Fiber-Base Duality and Global Symmetry Enhancement

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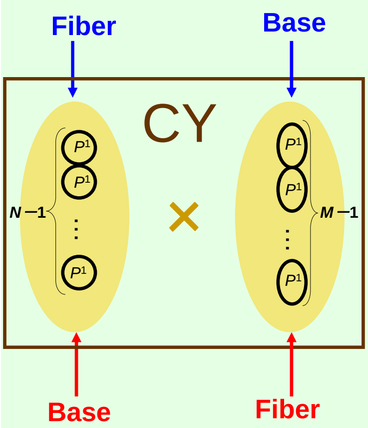
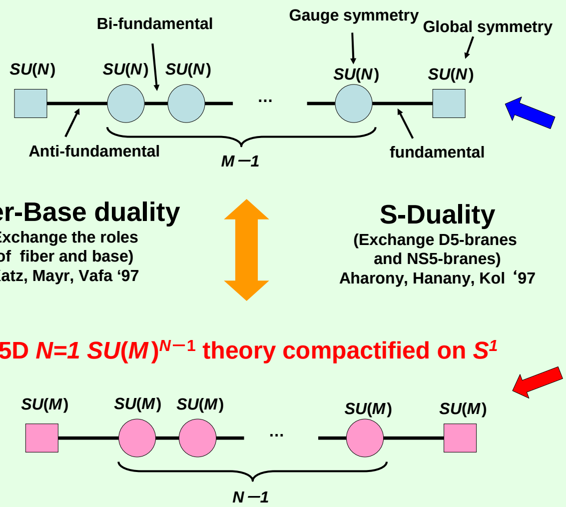
Based on Work in progress with E.Pomoni, V. Mitev, M. Taki

Background and Motivation 1 – Fiber-Base duality

Calabi-Yau compactification
Fiber – gauge group
Base – Quiver shape



5D $N=1$ $SU(N)^{M-1}$ theory compactified on S^1

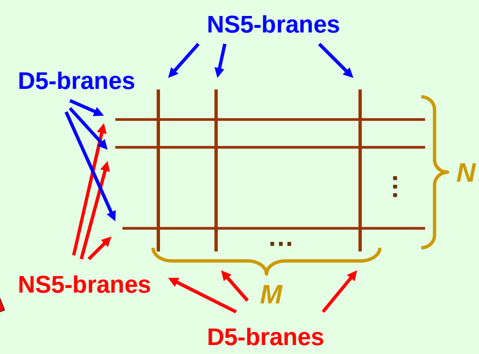


Fiber-Base duality
(Exchange the roles of fiber and base)
Katz, Mayr, Vafa '97

S-Duality
(Exchange D5-branes and NS5-branes)
Aharony, Hanany, Kol '97

5D $N=1$ $SU(M)^{N-1}$ theory compactified on S^1

5-brane picture
D5-brane – gauge group
NS5-brane– Quiver shape



Question 1
New "symmetry" seems to be implied for $M = N$ (= 2).
What kind of symmetry?

Background and Motivation 2 – Global symmetry enhancement

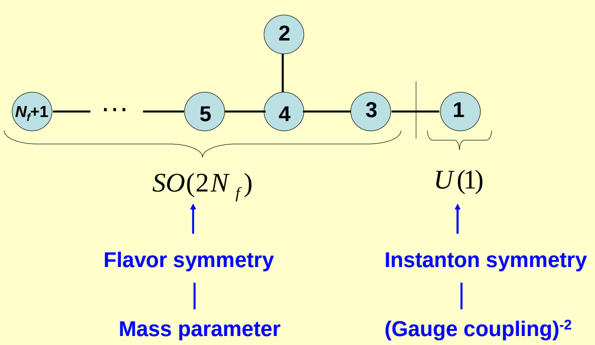
Five dimensional $N=1$ $SU(2)$ theory with N_f flavor

Conjecture '06 Seiberg

1. UV fixed points exist for $N_f \leqq 7$ (N_f =8 is subtle)
(massless, infinite coupling)

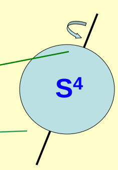
2. Global symmetry is enhanced to E_{N_f+1}

$$E_5 = SO(10),\quad E_4 = SU(5),\quad E_3 = SU(3)\times SU(2),$$
$$E_2 = SU(2)\times U(1),\quad E_1 = SU(2)$$



Check '12 Kim Kim Lee

Superconformal index
→ Partition function on $S^4 \times S^1$



$$I = \int da\; Z_{Nek}(u) Z_{Nek}(u^{-1})$$

u : Instanton factor

is written in terms of E_{N_f+1} characters
→ invariant under the Weyl symmetry of E_{N_f+1}

$$e.g.\quad u \leftrightarrow u^{-1} \quad \text{for pure SYM} \quad (N_f = 0)$$

Question 2

How about $Z_{Nek}(u)$?

Should be invariant because IR physics respect UV symmetry ??
Not invariant because it includes only positive power of instanton factor ??

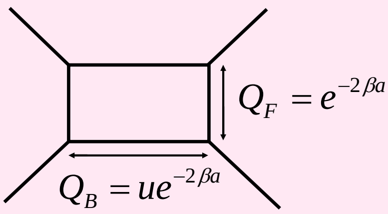
Claim

1. Flavor $SO(2N_f)$ (Weyl symmetry of) & Fiber-base duality ⇒ Enhanced E_{N_f+1} (Weyl symmetry of) (Answer to Question 1)

2. **Nekrasov partition function** is written in terms of **E_{N_f+1} characters** if we introduce **"invariant Coulomb moduli parameters"** (Answer to Question 2)

Pure $SU(2)$ SYM

Toric-web diagram



$(u = (\beta\Lambda)^4)$

β : circumference of the compactified S^1
a : Coulomb moduli parameter
u : Instanton factor

$Q_F \leftrightarrow Q_B$: **Fiber-Base Duality**

⇓

$$u \to u^{-1}, \quad e^{-\beta a} \to \sqrt{u}e^{-\beta a}$$

Introduce "invariant Coulomb moduli parameter"

$$\tilde{A} \equiv (Q_F Q_B)^{\frac{1}{4}} = u^{\frac{1}{4}} e^{-\beta a}$$

⇓

$$u \to u^{-1}, \quad \tilde{A} \to \tilde{A}$$

Weyl symmetry of **Enhanced $E_1 = SU(2)$ Symmetry**

In order to write Nekrasov partition function manifestly invariant under E_1 symmetry …

1. Use the variables $(u, \tilde{A}; q, t)$ instead of $(u, a; q, t)$ $\quad \left(q = e^{-\beta\varepsilon_1}, t = e^{+\beta\varepsilon_2}\right)$

2. Expand in terms of $\tilde{A}$ instead of u ($k \le n$ instanton correction is necessary to obtain the expansion up to $\tilde{A}^{2n}$)

$$Z_{Nek}(u, \tilde{A}; q, t) = 1 + \frac{t+q}{(1-t)(1-q)} \chi_{[1]}(u) \tilde{A}^2 + \frac{q+t+q^2+t^2+qt+q^2t+qt^2}{(1-q)(1+q)(1-t)(1+t)} \left[\frac{q^2+t^2}{qt} + \frac{1}{(1-q)(1-t)} \chi_{[2]}(u) \right] \tilde{A}^4 + \cdots \qquad \left(\chi_{[\ell]}(u) \equiv \sum_{m=0}^{\ell} u^{\ell-2m} \right)$$

$SU(2)$ with N_f=4 flavor

Flavor $SO(8)$ Weyl transformation
• $y_i \leftrightarrow y_j$ (other y fixed)
• $y_i \leftrightarrow y_j^{-1}$ (other y fixed)
$(1 \le i, j \le 4, i \ne j)$

&

Fiber-base duality
• $y_0 \leftrightarrow y_1, \quad y_2 \leftrightarrow y_3$
(y_4 and $\tilde{A}$ fixed)

⇒

Enhanced $E_5 = SO(10)$ Weyl transformation
• $y_I \leftrightarrow y_J$ (other y fixed)
• $y_I \leftrightarrow y_J^{-1}$ (other y fixed)
$(0 \le I, J \le 4, I \ne J)$

$$y_0 \equiv u = e^{2\pi i\tau}, \quad y_1 \equiv e^{-\frac{1}{2}\beta(m_1-m_2+m_3-m_4)}, \quad y_2 \equiv e^{-\frac{1}{2}\beta(-m_1+m_2+m_3-m_4)}, \quad y_3 \equiv e^{-\frac{1}{2}\beta(m_1+m_2-m_3-m_4)}, \quad y_4 \equiv e^{-\frac{1}{2}\beta(-m_1-m_2-m_3-m_4)}, \quad \tilde{A} \equiv u^{\frac{1}{2}} e^{-\beta a}$$

$$Z_{Nek}(y, \tilde{A}; q, t) = 1 - \frac{\sqrt{qt}}{(1-q)(1-t)} \chi^{E_5}_{16}(y) \tilde{A} + \frac{(q-q^3+t-t^3+q^3t^2+q^2t^3)\chi^{E_5}_{10}(y) + (qt+q^2t^2)\chi^{E_5}_{120}(y) + (q^2t+qt^2)\chi^{E_5}_{126}(y)}{(1-t)^2(1+t)(1-q)^2(1+q)} \tilde{A}^2 + \cdots \qquad \left(\chi_{10} = \sum_i y_i + y_i^{-1} \quad \vdots \right)$$

We did analogous analysis also for N_f=1,2,3. More flavor case (N_f≦7) will be also possible.

Higher rank generalization

- Fiber-Base duality: $SU(N)$ with $2N$ flavor ⇔ $SU(2)^{N-1}$ with 2+2 flavor
  → Expected enhanced global symmetry is $SU(2N)$ x $SU(2)$ x $SU(2)$

- Fiber-Base duality: $SU(N)^{M-1}$ with N+N flavor ⇔ $SU(M)^{N-1}$ with M+M flavor
  → Expected enhanced global symmetry is $SU(N)$ x $SU(N)$ x $SU(M)$ x $SU(M)$

Nekrasov partition function is written in terms of the character of this enhanced symmetry (partially done)

Message: Fiber-Base Duality tells us about Global Symmetry Enhancement!