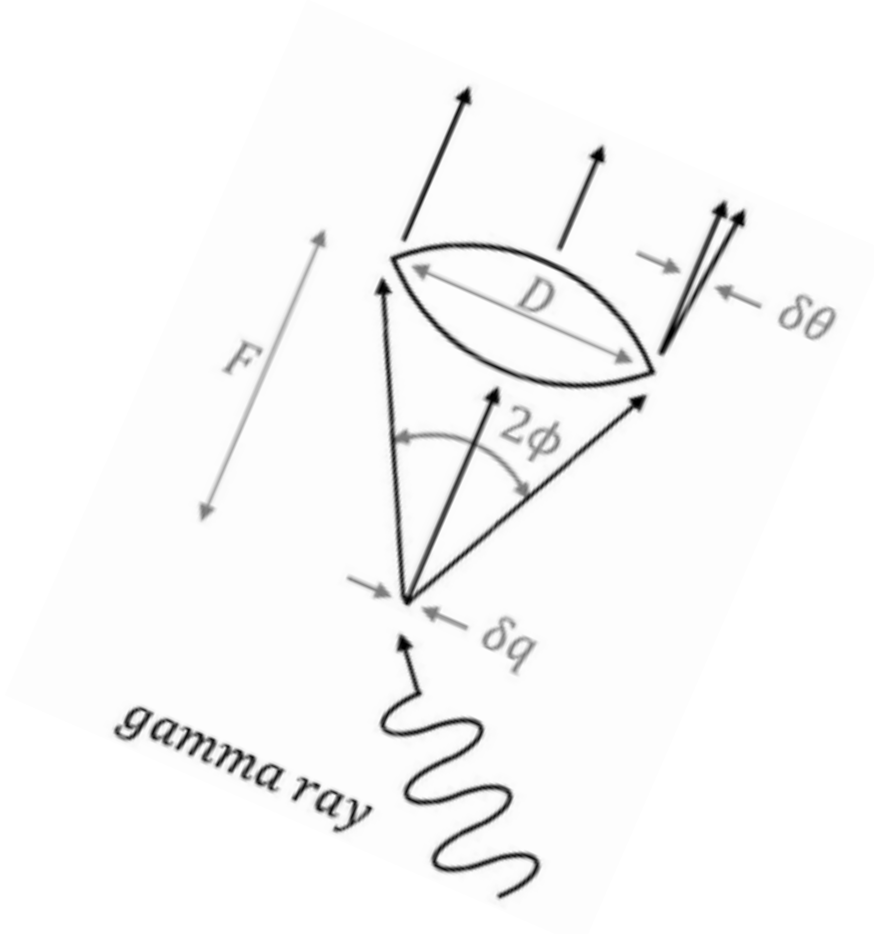
Information and disturbance in operational probabilistic theories

G.M. D'Ariano, P. Perinotti, Alessandro Tosini

G.M. D'Ariano, P. Perinotti, AT, arXiv:1907.07043 (2019)

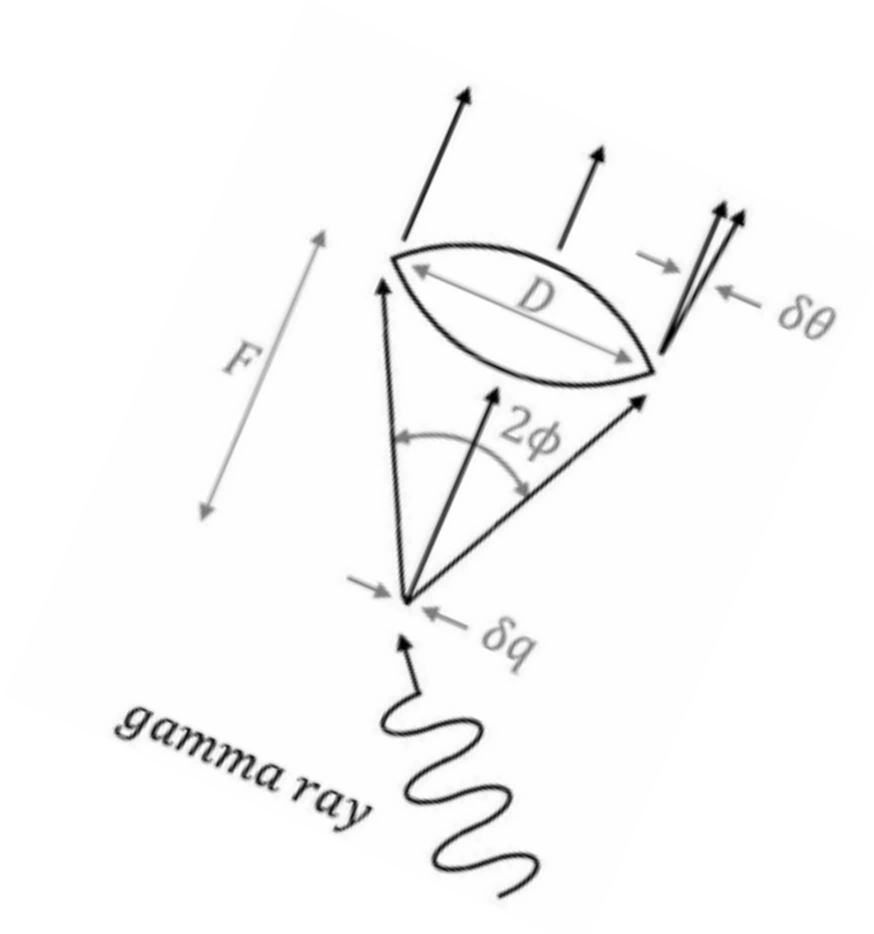
Alessandro Tosini, QUIT group, Pavia University

QICF20
14-18 September 2020 Kyoto



'27 Heisenberg's measurement uncertainty

W. Heisenberg, Zeitschrift für Physik 43, 172 (1927)



'27 Heisenberg's measurement uncertainty

W. Heisenberg, Zeitschrift für Physik 43, 172 (1927)

No-information without disturbance

If an instrument leaves unchanged all states of the system the associated observable is trivial (do not provide information).

P. Busch, P. Lahti, and R. F. Werner, Rev. Mod. Phys. 86, 1261 (2014)

P. Busch, "no information without disturbance": Quantum limitations of measurement," in Quantum Reality, Relativistic Causality, and Closing the Epistemic Circle: Essays in Honour of Abner Shimony (Springer Netherlands, Dordrecht, 2009) pp. 229–256

No-information without disturbance

“Quantum” | “Classical”

No-information without disturbance

"Quantum" | "Classical"

How "quantum" is no-information without disturbance?

J. Barrett, Physical Review A 75, 032304 (2007)

G. M. D'Ariano, G. Chiribella, and P. Perinotti, Cambridge University Press (2017)

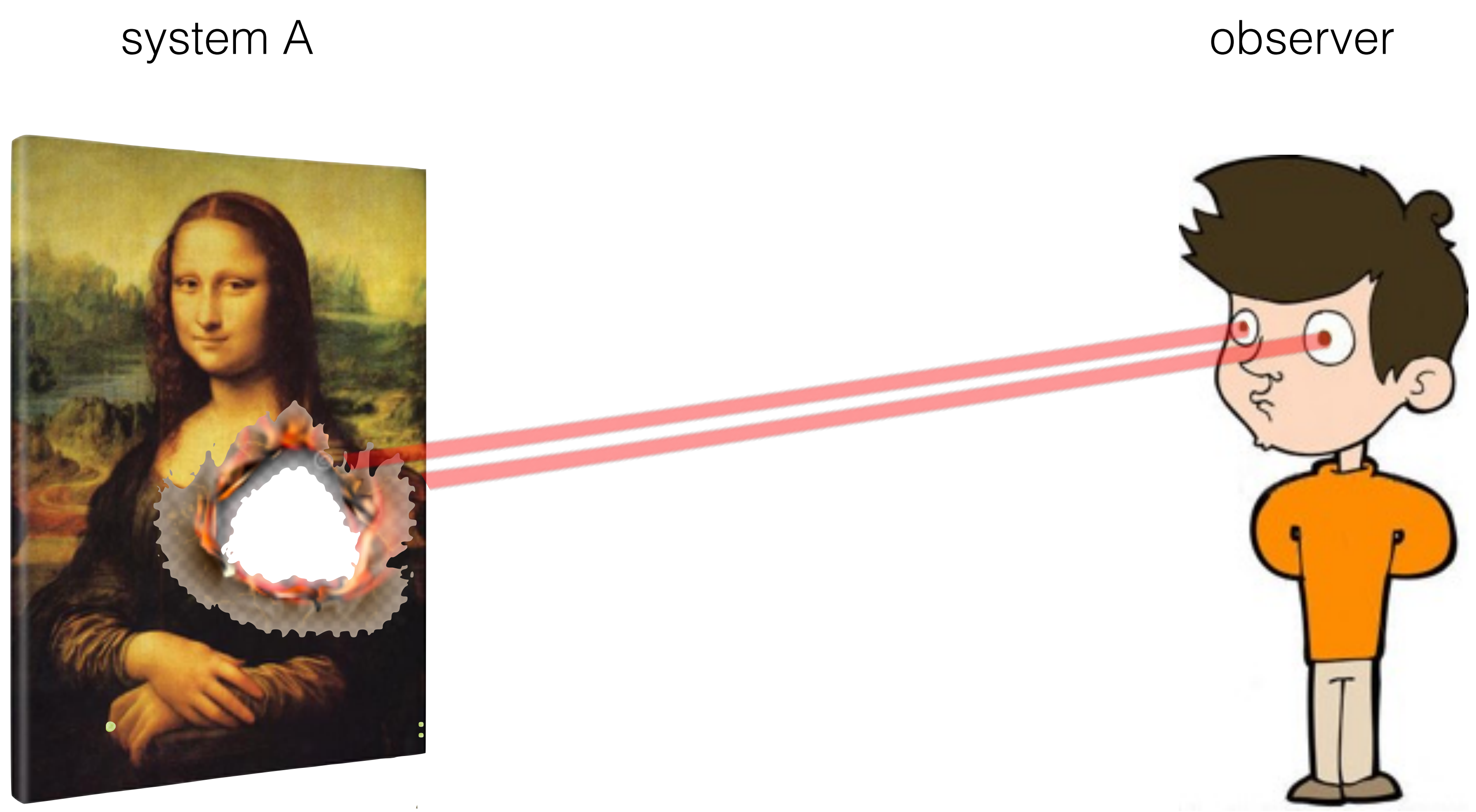
T. Heinosaari, L. Leppajarvi, and M. Plavala, arXiv:1808.07376 (2018)

Disturb the system itself

system A

observer

Disturb the system itself



Disturb the correlations with other systems

system A

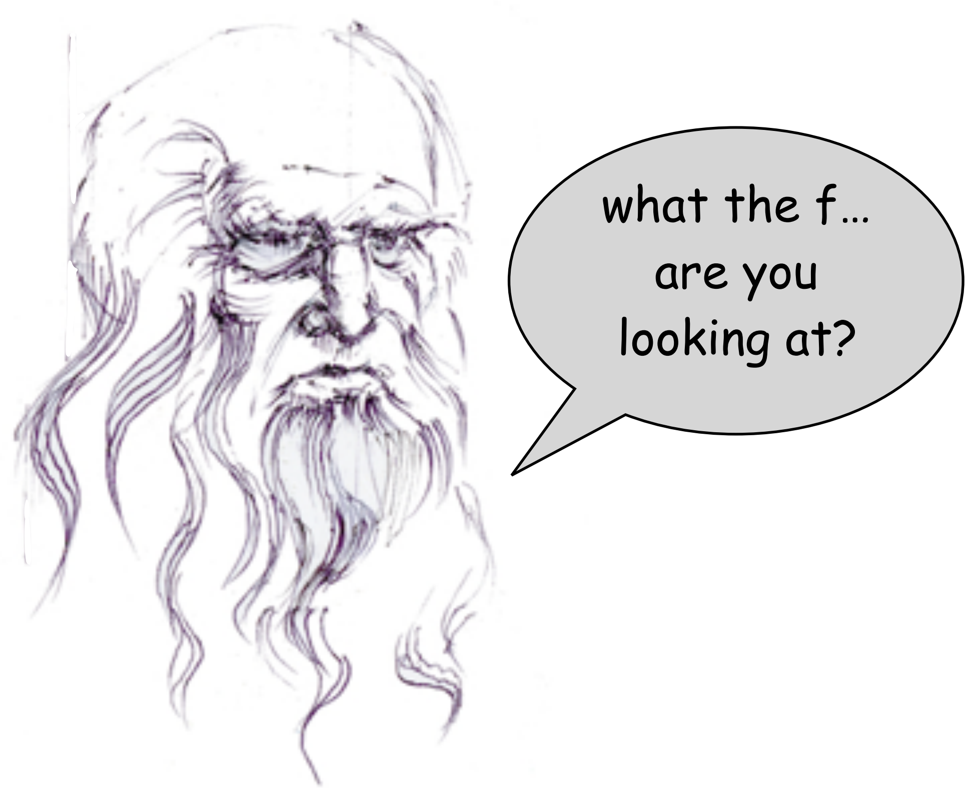
observer

Disturb the correlations with other systems

system A

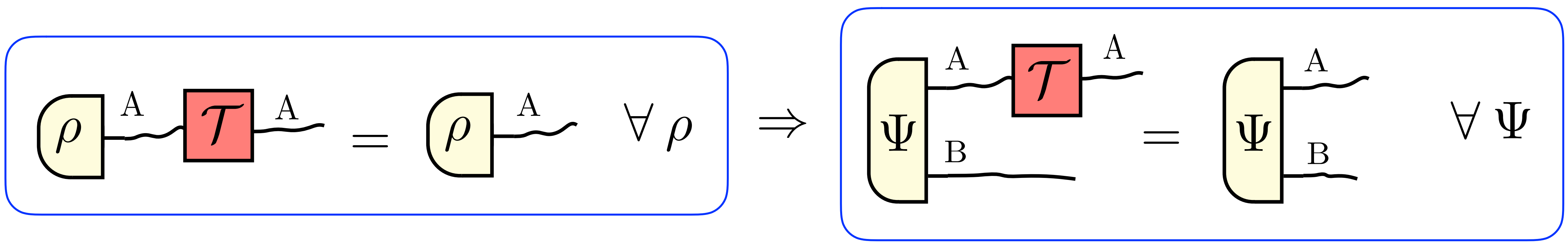
observer

system B



In **quantum theory**

if I do not disturb local states then I do not disturb at all



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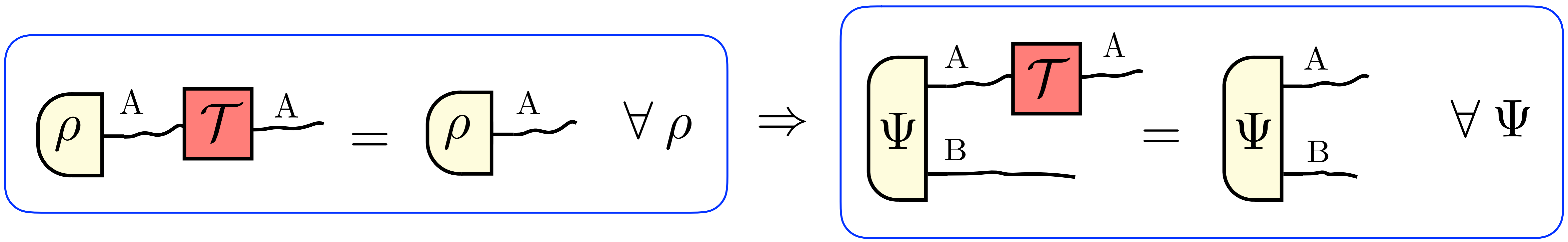
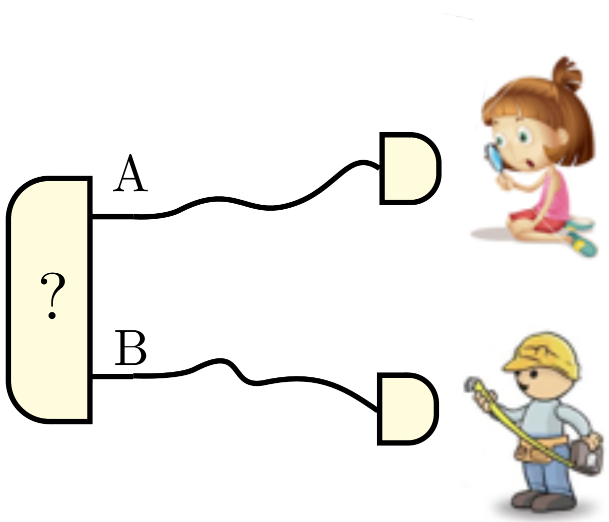
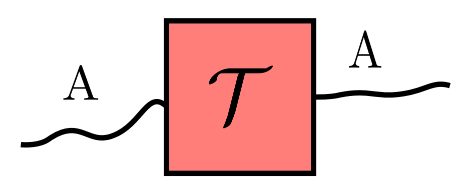


This is due to **LOCAL TOMOGRAPHY**



$\Rightarrow$

Local process tomography



fully characterised by its local action

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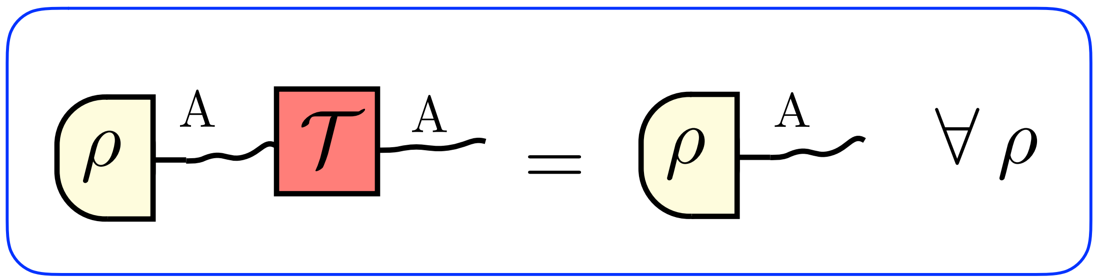

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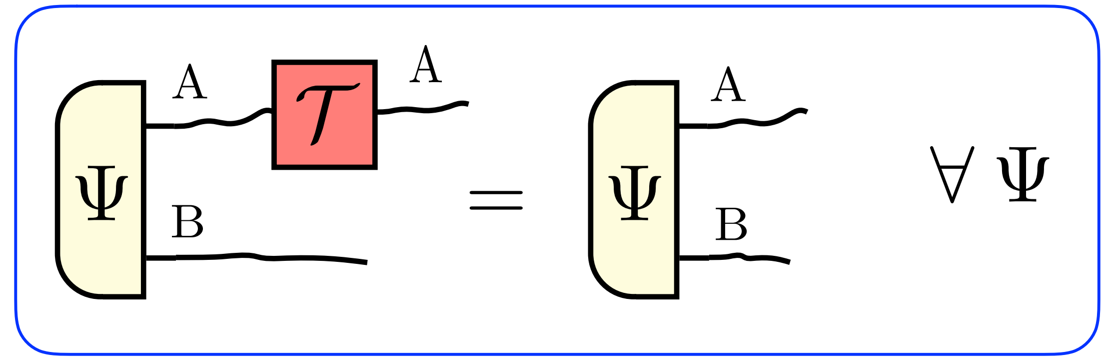

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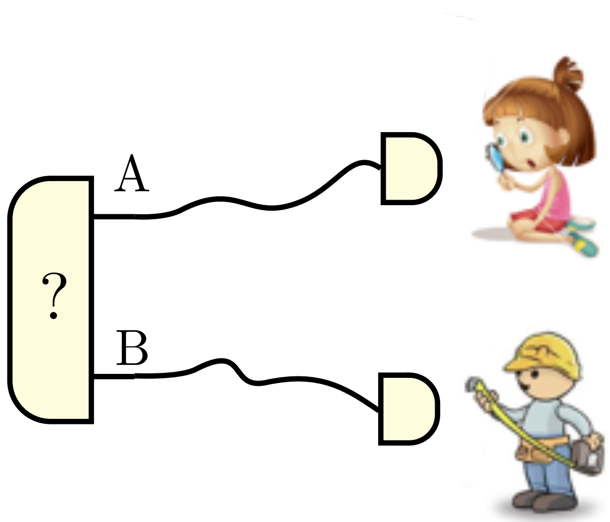

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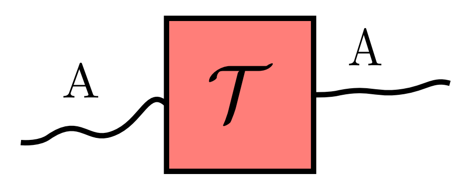

fully characterised by its local action

However, in general, disturbance cannot be checked locally

Outline

1. Framework: operational probabilistic theories

Definition of *non-disturbing test*

Definition of *no-information test*

2. No-information without disturbance

Necessary and sufficient (and only sufficient) conditions

3. Information without disturbance

Structure theorem for probabilistic theories

Theory with full-information without disturbance $\Longrightarrow$ all systems are classical

4. Overview: *no-information without disturbance* vs *local tomography* and *purification*

Operational probabilistic theories

Hardy, L. quant-ph/0101012 (2001)

CDP, Phys. Rev. A 84, 012311 (2011)

Systems:

Operational probabilistic theories

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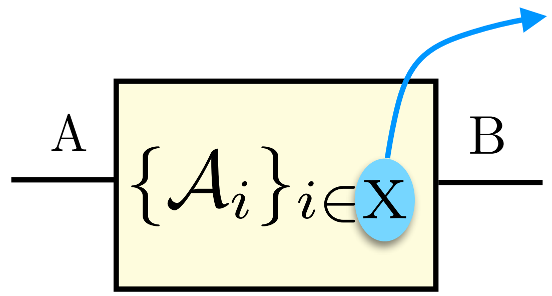
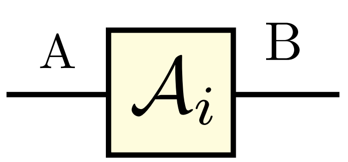
Test:



finite set of outcomes

collection of **events:**



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"quantum instrument"
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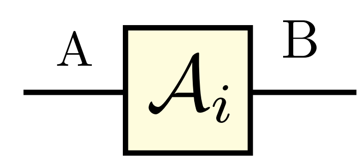

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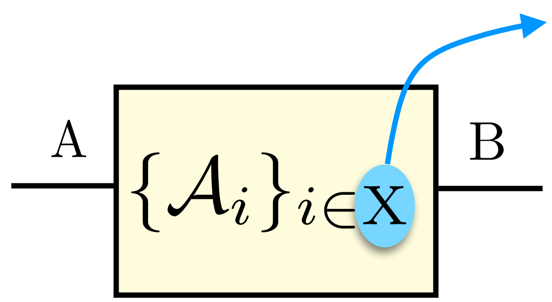

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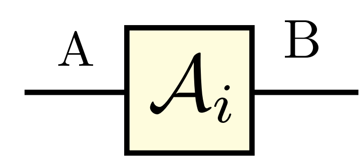
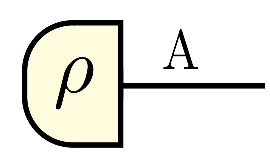
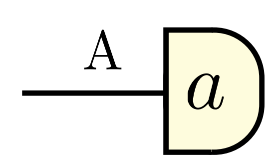


Preparation test: collection of **states**

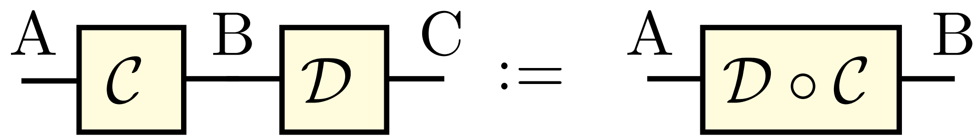


Observation test: collection of **effects**
(measurement)



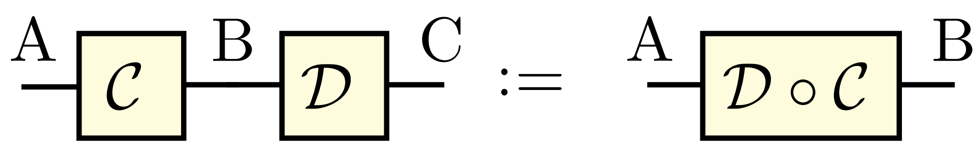
in sequence

Composition:

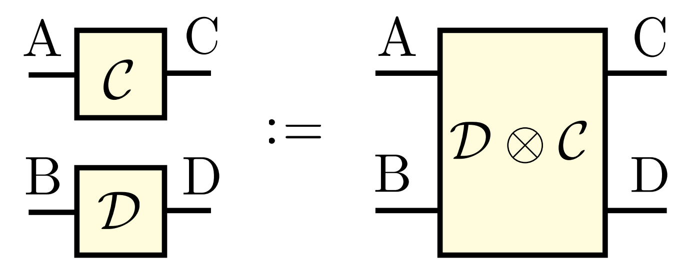


Composition:

in sequence



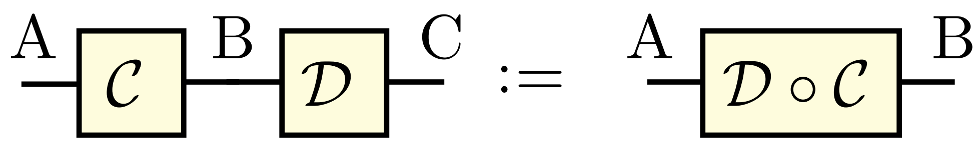
in parallel

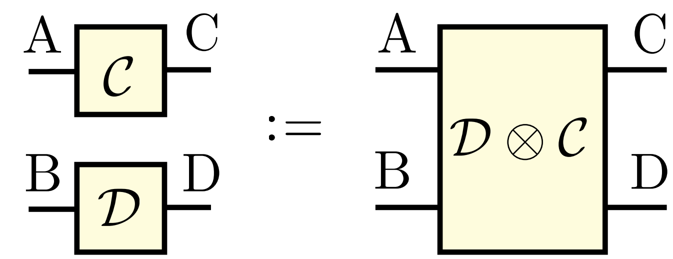


in sequence

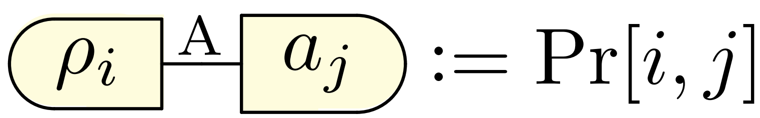
in parallel

Composition:





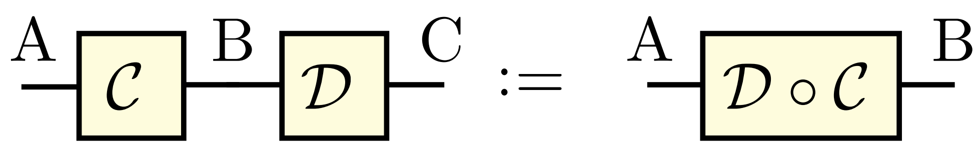
Probabilistic structure:

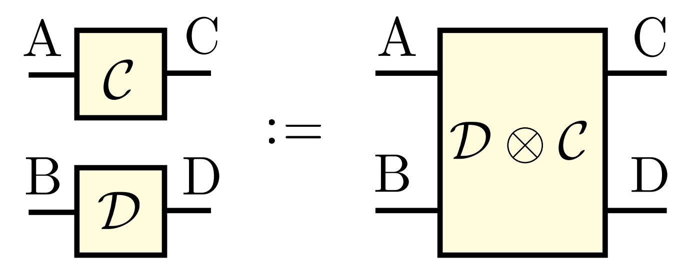


in sequence

in parallel

Composition:





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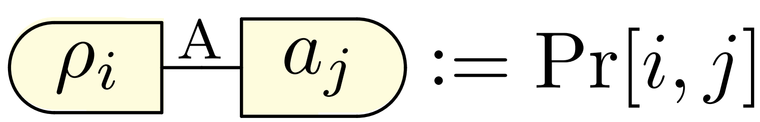
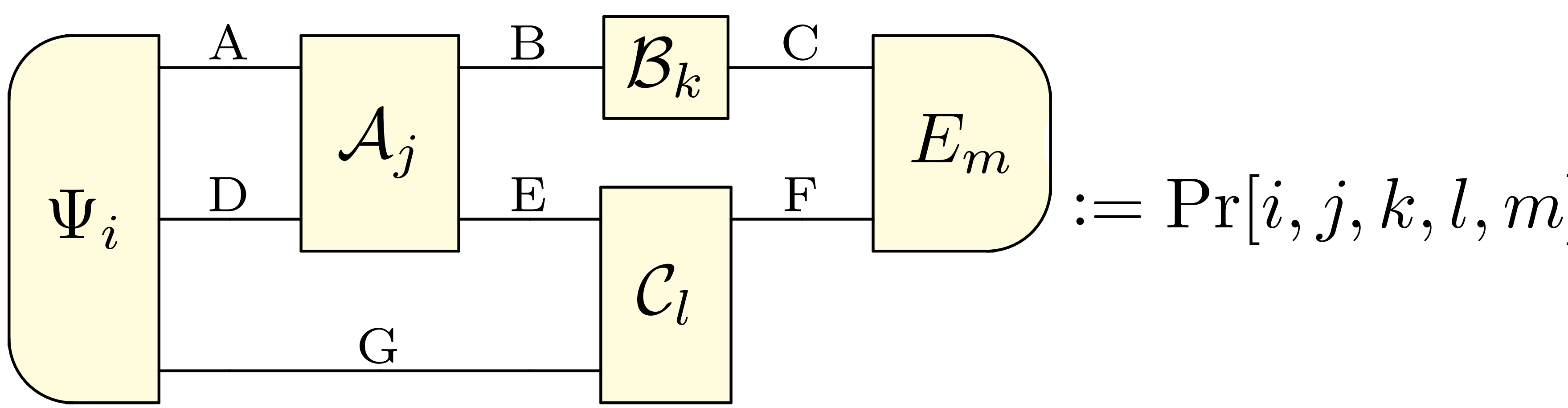
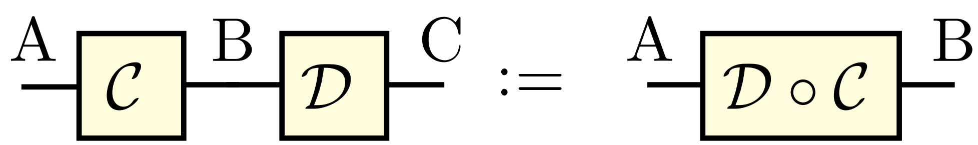




Composition:

in sequence



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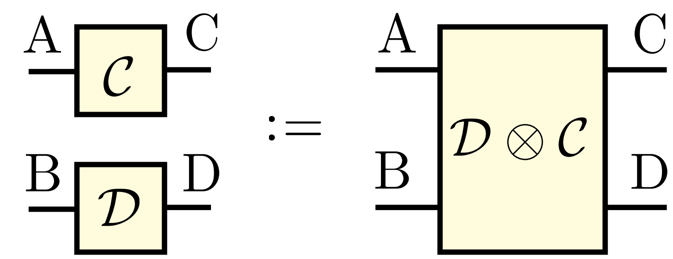

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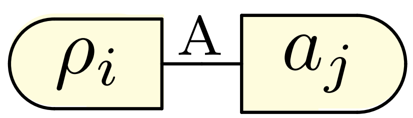
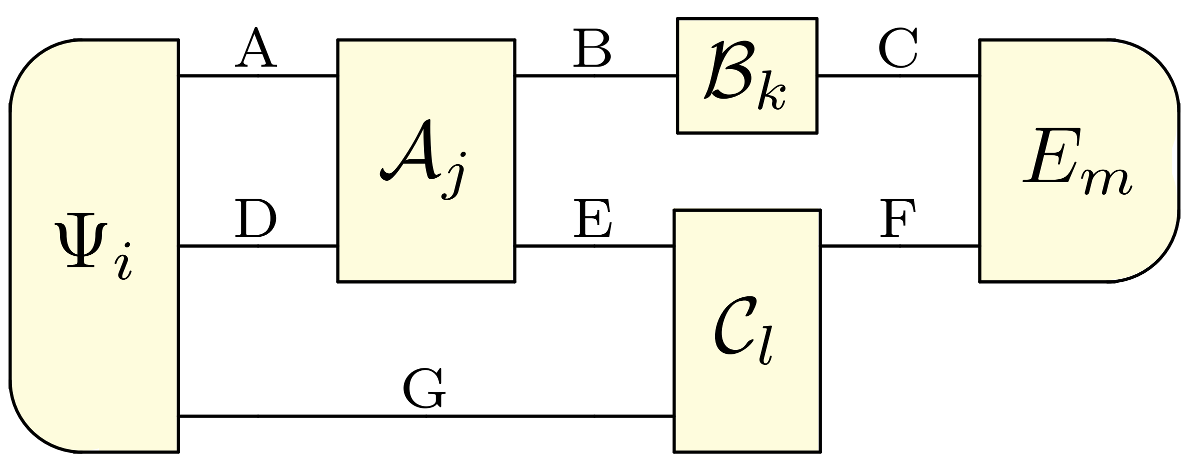


$:= \Pr[i, j]$

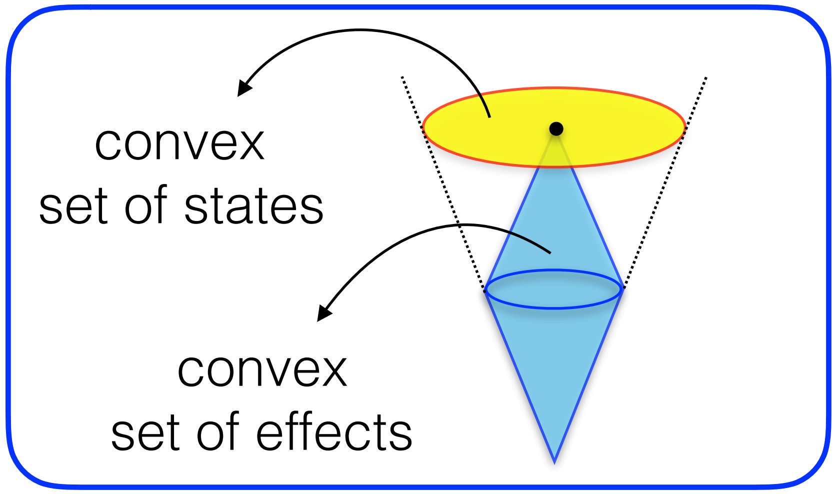


$:= \Pr[i, j, k, l, m]$

Some properties of quantum theory

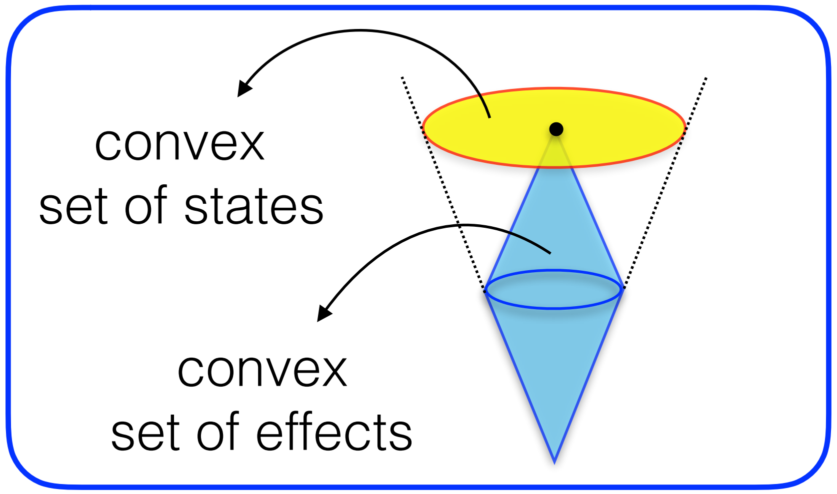
Some properties of quantum theory

Convexity

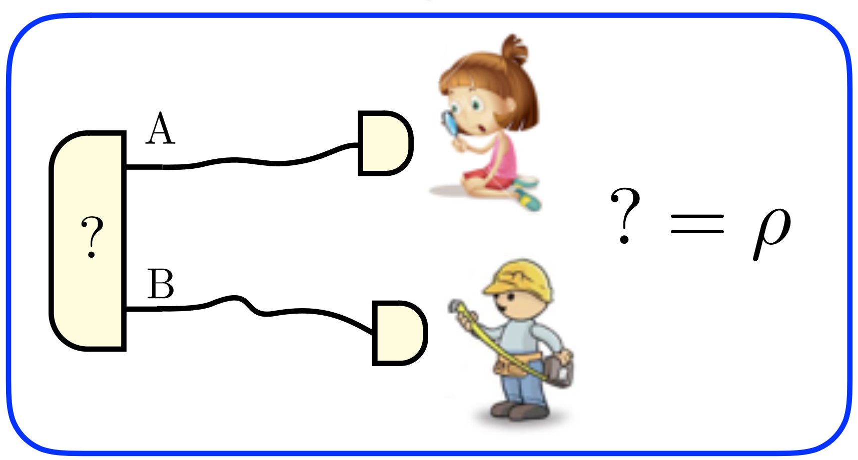


Some properties of quantum theory

Convexity



Local tomography



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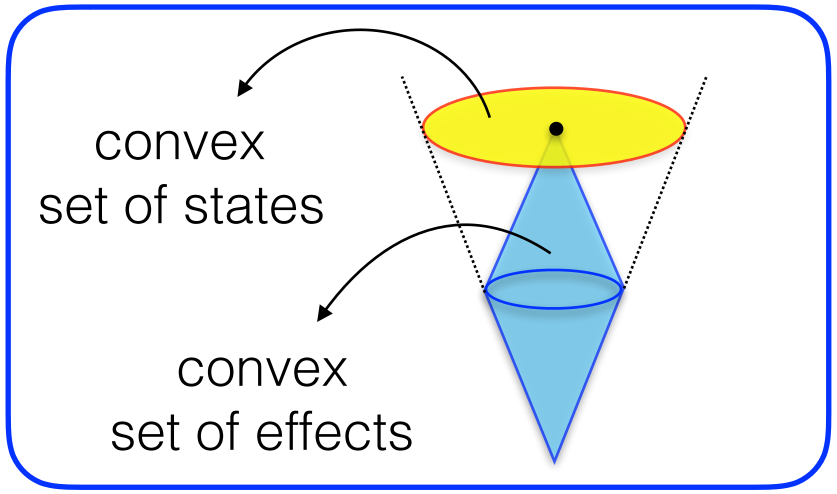

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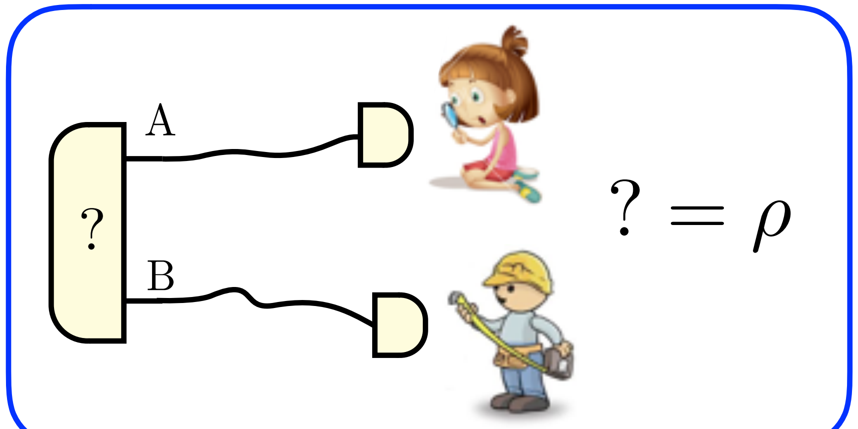


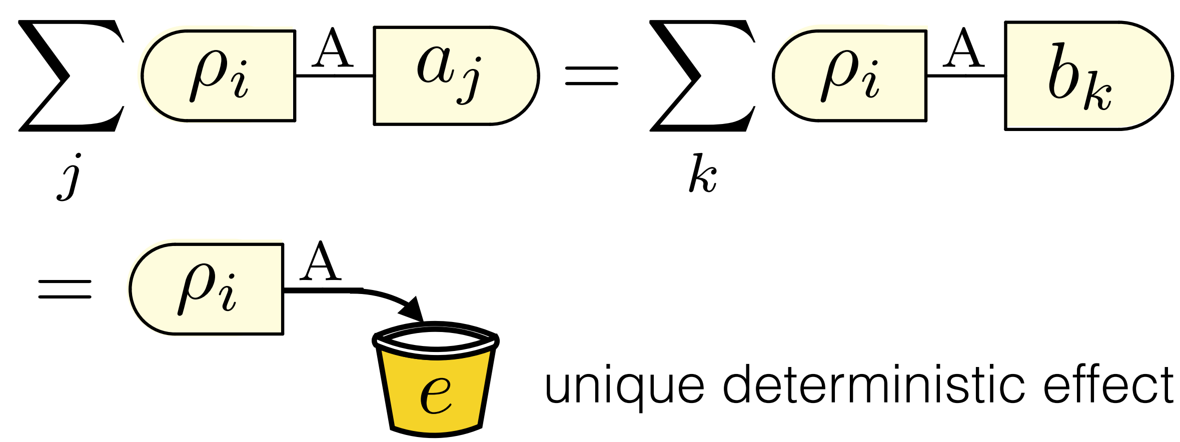
Causality

Prob. of preparations is independent of the choice of observations

$\{\rho_i\}$ prep. test

$\{a_j\}$ obs. test

$\{b_k\}$ obs. test



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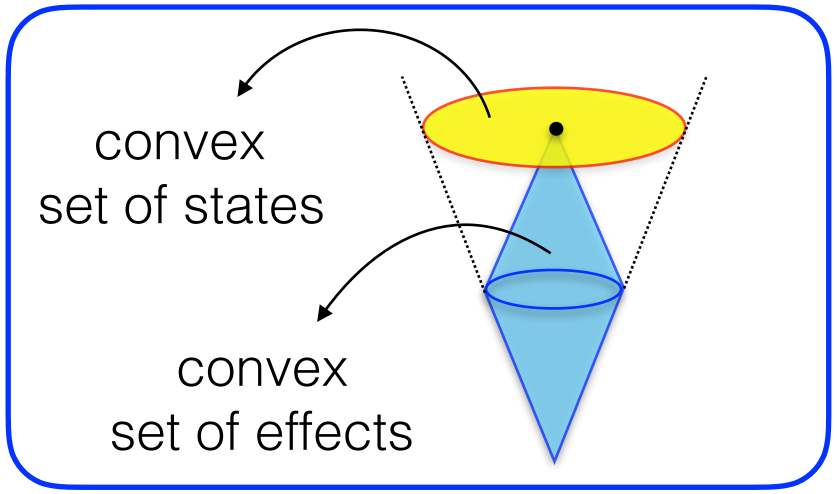

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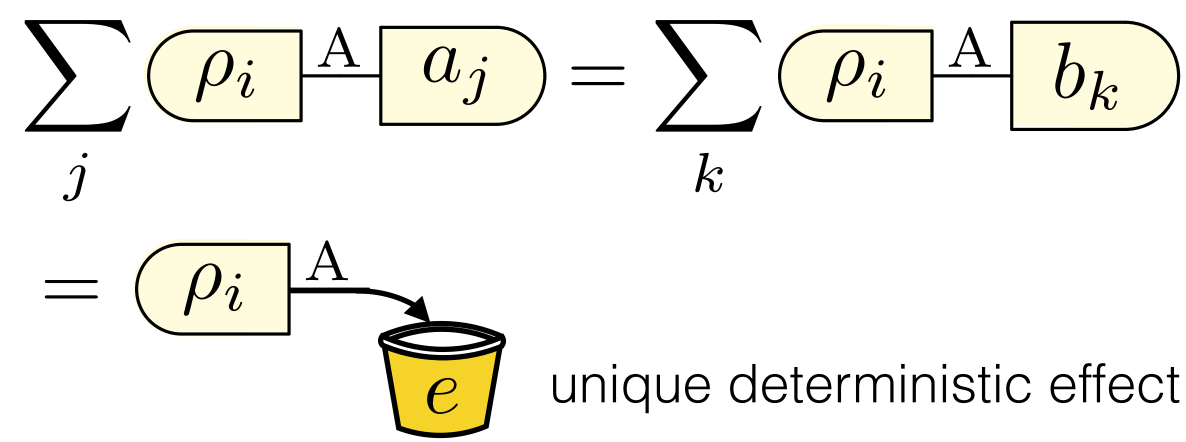
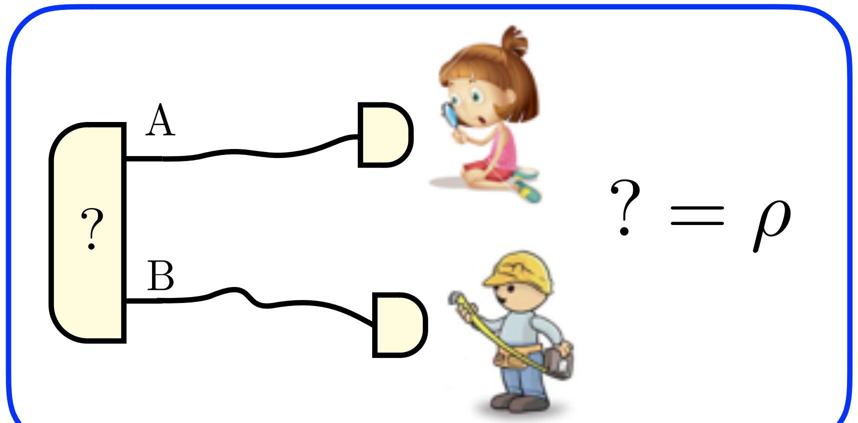
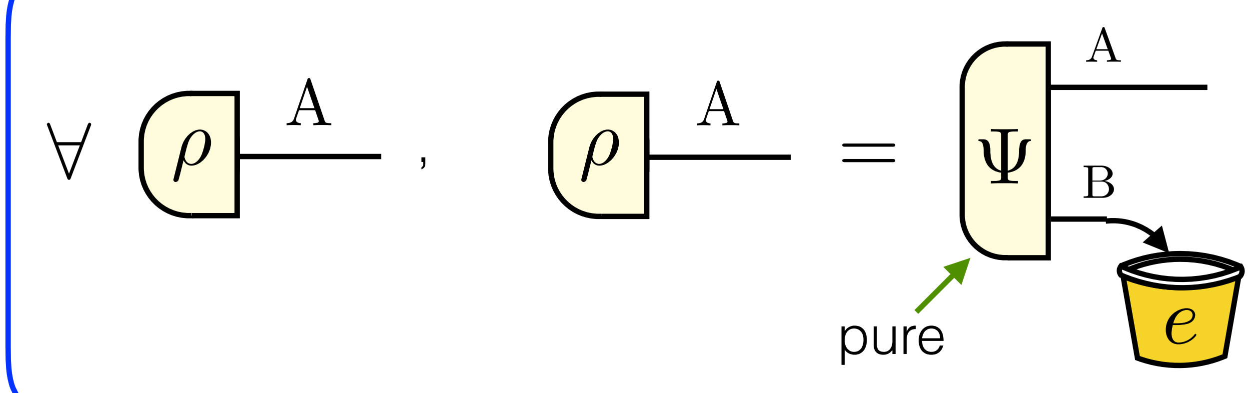


Local tomography



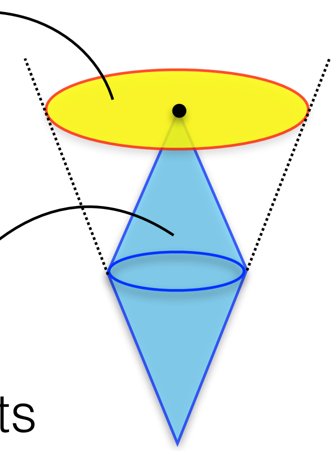
Purification



Some properties of quantum theory

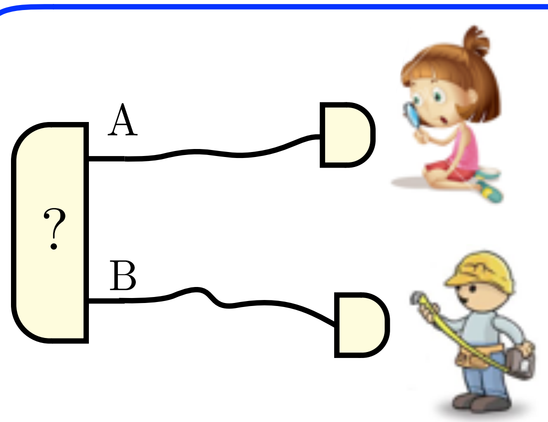
Convexity

convex
set of states



convex
set of effects

Local tomography



$? = \rho$

Causality

Prob. of preparations is independent of the choice of observations

$\{\rho_i\}$ prep. test

$\{a_j\}$ obs. test

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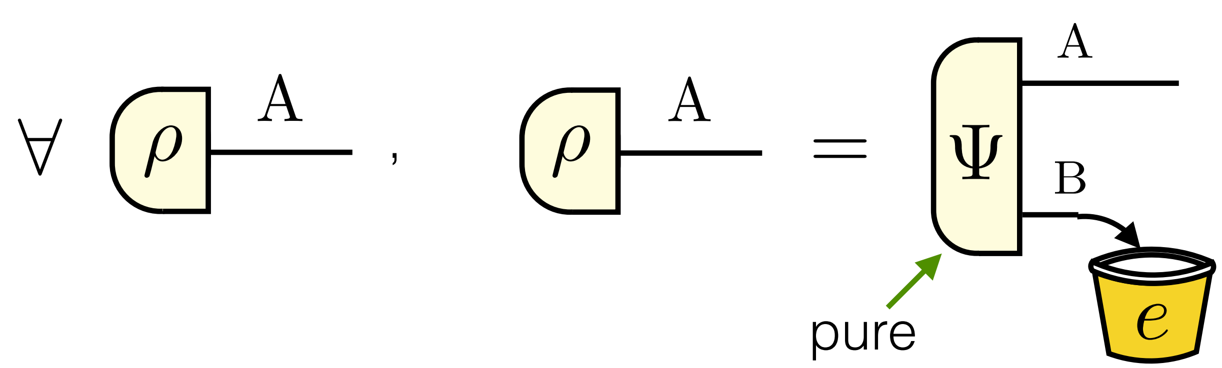
$$\sum_j \rho_i \xrightarrow{A} a_j = \sum_k \rho_i \xrightarrow{A} b_k$$

$$= \rho_i \xrightarrow{A} e$$

unique deterministic effect

Purification



....not assumed in the following

Non-disturbing test

Consider a test of a theory

Non-disturbing test

Consider a test of a theory

$$\overset{\mathrm{A}}{\longrightarrow}\boxed{\{\mathcal{A}_i\}_{i\in\mathrm{X}}}\overset{\mathrm{A}}{\longrightarrow}$$

When the test is non-disturbing?

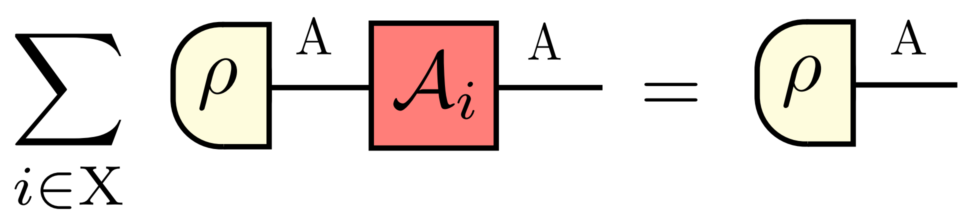
Non-disturbing test

Consider a test of a theory

When the test is non-disturbing?

Usual definition (“quantum” definition)

$\{\mathcal{A}_i\}_{i\in \mathrm{X}}$ is non-disturbing if



$\forall\, \rho \in \mathrm{St(A)}$

set of states of system A

Non-disturbing test

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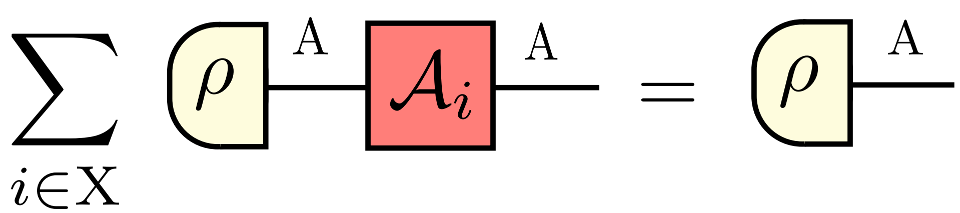

$\forall \rho \in \mathrm{St}(\mathrm{A})$

set of states of system A

this is inconsistent for theories without **LOCAL TOMOGRAPHY**

An example: Fermions

Fock space: $\mathcal{F} = \mathcal{F}_e \bigoplus \mathcal{F}_o$

even sector

odd sector

PARITY SUPERSELECTION

No even-odd superimposition

An example: Fermions

Fock space: $\mathcal{F} = \mathcal{F}_e \oplus \mathcal{F}_o$

even sector

odd sector

PARITY SUPERSELECTION

No even-odd superimposition

1 local mode

$$|0\rangle$$

$$|1\rangle$$

$$\cancel{\alpha\,|0\rangle + \beta\,|1\rangle}$$

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2 local modes

$$\alpha|10\rangle + \beta|01\rangle$$
$$\alpha|00\rangle + \beta|11\rangle$$
$$\cancel{\alpha\,|00\rangle + \beta\,|10\rangle}$$

An example: Fermions

Fock space: $\mathcal{F} = \mathcal{F}_e \oplus \mathcal{F}_o$

$\mathcal{F}_e$: even sector

$\mathcal{F}_o$: odd sector

PARITY SUPERSELECTION

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Any state is of the form:

$$\rho = \left(\begin{array}{c|c} p\rho_e & 0 \\ \hline 0 & (1-p)\rho_o \end{array}\right)$$

An example: Fermions

Fock space: $\mathcal{F} = \mathcal{F}_e \oplus \mathcal{F}_o$

even sector

odd sector

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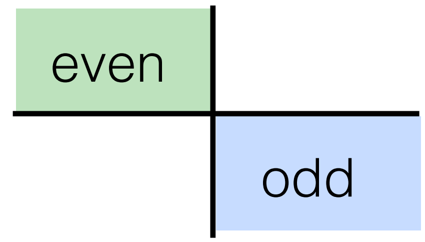
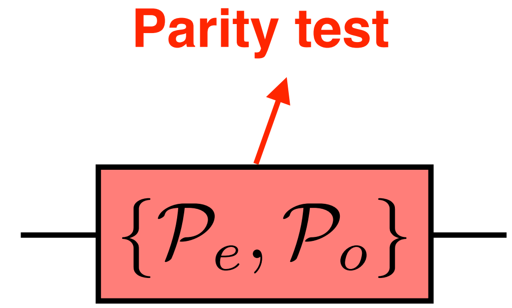
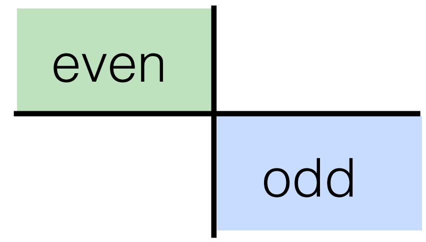
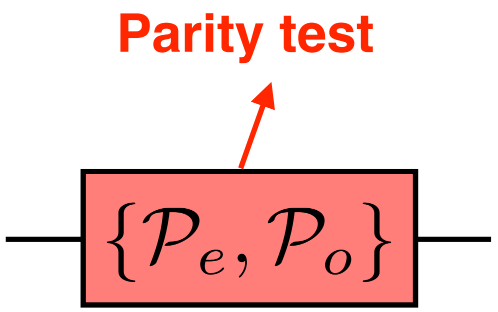
PARITY SUPERSELECTION => **non-local tomography**

G. M. D'Ariano, F. Manessi, P. Perinotti and A. Tosini, IJMPA (2014)

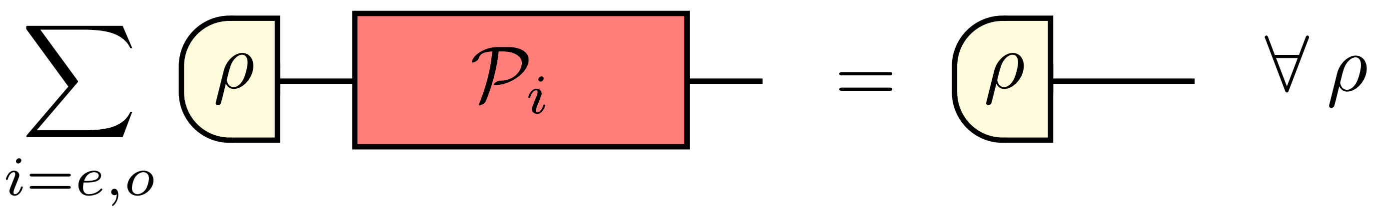

even
odd

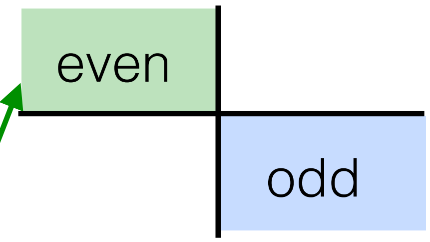

Parity test
$\{\mathcal{P}_e, \mathcal{P}_o\}$

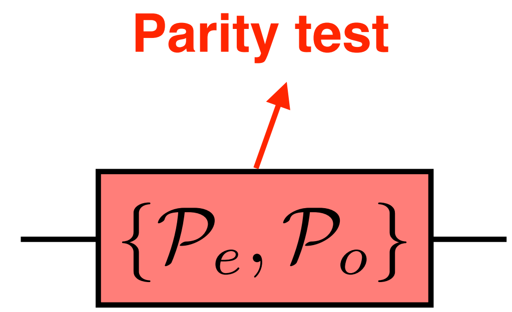




Parity test do not disturb locally



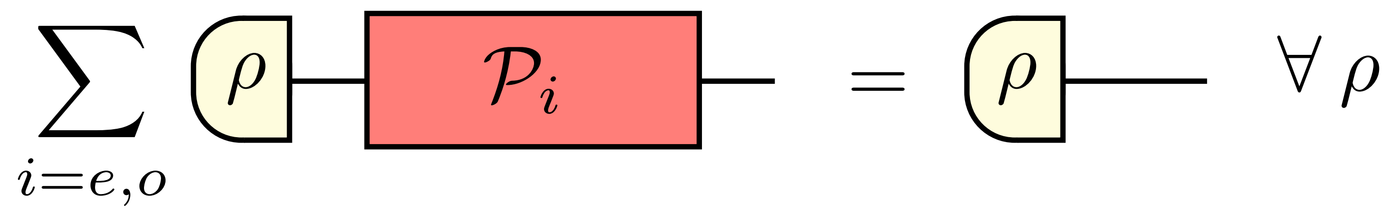


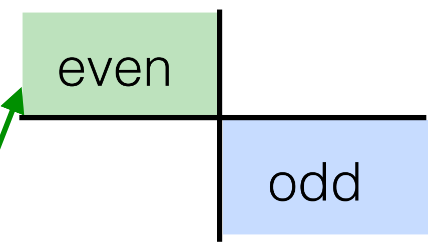


mixed

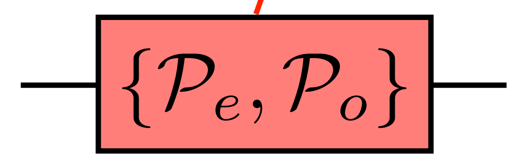
$$\rho = \frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1|)$$

Parity test do not disturb locally





Parity test



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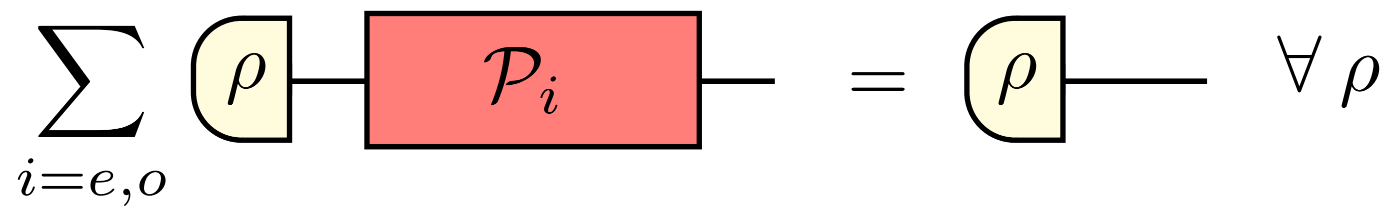
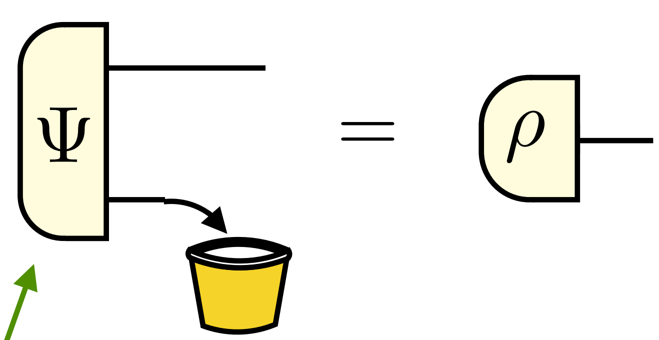
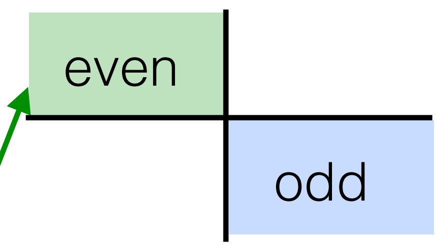


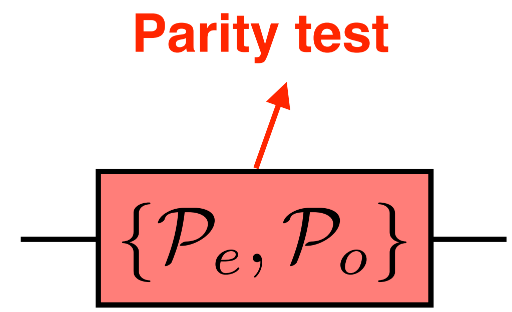
Purification of ρ

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$



pure





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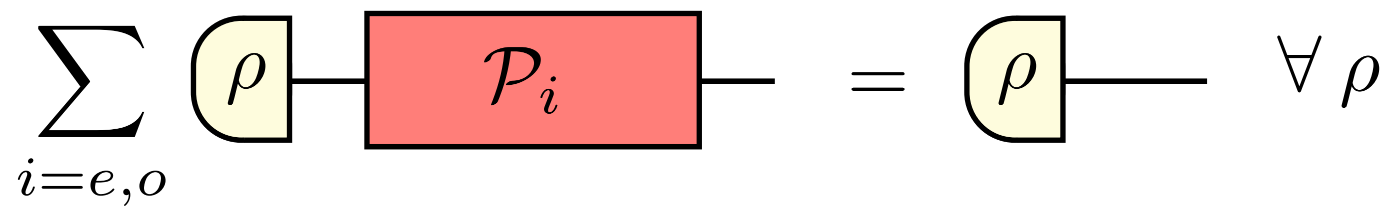

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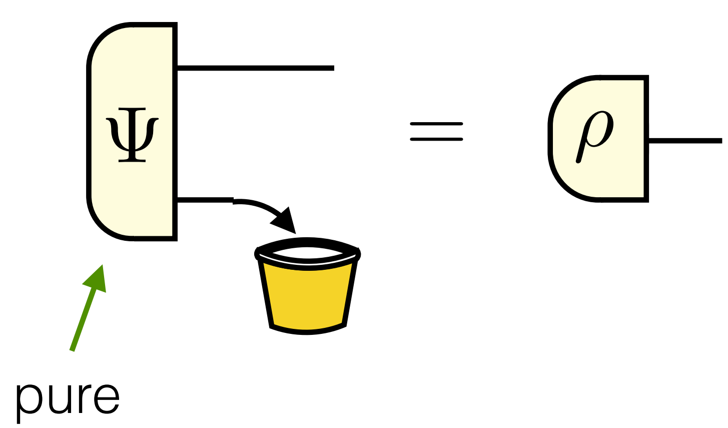
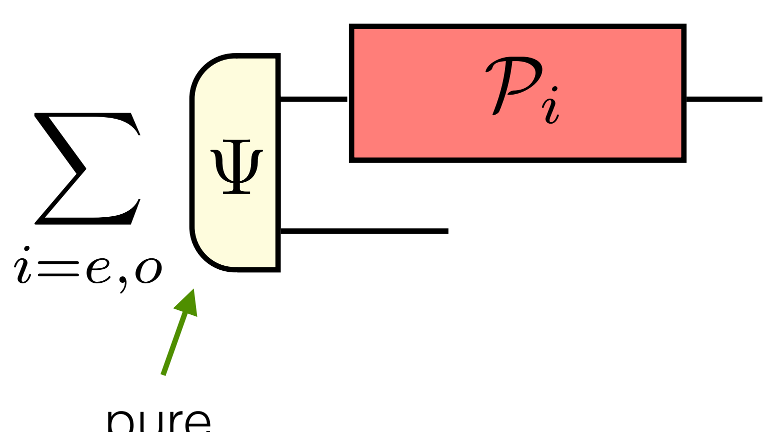


Parity test disturbs purifications



$$= \frac{1}{2}(|00\rangle\langle 00| + |11\rangle\langle 11|)$$

mixed

Consider a test of a theory

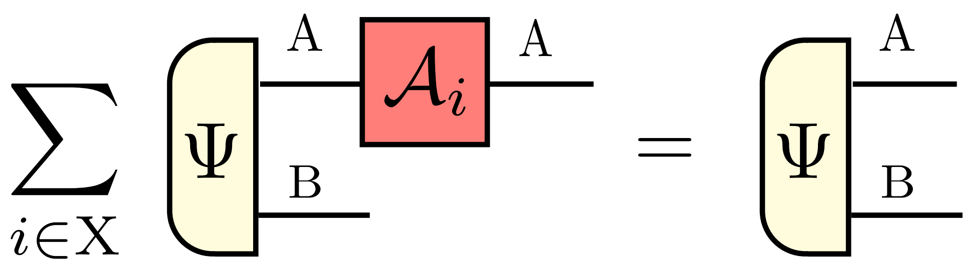
When the test is non-disturbing?

Consider a test of a theory

When the test is non-disturbing?

Definition (non-disturbing test):

The test $\{\mathcal{A}_i\}_{i\in \mathrm{X}}$ is non-disturbing if



$\forall\, \Psi \in \mathrm{St}(\mathrm{AB})$

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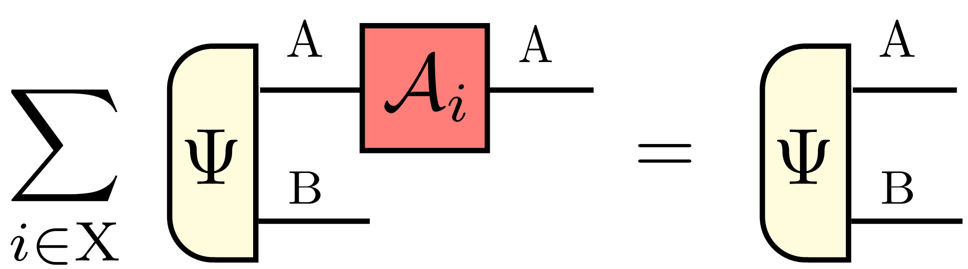
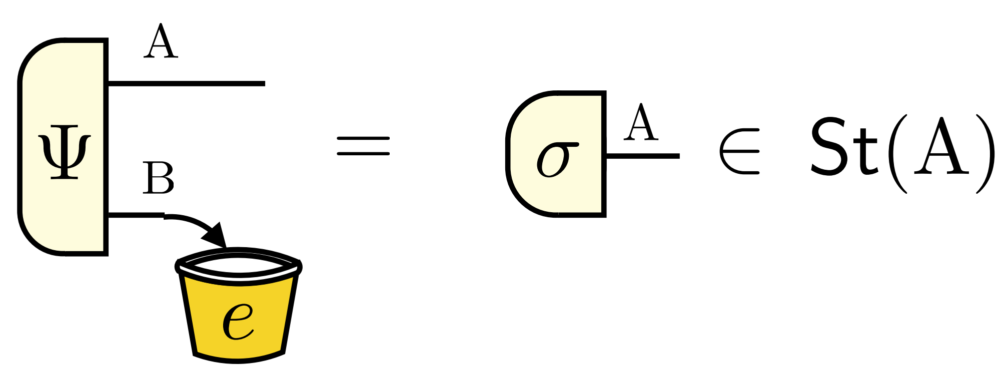


$\forall\, \Psi \in \mathsf{St}(\mathrm{AB})$

$\forall\, \Psi$ *dilation of a state* in $\mathsf{St}(\mathrm{A})$



a deterministic effect

(not unique for *non-causal* theories)

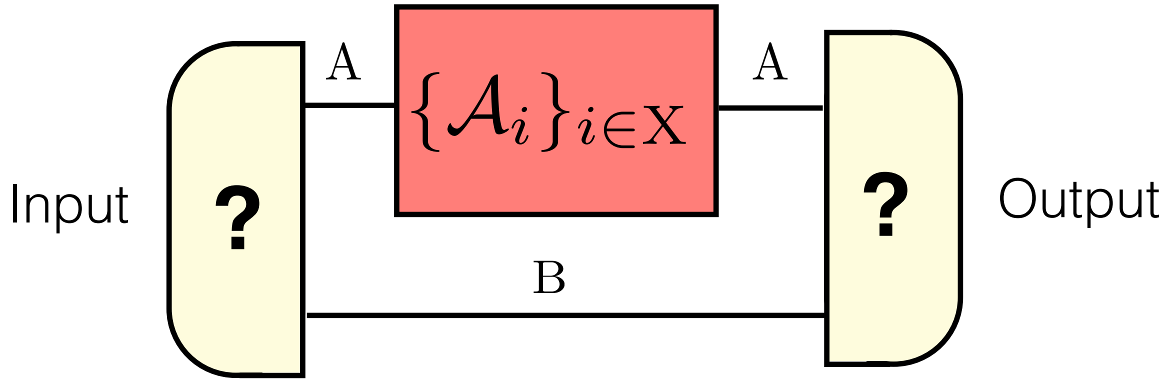
No-information test

Consider a test of a theory

when the test provides information?

No-information test

Consider a test of a theory



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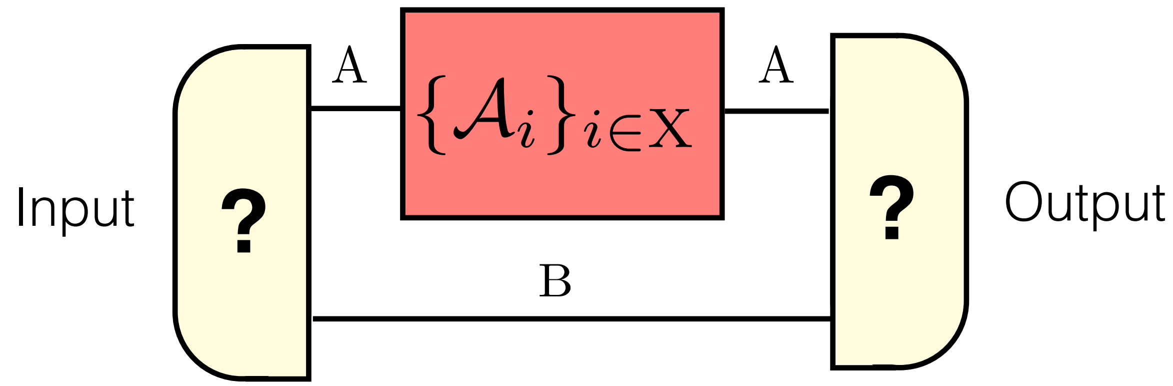


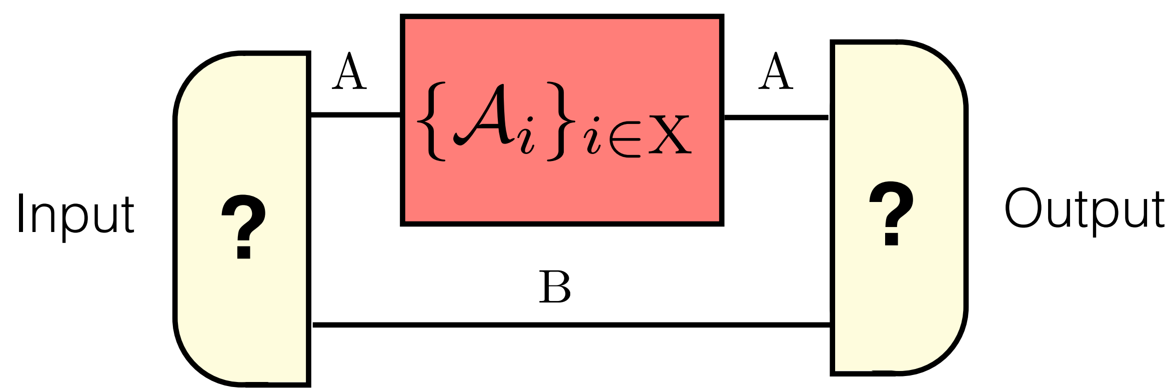
when the test $\boxed{\text{provides information?}}$

on the input

on the output

No-information test

Consider a test of a theory



when the test provides information?

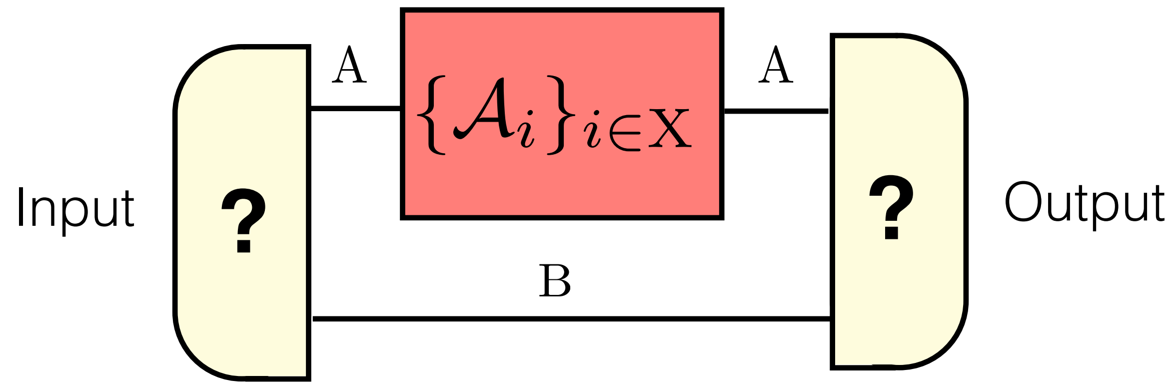
on the input

on the output

Focus on the Input

No-information test

Consider a test of a theory



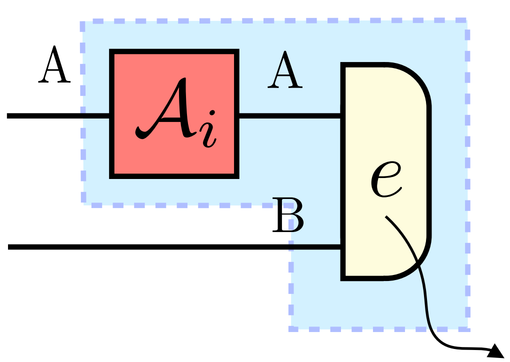
when the test $\boxed{\text{provides information?}}$

on the input

on the output

Focus on the Input

I build up a measurement using the test

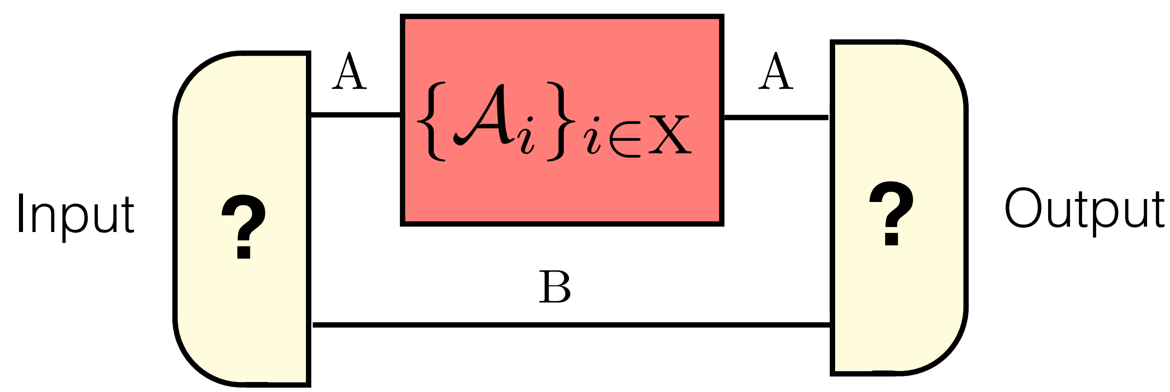


a deterministic effect

not unique in general

No-information test

Consider a test of a theory



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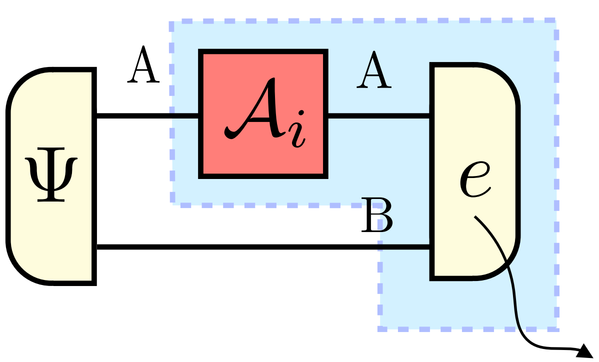

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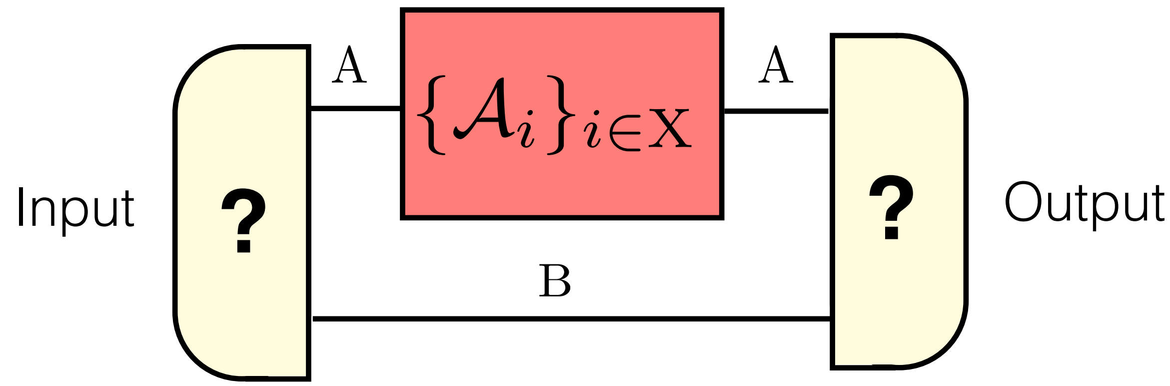

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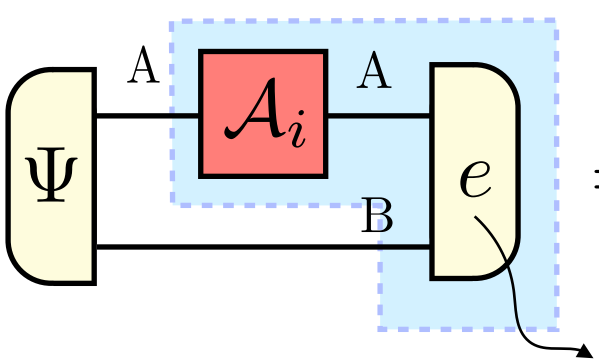


$$= p_i^e(\Psi)$$

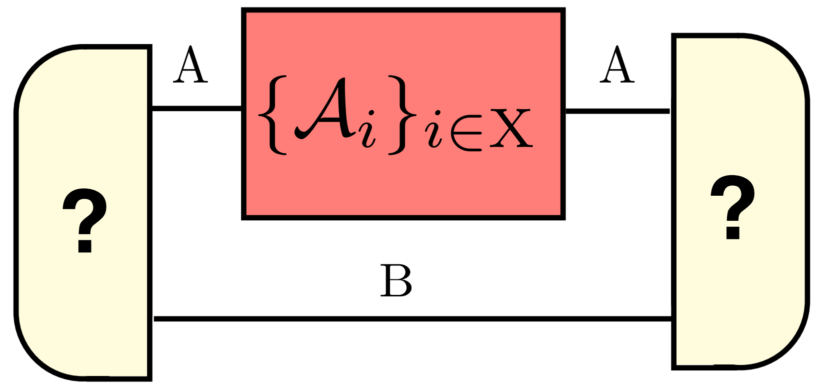
a deterministic effect

not unique in general

No-information test

Consider a test of a theory

Input



Output

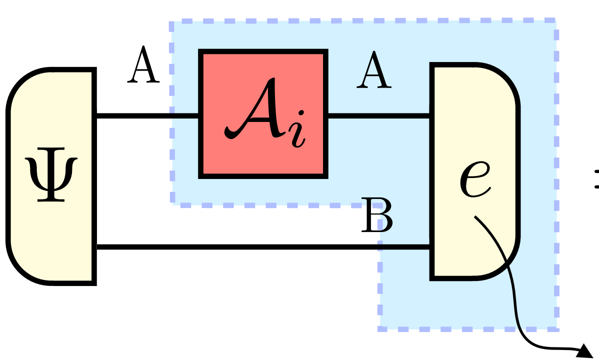
when the test provides information?

on the input

on the output

Focus on the Input

I build up a measurement using the test



$$= p_i^e(\Psi)$$

Info on the input if for some deterministic effect

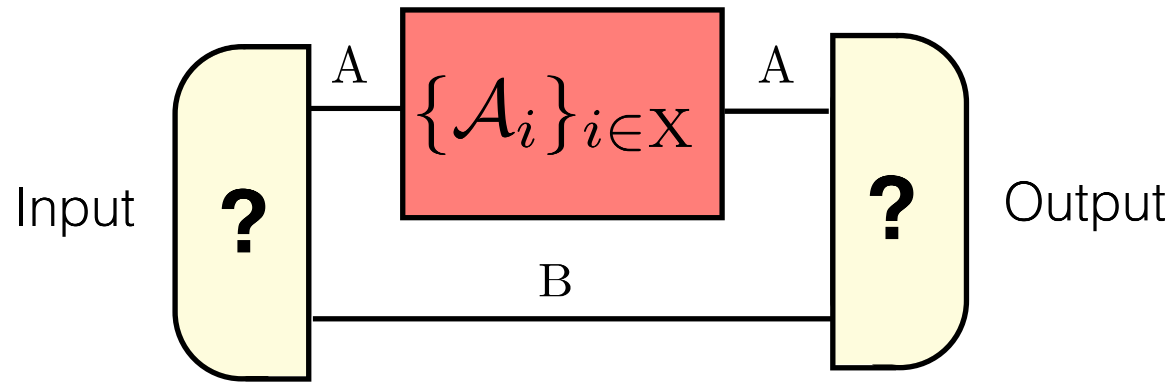
outcome prob. depends on the state

a deterministic effect

not unique in general

No-information test

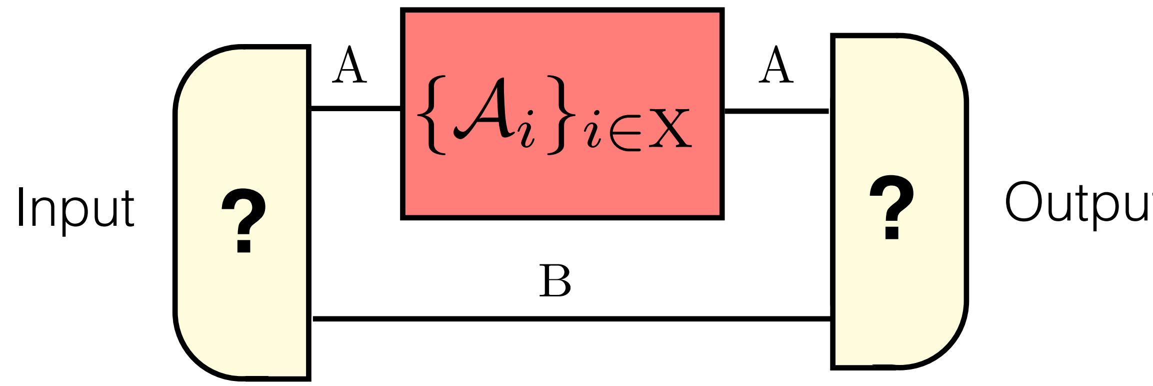
Consider a test of a theory



$\{\mathcal{A}_i\}_{i\in\mathrm{X}}$ provides information if

No-information test

Consider a test of a theory



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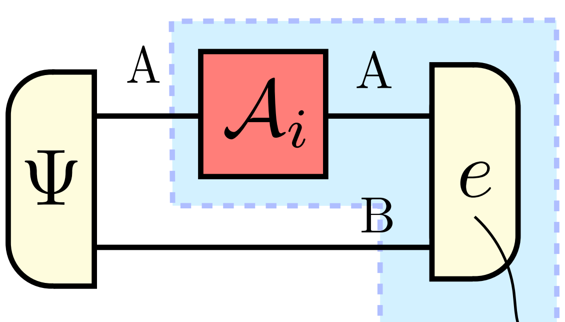
outcome prob. depends on the state

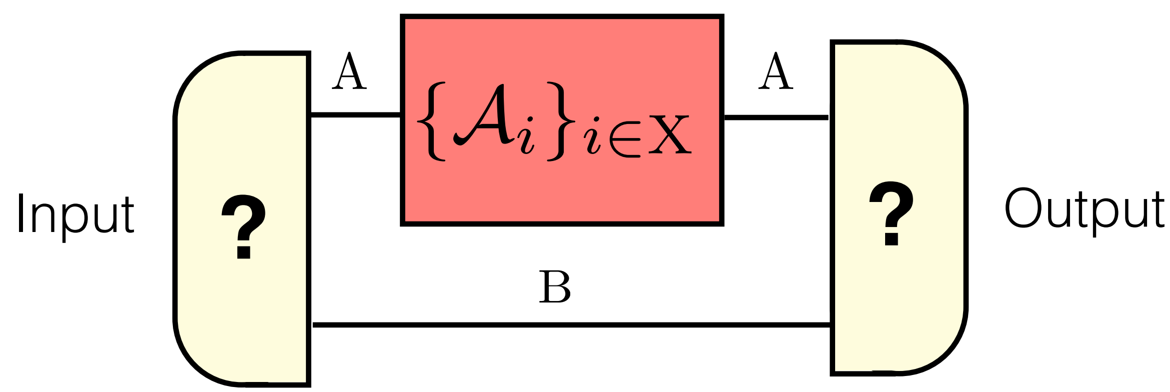


$= p_i^e(\Psi)$

exists a
deterministic effect

No-information test

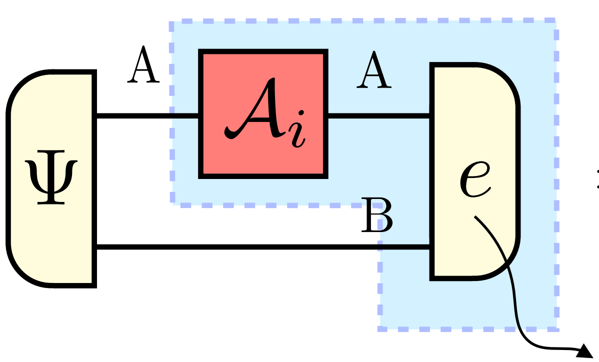
Consider a test of a theory



$\{\mathcal{A}_i\}_{i\in \mathrm{X}}$ provides information if

information on the input

outcome prob. depends on the state



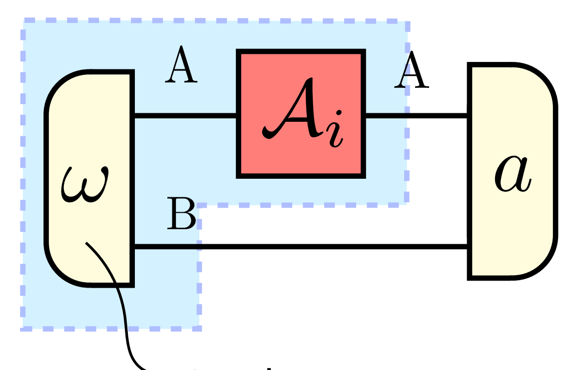
$= p_i^e(\Psi)$

exists a
deterministic effect

or

information on the output

outcome prob. depends on the effect



$= q_i^\omega(a)$

exists a
deterministic state

Consider a test of a theory

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Definition: A test $\{\mathcal{A}_i\}_{i\in\mathrm{X}}$ does not provide information if

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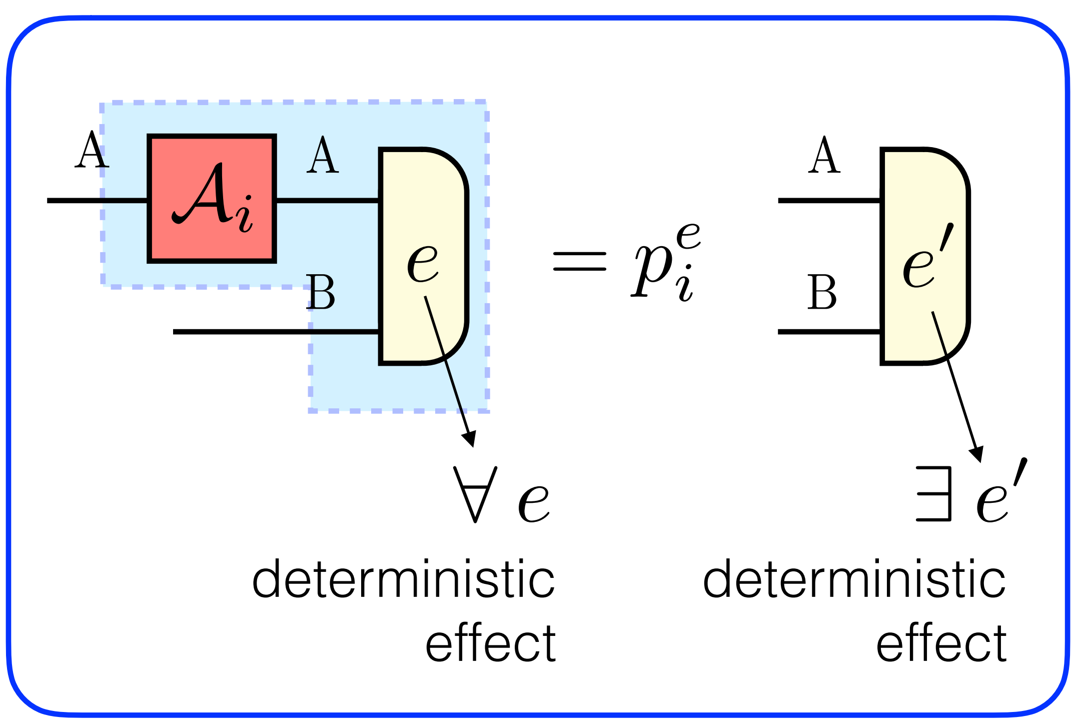

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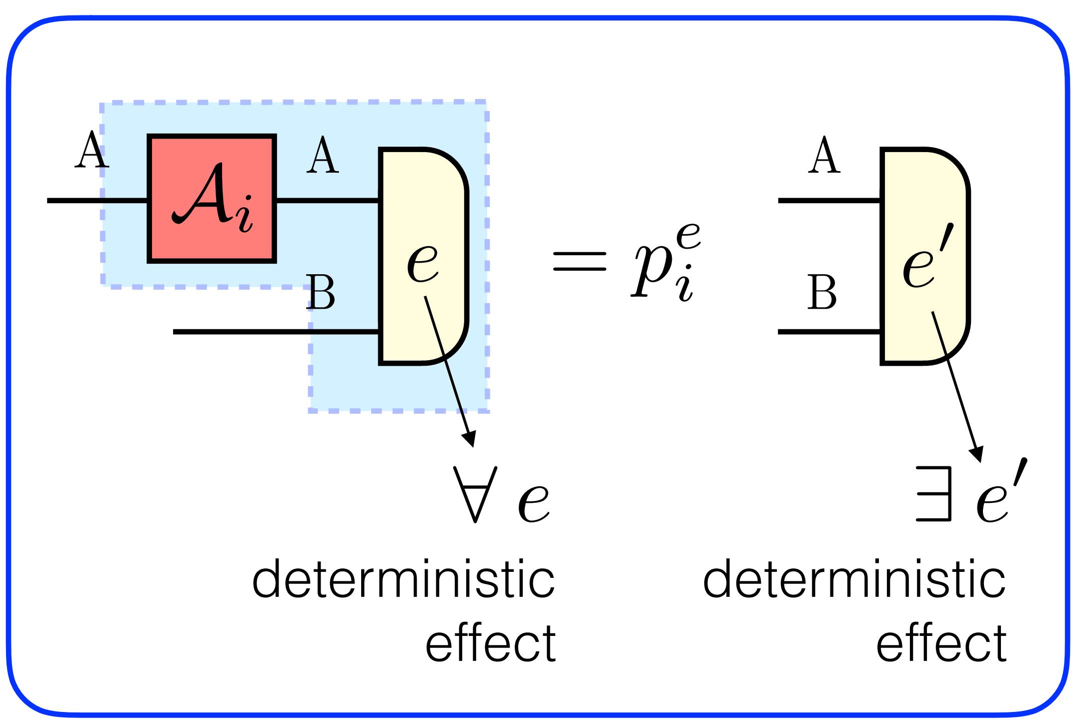
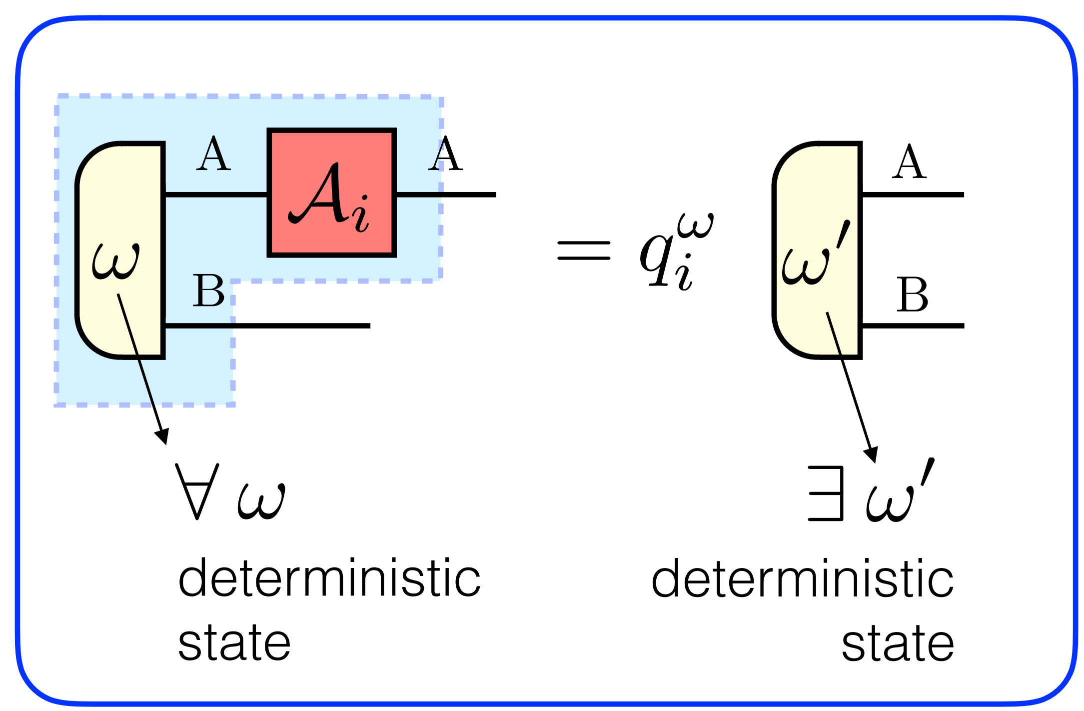


and

no-information on the output



No-info without disturbance

Definition: a theory satisfies no-information without disturbance (NIWD) if

$$\{\mathcal{A}_i\}_{i\in\mathrm{X}} \ \textbf{non-disturbing} \Rightarrow \{\mathcal{A}_i\}_{i\in\mathrm{X}} \ \textbf{no-information}$$

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Necessary and sufficient condition for NIWD

Theorem: NIWD $\Longleftrightarrow$ the identity transformation is **atomic** for every system of the theory

$$\boxed{\mathcal{I}_{\mathrm{A}} = \sum_i \mathcal{A}_i \ \Rightarrow \ \mathcal{A}_i \propto \mathcal{I}_{\mathrm{A}}}$$

Other necessary and sufficient condition for NIWD

Proposition: NIWD $\iff \forall$ system there exists an atomic transformation which is either left- or right-reversible

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Corollary: Non-local boxes (PR-boxes) satisfy no-information without disturbance

Sketch of the proof

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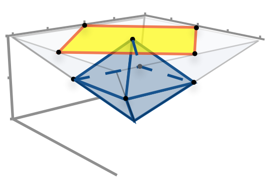
Sketch of the proof

Elementary system: S



Arbitrary system $\mathrm{S}^{\otimes N}$

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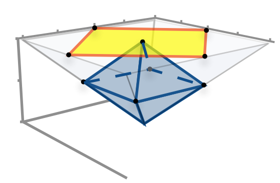

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G. M. D'Ariano and A. Tosini, Quantum Information Processing 9, 95 (2010)

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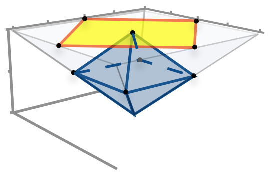

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S. W. Al-Safi and A. J. Short, J. Phys. A: Mathematical and Theoretical 47, 325303 (2014)

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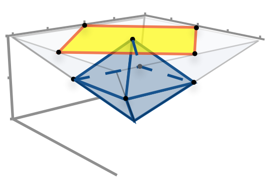

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Parallel composition is atomic due to local tomography

G. M. D'Ariano, F. Manessi, and P. Perinotti, Phys. Scr. 2014, 014013 (2014)

Only sufficient conditions for NIWD

Proposition: Any convex theory with purification satisfies NIWD

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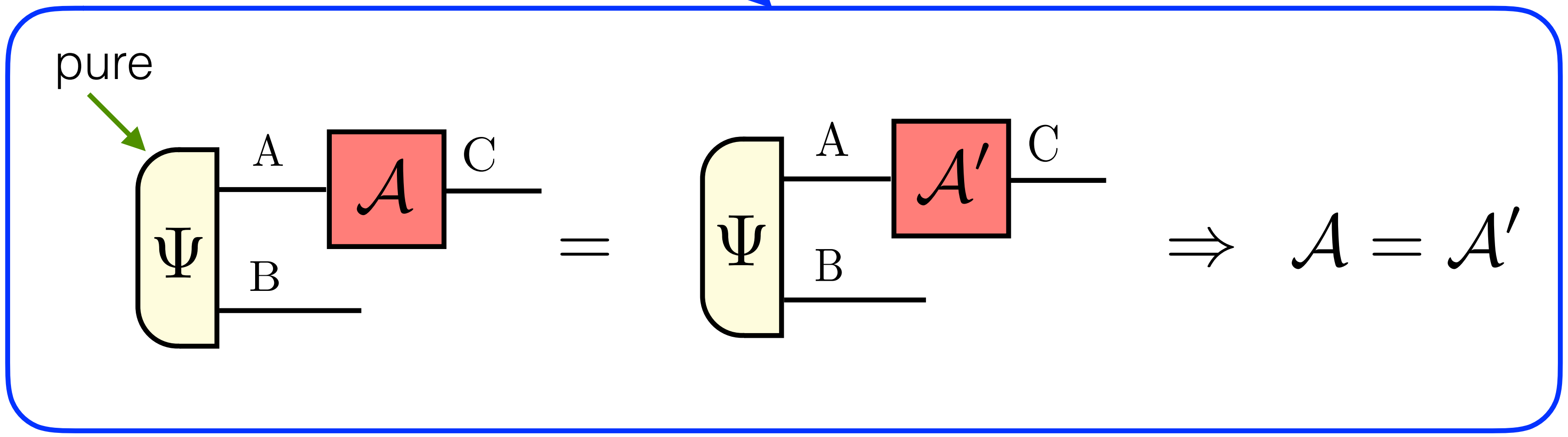
Proposition: If for every system there exists a pure faithful state then the theory satisfies NIWD



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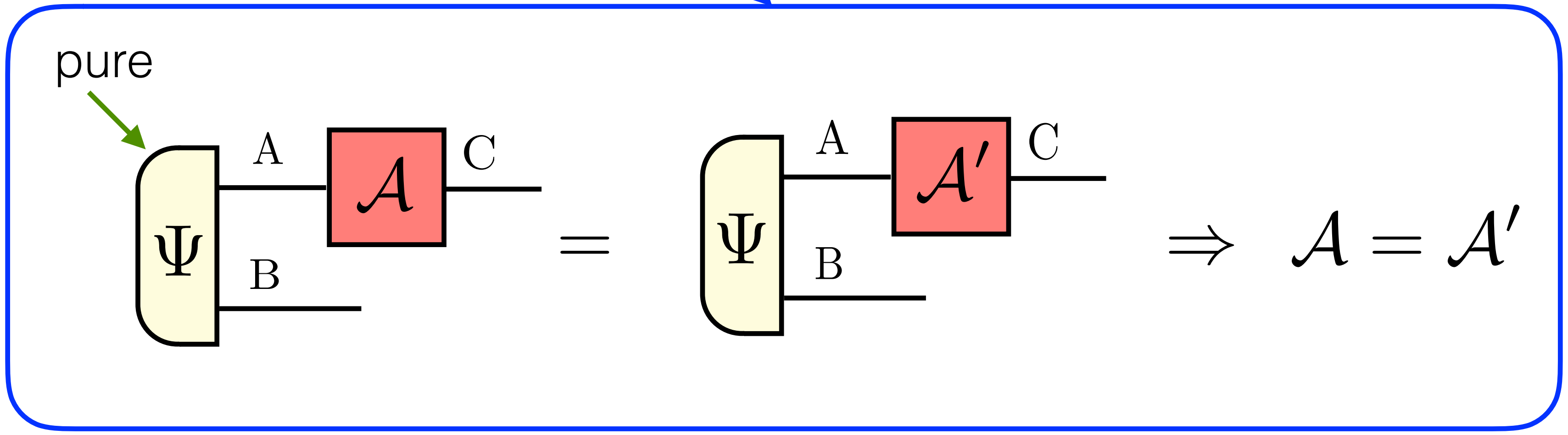

Corollary: Fermionic quantum theory satisfies NIWD

Only sufficient conditions for NIWD

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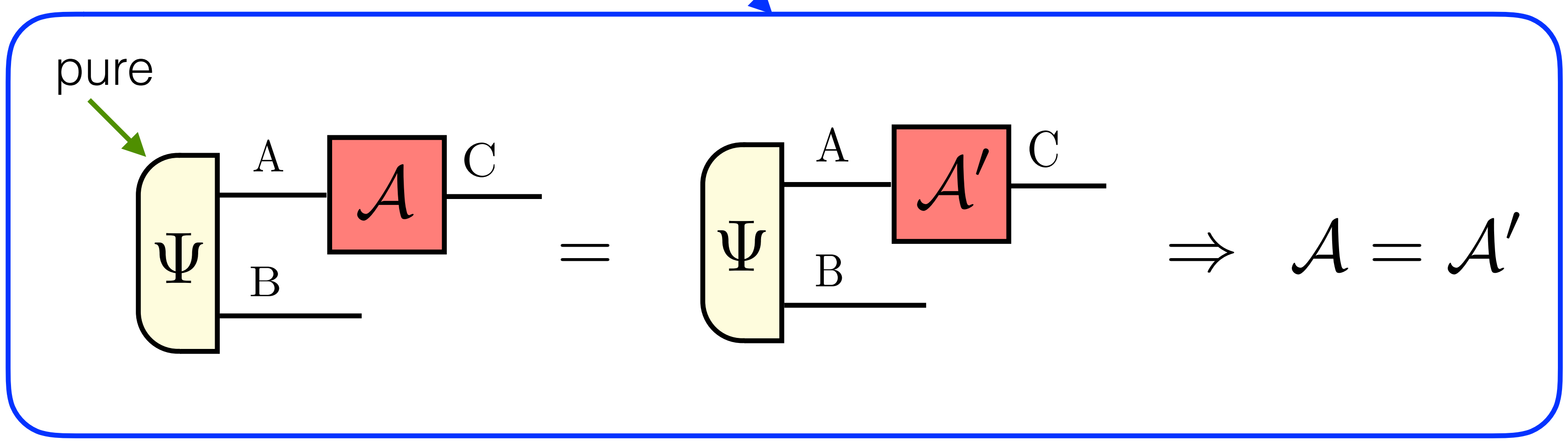
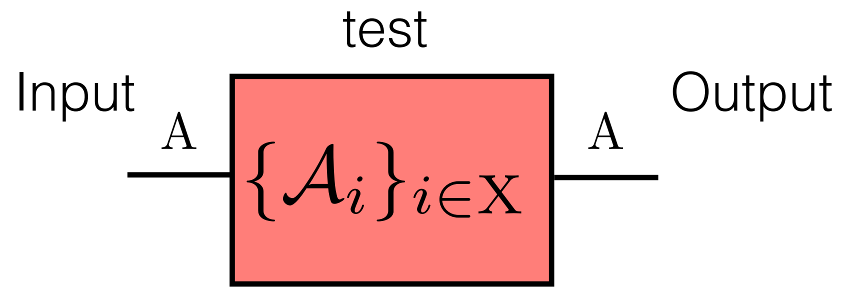


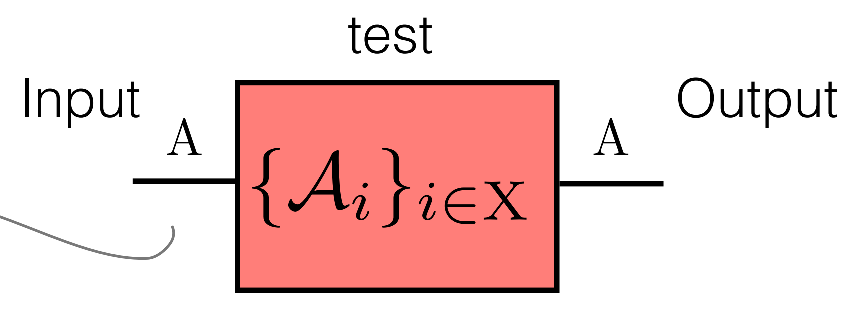
Corollary: Fermionic quantum theory satisfies NIWD

Corollary: Real quantum theory satisfies NIWD

Generalization: restricted resources

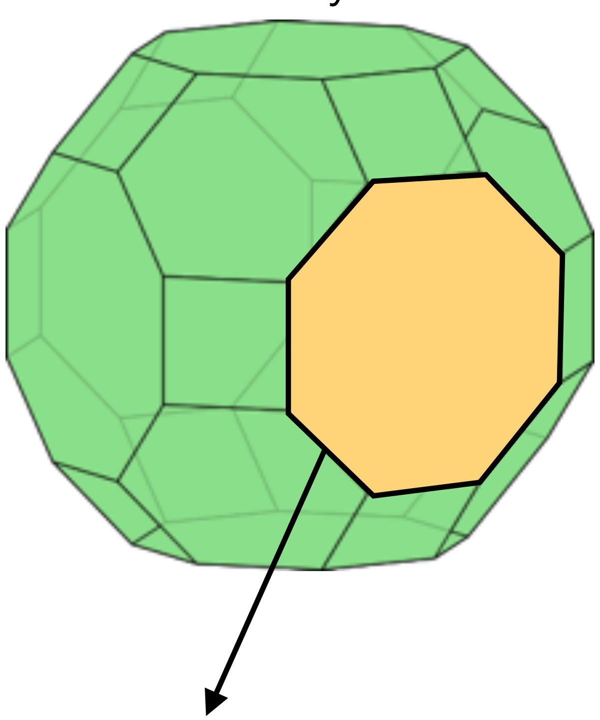


Generalization: restricted resources



Example of restricted input

States of system A



Non-disturbance here?

No-information here?

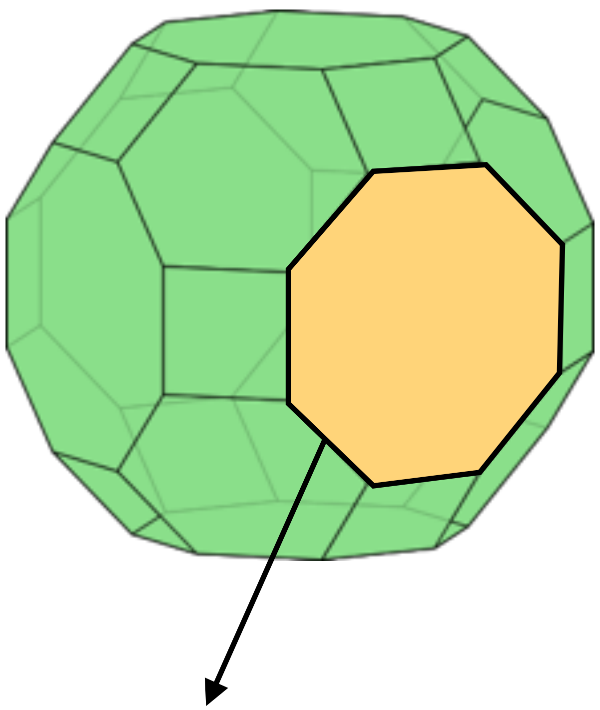
Generalization: restricted resources

test

Input A $\{\mathcal{A}_i\}_{i\in \mathrm{X}}$ A Output

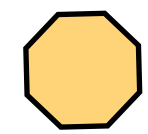
Example of restricted input

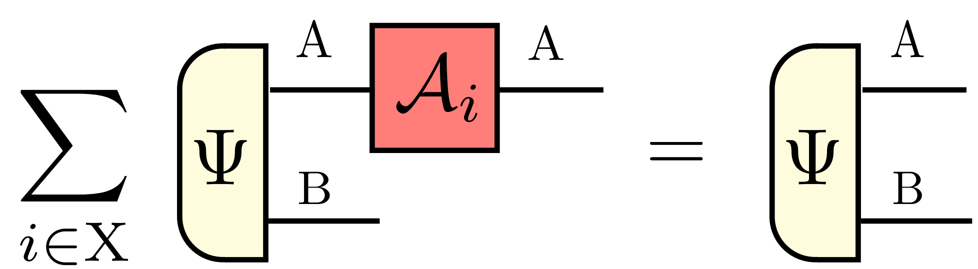
States of system A

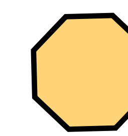


Non-disturbance here?

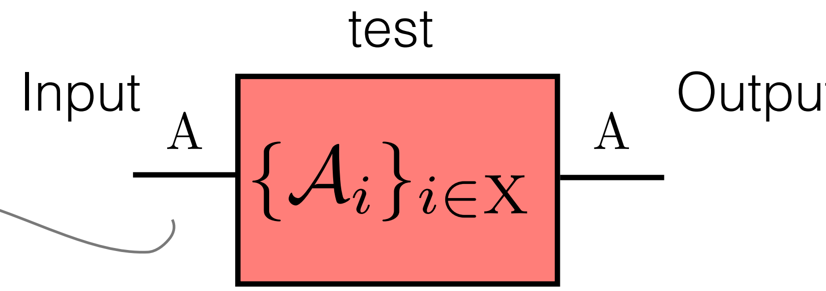
No-information here?

$\{\mathcal{A}_i\}_{i\in \mathrm{X}}$ is non-disturbing upon input of 

if 

$\forall\ \Psi$ *dilation of a state* in 

Generalization: restricted resources



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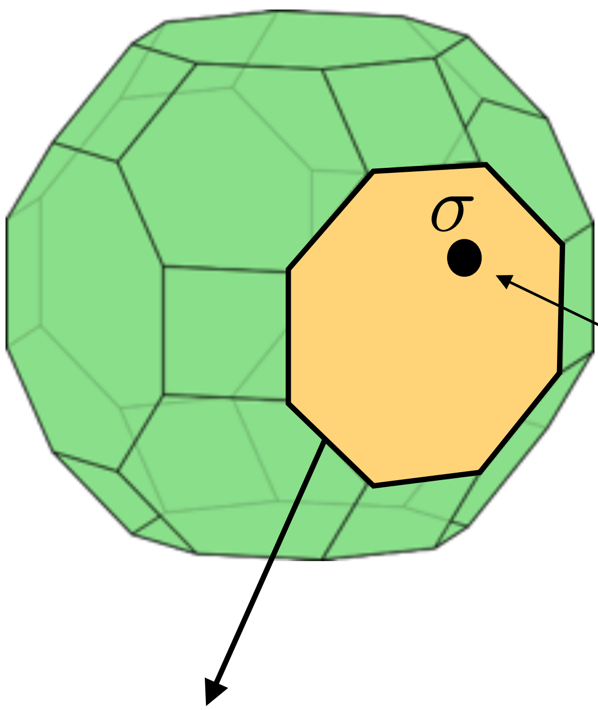
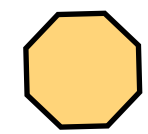
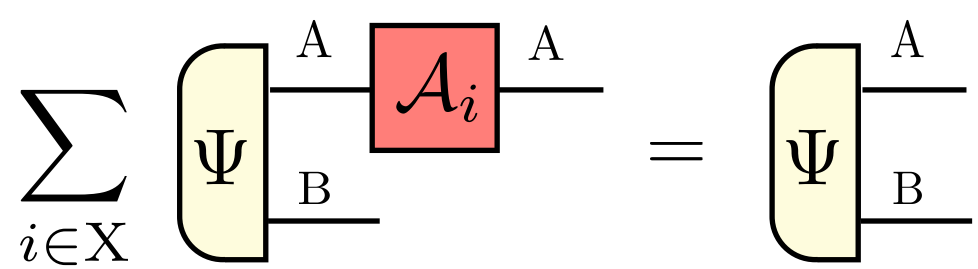
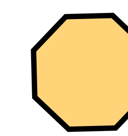
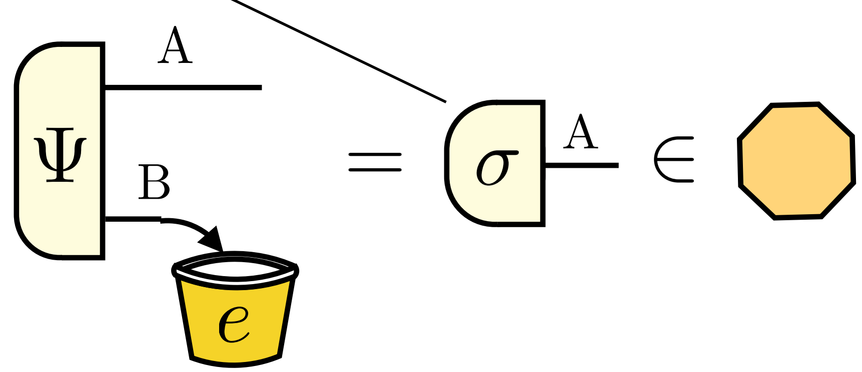


$\{\mathcal{A}_i\}_{i\in\mathrm{X}}$ is non-disturbing upon input of 

if 

$\forall\ \Psi$ *dilation of a state* in 

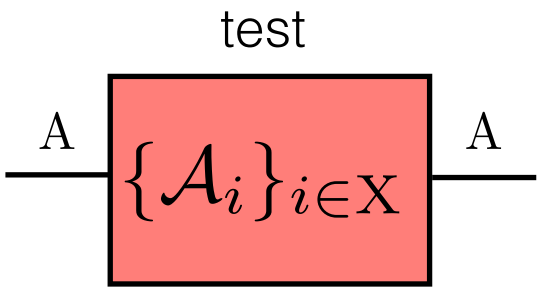


Non-disturbance here?

No-information here?

Deterministic effect
not unique for *non-causal* theories

Generalization: restricted resources



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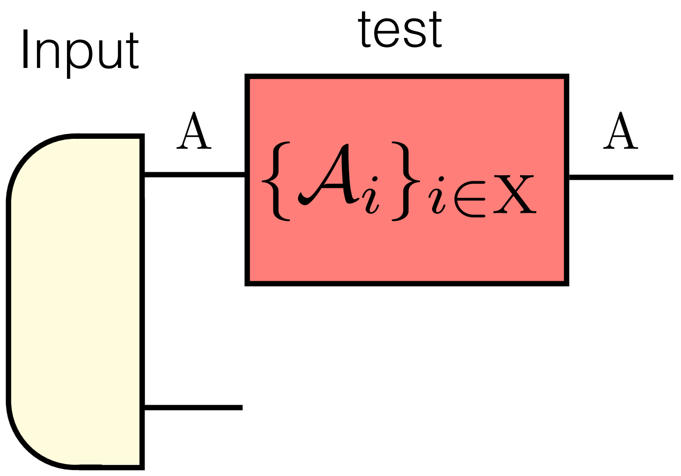
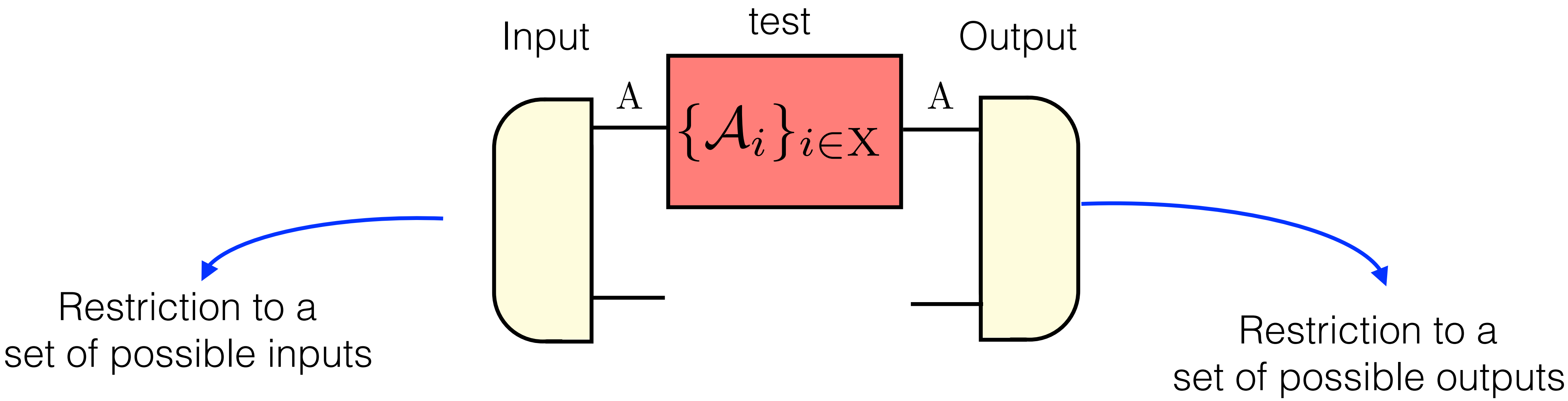


Restriction to a set of possible inputs

$X \subset$ Dilations of $\mathsf{St}(\mathrm{A})$

Generalization: restricted resources



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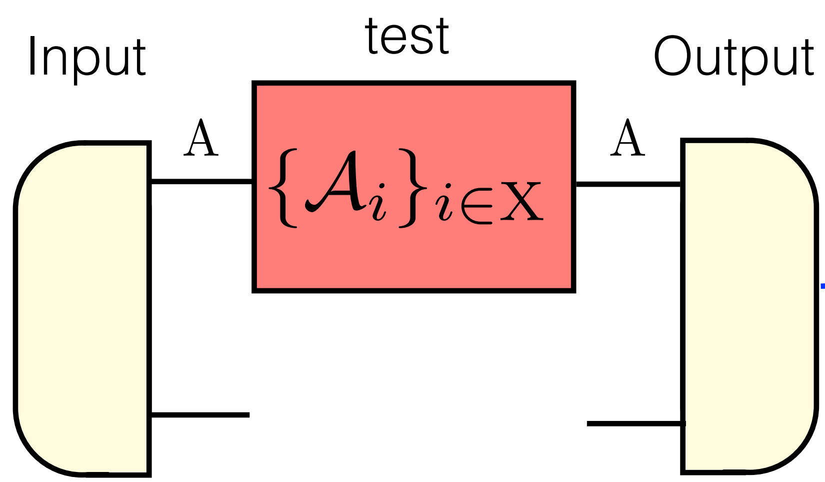

Restriction to a
set of possible inputs

$X \subset$ Dilations of $\mathsf{St}(\mathrm{A})$

Restriction to a
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$Y \subset$ Dilations of $\mathsf{Eff}(\mathrm{A})$

No-information without disturbance **upon input of *X*** and **upon output of *Y***

$\{\mathcal{A}_i\}_{i\in\mathrm{X}}$ non-disturbing **upon input of *X*** and **upon output of *Y*** $\Rightarrow$ $\{\mathcal{A}_i\}_{i\in\mathrm{X}}$ no-information **upon input of *X*** and **upon output of *Y***

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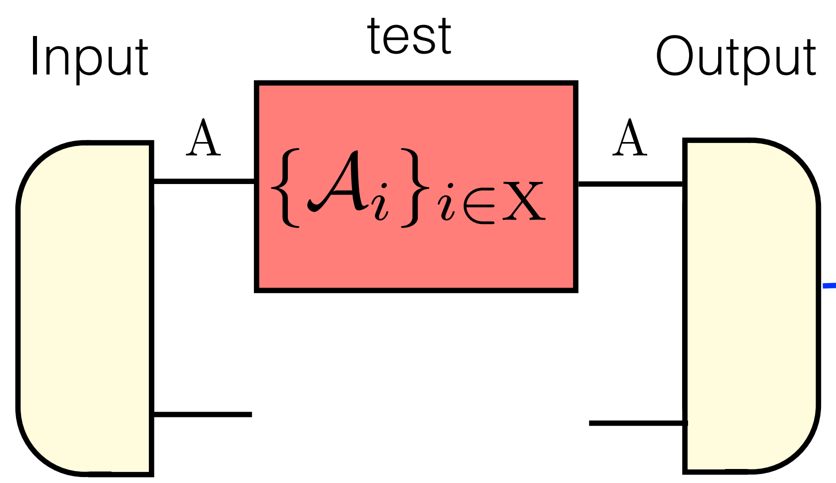

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necessary and sufficient (and only sufficient) conditions analogous to that for no-information without disturbance

Information without disturbance

If the identity map is not atomic?

Information without disturbance

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Theorem: for every system the atomic decomposition of the identity is unique, and

$$\mathcal{I}_{\mathrm{A}} = \sum_i \mathcal{A}_i \quad \Rightarrow \quad \mathcal{A}_i \mathcal{A}_j = \delta_{ij} \mathcal{A}_i$$

"orthogonal projectors"

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➡ The information that can be extracted without disturbance is *"classical"* information

$$\mathsf{St}(\mathrm{A}) = \begin{pmatrix} i=1 & & & \\ & i=2 & & \\ & & \ddots & \\ & & & \blacksquare \end{pmatrix}$$

Each block satisfies NIWD

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Systems A,B

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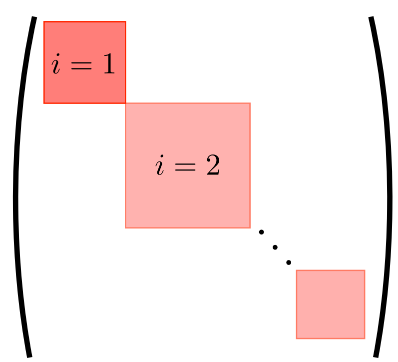

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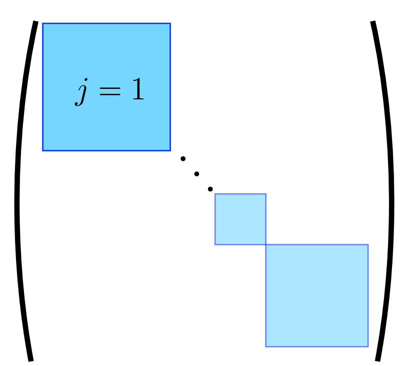

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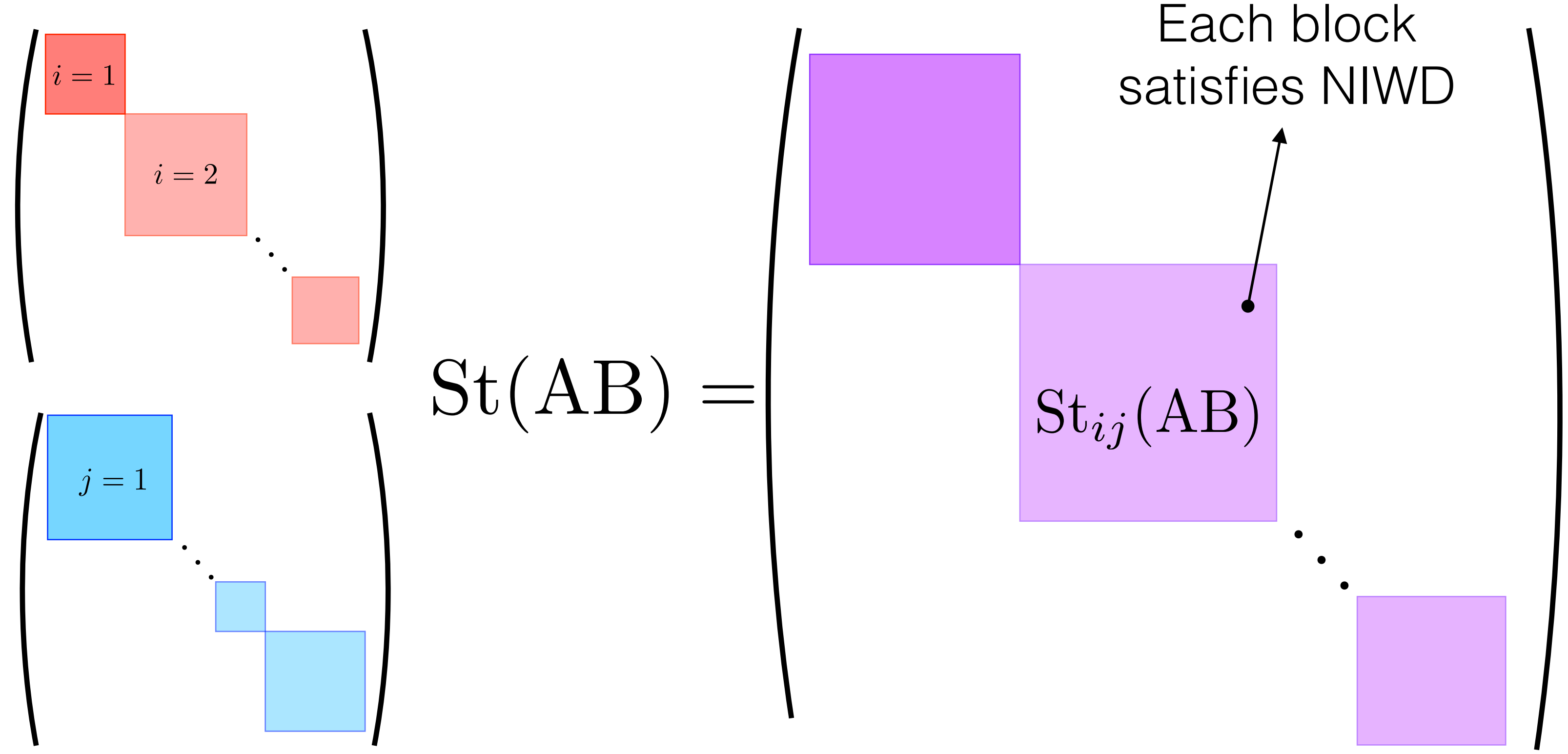

Block structure due to information without disturbance

$$\mathrm{St}(\mathrm{A}) = \begin{pmatrix} \mathbf{A_1} & \\ & \mathbf{A_2} \end{pmatrix}$$

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$$\Rightarrow \quad \mathrm{St}(\mathrm{AB}) = \begin{pmatrix} \mathbf{A_1B_1} & & & \\ & \mathbf{A_1B_2} & & \\ & & \mathbf{A_2B_1} & \\ & & & \mathbf{A_2B_2} \end{pmatrix}$$

Block structure due to information without disturbance

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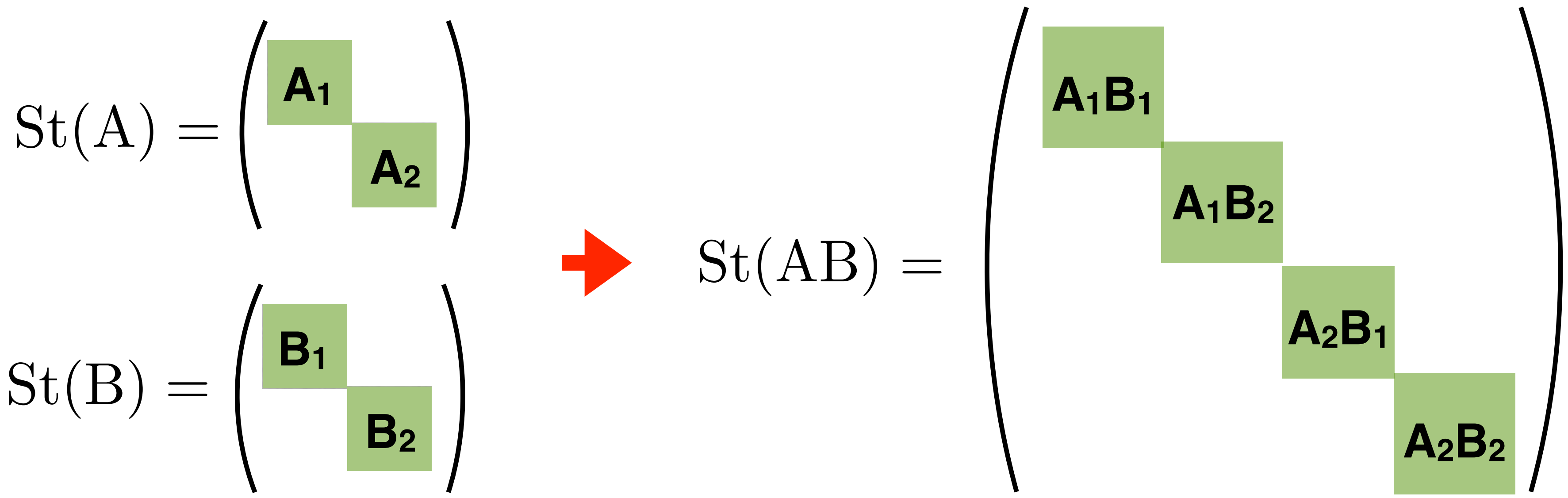
Other block structures: *e.g. Fermions superselection*

$$\mathrm{St(A)} = \begin{pmatrix} \mathbf{E} & \\ & \mathbf{O} \end{pmatrix}$$

$$\mathrm{St(B)} = \begin{pmatrix} \mathbf{E} & \\ & \mathbf{O} \end{pmatrix}$$

Block structure due to information without disturbance



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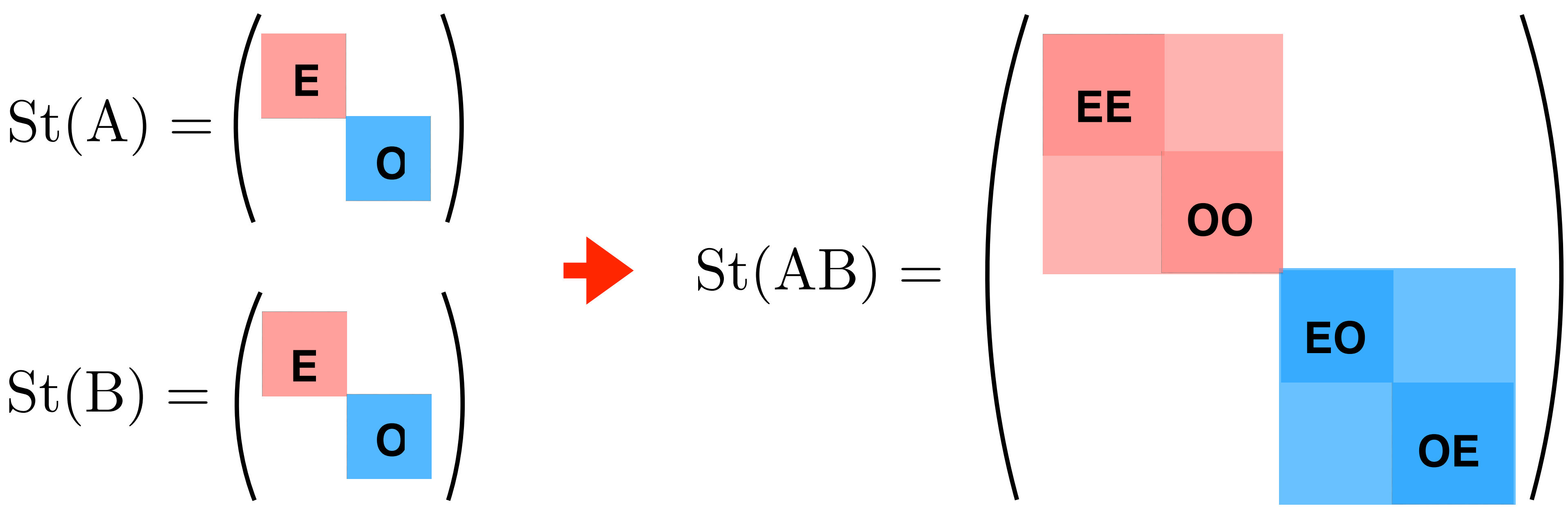

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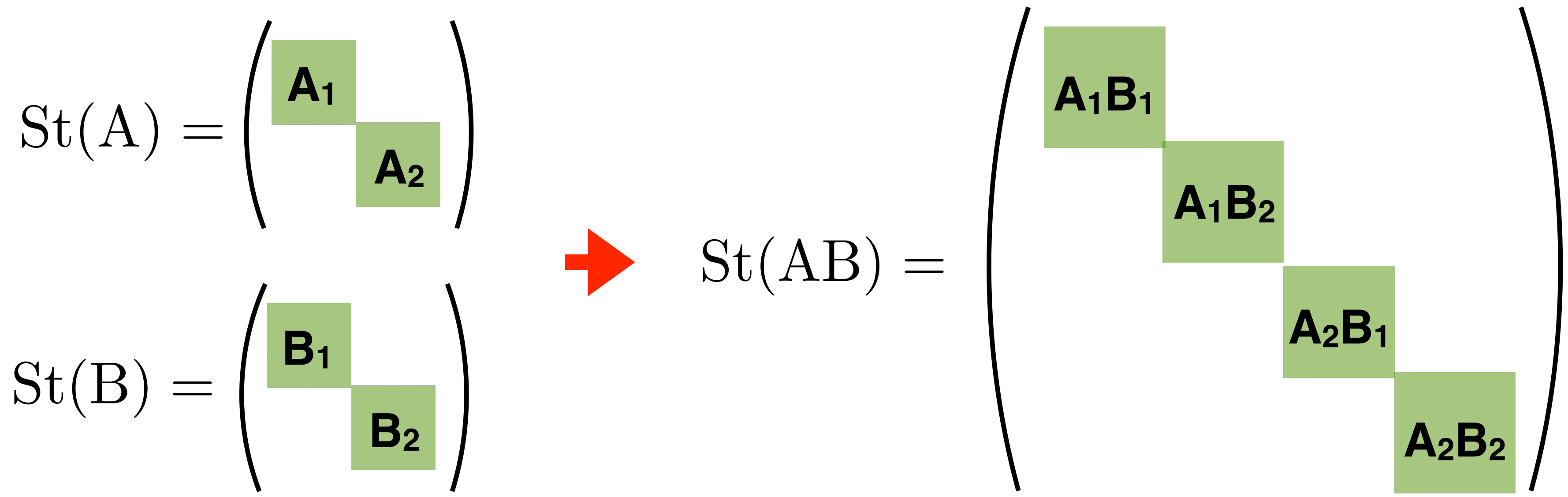

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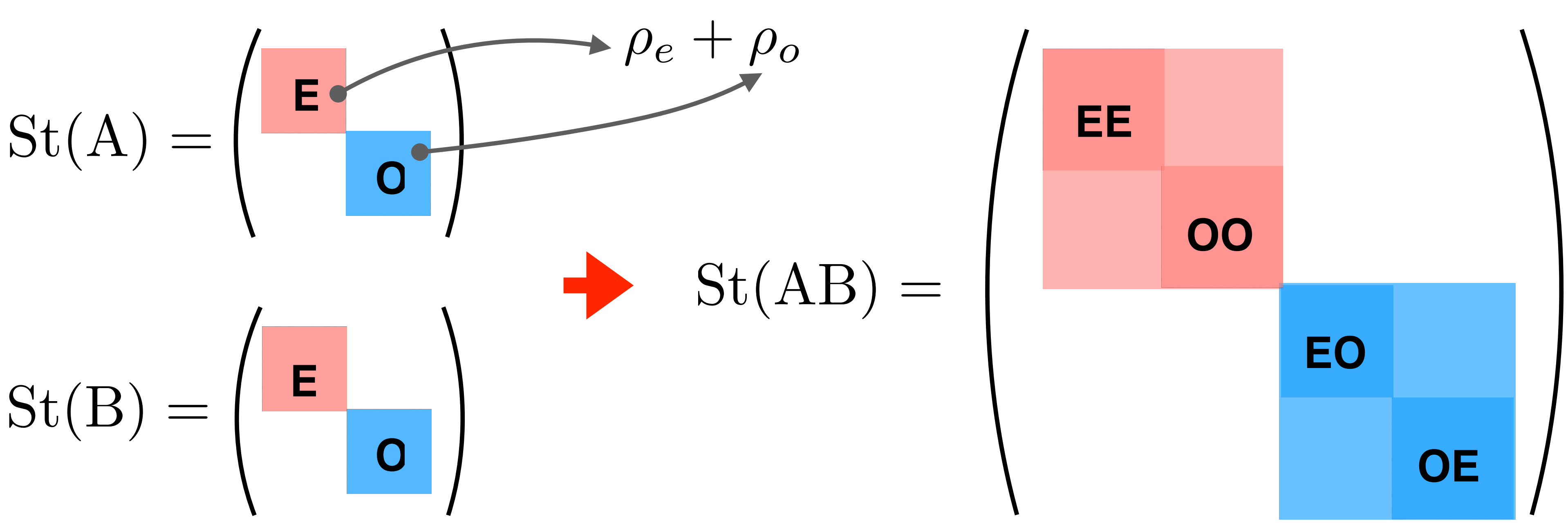

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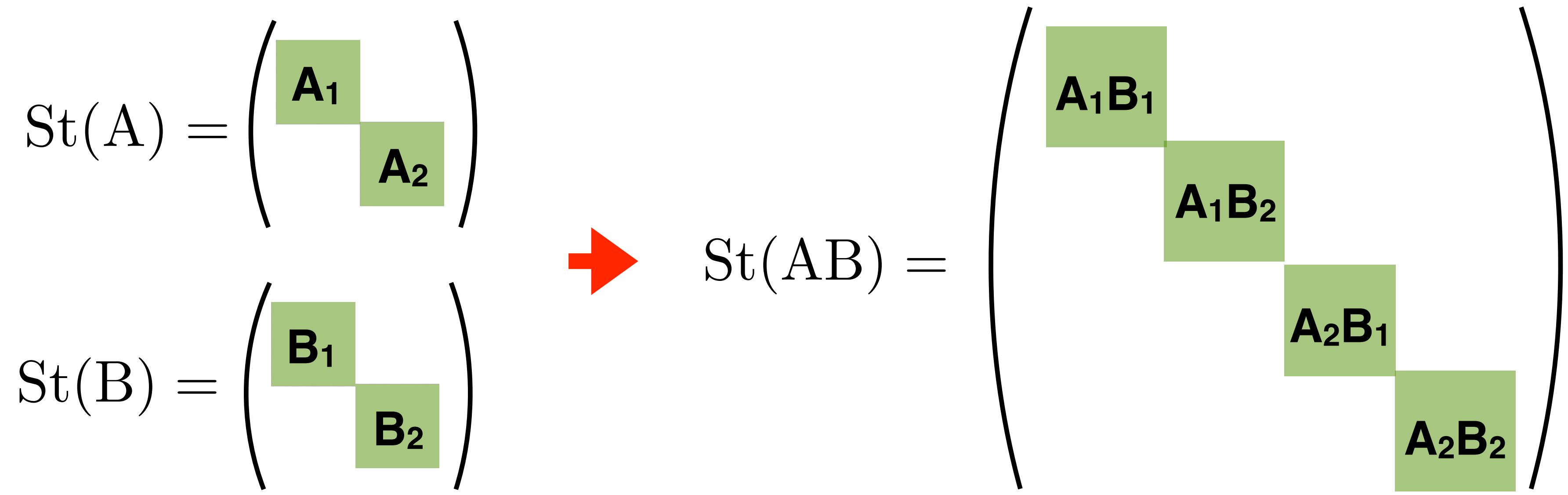

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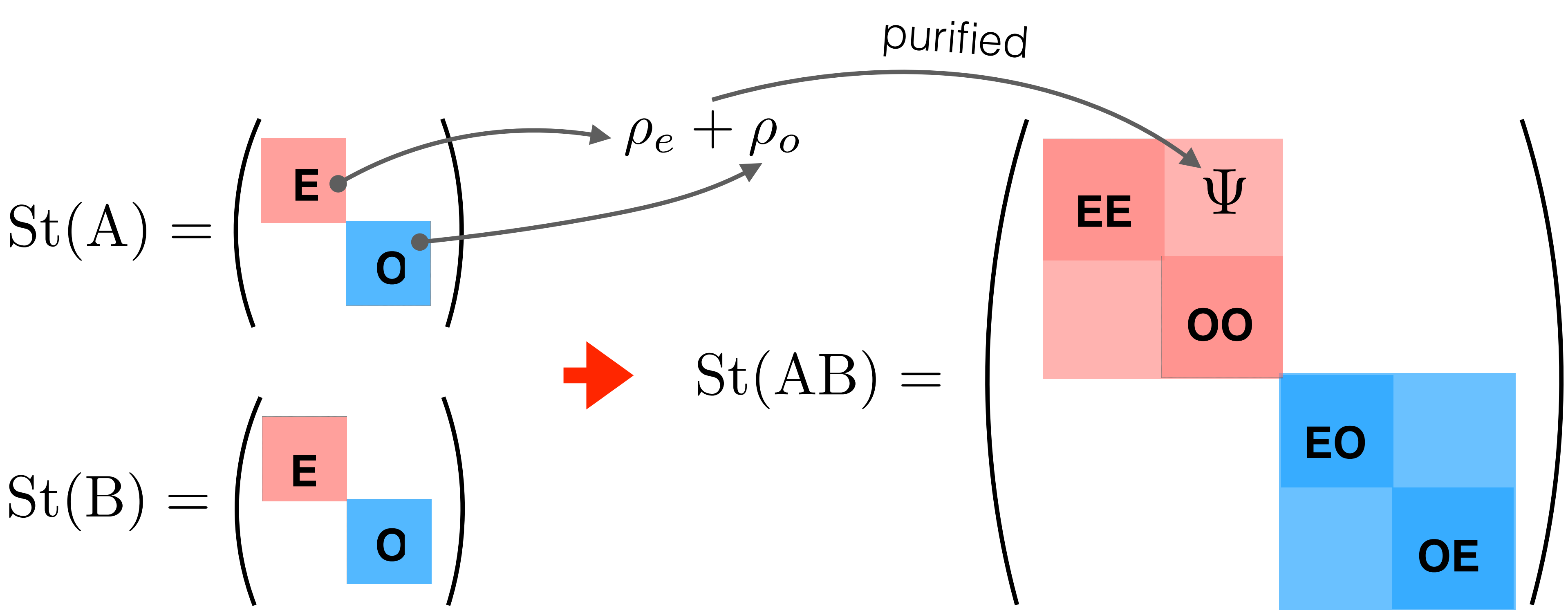

Full information without disturbance

Full information without disturbance

Definition: a theory satisfies full-information without disturbance if

for every test $\;\xrightarrow{\mathrm{A}}\boxed{\{\mathcal{B}_j\}_{j\in\mathrm{Y}}}\xrightarrow{\mathrm{A}}$

there exists a non-disturbing test $\;\xrightarrow{\mathrm{A}}\boxed{\{\mathcal{A}_i\}_{i\in\mathrm{X}}}\xrightarrow{\mathrm{A}}$

such that $\;\xrightarrow{\mathrm{A}}\boxed{\mathcal{B}_j}\xrightarrow{\mathrm{A}}$

$$= \sum_i p(j|i) \;\xrightarrow{\mathrm{A}}\boxed{\mathcal{R}}\xrightarrow{\mathrm{A}}\boxed{\mathcal{A}_i}\xrightarrow{\mathrm{A}}\boxed{\mathcal{V}}\xrightarrow{\mathrm{A}}\;,$$

reversible pre- and post-processing

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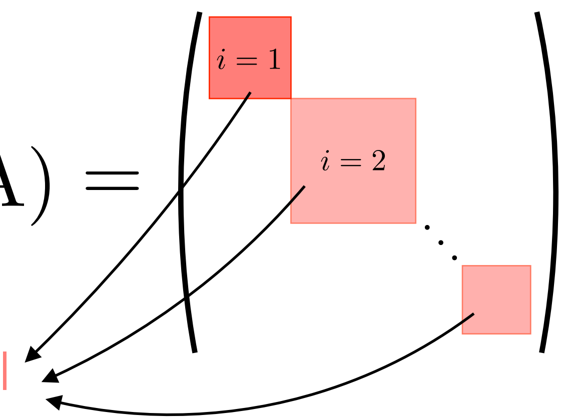

III. Prove that all blocks are 1-dimensional

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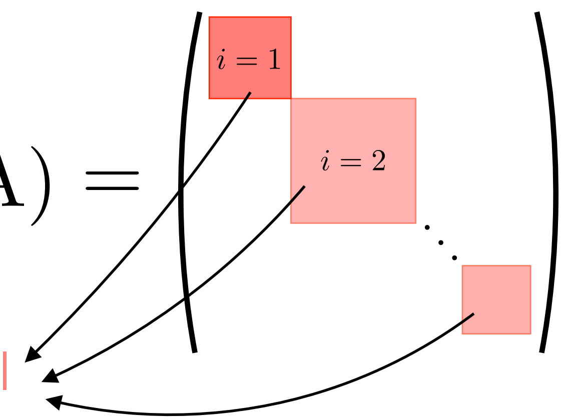
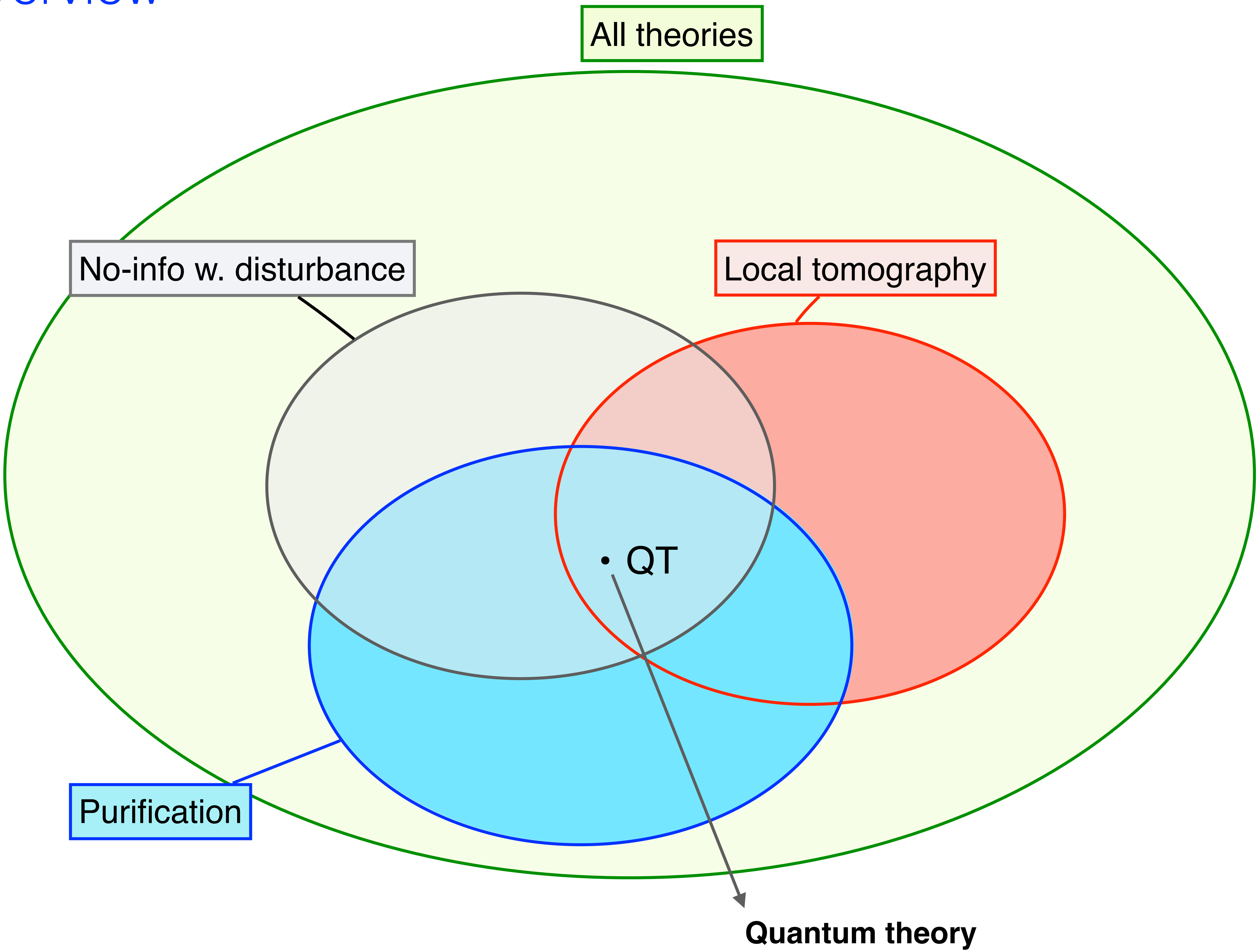


III. Prove that all blocks are 1-dimensional

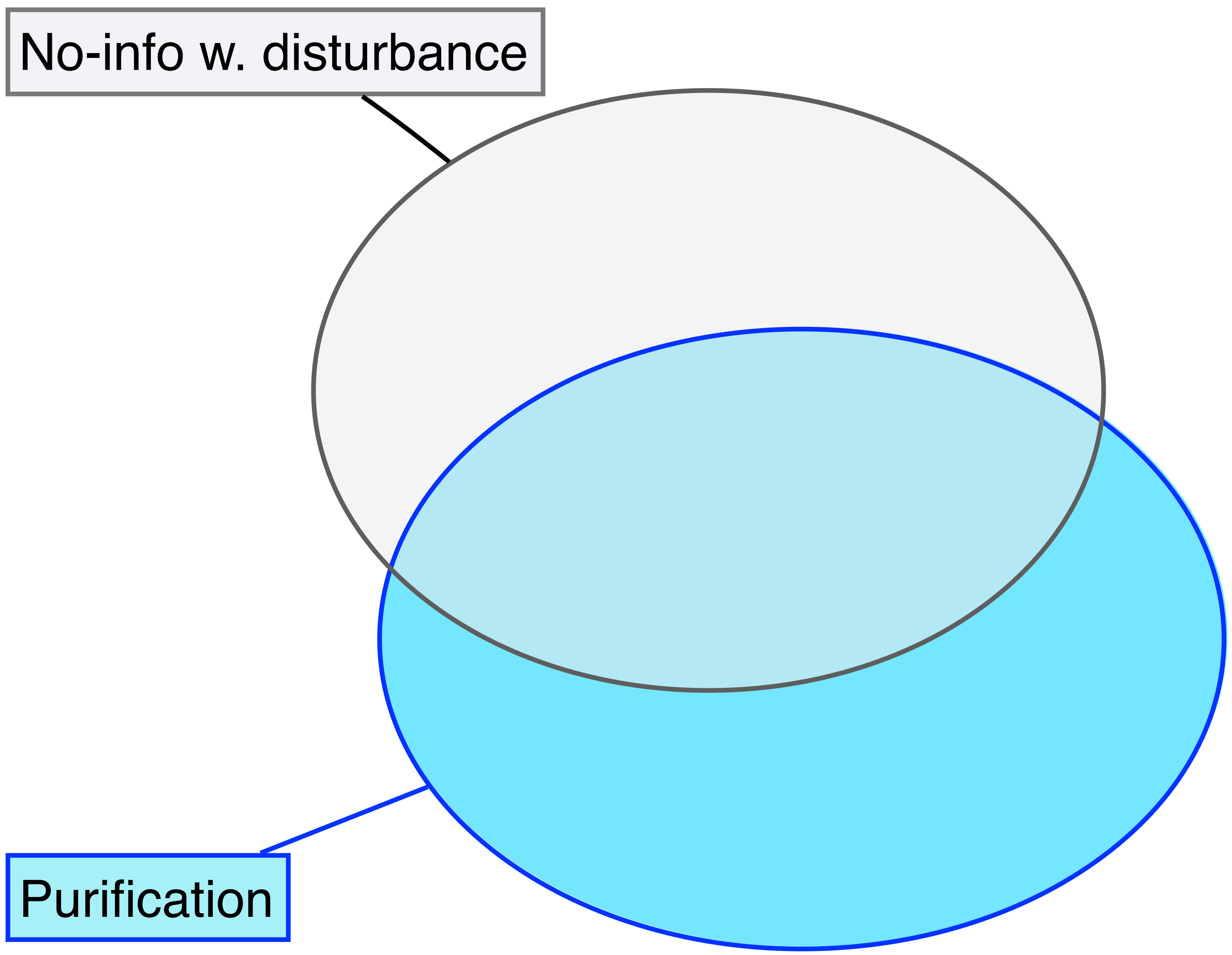
Classical system: the base of the cone of states and effects is a SIMPLEX

Overview


All theories
No-info w. disturbance
Local tomography
QT
Purification
Quantum theory

Purification vs NIWD

Proposition: Purification and NIWD are independent properties



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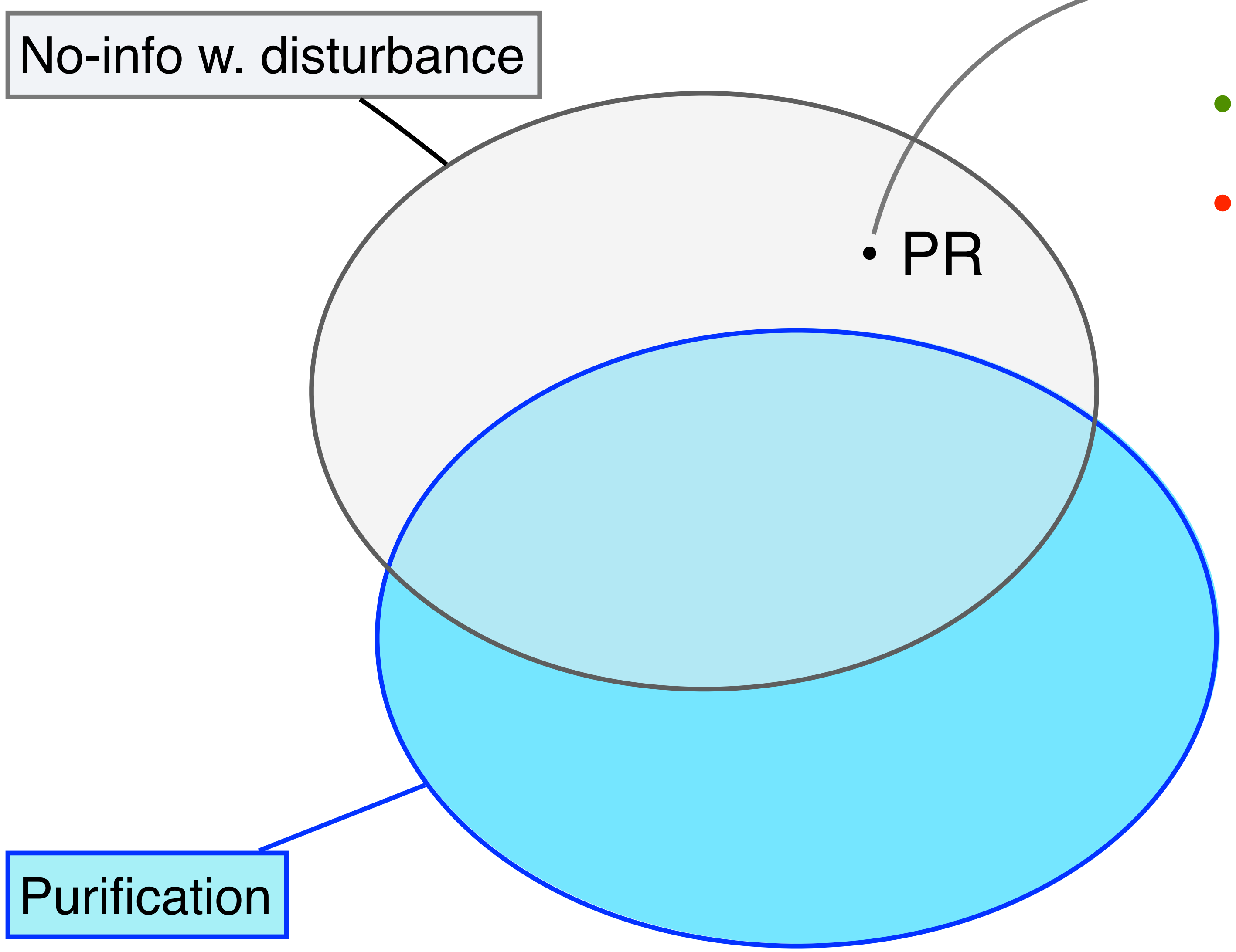

Non-local boxes theory (PR-boxes)

- We proved that it satisfies NIWD
- Violates purification

Purification vs NIWD

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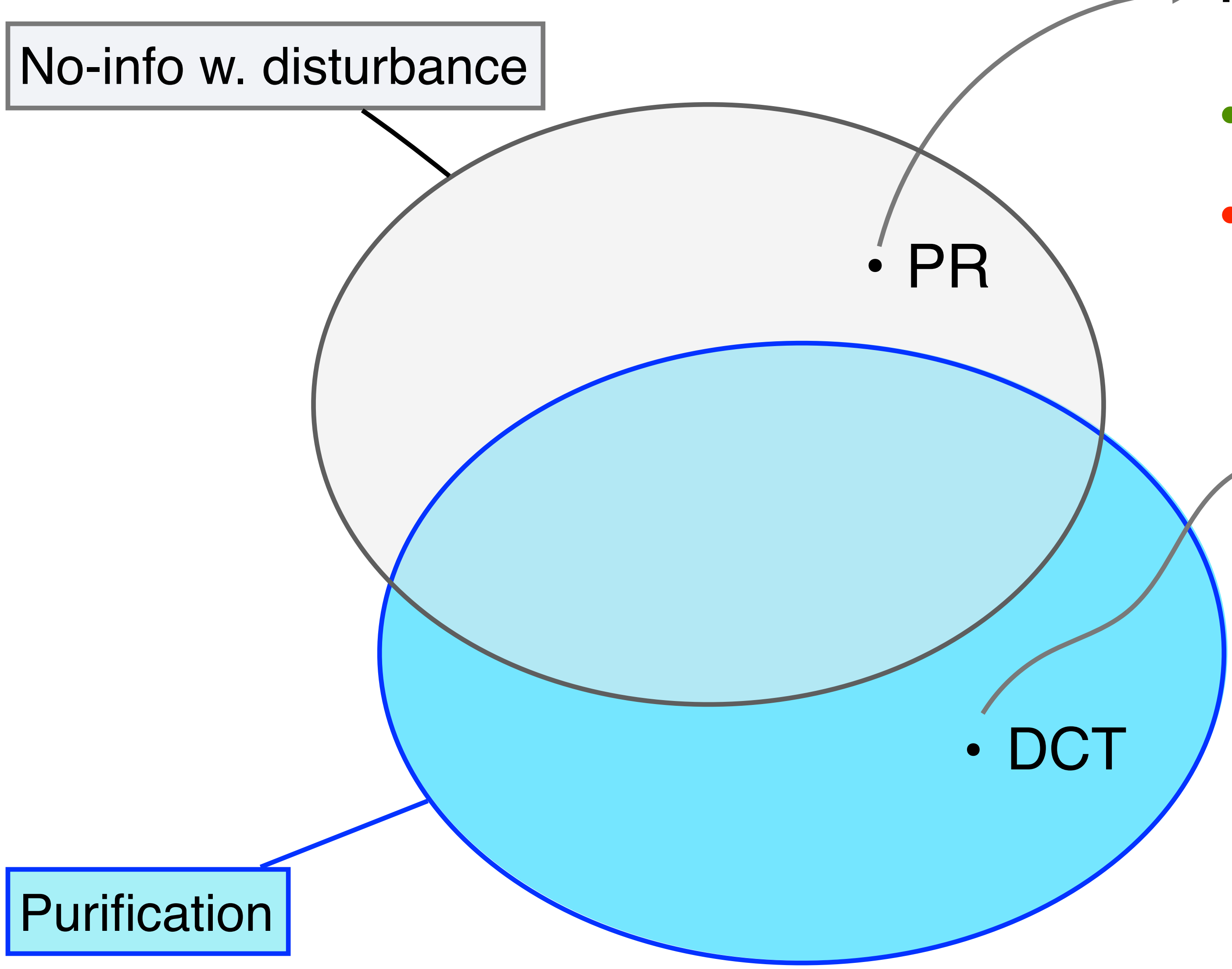

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Deterministic classical theory

- it satisfies purification
- Violates NIWD

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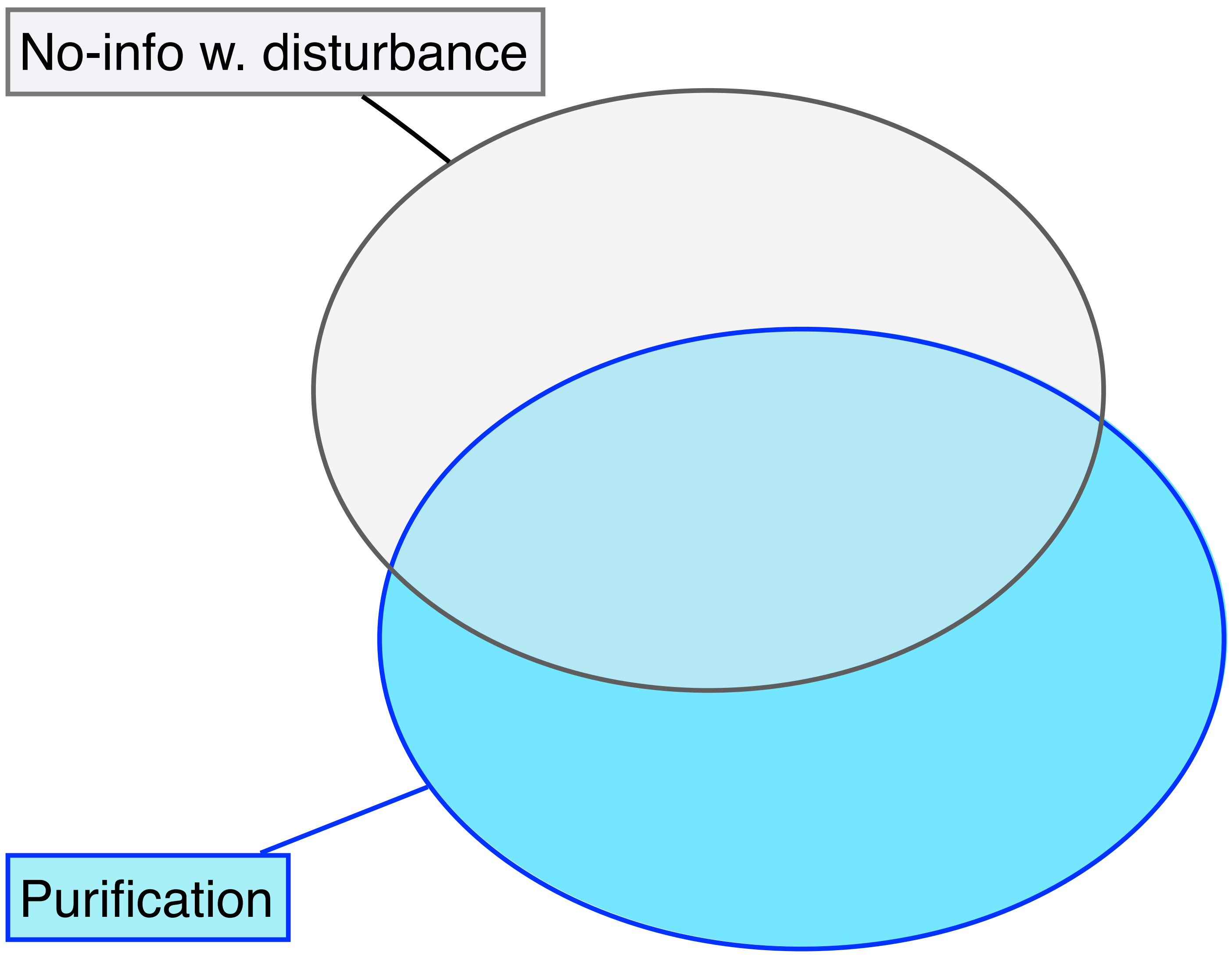

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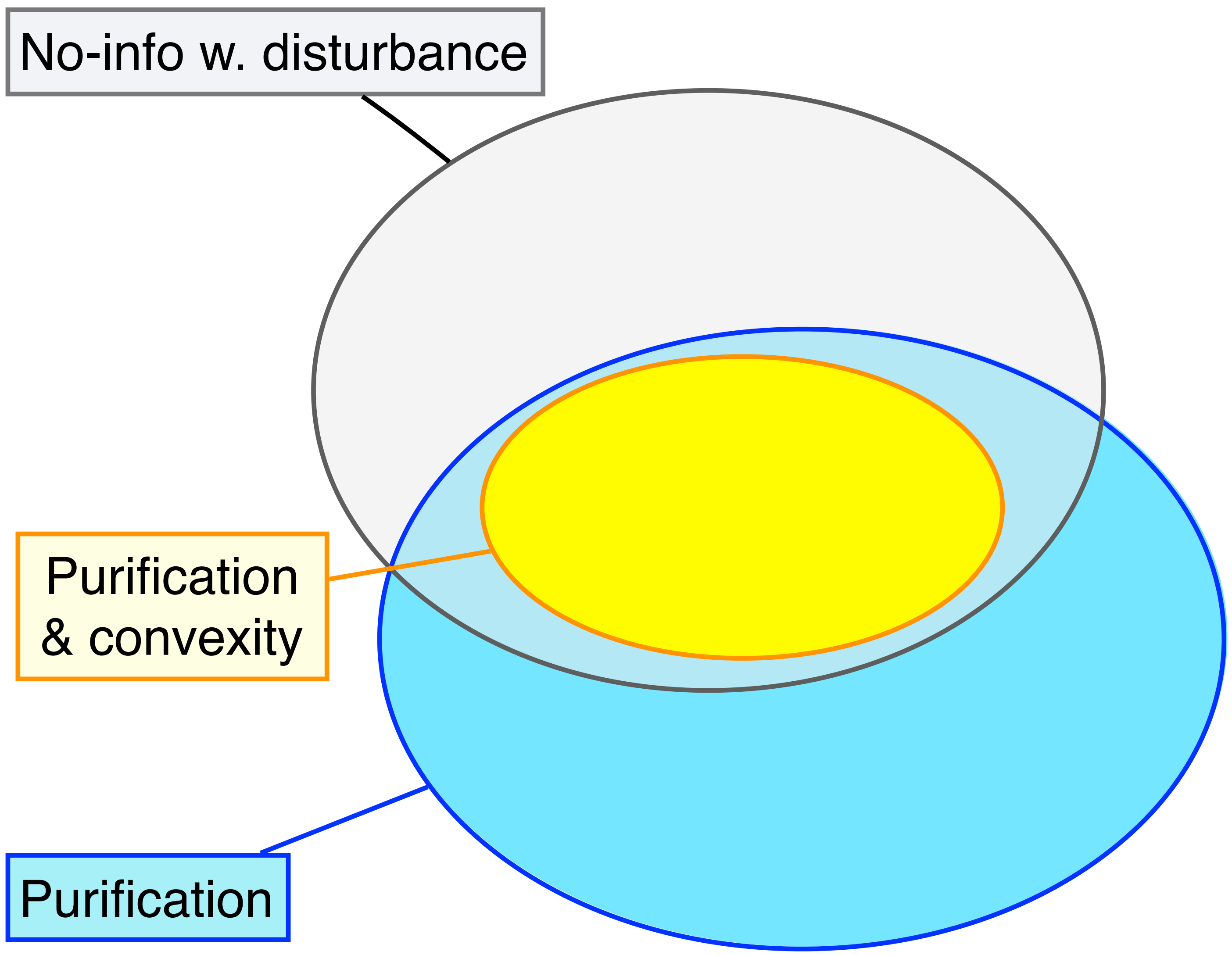

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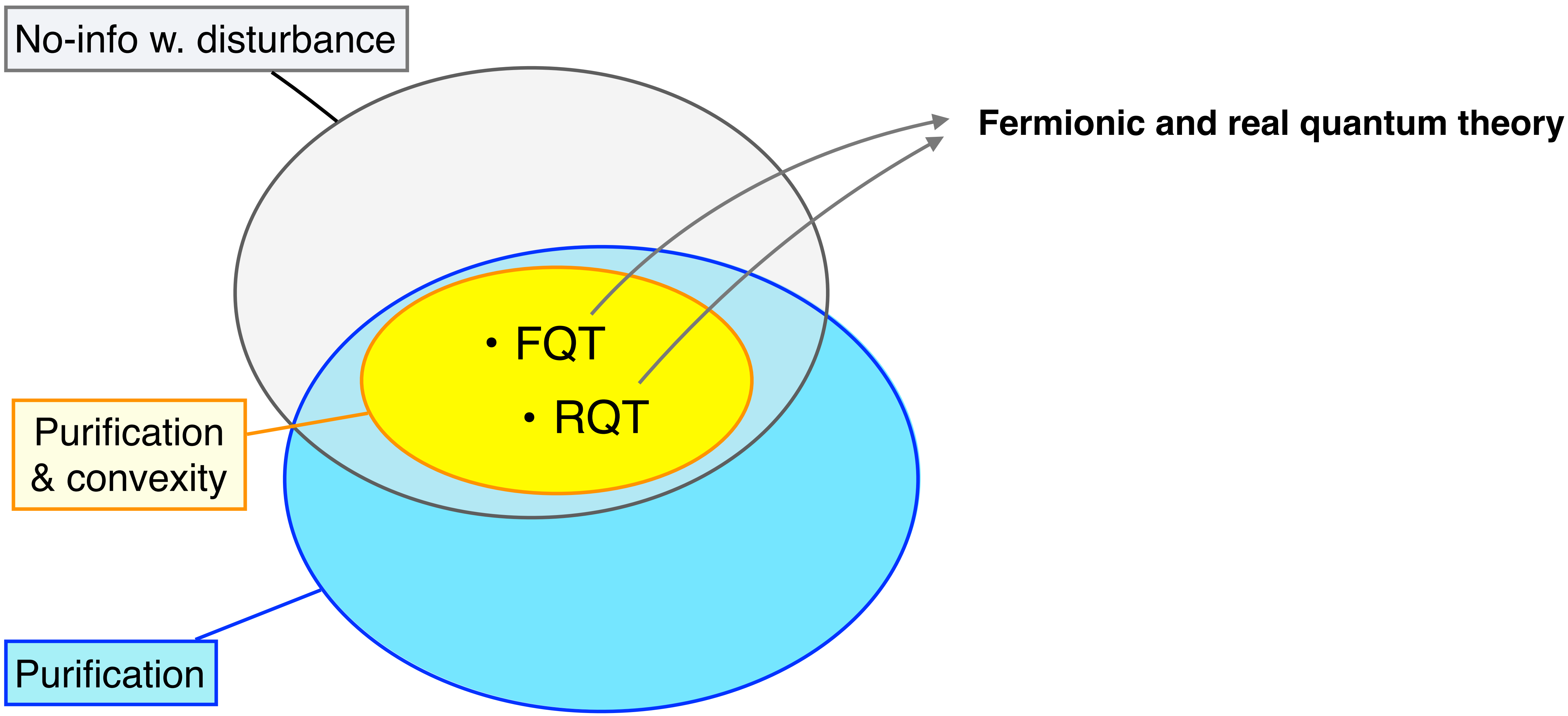

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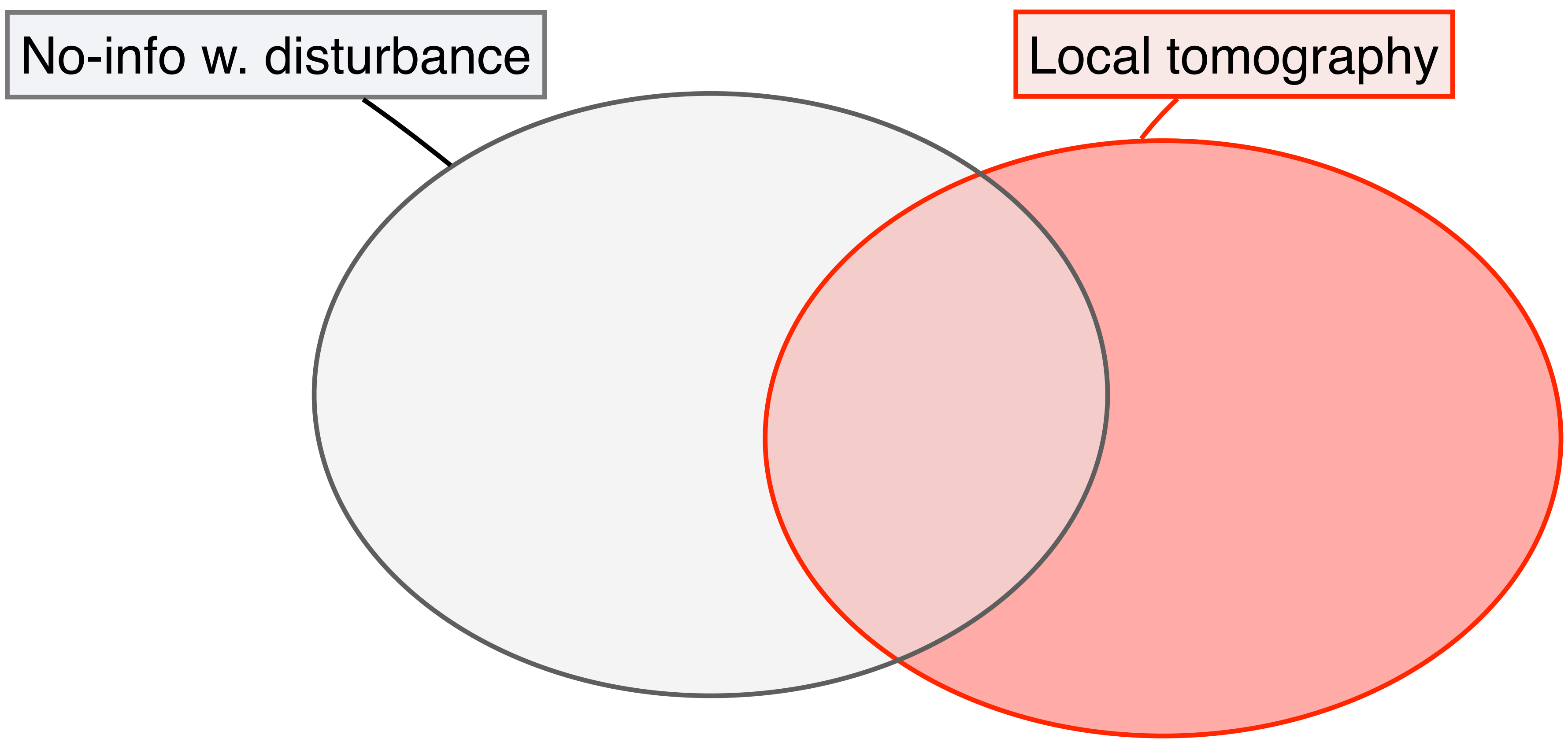

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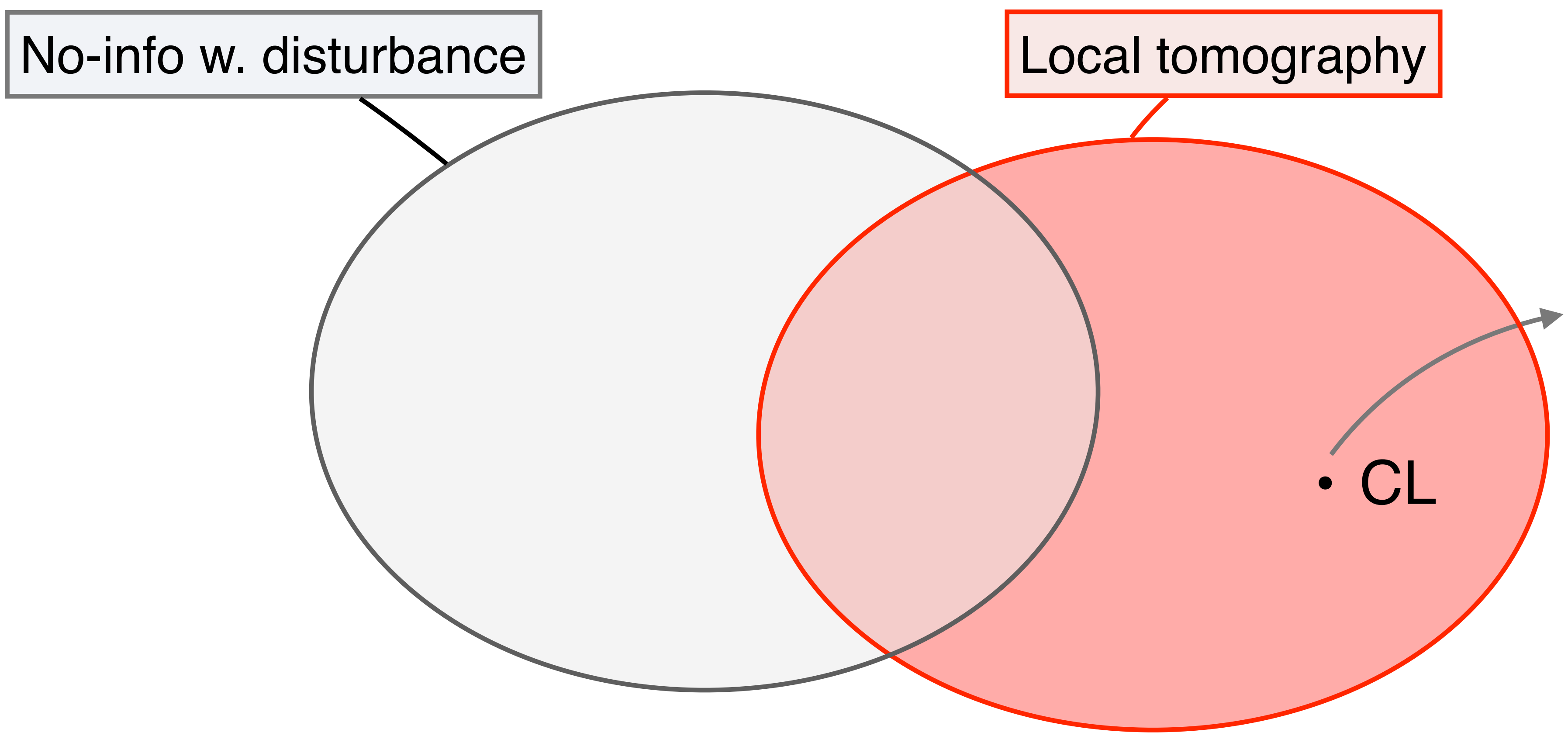

Classical theory

- Satisfies local tomography
- Violates NIWD:

  Full-information without disturbance

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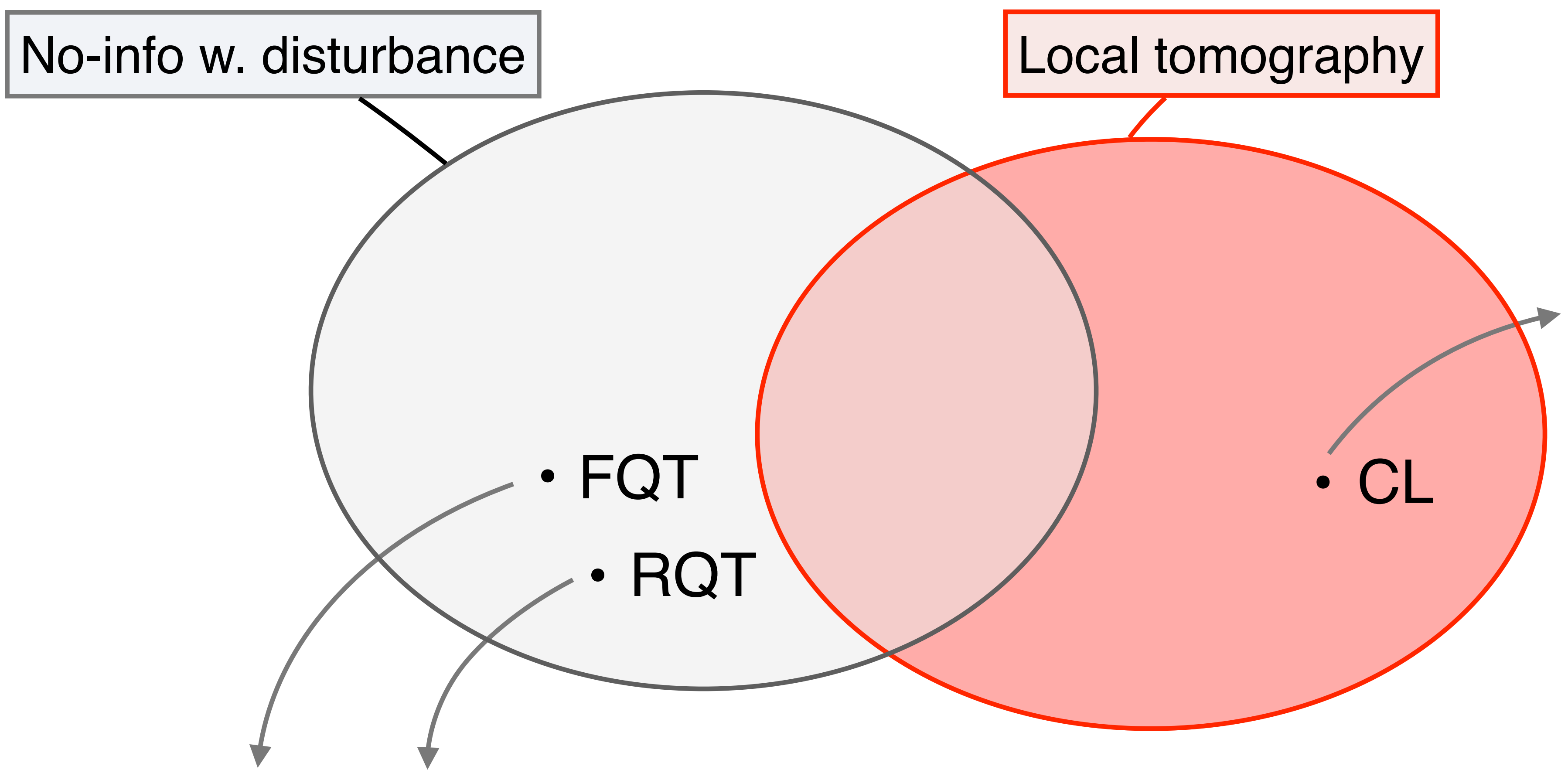

Classical theory

- Satisfies local tomography
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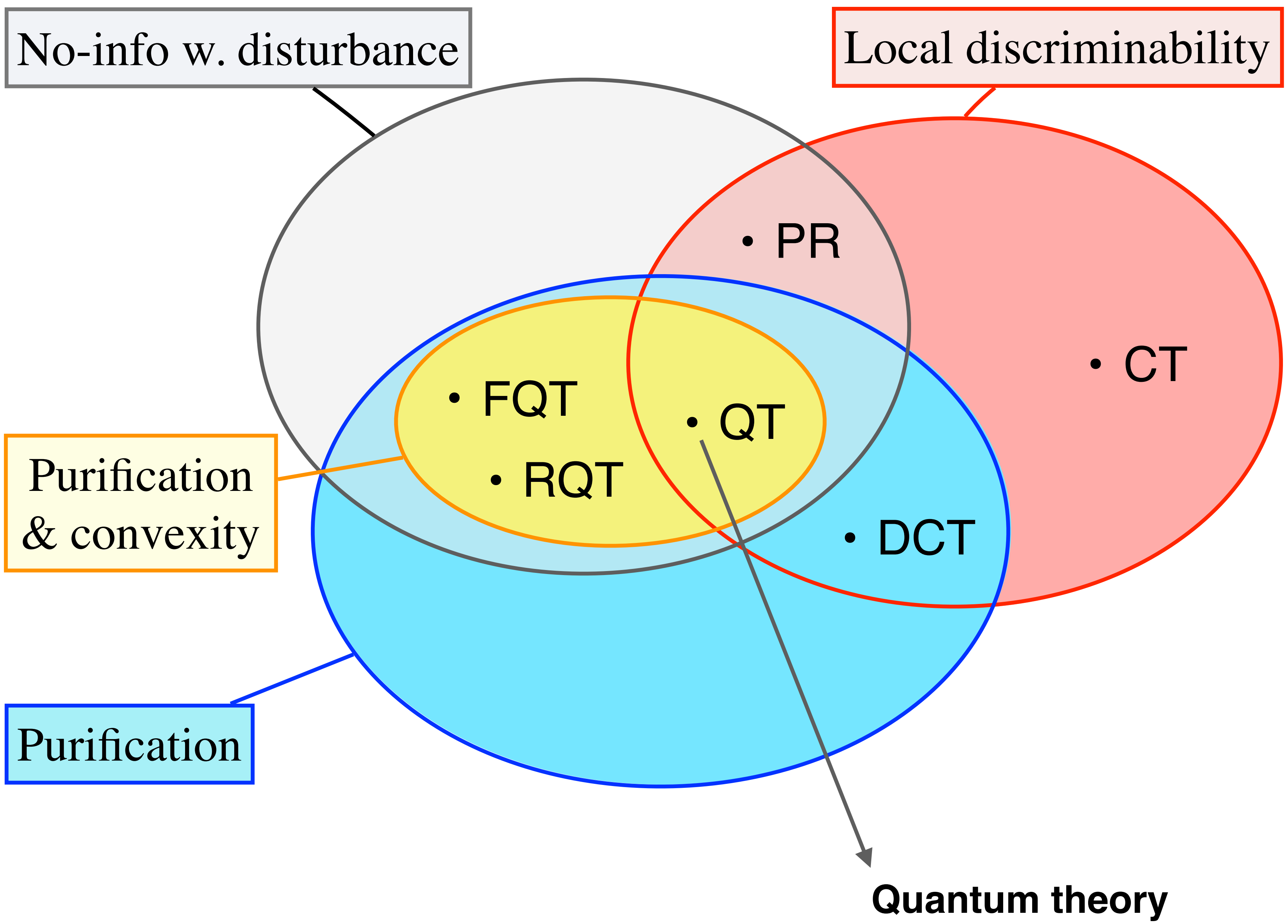
  Full-information without disturbance

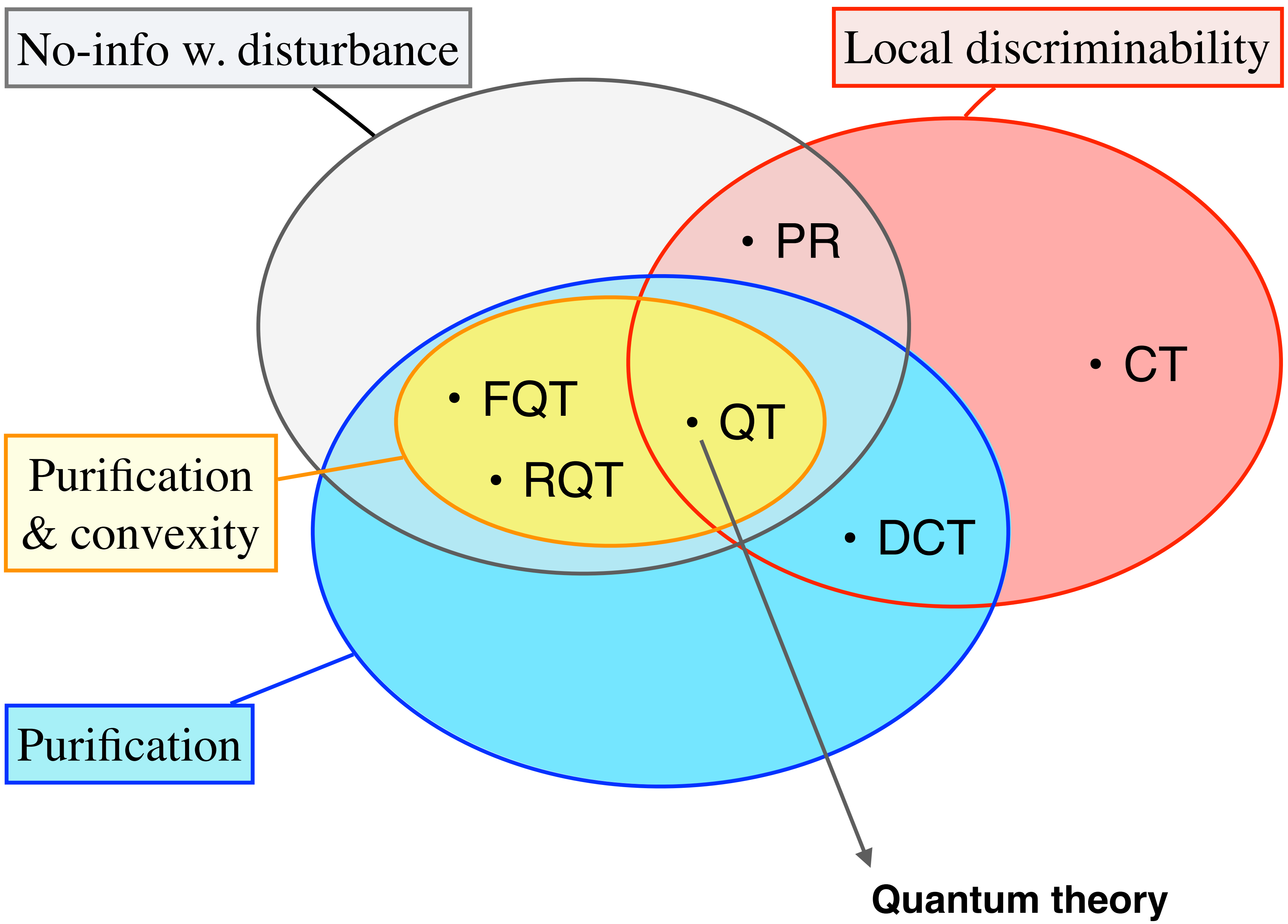
Fermionic and real quantum theory

- Satisfy convexity and purification $\Longrightarrow$ satisfy NIWD
- Both bilocal-tomographic $\Longrightarrow$ violate local tomography

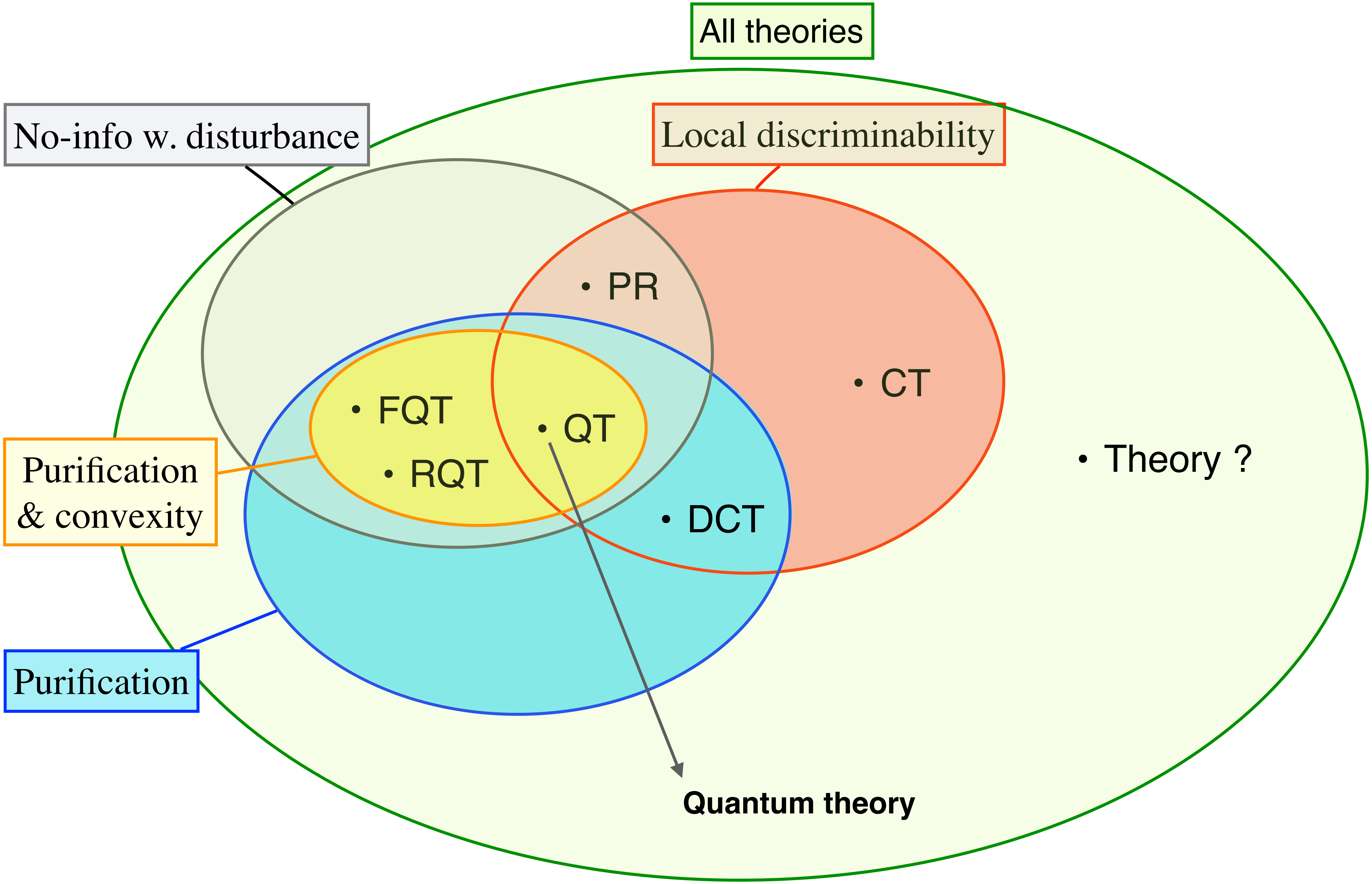
L. Hardy and W. K. Wootters, Foundations of Physics 42, 454 (2012)
G. M. D'Ariano, F. Manessi, P. Perinotti, and A. Tosini, IJMP A 29, 1430025 (2014)



No-info w. disturbance
Local discriminability
PR
CT
FQT
QT
RQT
DCT
Purification & convexity
Purification
Quantum theory



NIWD can be satisfied in the absence
of most of quantum features



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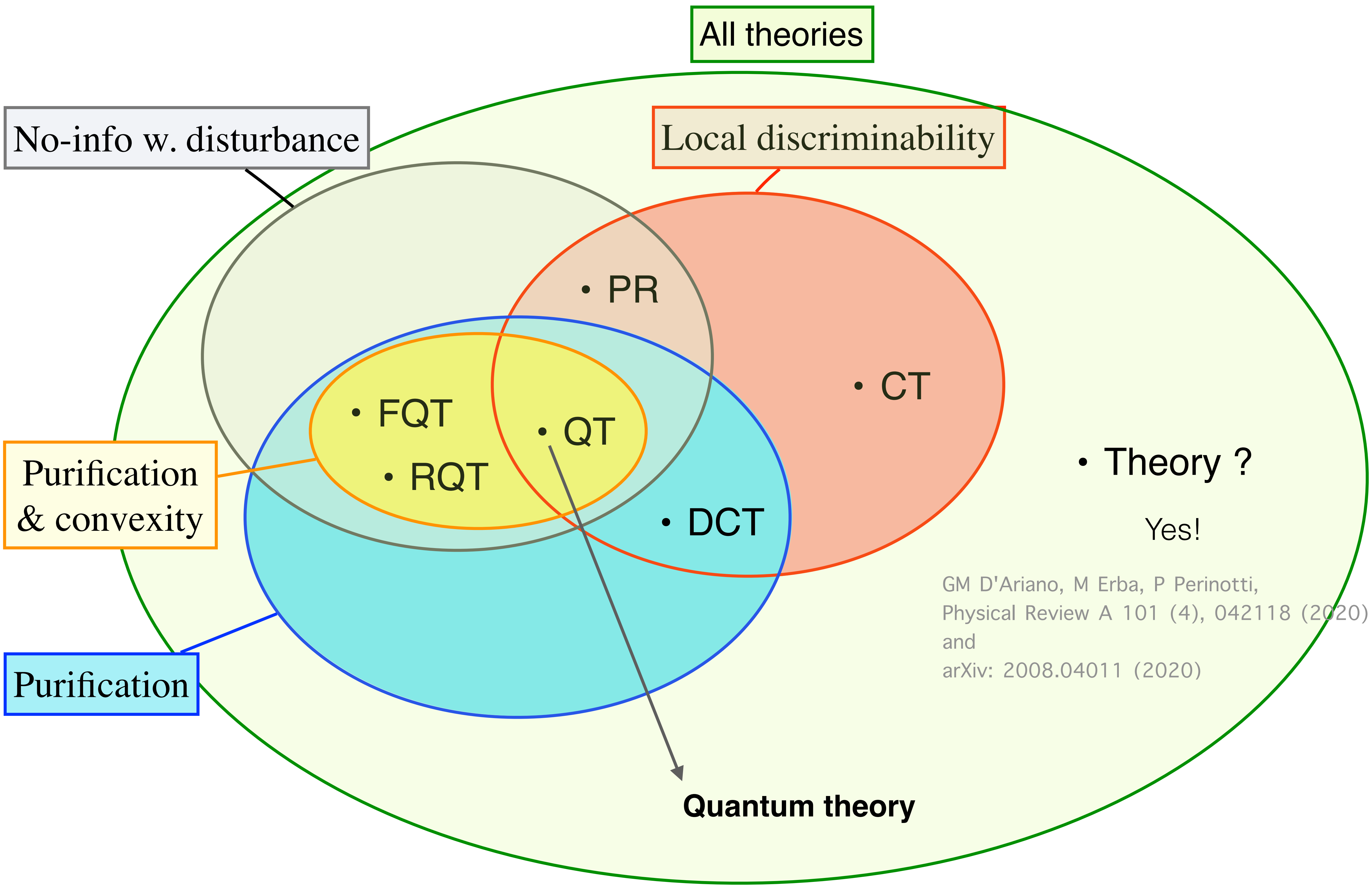

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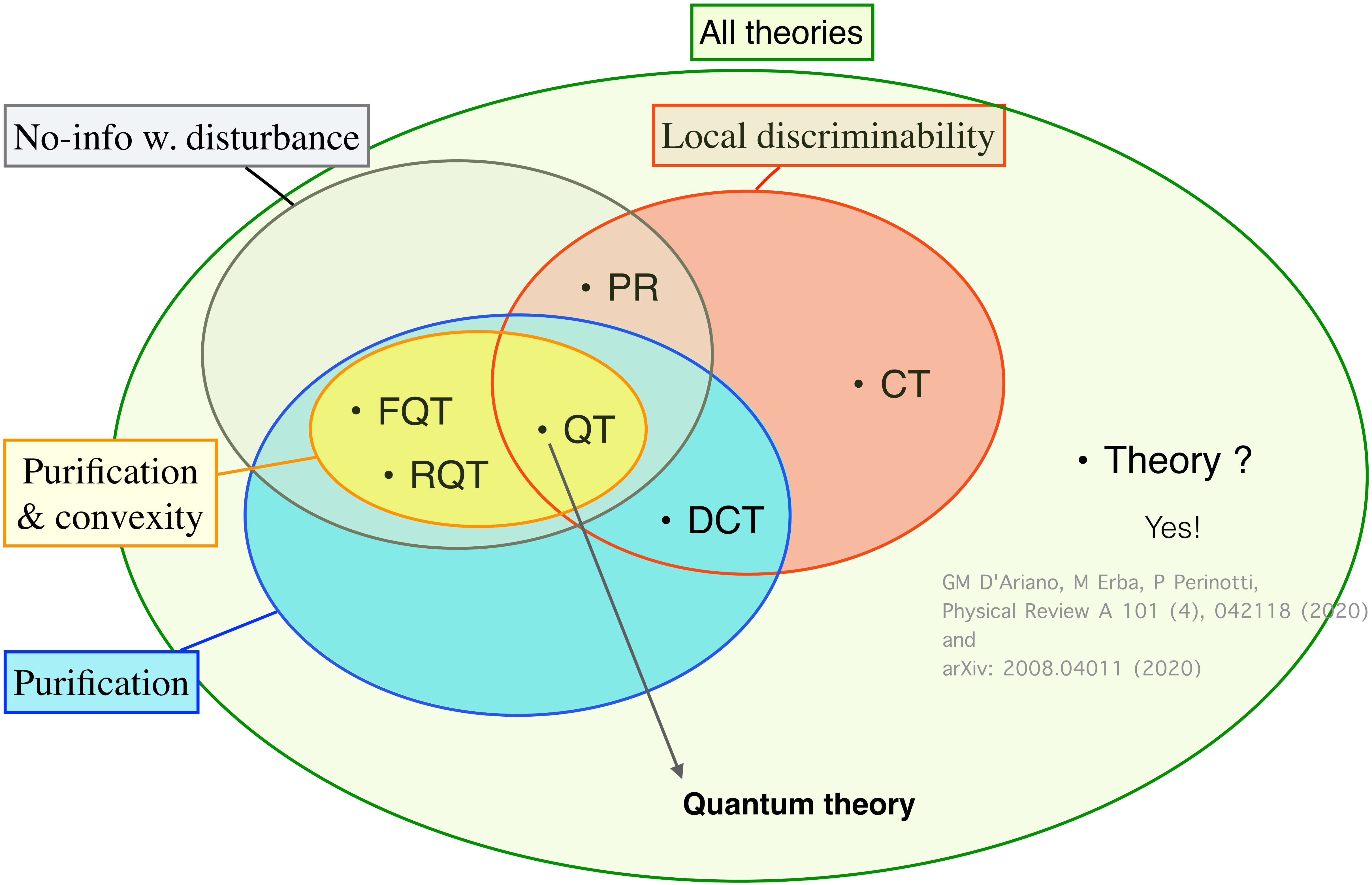

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Thanks