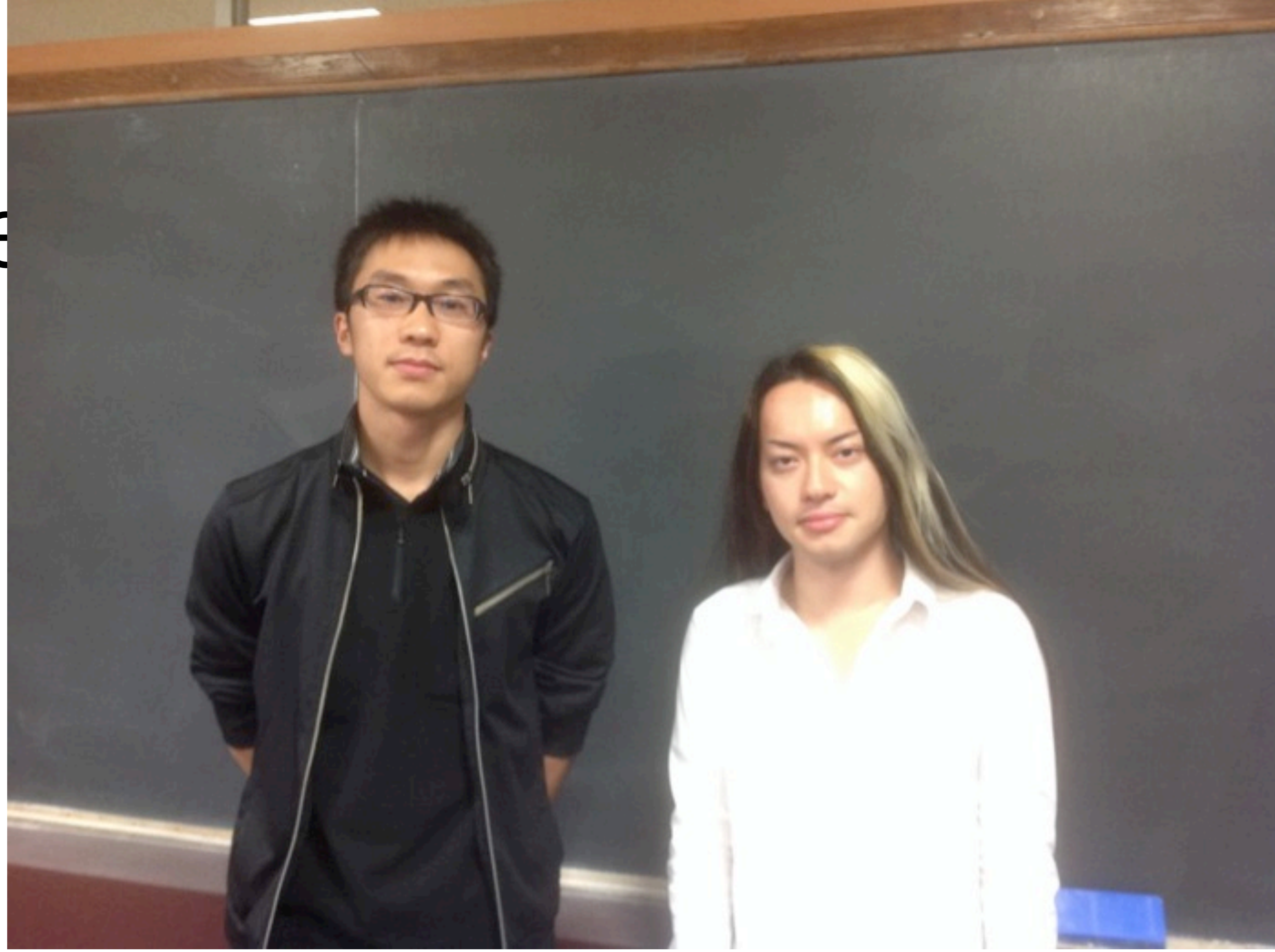
collaboration with

Hironori Mori and Takeshi Morita

"Strings and Fields"

@ Yukawa Institute for Theoretical Physics

Supe … $\times S^1$



Hironori Mori　　Takeshi Morita

"Strings and Fields"

@ Yukawa Institute for Theoretical Physics

Introduction

“What is the (3d) mirror symmetry ?”

$$(\mathit{Theory}\ A) = (\mathit{Theory}\ B)$$

They look totally different.

Introduction

"What is the (3d) mirror symmetry ?"

SQED

XYZ-model

$$(\,Theory\ A)$$

$$(\,Theory\ B)$$

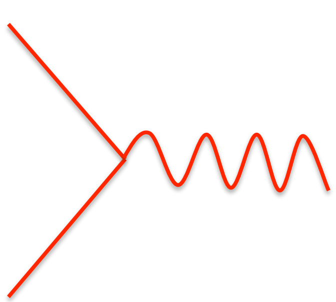
Introduction

"What is the (3d) mirror symmetry ?"

SQED

gauge interaction

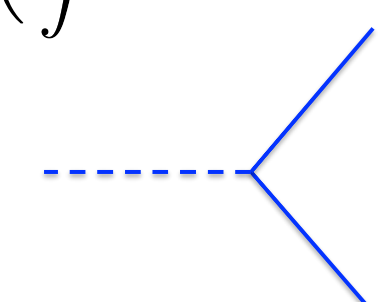
$$\mathcal{L}_{SQED} = \begin{array}{l} +\mathcal{L}_{vec}(A_\mu, \sigma, \overline{\lambda}, \lambda, D) \\ +\mathcal{L}_{Q}(\phi_Q, \psi_Q, F_Q, +c.c) \\ +\mathcal{L}_{\tilde{Q}}(\phi_{\tilde{Q}}, \psi_{\tilde{Q}}, F_{\tilde{Q}}, +c.c) \end{array}$$



XYZ-model

Yukawa interaction

$$\mathcal{L}_{XYZ} = \begin{array}{l} +\mathcal{L}_{X}(\phi_X, \psi_X, F_X, +c.c) \\ +\mathcal{L}_{Y}(\phi_Y, \psi_Y, F_Y, +c.c) \\ +\mathcal{L}_{Z}(\phi_Z, \psi_Z, F_Z, +c.c) \\ +\left(\int d^2\theta \; XYZ + c.c\right) \end{array}$$



Why is it believed so ?

- Moduli spaces are in agreement
- Global symmetries are same
- Stringy origin (brane construction)
- **Direct check based on exact calculation**

· Direct check based on exact calculation

Exact partition function on $\mathbb{S}^3_b$

SQED = XYZ-model

⇒ "Fourier transform of double sine function"

Exact Superconformal index on $\mathbb{S}^2 \times S^1$

SQED = XYZ-model

⇒ "Ramanujan's sum + q-binomial theorem"

Exact Superconformal index on $\mathbb{RP}^2 \times S^1$

SQED = XYZ-model

New result !

⇒ "q-binomial theorem" itself

Plan

- Superconformal index on $\mathbb{M}^2 \times S^1$
- $\mathbb{M}^2 = \mathbb{S}_b^2$ & mirror symmetry
- $\mathbb{M}^2 = \mathbb{RP}_b^2$ & mirror symmetry

● Superconformal index on $\mathbb{M}^2 \times S^1$

Refinement of

$$\mathrm{Tr}_{\mathcal{H}}(-1)^{\hat{F}}$$

※↑ counts # of BPS state of the theory

● Superconformal index on $\mathbb{M}^2 \times S^1$

Definition

$$\mathcal{I}^{\mathbb{M}^2}_{\text{Theory}}(x, y, \alpha_i) := \mathrm{Tr}_{\mathcal{H}_{\mathbb{M}^2}} (-1)^{\hat{F}} x^{-\hat{j}_3} y^{-(\hat{R}-\hat{j}_3)} \prod_{i:\text{Flavors}} \alpha_i^{\hat{f}_i}$$

- $\mathcal{H}_{\mathbb{M}^2}$: Hilbert space of the Theory on $\mathbb{M}^2$
- $\hat{F}$: Fermion number operator
- $\hat{R}$: R-charge operator
- $\hat{j}_3$: 3rd comp of orbital angular operator
- $\hat{f}_i$: i th Flavor-charge operator

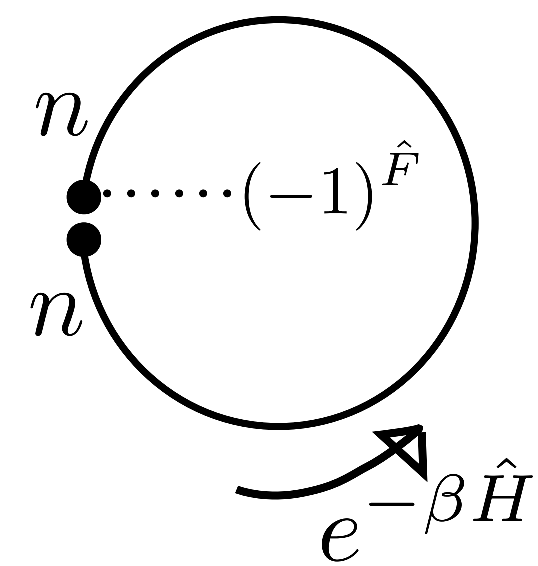
※↑ counts # of BPS state of the theory weighted by symmetries' fugacities

● Superconformal index on $\mathbb{M}^2 \times S^1$

Definition

$$\mathrm{Tr}_{\mathcal{H}} \quad (-1)^{\hat{F}}$$

$$\sim \sum_n \langle n|(-1)^{\hat{F}} e^{-\beta \hat{H}} |n\rangle$$

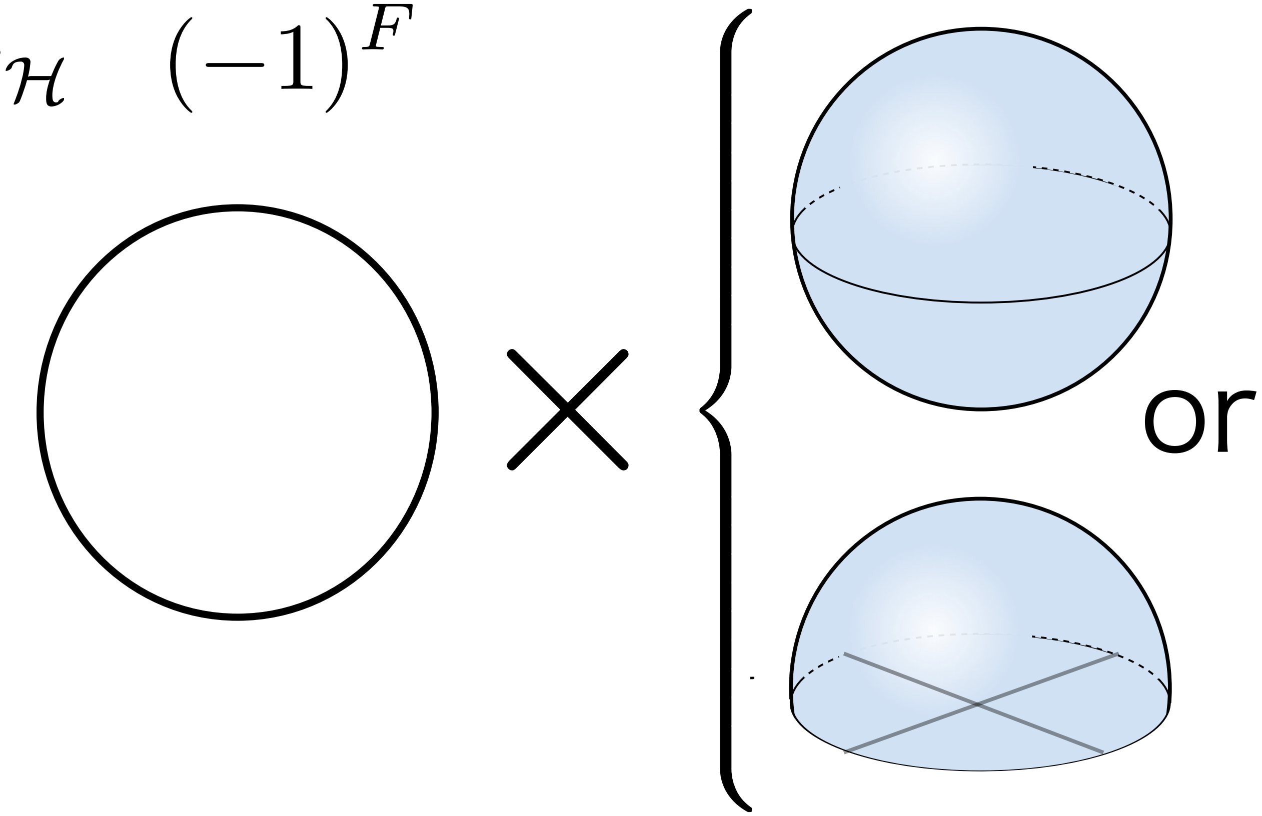


● Superconformal index on $\mathbb{M}^2 \times S^1$

Definition

$$\mathrm{Tr}_{\mathcal{H}} \quad (-1)^{\hat{F}}$$

$\sim$

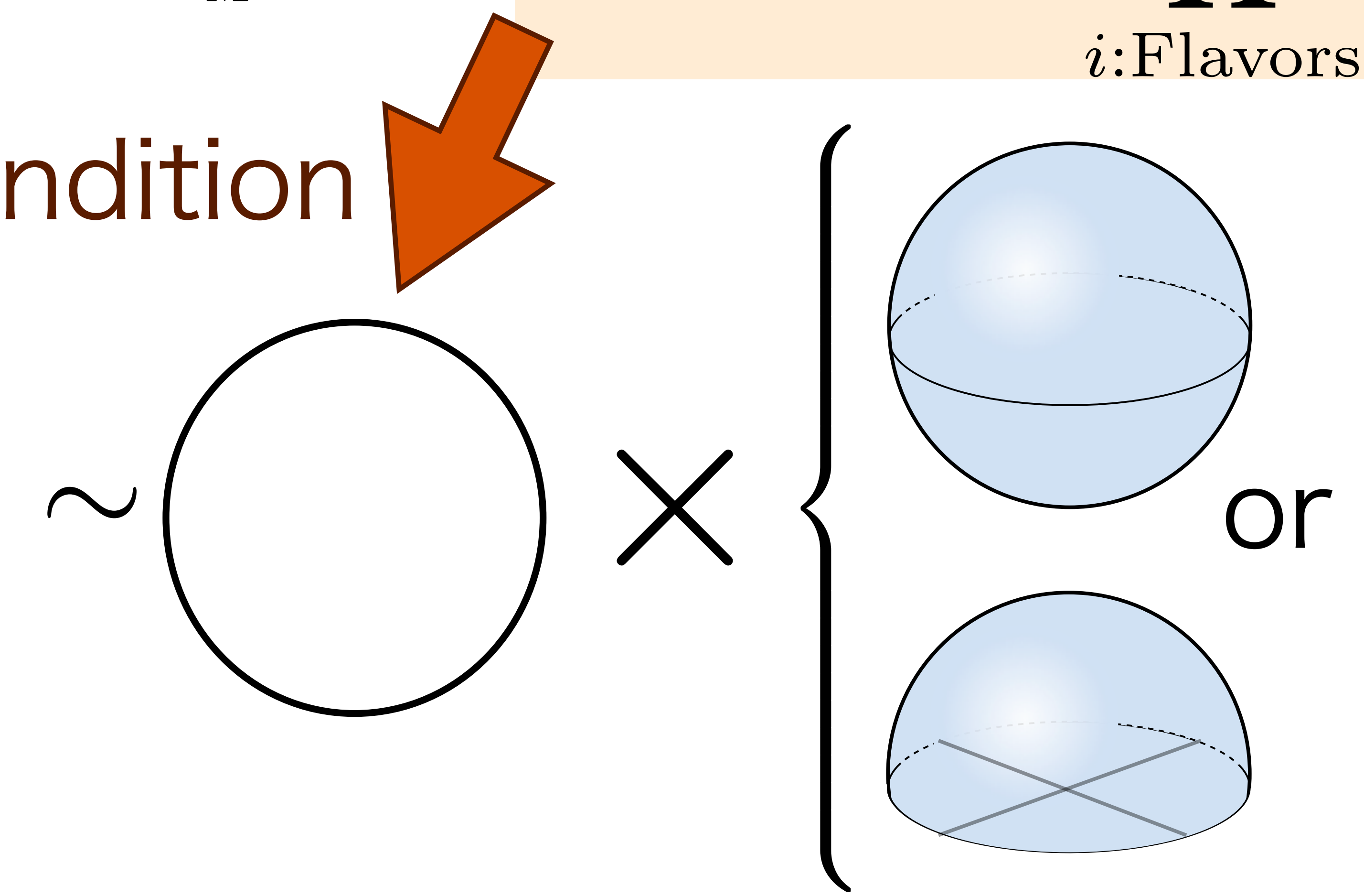


● Superconformal index on $\mathbb{M}^2 \times S^1$

Definition

$$\mathrm{Tr}_{\mathcal{H}_{\mathbb{M}^2}} (-1)^{\hat{F}} x^{-\hat{j}_3} y^{-(\hat{R}-\hat{j}_3)} \prod_{i:\mathrm{Flavors}} \alpha_i^{\hat{f}_i}$$

Boundary condition

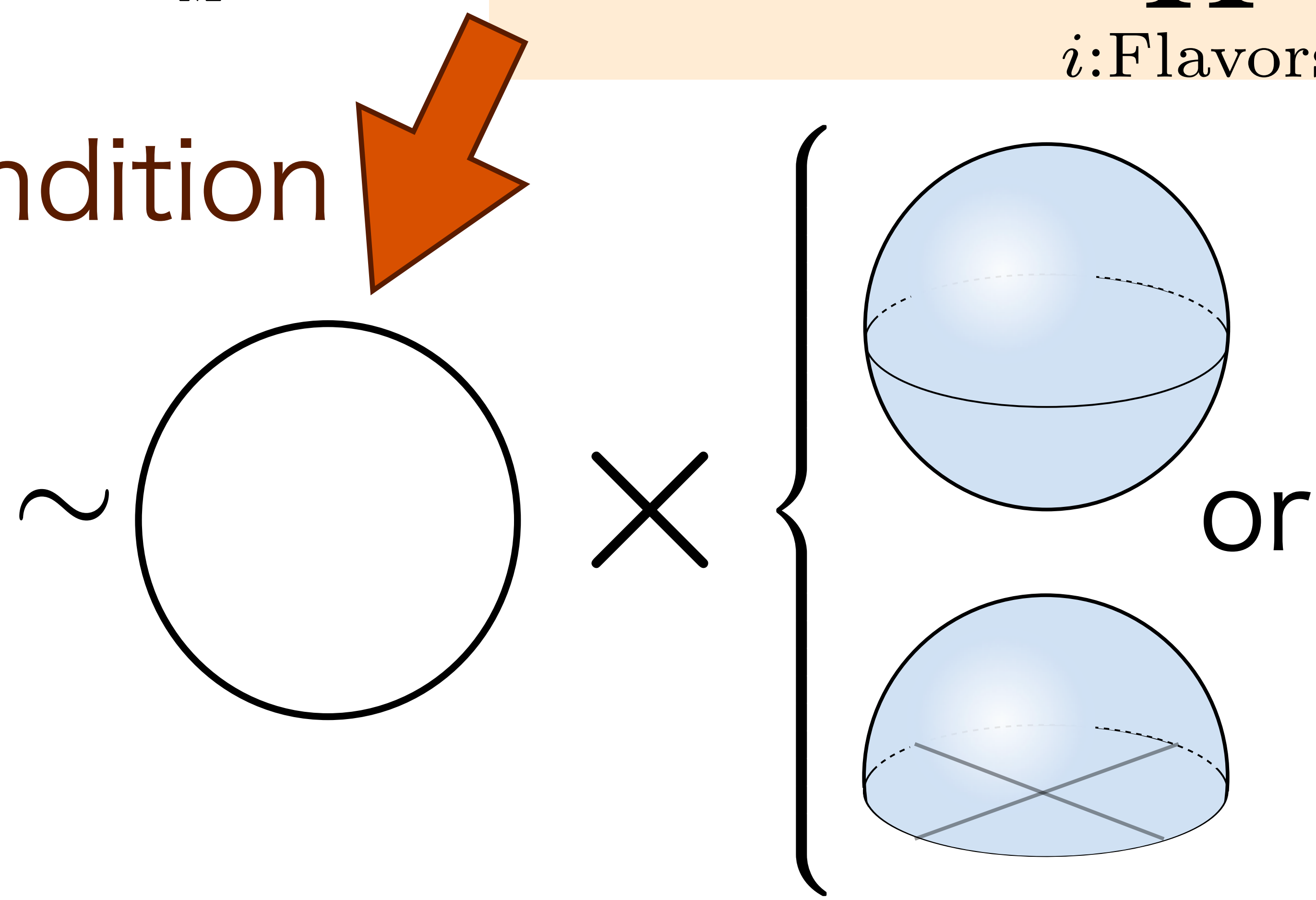


● Superconformal index on $\mathbb{M}^2 \times S^1$

Definition

$$\mathcal{I}^{\mathbb{M}^2}_{\text{Theory}}(x, y, \alpha_i) := \text{Tr}_{\mathcal{H}_{\mathbb{M}^2}} (-1)^{\hat{F}} x^{-\hat{j}_3} y^{-(\hat{R}-\hat{j}_3)} \prod_{i:\text{Flavors}} \alpha_i^{\hat{f}_i}$$

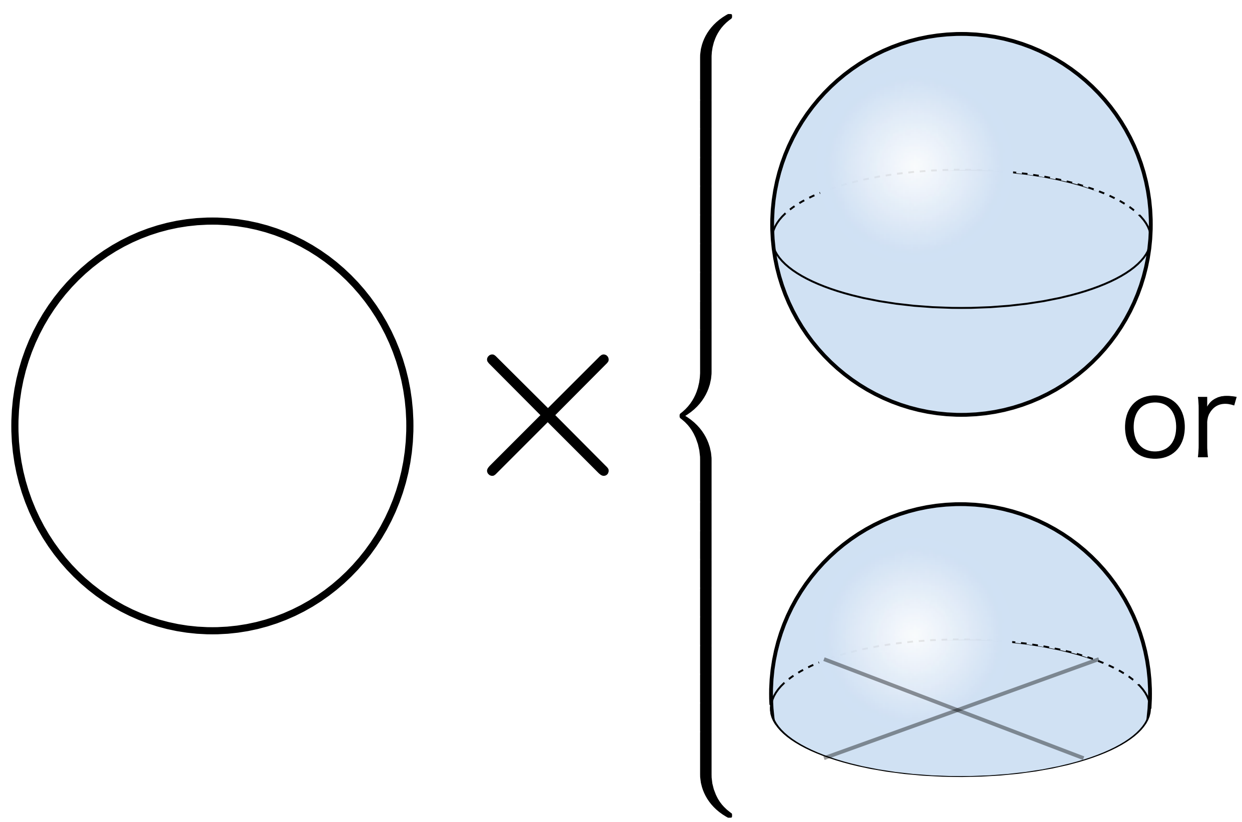
Boundary condition



$\mathcal{I}^{\mathbb{M}^2}_{\text{Theory}}(x, y, \alpha_i)$ is just a Euclidean path integral on $\mathbb{M}^2 \times S^1$ with boundary condition along S^1.

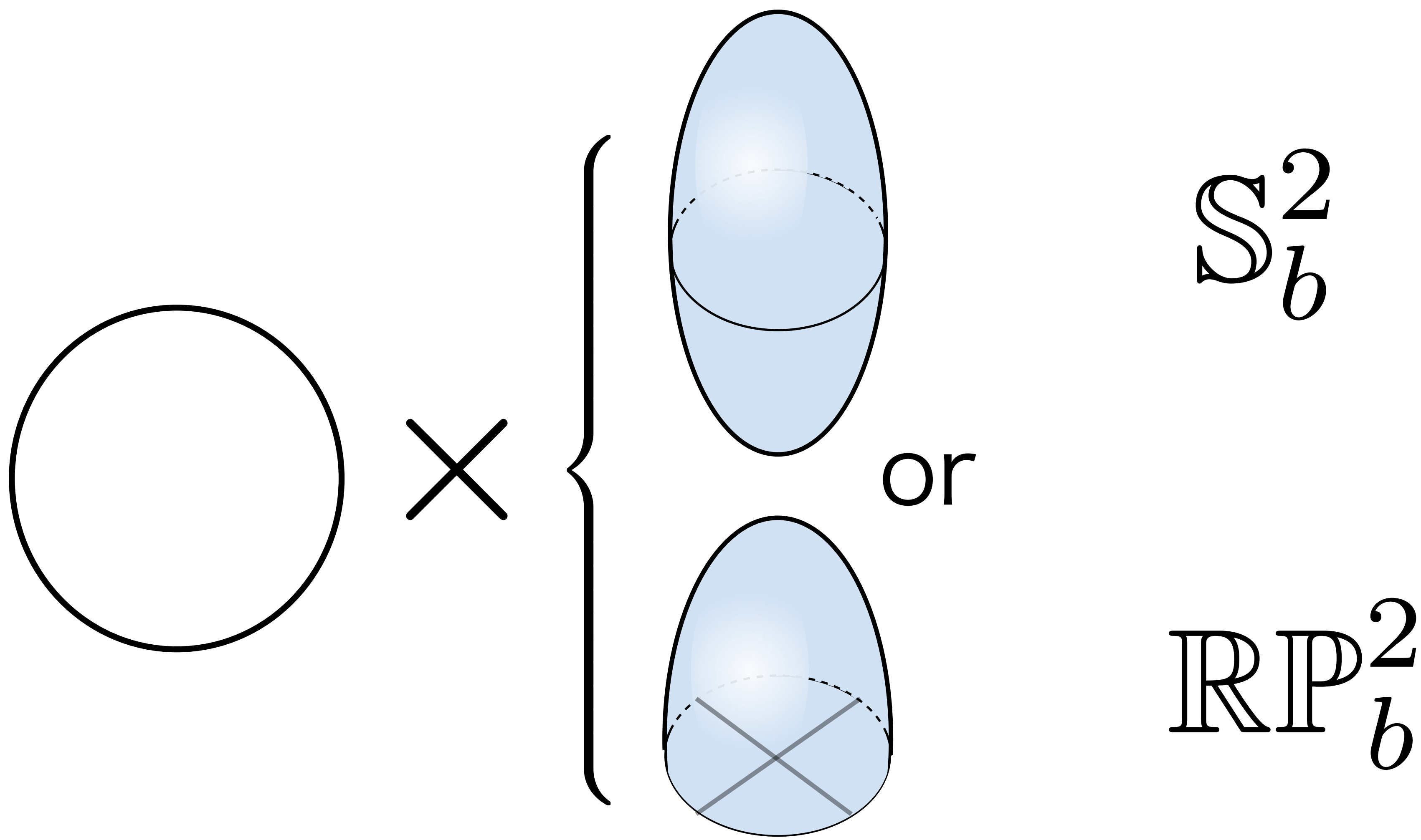
● Superconformal index on $\mathbb{M}^2 \times S^1$

Generalization



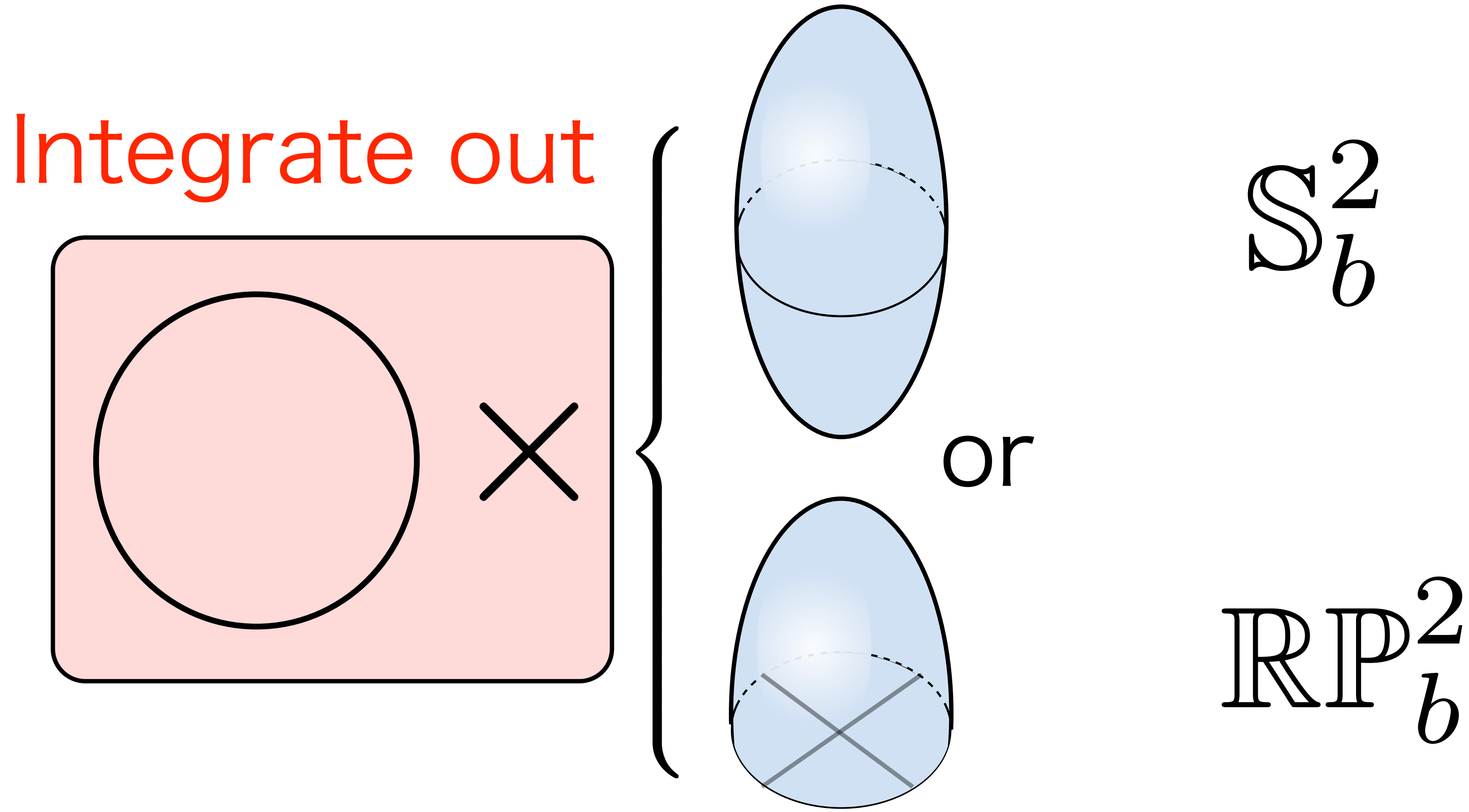
● Superconformal index on $\mathbb{M}^2 \times S^1$

Generalization



● Superconformal index on $\mathbb{M}^2 \times S^1$

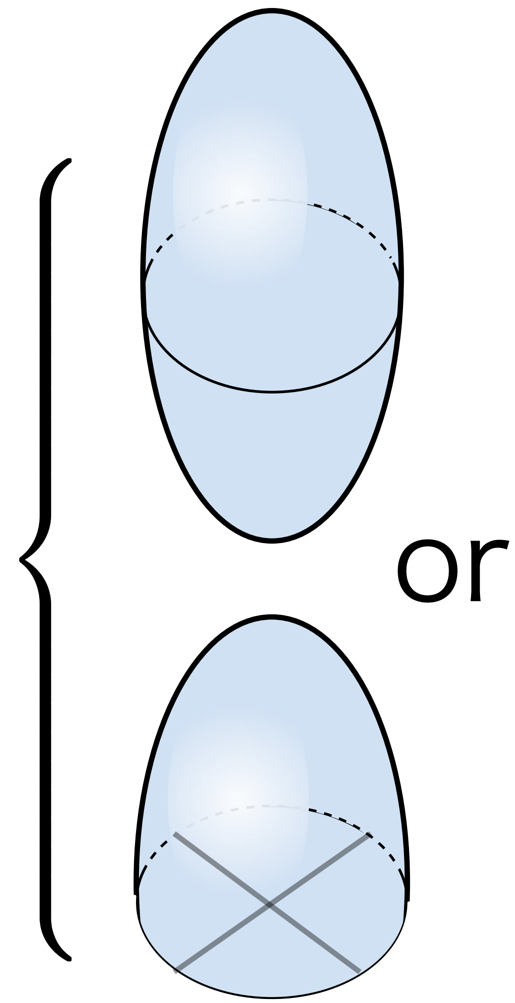
Idea (short-cut)



- Superconformal index on $\mathbb{M}^2 \times S^1$

Idea (short-cut)

$\sum$ KK fields on



$\mathbb{S}_b^2$

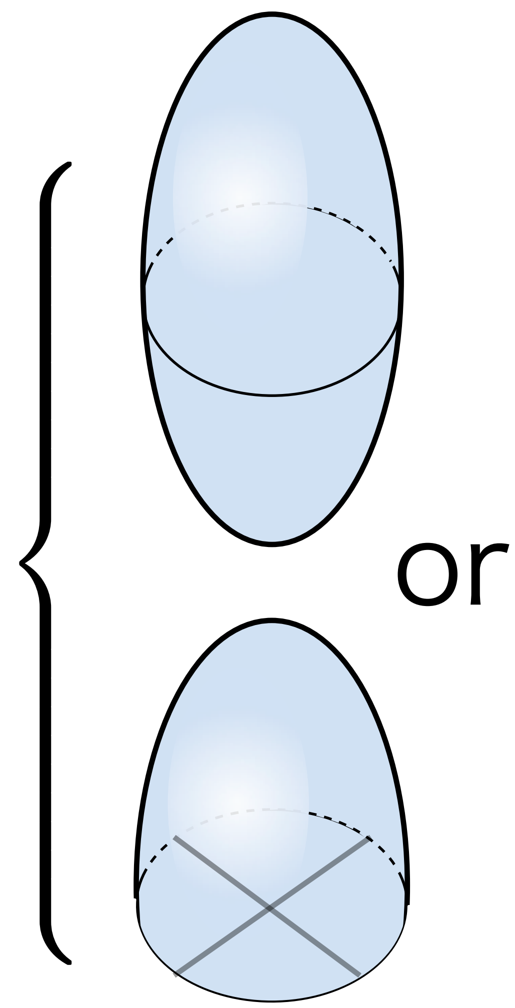
$\mathbb{RP}_b^2$

⇒ We can read off SCI from 2d partition functions $Z[\mathbb{M}^2]$.

● Superconformal index on $\mathbb{M}^2 \times S^1$

Idea (short-cut)

$\sum$ KK fields on



$\mathbb{S}_b^2$

$\mathbb{RP}_b^2$

Known fact [Gomis, Lee]

$$Z[\mathbb{S}_b^2] = Z[\mathbb{S}_{b=1}^2]$$

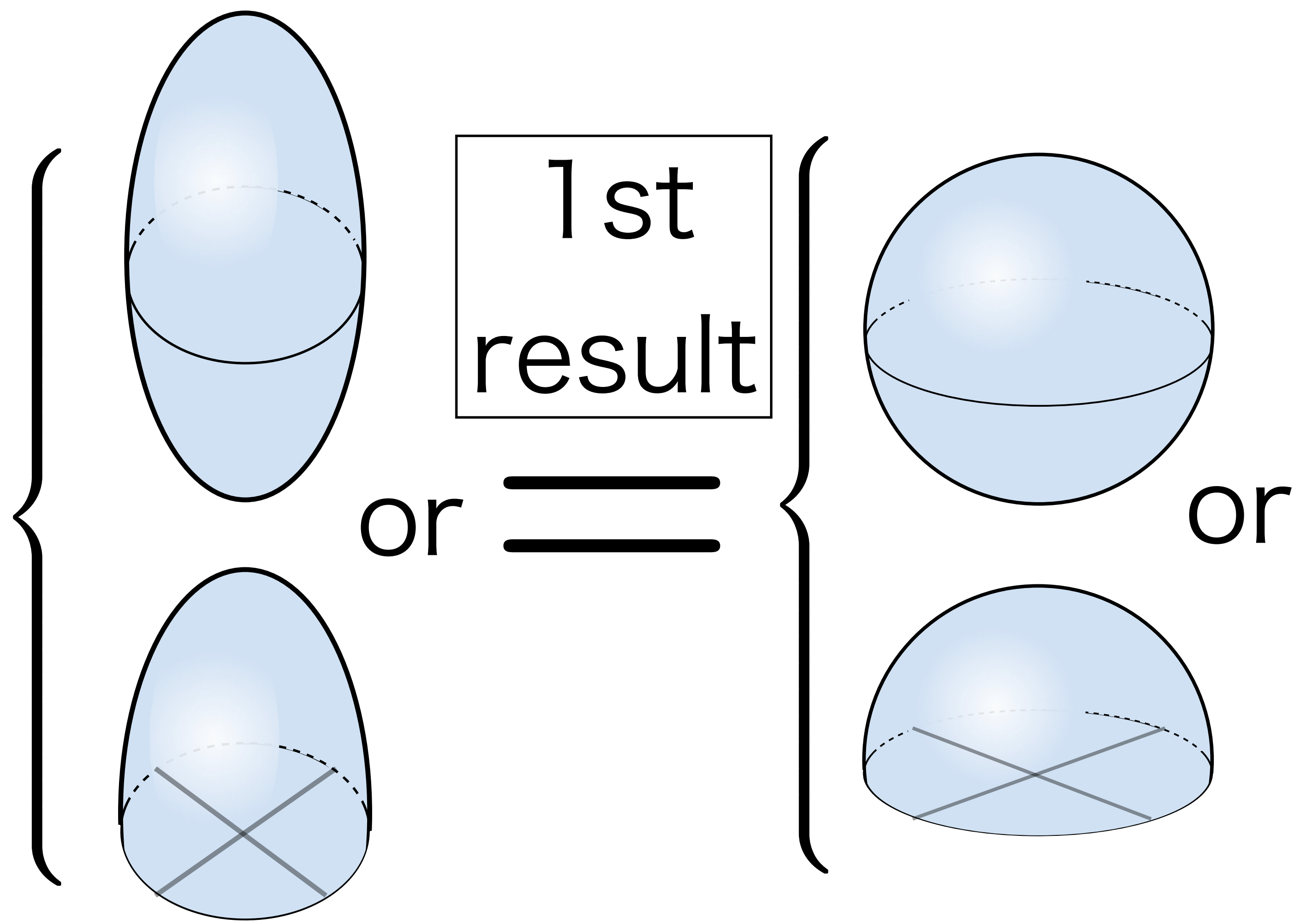
Known fact [Kim, Lee, Yi]

$$Z[\mathbb{RP}_b^2] = Z[\mathbb{RP}_{b=1}^2]$$

● Superconformal index on $\mathbb{M}^2 \times S^1$

Idea (short-cut)

$\sum$ KK fields on



Known fact [Gomis, Lee]

$$Z[\mathbb{S}^2_b] = Z[\mathbb{S}^2_{b=1}]$$

Known fact [Kim, Lee, Yi]

$$Z[\mathbb{RP}^2_b] = Z[\mathbb{RP}^2_{b=1}]$$

Plan

- ~~Superconformal index on $\mathbb{M}^2 \times S^1$~~ √

- $\mathbb{M}^2 = \mathbb{S}_b^2$ & mirror symmetry

- $\mathbb{M}^2 = \mathbb{RP}_b^2$ & mirror symmetry

● $\mathbb{M}^2 = \mathbb{S}_b^2$ & mirror symmetry ⚠REVIEW

We can check our method's validity.

⇒ It reproduces known SCI.

$\boxed{\text{SQED}} = \boxed{\text{XYZ-model}}$ is checked already.

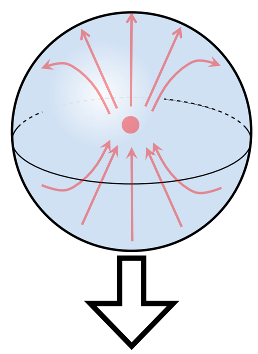
[Krattenthaler, Spiridonov, Vartanov]

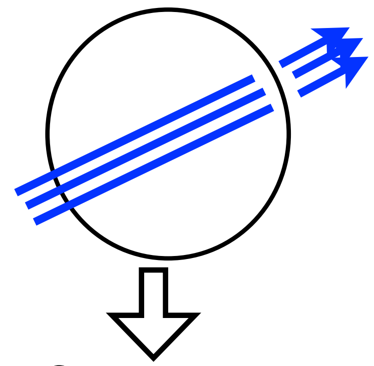
● $\mathbb{M}^2 = \mathbb{S}_b^2$ & mirror symmetry ⚠REVIEW

Contribution rules

$$(z;q)_\infty = \prod_{k=0}^{\infty}(1-zq^k)$$

SQED





$$\mathcal{L}_{vec} \sim \sum_{s:\text{monopoles}} \int_0^{2\pi} \frac{d\theta}{2\pi} \; ①$$

trivial

$$\mathcal{L}_Q \sim (x^{\frac{1}{2}} e^{+i\theta})^{|s|} \frac{(e^{+i\theta} x^{2|s|+1+\Delta}; x^2)_\infty}{(e^{-i\theta} x^{2|s|+1-\Delta}; x^2)_\infty}$$

$$\mathcal{L}_{\tilde{Q}} \sim (x^{\frac{1}{2}} e^{-i\theta})^{|s|} \frac{(e^{-i\theta} x^{2|s|+1+\Delta}; x^2)_\infty}{(e^{+i\theta} x^{2|s|+1-\Delta}; x^2)_\infty}$$

non-trivial

XYZ-model

$$\mathcal{L}_X \sim \frac{(x^{2-\Delta}; x^2)_\infty}{(x^{\Delta}; x^2)_\infty}$$

$$\mathcal{L}_Y \sim \frac{(x^{2-\Delta}; x^2)_\infty}{(x^{\Delta}; x^2)_\infty}$$

$$\mathcal{L}_Z \sim \frac{(x^{1+2\Delta}; x^2)_\infty}{(x^{1-2\Delta}; x^2)_\infty}$$

non-trivial

$$\left(\int d^2\theta \; XYZ + c.c\right) \sim ①$$

trivial

● $\mathbb{M}^2 = \mathbb{S}_b^2$ & mirror symmetry ⚠REVIEW

Contribution rules

$$(z;q)_\infty = \prod_{k=0}^{\infty}(1 - zq^k)$$

SQED

XYZ-model

$$\sum_{s:\text{monopoles}} \int_0^{2\pi} d\theta x^{|s|} \frac{(e^{-i\theta}x^{2|s|+1+\Delta};x^2)_\infty}{(e^{+i\theta}x^{2|s|+1-\Delta};x^2)_\infty} \frac{(e^{+i\theta}x^{2|s|+1+\Delta};x^2)_\infty}{(e^{-i\theta}x^{2|s|+1-\Delta};x^2)_\infty} \overset{?}{=} \frac{(x^{2-\Delta};x^2)_\infty^2}{(x^{\Delta};x^2)_\infty^2} \frac{(x^{1+2\Delta};x^2)_\infty}{(x^{1-2\Delta};x^2)_\infty}$$

● $\mathbb{M}^2 = \mathbb{S}_b^2$ & mirror symmetry ⚠REVIEW

Numerical check

SQED

$$\text{In[21]:= Series}\left[\text{Sum}\left[x^{\text{Abs}[k]/2}\,\frac{(\text{QPochhammer}[x^{2(\text{Abs}[k]+j+1)},\,x^2])}{(\text{QPochhammer}[x^{-1+2(\text{Abs}[k]+j+1)},\,x^2])}\,\frac{(\text{QPochhammer}[x^{1-2j},\,x^2])}{(\text{QPochhammer}[x^2,\,x^2])}\,\frac{x^{j(j+1)}\,(-1)^j}{(\text{QPochhammer}[x^2,\,x^2,\,j])},\,\{j,0,10,1\},\,\{k,-10,10,1\}\right],\,\{x,0,5\}\right]$$

$$\text{Out[21]= } 1+2\sqrt{x}+3x+2x^{3/2}+x^2+2x^{5/2}+4x^3+4x^{7/2}+2x^{9/2}+2x^5+O[x]^{11/2}$$

Numerical error :)

XYZ-model

$$\text{In[22]:= Series}\left[\frac{(\text{QPochhammer}[x^{3/2},\,x^2])^2}{(\text{QPochhammer}[x^{1/2},\,x^2])^2},\,\{x,0,5\}\right]$$

$$\text{Out[22]= } 1+2\sqrt{x}+3x+2x^{3/2}+x^2+2x^{5/2}+4x^3+4x^{7/2}+2x^{9/2}+3x^5+O[x]^{11/2}$$

● $\mathbb{M}^2 = \mathbb{S}_b^2$ & mirror symmetry ⚠REVIEW

Contribution rules

$$(z;q)_\infty = \prod_{k=0}^{\infty}(1-zq^k)$$

SQED | XYZ-model

$$\sum_{s:\text{monopoles}} \underbrace{\int_0^{2\pi} d\theta x^{|s|} \frac{(e^{-i\theta}x^{2|s|+1+\Delta};x^2)_\infty}{(e^{+i\theta}x^{2|s|+1-\Delta};x^2)_\infty} \frac{(e^{+i\theta}x^{2|s|+1+\Delta};x^2)_\infty}{(e^{-i\theta}x^{2|s|+1-\Delta};x^2)_\infty}}_{\text{Picking up residue}} = \frac{(x^{2-\Delta};x^2)_\infty^2}{(x^\Delta;x^2)_\infty^2} \frac{(x^{1+2\Delta};x^2)_\infty}{(x^{1-2\Delta};x^2)_\infty}$$

Picking up residue

In fact

⇒Ramanujan's sum formula

\+ q-binomial formula

[Krattenthaler, Spiridonov, Vartanov]

Plan

- Superconformal index on $\mathbb{M}^2 \times S^1$
- $\mathbb{M}^2 = \mathbb{S}_b^2$ & mirror symmetry ⚠REVIEW
- $\mathbb{M}^2 = \mathbb{RP}_b^2$ & mirror symmetry

● $\mathbb{M}^2 = \mathbb{RP}^2_b$ & mirror symmetry

We know our method's validity.

⇒ This case produces

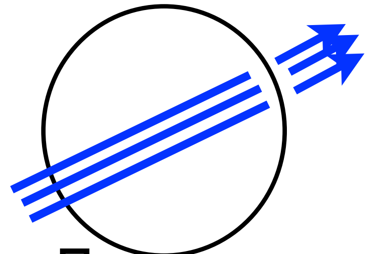
completely new result !

$\boxed{\mathrm{SQED}} = \boxed{\mathrm{XYZ\text{-}model}}$ is, of course, nontrivial.

● $\mathbb{M}^2 = \mathbb{RP}^2_b$ & mirror symmetry

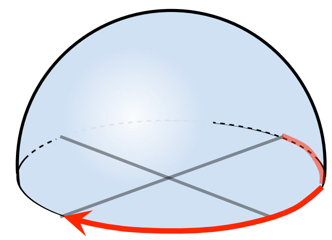
Our 1st attempt

$$(z;q)_\infty = \prod_{k=0}^{\infty}(1 - zq^k)$$



non-trivial !

SQED



$$\mathcal{L}_{vec} \sim \begin{cases} \int_0^{2\pi} \frac{d\theta_+}{2\pi} x^{+\frac{1}{4}} \frac{(x^4;x^4)_\infty}{(x^2;x^4)_\infty} \\ + \\ \int_0^{2\pi} \frac{d\theta_-}{2\pi} x^{+\frac{1}{4}} \frac{(x^4;x^4)_\infty}{(x^2;x^4)_\infty} \end{cases}$$

$$\mathcal{L}_Q \sim \begin{cases} (x^{\Delta-1}e^{+i\theta_+})^{+\frac{1}{4}} \frac{(e^{+i\theta_+}x^{(2-\Delta)};x^4)_\infty}{(e^{-i\theta_+}x^{\Delta};x^4)_\infty} \\ (x^{\Delta-1}e^{-i\theta_-})^{-\frac{1}{4}} \frac{(e^{+i\theta_-}x^{(4-\Delta)};x^4)_\infty}{(e^{-i\theta_-}x^{(2+\Delta)};x^4)_\infty} \end{cases}$$

$$\mathcal{L}_{\tilde{Q}} \sim \begin{cases} (x^{\Delta-1}e^{-i\theta_+})^{+\frac{1}{4}} \frac{(e^{-i\theta_+}x^{(2-\Delta)};x^4)_\infty}{(e^{+i\theta_+}x^{\Delta};x^4)_\infty} \\ (x^{\Delta-1}e^{+i\theta_-})^{-\frac{1}{4}} \frac{(e^{-i\theta_-}x^{(4-\Delta)};x^4)_\infty}{(e^{+i\theta_-}x^{(2+\Delta)};x^4)_\infty} \end{cases}$$

non-trivial

XYZ-model

$$\mathcal{L}_X \sim (x^{2-\Delta})^{+\frac{1}{4}} \frac{(x^{1+\Delta};x^4)_\infty}{(x^{1-\Delta};x^4)_\infty}$$

$$\mathcal{L}_Y \sim (x^{2-\Delta})^{+\frac{1}{4}} \frac{(x^{1+\Delta};x^4)_\infty}{(x^{1-\Delta};x^4)_\infty}$$

$$\mathcal{L}_Z \sim (x^{2\Delta-1})^{+\frac{1}{4}} \frac{(x^{2-2\Delta};x^4)_\infty}{(x^{2\Delta};x^4)_\infty}$$

non-trivial

$$\left(\int d^2\theta\, XYZ + c.c\right) \sim ①$$

trivial

● $\mathbb{M}^2 = \mathbb{RP}^2_b$ & mirror symmetry

Our 1st attempt

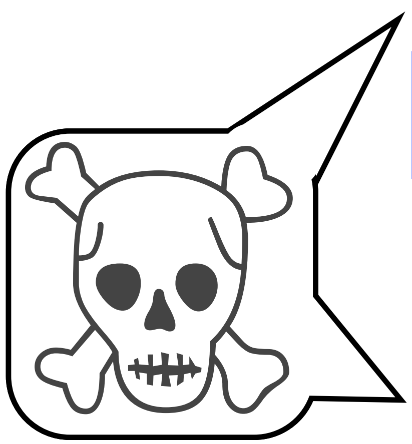
$$(z;q)_\infty = \prod_{k=0}^{\infty}(1 - zq^k)$$

SQED

$$\text{Simplify}\Big[\text{Series}\Big[x^{1/4}\frac{\text{QPochhammer}[x^4, x^4]}{\text{QPochhammer}[x^2, x^4]}\Big(x^{\frac{1}{2}(1/2-1)}\frac{\text{QPochhammer}[x^{2(1-1/2)}, x^4]\,\text{QPochhammer}[x^2, x^4]}{\text{QPochhammer}[x^{2\times 1/2}, x^4]\,\text{QPochhammer}[x^4, x^4]}\text{QHypergeometricPFQ}[\{x^{2(1/2+1)}, x^{2\times 1/2}\}, \{x^2\}, x^4,$$
$$x^{\frac{-1}{2}(1/2-1)}\frac{\text{QPochhammer}[x^{2(1-1/2)}, x^4]\,\text{QPochhammer}[x^6, x^4]}{\text{QPochhammer}[x^{2(2+1/2)}, x^4]\,\text{QPochhammer}[x^4, x^4]}$$
$$\text{QHypergeometricPFQ}[\{x^{2(1/2+1)}, x^{2(2+1/2)}\}, \{x^6\}, x^4, x^{2(1-1/2)}]\Big), \{x, 0, 10\}\Big]\Big]$$

Some missings?

$$1 + \sqrt{x} + x + x^{5/2} + x^3 - x^4 + 2x^5 + x^{11/2} - x^6 - x^{13/2} + x^7 + 2x^{15/2} - x^8 - 2x^{17/2} + x^9 + 3x^{19/2} + x^{10} + O[x]^{41/4}$$



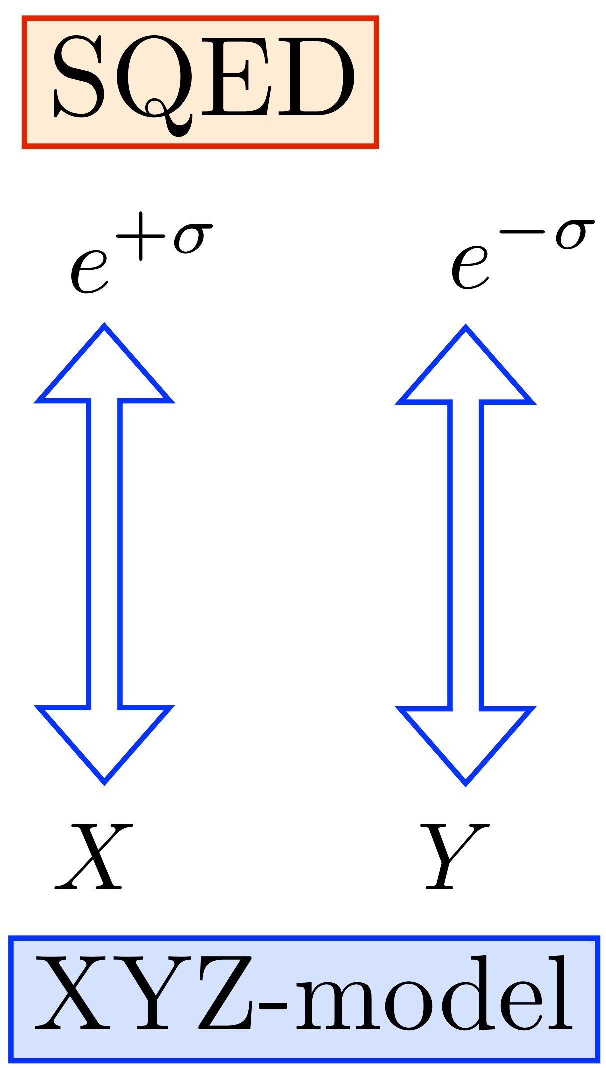
XYZ-model

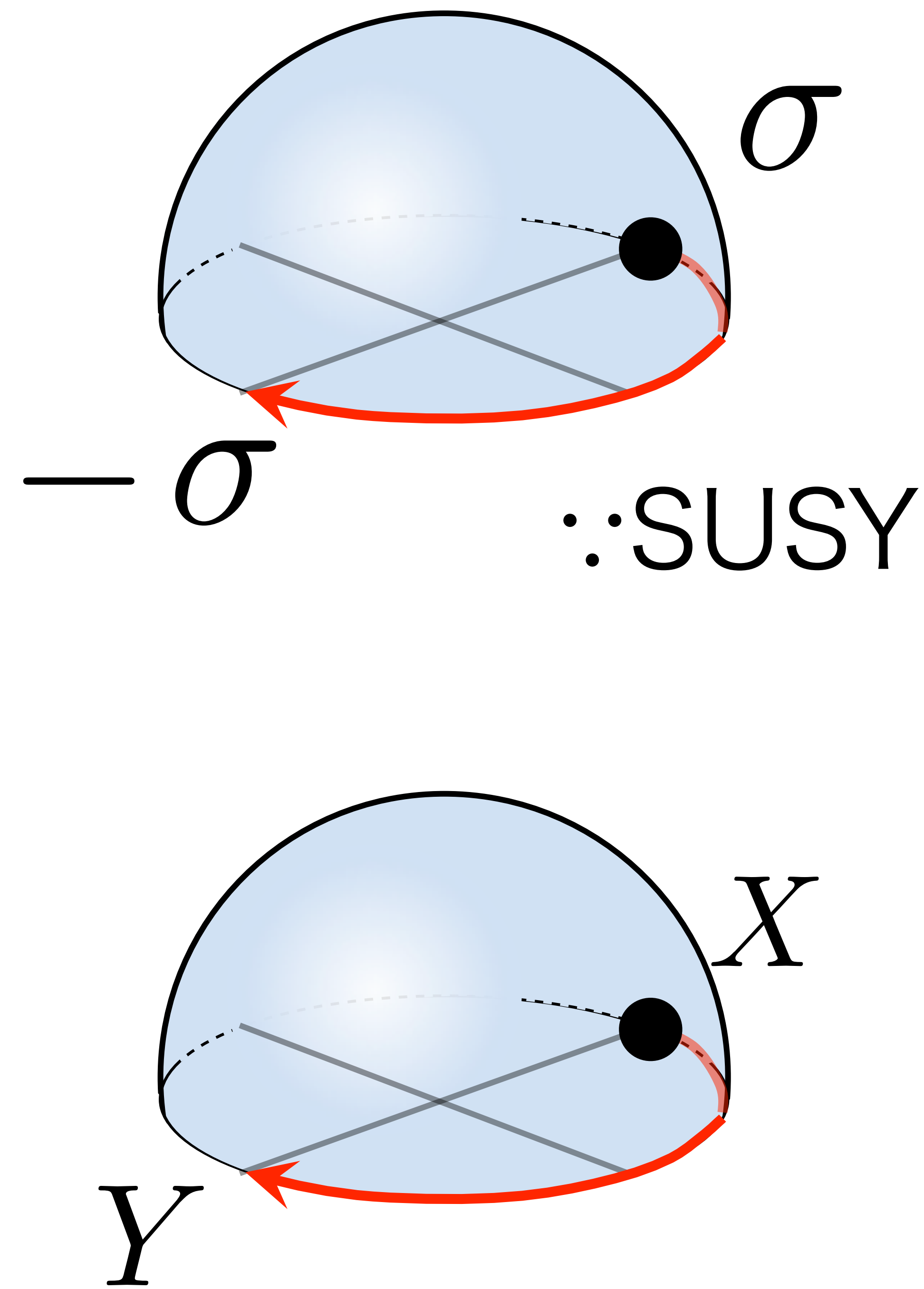
$$\text{Simplify}\Big[\text{Series}\Big[\Big(x^{\frac{2-1/2}{4}}\frac{\text{QPochhammer}[x^{1+1/2}, x^4]}{\text{QPochhammer}[x^{1-1/2}, x^4]}\Big)^2\Big(x^{\frac{2\times 1/2-1}{4}}\frac{\text{QPochhammer}[x^{2-2\times 1/2}, x^4]}{\text{QPochhammer}[x^{2\times 1/2}, x^4]}\Big), \{x, 0, 10\}\Big]\Big]$$

$$x^{3/4} + 2x^{5/4} + 3x^{7/4} + 2x^{9/4} + x^{11/4} + 2x^{21/4} + 4x^{23/4} + 4x^{25/4} - 4x^{29/4} - 4x^{31/4} - 2x^{33/4} + 2x^{37/4} + 7x^{39/4} + 10x^{41/4} + O[x]^{43/4}$$

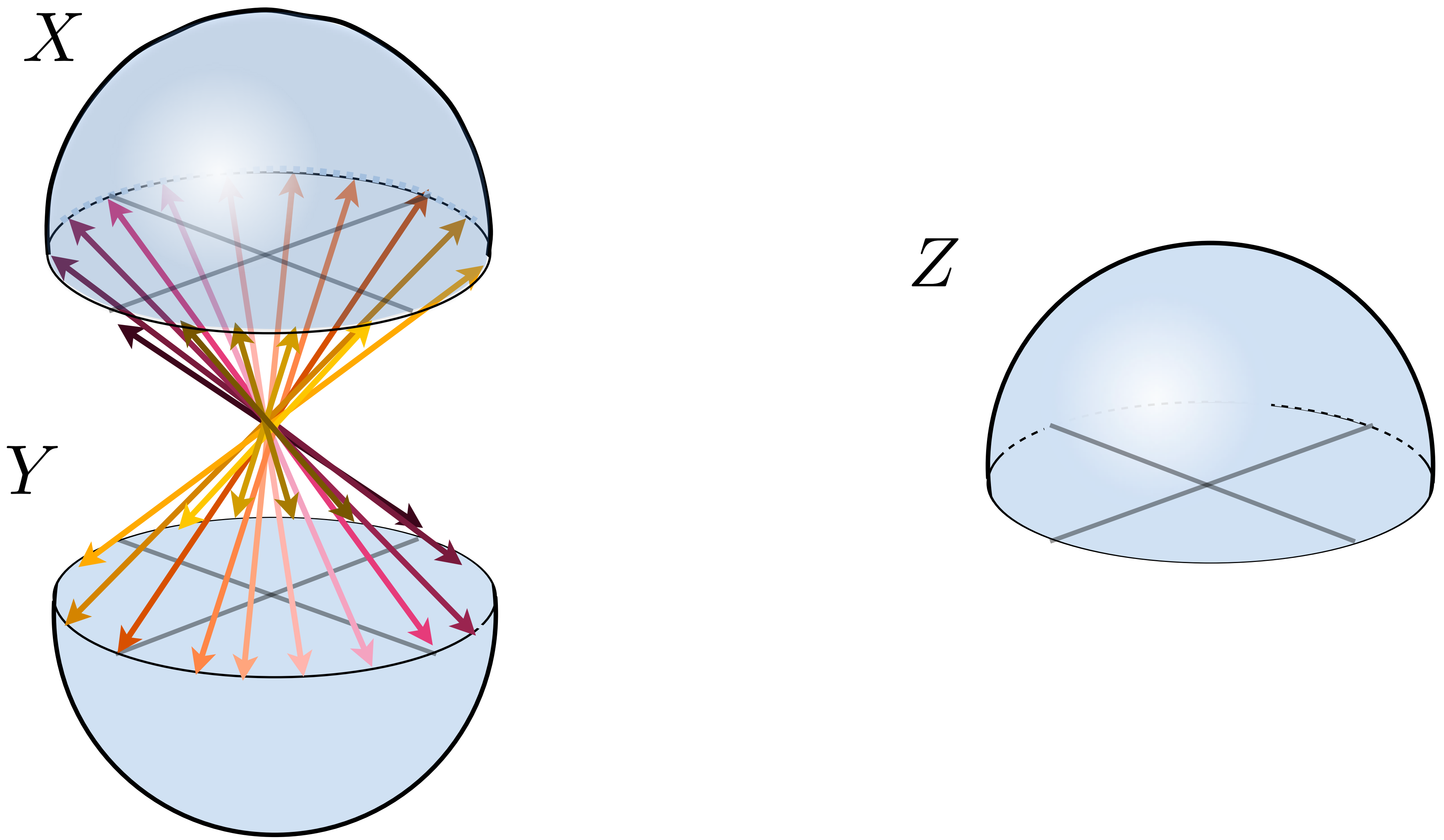
● $\mathbb{M}^2 = \mathbb{RP}^2_b$ & mirror symmetry

Now, σ has a profound effect.

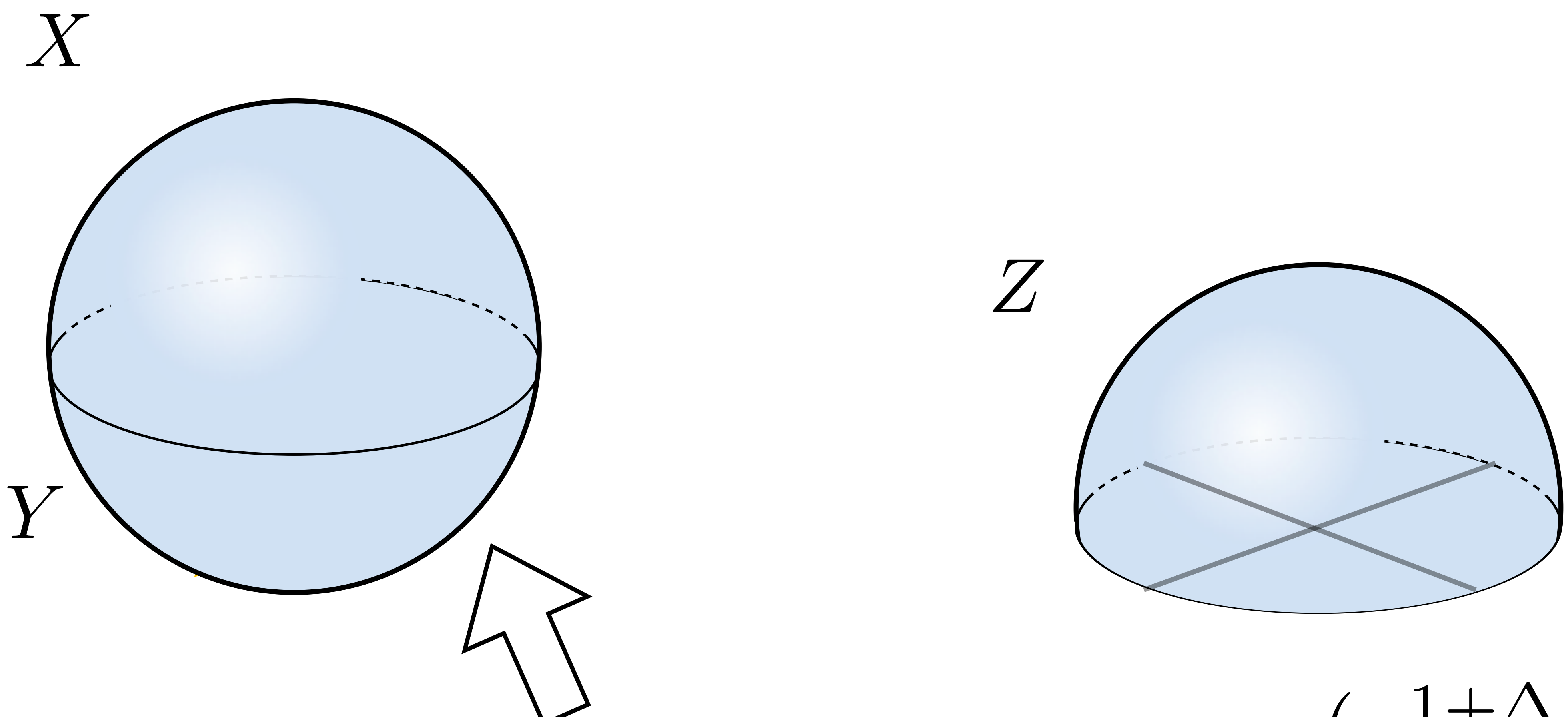




● $\mathbb{M}^2 = \mathbb{RP}^2_b$ & mirror symmetry



● $\mathbb{M}^2 = \mathbb{RP}^2_b$ & mirror symmetry

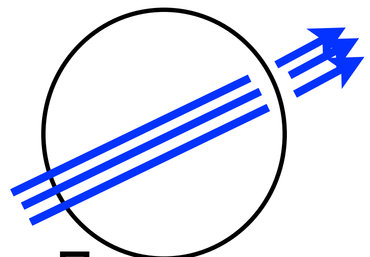


We should use $\mathbb{S}^2$ contribution $\dfrac{(x^{1+\Delta}; x^2)_\infty}{(x^{1-\Delta}; x^2)_\infty}$!

● $\mathbb{M}^2 = \mathbb{RP}^2_b$ & mirror symmetry

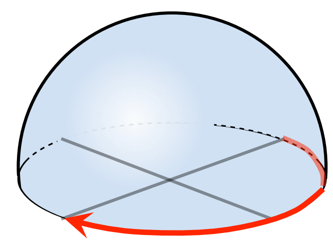
Our 1st attempt

$$(z;q)_\infty = \prod_{k=0}^{\infty}(1 - zq^k)$$



SQED

$$\mathcal{L}_{vec} \sim \begin{cases} \int_0^{2\pi} \frac{d\theta_+}{2\pi} x^{+\frac{1}{4}} \frac{(x^4;x^4)_\infty}{(x^2;x^4)_\infty} \\ + \\ \int_0^{2\pi} \frac{d\theta_-}{2\pi} x^{+\frac{1}{4}} \frac{(x^4;x^4)_\infty}{(x^2;x^4)_\infty} \end{cases}$$



$$\mathcal{L}_Q \sim \begin{cases} (x^{\Delta-1}e^{+i\theta_+})^{+\frac{1}{4}} \frac{(e^{+i\theta_+}x^{(2-\Delta)};x^4)_\infty}{(e^{-i\theta_+}x^{\Delta};x^4)_\infty} \\ (x^{\Delta-1}e^{-i\theta_-})^{-\frac{1}{4}} \frac{(e^{+i\theta_-}x^{(4-\Delta)};x^4)_\infty}{(e^{-i\theta_-}x^{(2+\Delta)};x^4)_\infty} \end{cases}$$

$$\mathcal{L}_{\tilde{Q}} \sim \begin{cases} (x^{\Delta-1}e^{-i\theta_+})^{+\frac{1}{4}} \frac{(e^{-i\theta_+}x^{(2-\Delta)};x^4)_\infty}{(e^{+i\theta_+}x^{\Delta};x^4)_\infty} \\ (x^{\Delta-1}e^{+i\theta_-})^{-\frac{1}{4}} \frac{(e^{-i\theta_-}x^{(4-\Delta)};x^4)_\infty}{(e^{+i\theta_-}x^{(2+\Delta)};x^4)_\infty} \end{cases}$$

XYZ-model

$$\mathcal{L}_X \sim (x^{2-\Delta})^{+\frac{1}{4}} \frac{(x^{1+\Delta};x^4)_\infty}{(x^{1-\Delta};x^4)_\infty}$$

$$\mathcal{L}_Y \sim (x^{2-\Delta})^{+\frac{1}{4}} \frac{(x^{1+\Delta};x^4)_\infty}{(x^{1-\Delta};x^4)_\infty}$$

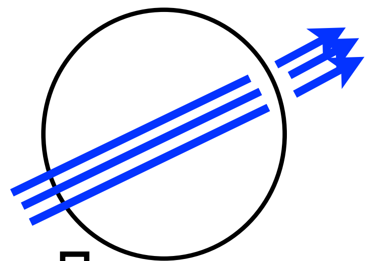
$$\mathcal{L}_Z \sim (x^{2\Delta-1})^{+\frac{1}{4}} \frac{(x^{2-2\Delta};x^4)_\infty}{(x^{2\Delta};x^4)_\infty}$$

$$\left(\int d^2\theta\, XYZ + c.c\right) \sim ①$$

● $\mathbb{M}^2 = \mathbb{RP}^2_b$ & mirror symmetry

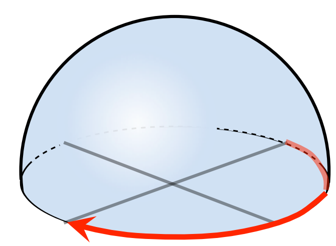
Precise contributions

$$(z;q)_\infty = \prod_{k=0}^{\infty}(1 - zq^k)$$



SQED

$$\mathcal{L}_{vec} \sim \begin{cases} \int_0^{2\pi} \frac{d\theta_+}{2\pi} x^{+\frac{1}{4}} \frac{(x^4;x^4)_\infty}{(x^2;x^4)_\infty} \\ + \\ \int_0^{2\pi} \frac{d\theta_-}{2\pi} x^{+\frac{1}{4}} \frac{(x^4;x^4)_\infty}{(x^2;x^4)_\infty} \end{cases}$$



$$\mathcal{L}_Q \sim \begin{cases} (x^{\Delta-1}e^{+i\theta_+})^{+\frac{1}{4}} \frac{(e^{+i\theta_+}x^{(2-\Delta)};x^4)_\infty}{(e^{-i\theta_+}x^{\Delta};x^4)_\infty} \\ (x^{\Delta-1}e^{-i\theta_-})^{-\frac{1}{4}} \frac{(e^{+i\theta_-}x^{(4-\Delta)};x^4)_\infty}{(e^{-i\theta_-}x^{(2+\Delta)};x^4)_\infty} \end{cases}$$

$$\mathcal{L}_{\tilde{Q}} \sim \begin{cases} (x^{\Delta-1}e^{-i\theta_+})^{+\frac{1}{4}} \frac{(e^{-i\theta_+}x^{(2-\Delta)};x^4)_\infty}{(e^{+i\theta_+}x^{\Delta};x^4)_\infty} \\ (x^{\Delta-1}e^{+i\theta_-})^{-\frac{1}{4}} \frac{(e^{-i\theta_-}x^{(4-\Delta)};x^4)_\infty}{(e^{+i\theta_-}x^{(2+\Delta)};x^4)_\infty} \end{cases}$$

XYZ-model

$$\left.\begin{array}{l}\mathcal{L}_X \\ \mathcal{L}_Y\end{array}\right\} \sim \frac{(x^{1+\Delta};x^2)_\infty}{(x^{1-\Delta};x^2)_\infty}$$

$$\mathcal{L}_Z \sim (x^{2\Delta-1})^{+\frac{1}{4}} \frac{(x^{2-2\Delta};x^4)_\infty}{(x^{2\Delta};x^4)_\infty}$$

$$\left(\int d^2\theta\, XYZ + c.c\right) \sim ①$$

● $\mathbb{M}^2 = \mathbb{RP}^2_b$ & mirror symmetry

SQED

$$\text{Simplify}\Big[\text{Series}\Big[x^{1/4}\frac{\text{QPochhammer}[x^4, x^4]}{\text{QPochhammer}[x^2, x^4]}$$

$$\Big(x^{\frac{1}{2}(1/2-1)}\frac{\text{QPochhammer}[x^{2(1-1/2)}, x^4]\,\text{QPochhammer}[x^2, x^4]}{\text{QPochhammer}[x^{2\times 1/2}, x^4]\,\text{QPochhammer}[x^4, x^4]}\text{QHypergeometricPFQ}[\{x^{2(1/2+1)}, x^2$$

$$x^{\frac{-1}{2}(1/2-1)}\frac{\text{QPochhammer}[x^{2(1-1/2)}, x^4]\,\text{QPochhammer}[x^6, x^4]}{\text{QPochhammer}[x^{2(2+1/2)}, x^4]\,\text{QPochhammer}[x^4, x^4]}$$

$$\text{QHypergeometricPFQ}[\{x^{2(1/2+1)}, x^{2(2+1/2)}\}, \{x^6\}, x^4, x^{2(1-1/2)}]\Big), \{x, 0, 10\}\Big]\Big]$$

$$1+\sqrt{x}+x+x^{5/2}+x^3-x^4+2x^5+x^{11/2}-x^6-x^{13/2}+x^7+2x^{15/2}-x^8-2x^{17/2}+x^9+3x^{19/2}+x^{10}+O[x]^{41/4}$$

XYZ-model

$$\text{Simplify}\Big[\text{Series}\Big[\frac{\text{QPochhammer}[x^{1+1/2}, x^2]}{\text{QPochhammer}[x^{1-1/2}, x^2]}\Big(x^{\frac{2\times 1/2-1}{4}}\frac{\text{QPochhammer}[x^{2-2\times 1/2}, x^4]}{\text{QPochhammer}[x^{2\times 1/2}, x^4]}\Big), \{x, 0, 10\}\Big]\Big]$$

$$1+\sqrt{x}+x+x^{5/2}+x^3-x^4+2x^5+x^{11/2}-x^6-x^{13/2}+x^7+2x^{15/2}-x^8-2x^{17/2}+x^9+3x^{19/2}+x^{10}+O[x]^{21/2}$$

Agree!

Meaning of agreement in mathematics?

● $\mathbb{M}^2 = \mathbb{RP}^2_b$ & mirror symmetry

森田様、

いきなりのメールを失礼致します。
阪大理学研究科素粒子論研究室D1の森と申します。

現在進めている研究で調べ物をしていたところ、森田様の論文に行き着きました。
そこでお伺いしたいことがありメールを送らせていただきます。
我々の場の理論という分野での研究でq-Pochhammer記号とq-hypergeometric functionに関する等式が出てきました。
森田様に見ていただきたいのですが、その式をお送りしてもよろしいでしょうか？

よろしくお願い致します。

森 裕紀

I and Mori : “Is it possible to prove this?”

田中さん
（cc:森さん）

先程送っていただきました、最も一般化した式の証明が出来ました。きちんと成り立ってます！

森田

Morita : “I did !” ⇒ q-binomial formula

- What we have done

● What we have done

- Localization w/ U(1) gauge symmetry

- Application to check of mirror symmetry

● Interesting point

- Localization w/ U(1) gauge symmetry

Monopole ∞ sum→Holonomy Z2 sum = Clean!

- Application to check of mirror symmetry

X & Y → one dof on 2-sphere

● Problems?

・Localization w/ U(1) gauge symmetry

Monopole ∞ sum→Holonomy Z2 sum = Clean!

Foundation of regularization for Casimir energy

・Application to check of mirror symmetry

X & Y → one dof on 2-sphere!

One contribution is neglected in our calculation

● Developments?

- Localization w/ U(1) gauge symmetry

Monopole ∞ sum→Holonomy Z2 sum = Clean!

Foundation of regularization for Casimir energy

Non-Abelian?

- Application to check of mirror symmetry

X & Y → one dof on 2-sphere!

One contribution is neglected in our calculation

Wilson/Vortex loop?

Thank you very much !

- Localization w/ U(1) gauge symmetry

Monopole ∞ sum→Holonomy Z2 sum = Clean!

Foundation of regularization for Casimir energy

Non-Abelian?

- Application to check of mirror symmetry

X & Y → one dof on 2-sphere!

One contribution is neglected in our calculation

Wilson/Vortex loop?

q-binomial formula & SQED = XYZ-model

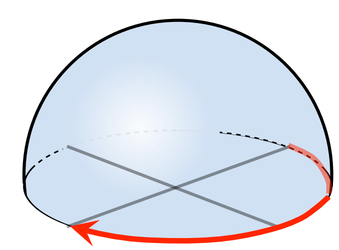
Sketch

$$\sum_{n\geq 0}\frac{(a;q)_n}{(q;q)_n}x^n = \frac{(ax;q)_\infty}{(x;q)_\infty}$$

dividing to even/odd parts

XYZ-model

$$= \sum_{n:even} + \sum_{n:odd}$$



SQED

Global U(1) correspondence

SQED ⟺ **XYZ-model**

SQED

$U(1)_J$ topological

$$J = *dA$$

⬇

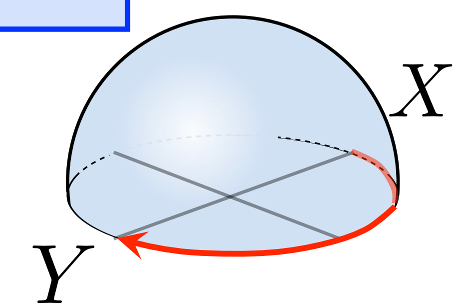
$B \wedge *J = B \wedge dA$ parity. ✕

$U(1)_A$

Q	$+1$
$\tilde{Q}$	$+1$

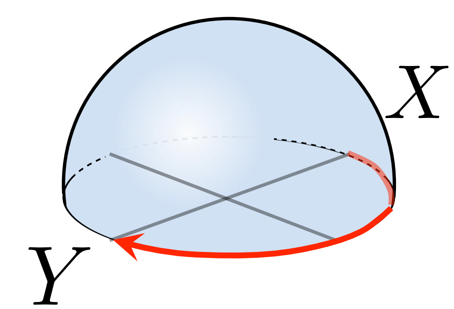
○

XYZ-model



$U(1)_V$

X	$+1$	$+1$
Y	-1	-1
Z	0	✕



$U(1)_A$

X	$+1$	$+1$
Y	$+1$	$+1$
Z	-2	○

Foundation of regularization for Casimir energy

$$\mathcal{Z}_{1-loop} = \prod \prod_n \frac{z_f + \pi i n}{z_b + \pi i n}$$

KK modes

$$= \prod \frac{2\sinh z_f}{2\sinh z_b}$$

$$= \prod e^{z_f - z_b} \frac{(1 - e^{-2z_f})}{(1 - e^{-2z_b})}$$

"Casimir energy" $= \infty$ finite

Our remedy : $$\prod_k f_k = \exp\left[\frac{d}{ds}\sum_k (f_n)^s\right]\Big|_{s=0}$$

One contribution is neglected in our calculation

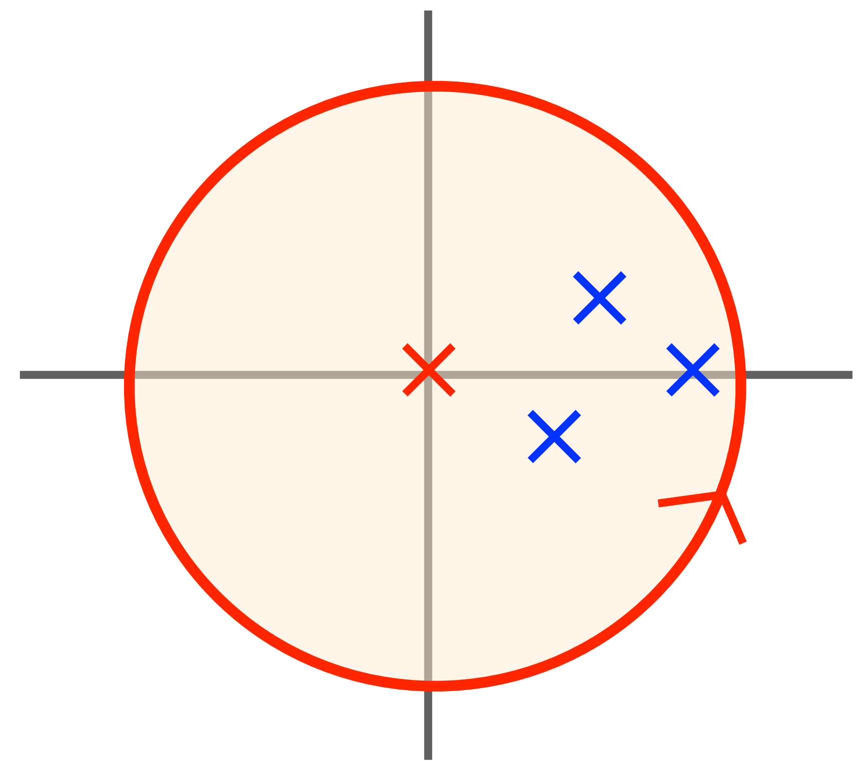
SQED

$$\mathcal{L}_{vec} \sim \begin{cases} \int_0^{2\pi} \frac{d\theta_+}{2\pi} x^{+\frac{1}{4}} \frac{(x^4;x^4)_\infty}{(x^2;x^4)_\infty} \\ \quad + \\ \int_0^{2\pi} \frac{d\theta_-}{2\pi} x^{+\frac{1}{4}} \frac{(x^4;x^4)_\infty}{(x^2;x^4)_\infty} \end{cases}$$

$$\mathcal{L}_Q \sim \begin{cases} (x^{\Delta-1} e^{+i\theta_+})^{+\frac{1}{4}} \frac{(e^{+i\theta_+} x^{(2-\Delta)};x^4)_\infty}{(e^{-i\theta_+} x^{\Delta};x^4)_\infty} \\ (x^{\Delta-1} e^{-i\theta_-})^{-\frac{1}{4}} \frac{(e^{+i\theta_-} x^{(4-\Delta)};x^4)_\infty}{(e^{-i\theta_-} x^{(2+\Delta)};x^4)_\infty} \end{cases}$$

$$\mathcal{L}_{\tilde{Q}} \sim \begin{cases} (x^{\Delta-1} e^{-i\theta_+})^{+\frac{1}{4}} \frac{(e^{-i\theta_+} x^{(2-\Delta)};x^4)_\infty}{(e^{+i\theta_+} x^{\Delta};x^4)_\infty} \\ (x^{\Delta-1} e^{+i\theta_-})^{-\frac{1}{4}} \frac{(e^{-i\theta_-} x^{(4-\Delta)};x^4)_\infty}{(e^{+i\theta_-} x^{(2+\Delta)};x^4)_\infty} \end{cases}$$

$e^{i\theta} := z$



× ⋯ We take.

× ⋯ We ignore.

$$d\theta = \frac{dz}{2\pi i} \times \boxed{\frac{1}{z}}$$

This corresponds to ×.