Is Unitarity an option?

Giacomo Mauro D'Ariano
Università degli Studi di Pavia

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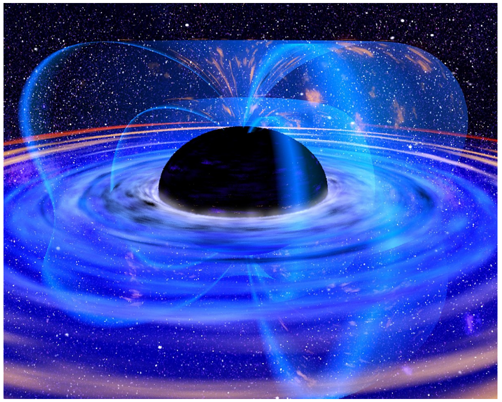
Index of the talk

- Motivations for the posed question "Is Unitarity an option?":
  - The "information paradox"
  - What we learn at school
- Quantum falsification tests
- Impossibility of falsification of:
  - purity of states
  - atomicity of transformations
  - max-entanglement of states
  - unitarity of transformations
  - pure realisation of mixed states and unitary realisations of transformations
- Customary and minimal mathematical axiomatisations of QT: the spurious postulate
- Conclusions: Copenhagen interpretation vs interpretations of the unnecessary postulate



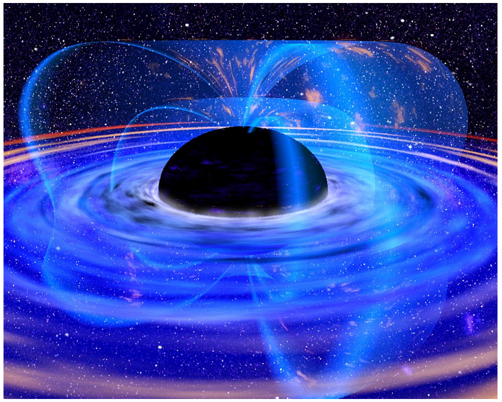
THE INFORMATION PARADOX

This puzzle has spawned many audacious ideas, beginning with Hawking's bold proposal that unitarity fails in quantum gravity.

unitarity can be temporarily violated during the black hole evaporation process, accommodating violations of monogamy of entanglement and the no-cloning principle, and allowing assumptions (1), (2), and (3) to be reconciled

S. Lloyd and J. Preskill, JHEP 08 2014 126

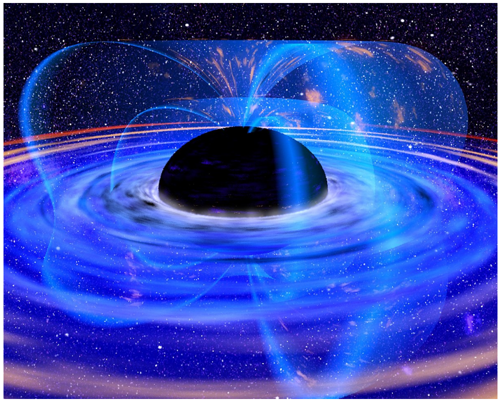
(1) An evaporating black hole scrambles quantum information without destroying it.
(2) A freely falling observer encounters nothing unusual upon crossing the event horizon of a black hole.
(3) An observer who stays outside a black hole detects no violations of relativistic effective quantum field theory.



THE INFORMATION PARADOX

Violation of unitarity by Hawking radiation does not violate energy-momentum conservation

H. Nikolic(Boskovic Inst., Zagreb) Feb 15, 2015



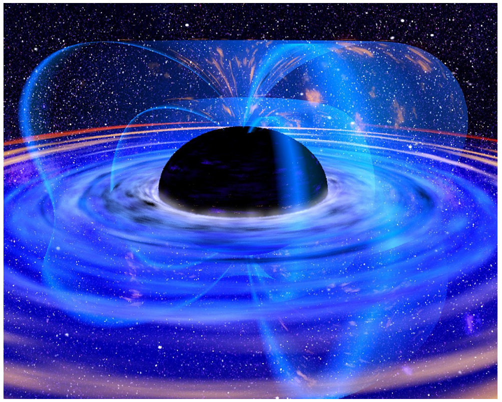
THE INFORMATION PARADOX

This is the essence of the black hole information paradox (BHIP): unlike any other classical or quantum system, black holes may not conserve information, thus violating unitarity.

Some physicists speculate that quantum gravity may actually be non-unitary

When this phenomenon is analyzed closer, we discover that it takes pure states to mixed states, a violation of unitarity, <u>a fundamental property of quantum physics</u>.

The Black Hole Information Paradox
Stefano Antonini, John Martyn, Gautam Nambiar,14/10/2018



THE INFORMATION PARADOX

Unitarity? Non consistent with AdS/CFT

Joe Polchinski, Simons Symposium, Caneel Bay 2/5/13

Why unitarity cannot be violated?

MOTIVATIONS FOR UNITARITY

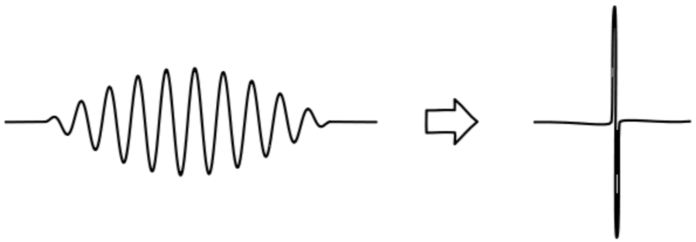
The “actual” quantum evolutions are reversible…
Information is conserved…
The “actual” quantum state is pure…

What we learn at school

Quantum “Mechanics” (non relativistic)

$$i\hbar\frac{\partial\psi(\boldsymbol{x},t)}{\partial t} = \left(-\frac{\hbar^2\nabla^2}{2m} + V(\boldsymbol{x})\right)\psi(\boldsymbol{x},t)$$

von Neumann collapse



$$i\hbar\frac{\partial}{\partial t}\Psi = \hat{H}\Psi$$

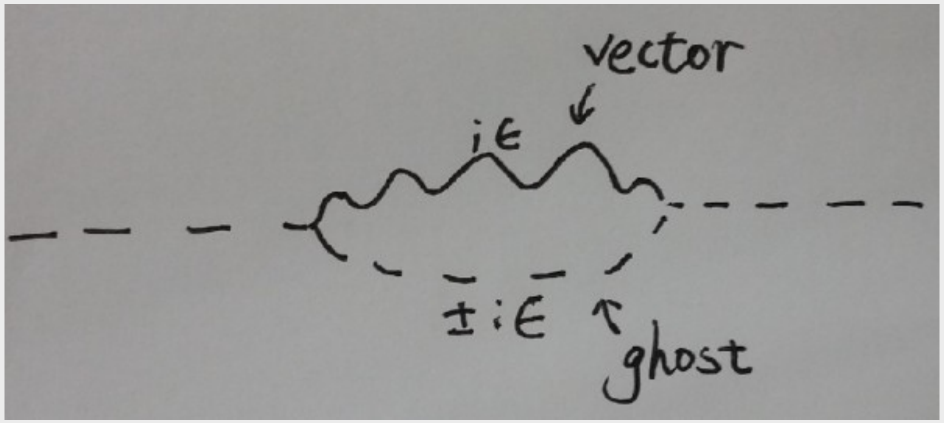
OK for Free Field Theory

(but what about collapse?)

Unitarity in interacting QFT

The necessity for Faddeev–Popov ghosts follows from the requirement that quantum field theories yield unambiguous, non-singular solutions. This is not possible in the path integral formulation when a gauge symmetry is present since there is no procedure for selecting among physically equivalent solutions related by gauge transformation. The path integrals overcount field configurations corresponding to the same physical state; the measure of the path integrals contains a factor which does not allow obtaining various results directly from the action.
It is possible, however, to modify the action, such that methods such as Feynman diagrams will be applicable by adding ghost fields which break the gauge symmetry.
The ghost fields do not correspond to any real particles in external states: they appear as virtual particles in Feynman diagrams – or as the absence of gauge configurations. However, they are a necessary computational tool to preserve unitarity.

Unitarity in quantum field theory?



Should the “actual” quantum evolution be reversible?

Should information be conserved?

Is the “actual” quantum state pure?

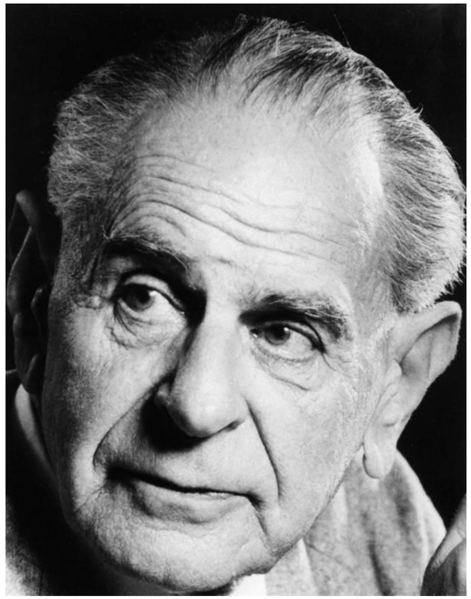
Should the "actual" quantum evolution be reversible?

Should information be conserved?

Is the "actual" quantum state pure?

UNFALSIFIABLE STATEMENTS

Quantum falsification tests



CONVENTIONS & NOTATIONS

Convenient rule of taking the trace Tr ϱ of the density matrix $\varrho \in \mathrm{St}(\mathrm{A})$ of system A as the preparation probability $p(\varrho) = \mathrm{Tr}\,\varrho$ of the state ϱ (unit-trace ϱ *deterministic* states).

In such a way, for example, the trace $\mathrm{Tr}[\mathcal{T}\varrho]$ will denote the joint probability of ϱ-preparation followed by the quantum operation $\mathcal{T}$.

This convention makes possible to regard *states* and *effects* just as special cases of transformations, from and to the trivial system I, respectively, with Hilbert space $\mathcal{H}_{\mathrm{I}} = \mathbb{C}$

$\mathcal{H}$	Hilbert space over $\mathbb{C}$
$\mathrm{Bnd}^+(\mathcal{H})$	bounded positive operators over $\mathcal{H}$
$\mathbb{U}(\mathcal{H})$	unitary group over $\mathcal{H}$
$\mathrm{T}(\mathcal{H})$	trace-class operators over $\mathcal{H}$
$\mathrm{T}^+(\mathcal{H})$	trace-class positive operators over $\mathcal{H}$
$\mathrm{T}^+_{\le 1}(\mathcal{H})$	positive sub-unit-trace operators over $\mathcal{H}$
$\mathrm{T}^+_{=1}(\mathcal{H})$	positive unit-trace operators over $\mathcal{H}$
$\mathrm{CP}_{\le}$	trace-non increasing completely positive map
$\mathrm{CP}_{=}$	trace-preserving completely positive map
$\mathsf{Conv}(\mathsf{S})$	convex hull of S
$\mathsf{Cone}(\mathsf{S})$	conic hull of S
$\mathsf{Cone}_{\le 1}(\mathsf{S})$	convex hull of $\{\mathsf{S} \cup 0\}$
$\mathrm{St}(\mathrm{A})$	set of states of system A
$\mathrm{St}_1(\mathrm{A})$	set of deterministic states of system A
$\mathrm{Eff}(\mathrm{A})$	set of effects of system A
$\mathrm{Eff}_1(\mathrm{A})$	set of deterministic effects of system A
$\mathrm{Trn}(\mathrm{A} \to \mathrm{B})$	set of transformations from system A to system B
$\mathrm{Trn}_1(\mathrm{A} \to \mathrm{B})$	set of deterministic transformations from system A to system B
	Special cases
	$\mathrm{T}(\mathbb{C}) = \mathbb{C}, \quad \mathrm{T}^+(\mathbb{C}) = \mathbb{R}^+, \quad \mathrm{T}^+_{\le 1}(\mathbb{C}) = [0,1], \quad \mathrm{T}^+_{=1}(\mathbb{C}) = \{1\}$
	$\mathrm{CP}(\mathrm{T}(\mathcal{H}) \to \mathrm{T}(\mathbb{C})) = \mathrm{P}(\mathrm{T}(\mathcal{H}) \to \mathrm{T}(\mathbb{C})) = \{\mathrm{Tr}[\cdot E],\ E \in \mathrm{Bnd}^+(\mathcal{H})\}$
	$\mathrm{CP}(\mathrm{T}(\mathbb{C}) \to \mathrm{T}(\mathcal{H})) = \mathrm{P}(\mathrm{T}(\mathbb{C}) \to \mathrm{T}(\mathcal{H})) = \mathrm{T}^+(\mathcal{H})$
	$\mathrm{CP}_{\le}(\mathrm{T}(\mathbb{C}) \to \mathrm{T}(\mathcal{H})) \equiv \mathrm{T}^+_{\le 1}(\mathcal{H})$
	$\mathrm{CP}_{\le}(\mathrm{T}(\mathcal{H}) \to \mathrm{T}(\mathbb{C})) \equiv \{\epsilon(\cdot) = \mathrm{Tr}[\cdot E],\ 0 \le E \le I\}$

The falsification test

Definition 1 (Falsifier). *The event F is a* falsifier *of hypothesis* Hyp *if F cannot happen for* Hyp $=$ TRUE.

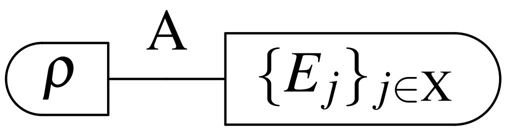
Accordingly we will call the binary test $\{F, F_?\}$ *a falsification test* for hypothesis Hyp, $F_?$ denoting the *inconclusive event*.[2]

Practically one is interested in *effective* falsification tests $\{F, F_?\}$ which are not singleton–the two singleton tests corresponding to $F = 0$ and $F_? = 0$ being the *inconclusive falsification test* and the *logical falsification*, respectively.

Suppose now that one wants to falsify a proposition about the state $\rho \in \mathrm{St}(\mathrm{A})$ of system A. In such case any effective falsification test can be achieved as a binary *observation test* of the form

$$\boxed{\{F, F_?\} \subset \mathrm{Eff}(A), \quad F_? := I_{\mathrm{A}} - F, \quad F > 0, F_? \geq 0,} \tag{0.1}$$

where with the symbol F ($F_?$) we denote both the event and its corresponding positive operator.



Example of falsification test

Consider the proposition

$$\mathrm{Hyp}: \quad \mathsf{Supp}\,\rho = \mathcal{K} \subset \mathcal{H}_{\mathrm{A}},\ \rho \in \mathrm{St}(\mathrm{A}), \qquad \dim \mathcal{H}_{\mathrm{A}} \geq 2 \tag{4.2}$$

where $\mathsf{Supp}\,\rho$ denotes the support of ρ. Then, any operator of the form

$$0 < F \leq I_{\mathrm{A}}, \quad \mathsf{Supp}\,F \subseteq \mathcal{K}^{\perp} \tag{4.3}$$

would have zero expectation for a state ρ satisfying Hyp (4.2), which means that occurrence of F would be a falsification of Hyp, namely

$$\mathrm{Tr}[\rho F] > 0 \Rightarrow \mathrm{Hyp} = \mathsf{FALSE}. \tag{4.4}$$

In this example we can see how the falsification test is not dichotomic, namely the occurrence of $F_?$ does not mean that $\mathrm{Hyp} = \mathsf{TRUE}$, since $F_?$ occurs if $\mathsf{Supp}\,F_? \cap \mathcal{K} \neq 0$. Eq. (4.3) provides the most general falsification test of Hyp (4.2), and the choice $\mathsf{Supp}\,F = \mathcal{K}^{\perp}$ provides the most efficient test since it maximises the falsification chance.

Unfalsifiability of purity of quantum states

Theorem 1 (Unfalsifiability of state-purity). *There exists no test falsifying purity of an unknown state of a given system* A.

Proof. In order to falsify the hypothesis

$$\mathrm{Hyp}: \ \rho \in \mathrm{PurSt}(\mathrm{A}), \tag{4.1}$$

we need a falsifier $F \in \mathrm{Eff}(\mathrm{A})$ satisfying

$$\mathrm{Tr}[F\rho] = 0, \ \forall \rho \in \mathrm{PurSt}(\mathrm{A}), \tag{4.2}$$

which means that

$$\forall \psi \in \mathcal{H}_{\mathrm{A}} : \ \langle \psi | F | \psi \rangle = 0, \tag{4.3}$$

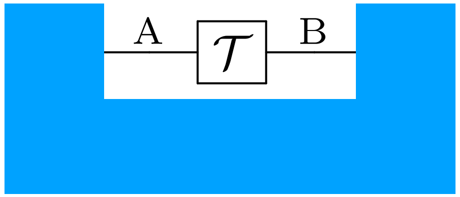
namely $F = 0$, which means that the test is inconclusive. ■

By the same argument one can easily prove the impossibility of falsifying purity even when $N > 1$ copies of the state are available.

Unfalsifiability of atomicity of a transformation of a quantum system

Theorem 2 (Unfalsifiability of atomicity of a transformation)**.** *There exists no test falsifying atomicity of an unknown transformation* $\mathcal{A} \in \mathrm{Trn}(\mathrm{A} \to \mathrm{B})$.

Proof. The most general scheme for testing a property of a transformation $\mathcal{T} \in \mathrm{Trn}(\mathrm{A} \to \mathrm{B})$ is the following



. (5.4)

We can use the maximally entangled state for $R = |\Phi\rangle\langle\Phi|$, thus exploiting the Choi-Jamiołkowski cone-isomorphism between transformations and bipartite states. One has

$$\textit{atomicity of } \mathcal{T} \equiv \textit{purity of state } (\mathcal{T} \otimes \mathcal{I}_{\mathrm{E}})R, \tag{5.5}$$

and falsifying atomicity of $\mathcal{T} \in \mathrm{Trn}(\mathrm{A} \to \mathrm{B})$ is equivalent to falsifying purity of $(\mathcal{T} \otimes \mathcal{I}_{\mathrm{E}})R$, which is impossible. ■

Unfalsifiability of max-entanglement of a pure bipartite state

Theorem 3 (Unfalsifiability of max-entanglement of a pure bipartite state.). *There exists no test falsifying max-entanglement of a pure bipartite state.*

Proof. Falsification of max-entanglement of state $|\Phi\rangle\langle\Phi|$ needs a falsifier $F \in \mathrm{Eff}(\mathrm{AB})$ satisfying

$$\mathrm{Tr}[F|\Phi\rangle\langle\Phi|] = 0,\ \forall|\Phi\rangle\langle\Phi| \text{ maximally entangled.} \tag{4.6}$$

In particular, since unitarity preserve max-entanglement, one has

$$\mathrm{Tr}[F(\mathcal{U}\otimes\mathcal{I}_{\mathrm{B}})|\Phi\rangle\langle\Phi|] = 0,\ \forall\mathcal{U} \in \mathrm{Trn}(\mathrm{A}) \text{ unitary.} \tag{4.7}$$

Notice that the trace in Eq. 4.7 cannot be negative for any F and $\mathcal{U}$. It follows that its average over the unitary group $G_{\mathrm{A}} = SU(d_{\mathrm{A}})$ must be zero, corresponding to[5]

$$0 = \int_{G_{\mathrm{A}}} d\mathcal{U}\,\mathrm{Tr}[F(\mathcal{U}\otimes\mathcal{I}_{\mathrm{A}})|\Phi\rangle\langle\Phi|] = \mathrm{Tr}[F(I_{\mathrm{A}}\otimes\mathrm{Tr}_{\mathrm{A}}[|\Phi\rangle\langle\Phi|])] = d_{\mathrm{A}}^{-1}\,\mathrm{Tr}[F(I_{\mathrm{A}}\otimes I_{\mathrm{A}})] = d_{\mathrm{A}}^{-1}\,\mathrm{Tr}[F] \tag{4.8}$$

which implies that $F = 0$, which contradicts the test effectiveness condition $F > 0$. ■

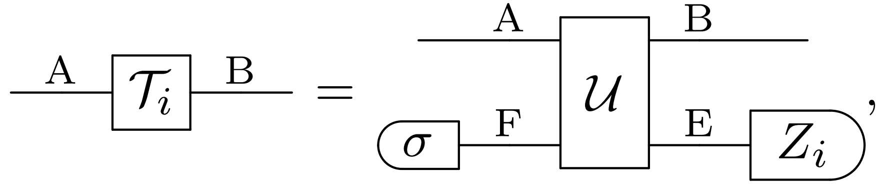
Unfalsifiability of unitarity of a quantum transformation

Theorem 4 (Unfalsifiability of unitarity of a transformation)**.** *There exists no test falsifying unitarity of a transformation* $\mathcal{T} \in \mathrm{Trn}(\mathrm{A})$.

Proof. The application of the operator to a fixed maximally-entangled state puts unitary transformations in one-to-one correspondence with maximally entangled states. Thus, being able to falsify maximal entanglement allows to falsify unitarity. ■

Unfalsifiability of unitary realization of a transformation

The impossibility of establishing the unitariety of transformation (Theorem 4) with input and output systems under our control excludes the possibility of falsifying that a transformation is actually achieved unitarily, according to the scheme

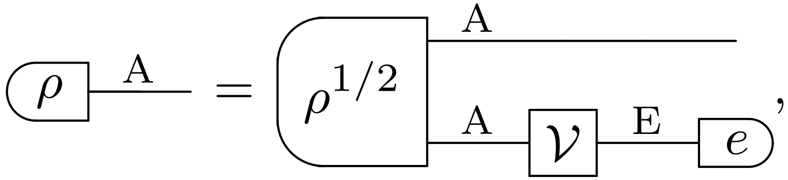


(5.10)

with $\{Z_i\}$ von Neuman measurement over the output environment E, and the input environment F is prepared in a state σ. Systems E, F, state σ, measurement Z_i, and unitary $\mathcal{U}$ are all not unique and unknown, hence the testing resorts to falsifying unitarity of $\mathcal{U}$, which is impossible, not even with control of input-output systems AF and BE.

Unfalsifiability of a mixed state being the marginalization of a pure one

Any purification pf the mixed state $\rho \in \mathrm{St}(\mathrm{A})$ can be written in the following diagrammatic form



(5.9)

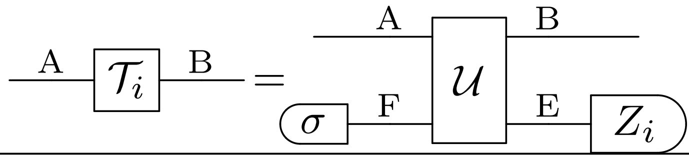
e denoting the deterministic effect, corresponding to perform no-measurement on the system E, and $\mathcal{V}$ being an isometry on the support of ρ. By writing $\mathcal{V}$ as $\mathcal{V} = \mathcal{U}\mathcal{V}_0$, with $\mathcal{U}$ unitary on E, the proof follows in a way similar to Eq. 5.8. This excludes the possibility of falsifying that a knowngly mixed state of a quantum system A actually is the marginal of a pure entangled state with an environment system E. Moreover, the system E is unknown (we just know that it must have dimension $d_{\mathrm{E}} \geq d_{\mathrm{A}}$).

About minimal mathematical axiomatization of QT

Customary mathematical axiomatisation of Quantum Theory

system	A	$\mathcal{H}_\mathrm{A}$
system composition	AB	$\mathcal{H}_\mathrm{AB} = \mathcal{H}_\mathrm{A} \otimes \mathcal{H}_\mathrm{B}$
deterministic pure state	$\sigma \in \mathrm{PurSt}_1(\mathrm{A})$	$\sigma = \lvert\psi\rangle\langle\psi\rvert,\ \psi \in \mathcal{H}_\mathrm{A},\ \lVert\psi\rVert = 1$
reversible transf.	$\mathcal{U} \in \mathrm{RevTrn}(\mathrm{A})$	$\mathcal{U}\sigma = U\lvert\psi\rangle\langle\psi\rvert U^\dagger,\ U \in \mathbb{U}(\mathrm{A})$
von Neumann-Lüders transformation	$\sigma \to \mathcal{Z}_i\sigma := Z_i\sigma Z_i$	$\{Z_i\}_{i\in\mathrm{X}} \subset \mathrm{Bnd}(\mathcal{H}_\mathrm{A})$ PVM
Born rule	$p(i\vert\psi) = \langle\psi\vert Z_i\vert\psi\rangle$	

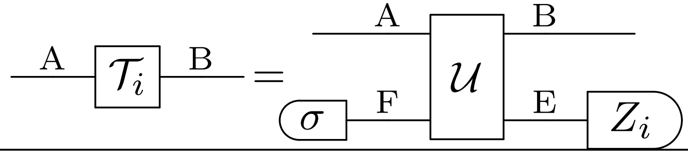
Theorems

trivial system	I	$\mathcal{H}_\mathrm{I} = \mathbb{C}$
deterministic states	$\rho \in \mathrm{St}_1(\mathrm{A}) \equiv \mathsf{Conv}(\mathrm{PurSt}_1(\mathrm{A}))$	$\rho \in \mathrm{T}^+_{=1}(\mathcal{H}_\mathrm{A})$
states	$\rho \in \mathrm{St}(\mathrm{A}) \equiv \mathsf{Cone}_{\leq 1}(\mathrm{PurSt}_1(\mathrm{A}))$	$\rho \in \mathrm{T}^+_{\leq 1}(\mathcal{H}_\mathrm{A})$
Transformation as unitary interaction + von Neumann observable on "meter"		$\mathcal{T}_i\rho = \mathrm{Tr}_\mathrm{E}[U(\rho \otimes \sigma)U^\dagger(I_\mathrm{B} \otimes Z_i)]$
transformation	$\mathcal{T} \in \mathrm{Trn}(\mathrm{A} \to \mathrm{B})$	$\mathcal{T} \in \mathrm{CP}_\leq(\mathrm{T}(\mathcal{H}_\mathrm{A}) \to \mathrm{T}(\mathcal{H}_\mathrm{B}))$
parallel composition	$\mathcal{T}_1 \in \mathrm{Trn}(\mathrm{A} \to \mathrm{B}),\ \mathcal{T}_2 \in \mathrm{Trn}(\mathrm{C} \to \mathrm{D})$	$\mathcal{T}_1 \otimes \mathcal{T}_2$
sequential composition	$\mathcal{T}_1 \in \mathrm{Trn}(\mathrm{A} \to \mathrm{B}),\ \mathcal{T}_2 \in \mathrm{Trn}(\mathrm{B} \to \mathrm{C})$	$\mathcal{T}_2\mathcal{T}_1$
effects	$\epsilon \in \mathrm{Eff}(\mathrm{A}) \equiv \mathrm{Trn}(\mathrm{A} \to \mathrm{I})$	$\epsilon(\cdot) = \mathrm{Tr}_\mathrm{A}[\cdot E],\ 0 \leq E \leq I_A$
	$\epsilon \in \mathrm{Eff}_1(\mathrm{A}) \equiv \mathrm{Trn}_1(\mathrm{A} \to \mathrm{I})$	$\epsilon = \mathrm{Tr}_\mathrm{A}$

Minimal mathematical axiomatisation of Quantum Theory

system	A	$\mathcal{H}_{\mathrm{A}}$
system composition	AB	$\mathcal{H}_{\mathrm{AB}} = \mathcal{H}_{\mathrm{A}} \otimes \mathcal{H}_{\mathrm{B}}$
transformation	$\mathcal{T} \in \mathrm{Trn}(\mathrm{A} \to \mathrm{B})$	$\mathcal{T} \in \mathrm{CP}_{\leq}(\mathrm{T}(\mathcal{H}_{\mathrm{A}}) \to \mathrm{T}(\mathcal{H}_{\mathrm{B}}))$
Born rule	$p(\mathcal{T}) = \mathrm{Tr}\,\mathcal{T}$	$\mathcal{T} \in \mathrm{Trn}(\mathrm{I} \to \mathrm{A})$

Theorems

trivial system	I	$\mathcal{H}_{\mathrm{I}} = \mathbb{C}$
reversible transf.	$\mathcal{U} = U \cdot U^{\dagger}$	$U \in \mathbb{U}(\mathcal{H}_{\mathrm{A}})$
determ. transformation	$\mathcal{T} \in \mathrm{Trn}_1(\mathrm{A} \to \mathrm{B})$	$\mathcal{T} \in \mathrm{CP}_{=}(\mathrm{T}(\mathcal{H}_{\mathrm{A}}) \to \mathrm{T}(\mathcal{H}_{\mathrm{B}}))$
parallel composition	$\mathcal{T}_1 \in \mathrm{Trn}(\mathrm{A} \to \mathrm{B}),\ \mathcal{T}_2 \in \mathrm{Trn}(\mathrm{C} \to \mathrm{D})$	$\mathcal{T}_1 \otimes \mathcal{T}_2$
sequential composition	$\mathcal{T}_1 \in \mathrm{Trn}(\mathrm{A} \to \mathrm{B}),\ \mathcal{T}_2 \in \mathrm{Trn}(\mathrm{B} \to \mathrm{C})$	$\mathcal{T}_2 \mathcal{T}_1$
states	$\rho \in \mathrm{St}(\mathrm{A}) \equiv \mathrm{Trn}(\mathrm{I} \to \mathrm{A})$	$\rho \in \mathrm{T}^{+}_{\leq 1}(\mathcal{H}_{\mathrm{A}})$
	$\rho \in \mathrm{St}_1(\mathrm{A}) \equiv \mathrm{Trn}_1(\mathrm{I} \to \mathrm{A})$	$\rho \in \mathrm{T}^{+}_{=1}(\mathcal{H}_{\mathrm{A}})$
	$\rho \in \mathrm{St}(\mathrm{I}) \equiv \mathrm{Trn}(\mathrm{I} \to \mathrm{I})$	$\rho \in [0,1]$
	$\rho \in \mathrm{St}_1(\mathrm{I}) \equiv \mathrm{Trn}(\mathrm{I} \to \mathrm{I})$	$\rho = 1$
effects	$\epsilon \in \mathrm{Eff}(\mathrm{A}) \equiv \mathrm{Trn}(\mathrm{A} \to \mathrm{I})$	$\epsilon(\cdot) = \mathrm{Tr}_{\mathrm{A}}[\cdot E],\ 0 \leq E \leq I_A$
	$\epsilon \in \mathrm{Eff}_1(\mathrm{A}) \equiv \mathrm{Trn}_1(\mathrm{A} \to \mathrm{I})$	$\epsilon = \mathrm{Tr}_{\mathrm{A}}$
Transformations as unitary interaction + von Neumann-Lüders		$\mathcal{T}_i \rho = \mathrm{Tr}_{\mathrm{E}}[U(\rho \otimes \sigma)U^{\dagger}(I_{\mathrm{B}} \otimes Z_i)]$

Quantum Theory:
no purification ontology

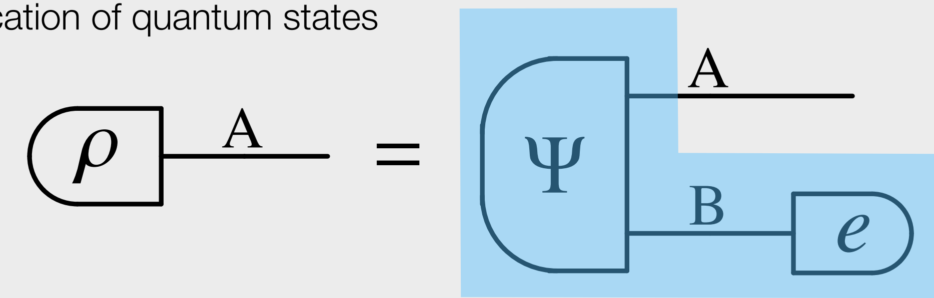
Commonplaces:

theory of “open systems”…

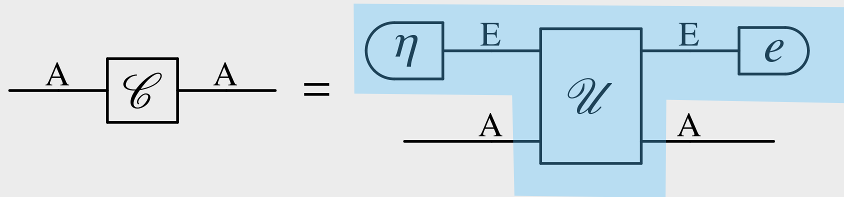
since “closed systems” (isolated systems) are supposedly in a pure state and undergo unitary transformations

Unfalsifiable ontologies

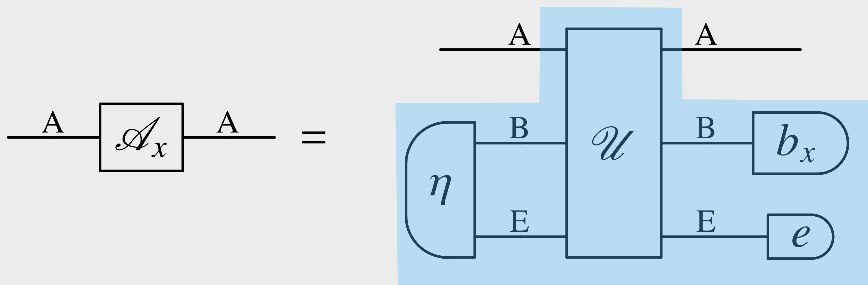
Purification of quantum states



Unitary purification of quantum channels



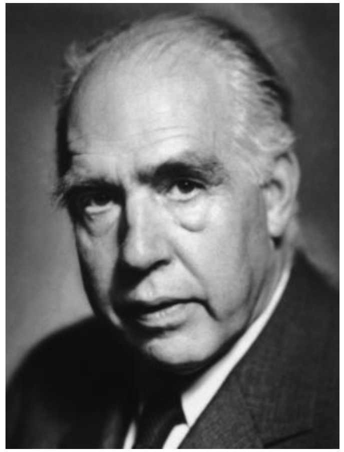
Unitary purification of quantum instruments



$\forall x \in \mathrm{X}$

Contrarily to the common belief

Quantum Theory is closed and logically consistent without the purification ontology



Consequences:

Popular interpretations (many world's, Rovelli's, …
are actually <u>interpretations of a spurious postulate</u>

They are still helpful tools for reasoning

<u>The interpretation of the strict-theory is Copenhagen's</u>

Conclusion:

we cannot say: "*information is conserved*"

we can say: "*things work as if information is conserved*"

Purification
is a powerful and elegant symmetry of the theory

It simplifies the theoretical evaluations, but …

Quantum Theory

is intrinsically probabilistic and irreversible

We are stubbornly determinists
and believe that probability and irreversibility are due to
lack of knowledge

“This is more or less what I wanted to say”

Thank you for your attention