Causal influence in operational probabilistic theories

Second Kyoto Workshop on Quantum Information, Computation, and Foundation

UNIVERSITÀ DI PAVIA

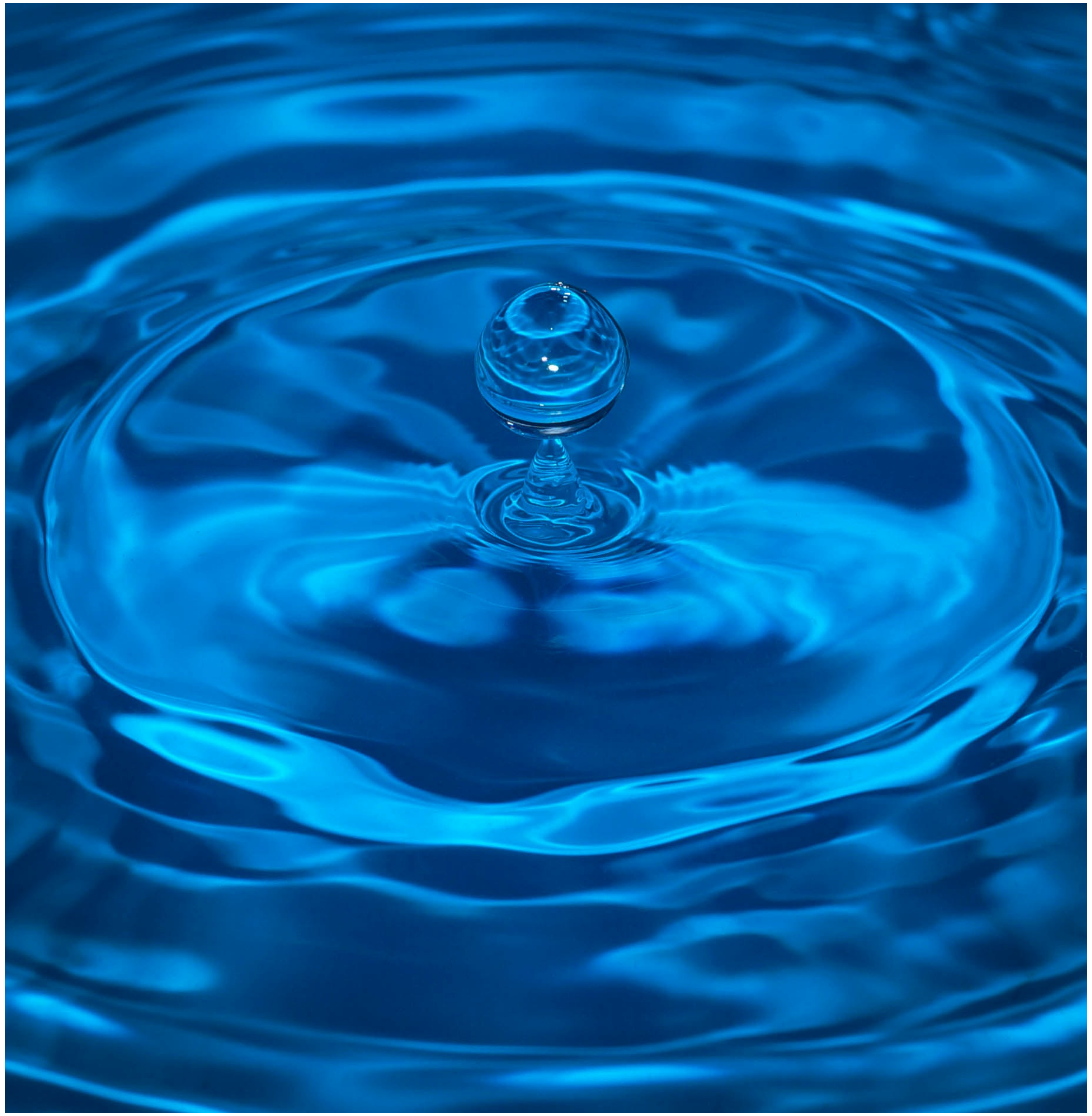
INFN Istituto Nazionale di Fisica Nucleare

QUit quantum information theory group

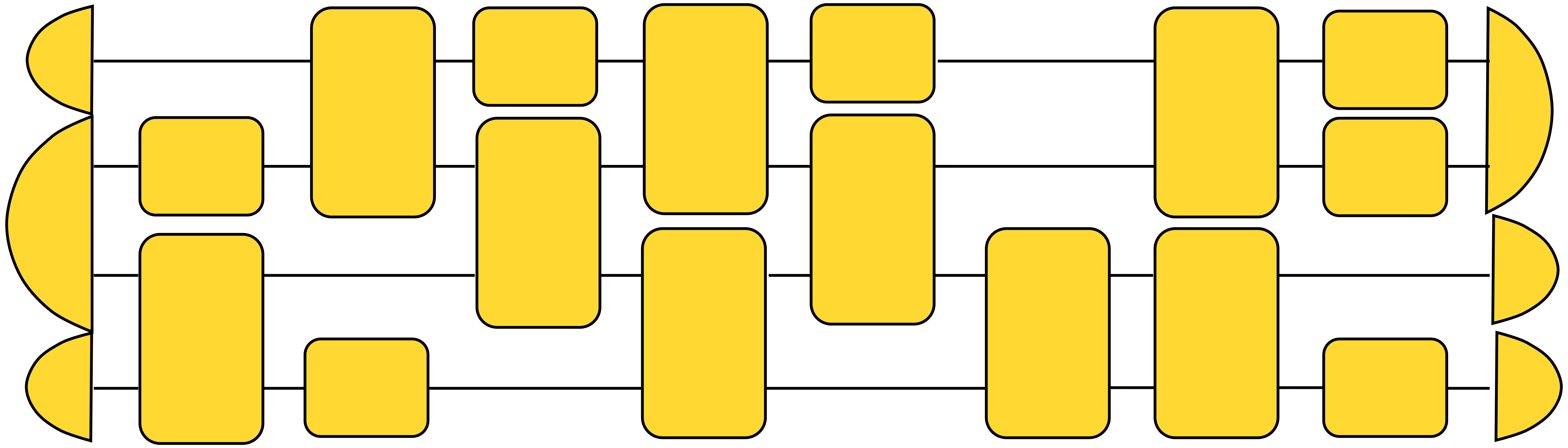
Paolo Perinotti - September 16th 2021

Summary

- OPTs
- Networks and causal cones
- Signalling vs causal influence
- Propagation of interventions
- Classical and Quantum theory
- No interaction without disturbance



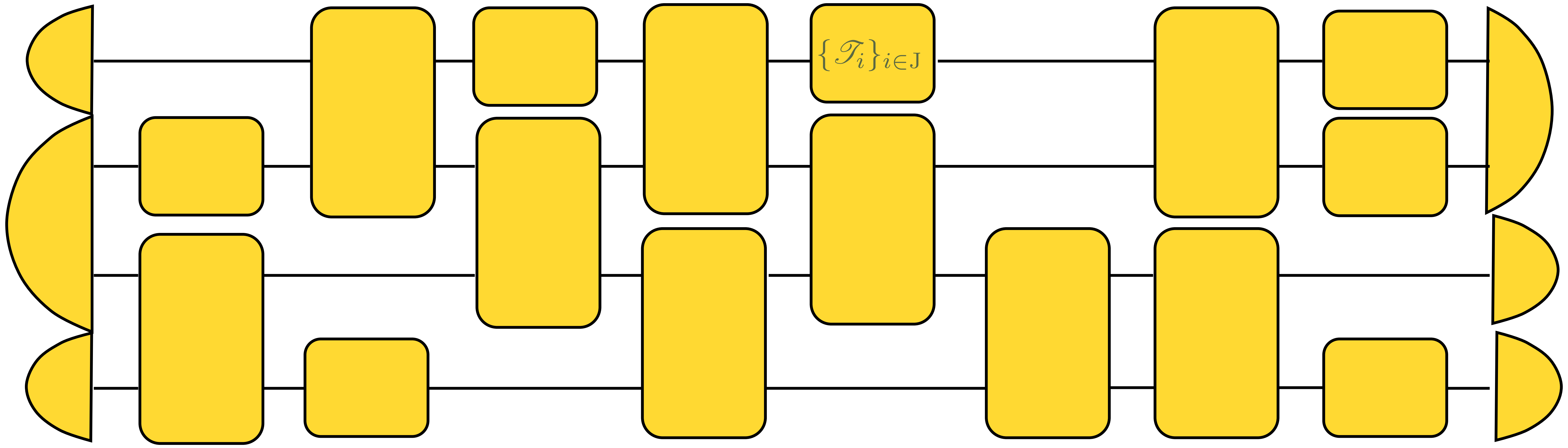
Operational Language



- Operational theory: tests with composition rules

G. Chiribella, G. M. D'Ariano, and PP, Phys. Rev. A **81**, 062348 (2010)

Operational Language



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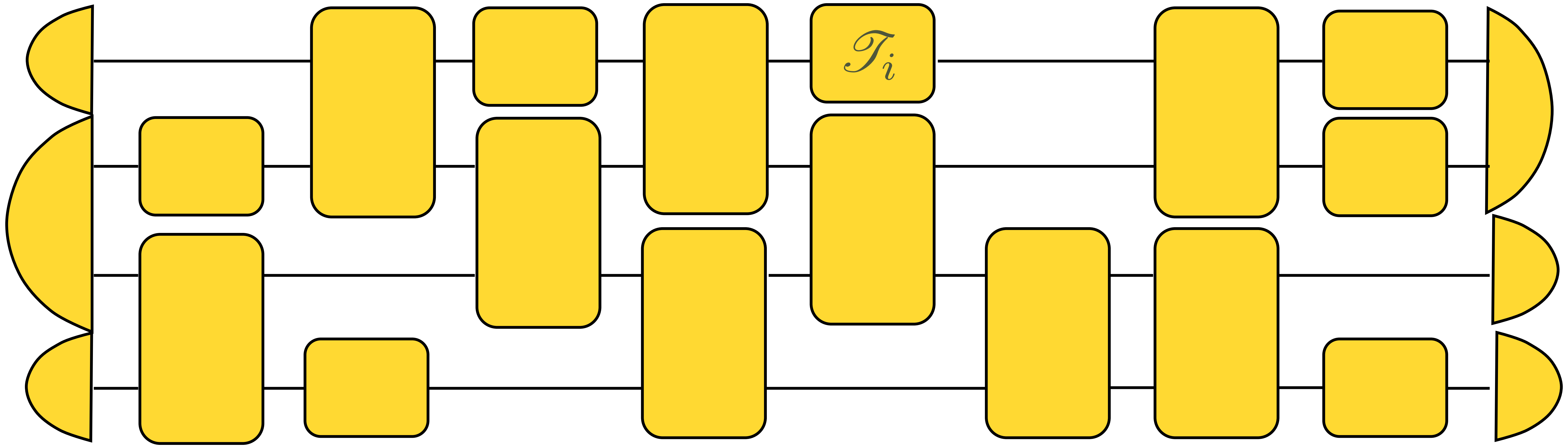

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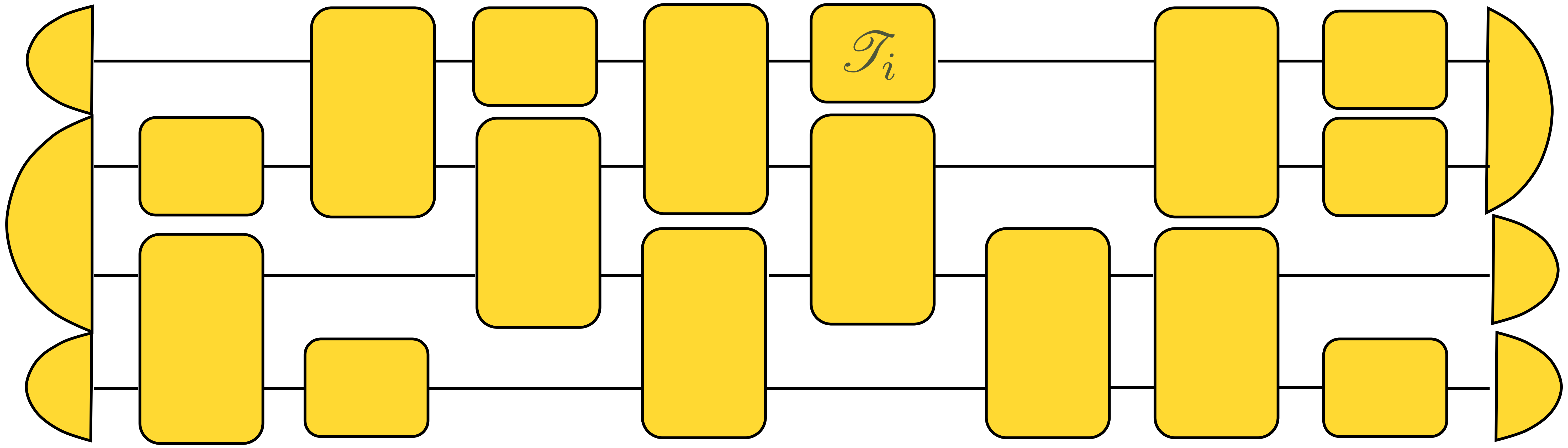

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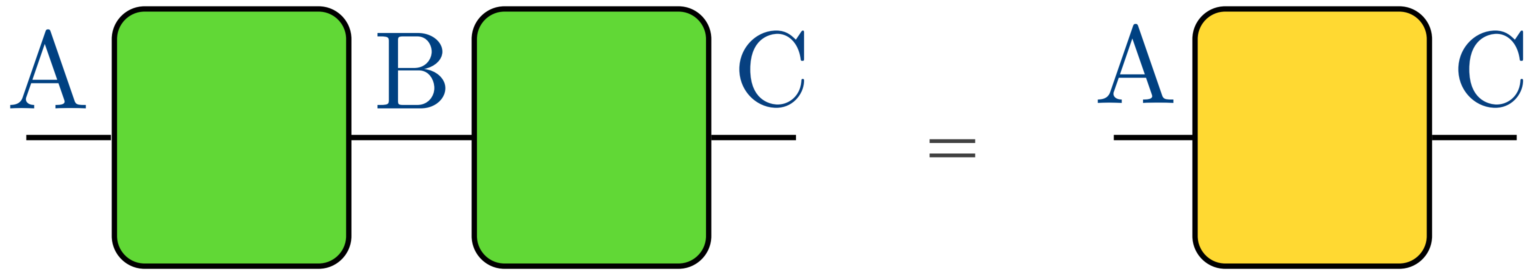
Sequential



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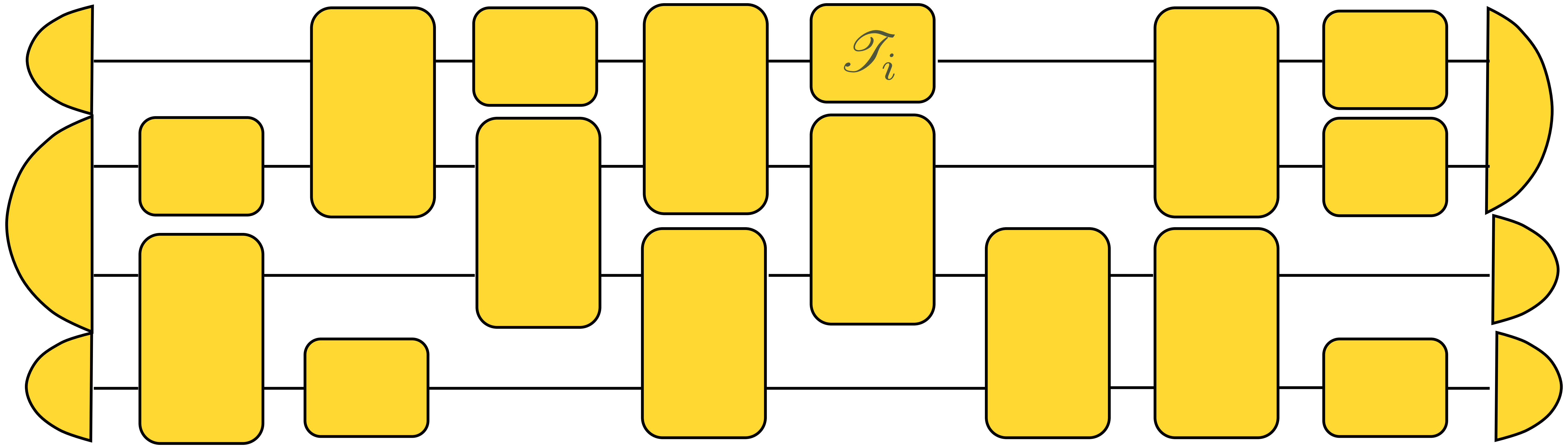

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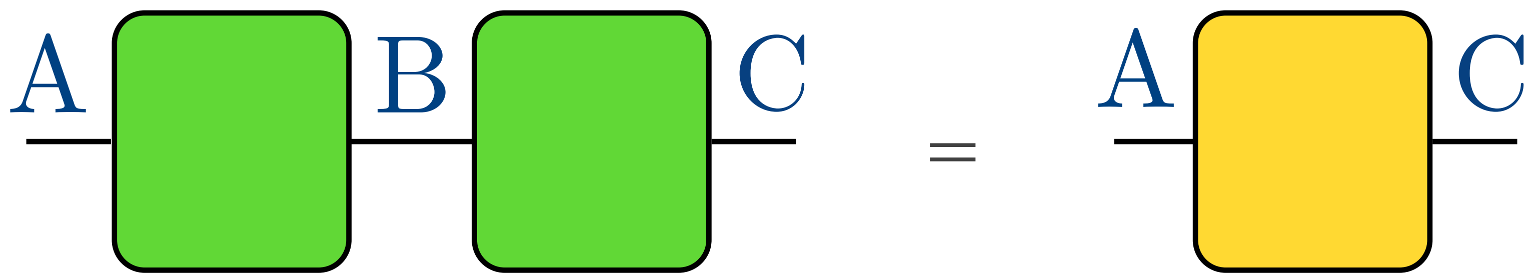
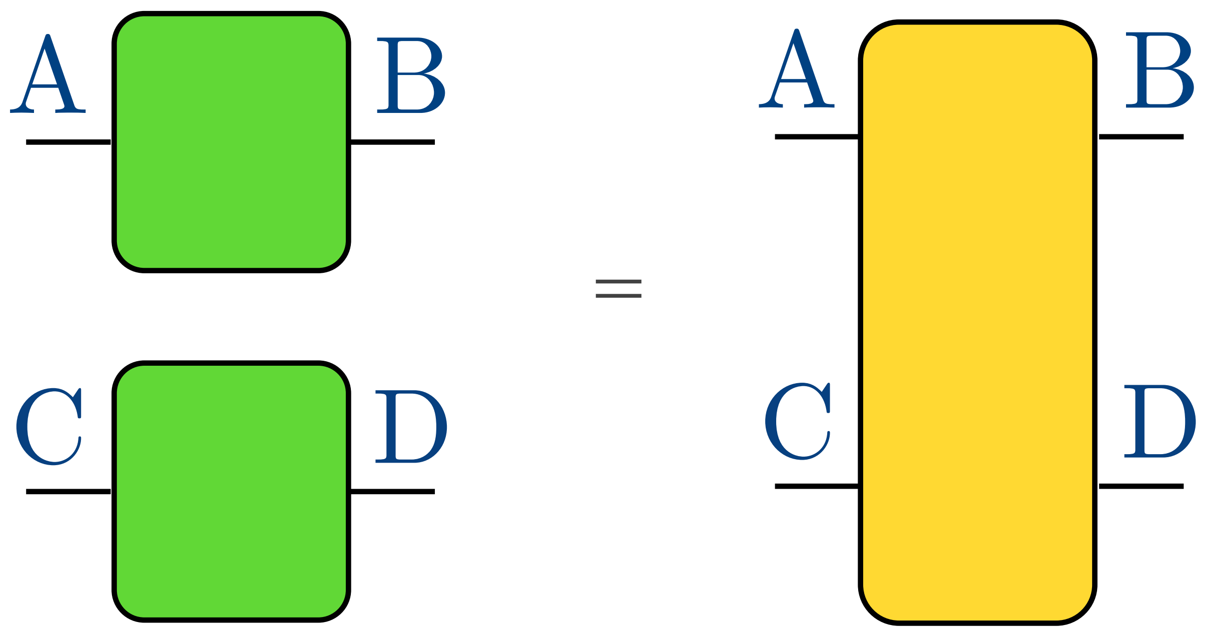


Parallel



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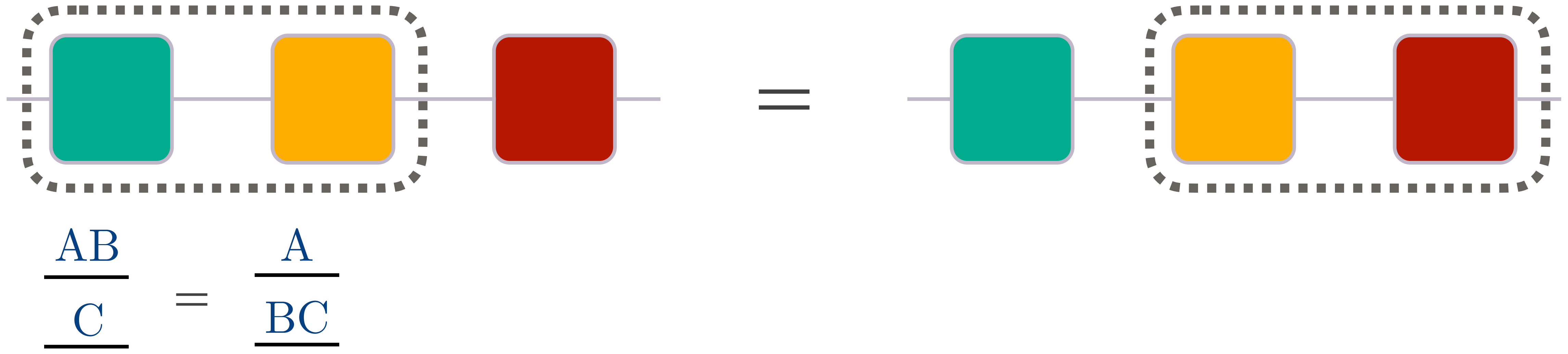
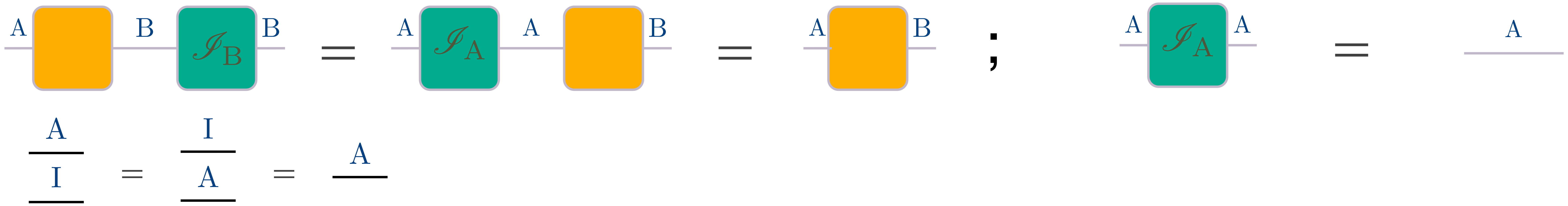
Operational Language

- Properties of composition rules:
  - Associativity



  - Unit



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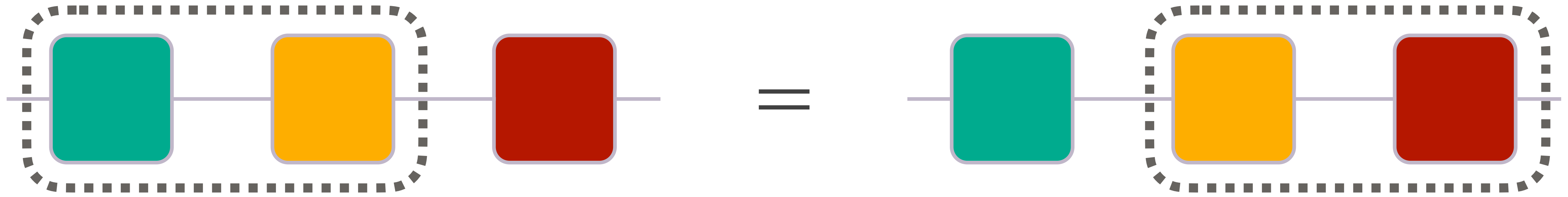
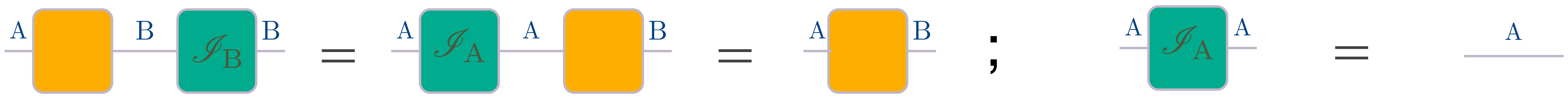


$$\frac{AB}{C} = \frac{A}{BC}$$

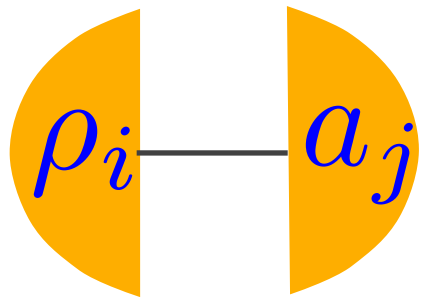
  - Unit



$$\frac{A}{I} = \frac{I}{A} = \underline{A}$$

Reversible event:

Probabilistic theories

Every test of type I→I is a probability distribution 

$= \Pr(a_j, \rho_i)$

States are functionals on effects and vice-versa

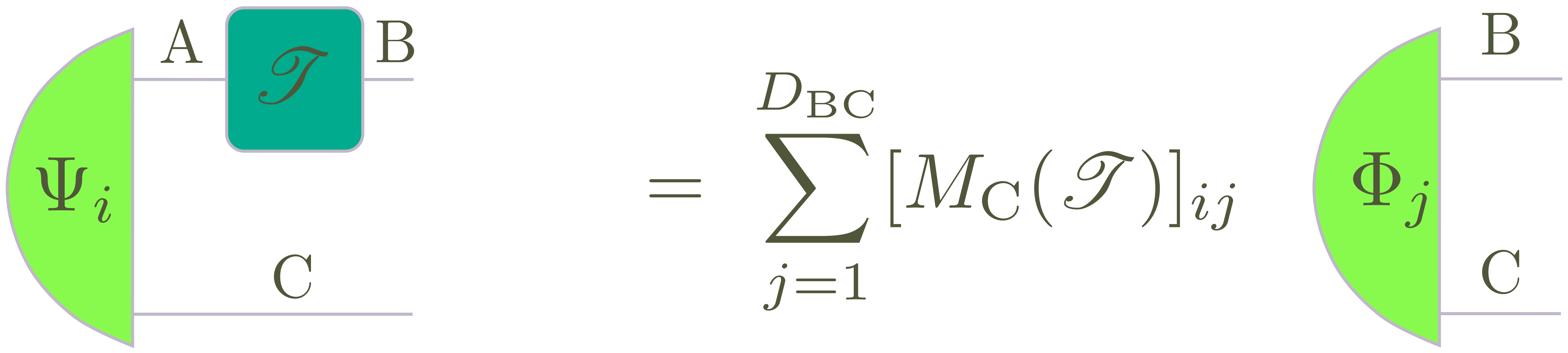
Real vector spaces $\mathsf{St}_{\mathbb{R}}(\mathrm{A}), \mathsf{Eff}_{\mathbb{R}}(\mathrm{A})$

G. Chiribella, G. M. D'Ariano, and PP, Phys. Rev. A **81**, 062348 (2010)

Transformations

A transformation $\mathscr{T} \in \mathsf{Transf}(\mathrm{A} \to \mathrm{B})$ induces a family of linear maps:

$\{M_{\mathrm{C}}(\mathscr{T})\}_{\mathrm{C}}$ representing $\mathscr{T} \otimes \mathscr{I}_{\mathrm{C}}$ on $\mathsf{St}(\mathrm{AC})_{\mathbb{R}}$



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Transformations

Indeed, it is not sufficient to know the linear map induced by $\mathscr{T}$ on $\mathsf{St}(\mathrm{A})_{\mathbb{R}}$

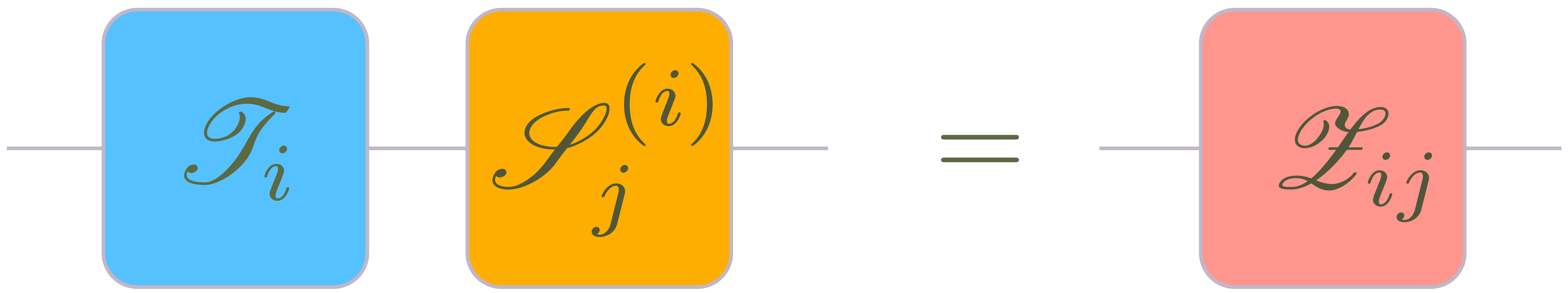
E.g.: transpose map in real quantum theory

- $\rho^T = \rho$ ➡ $\mathscr{T}(\rho) = \mathscr{I}(\rho) \quad \forall \rho$

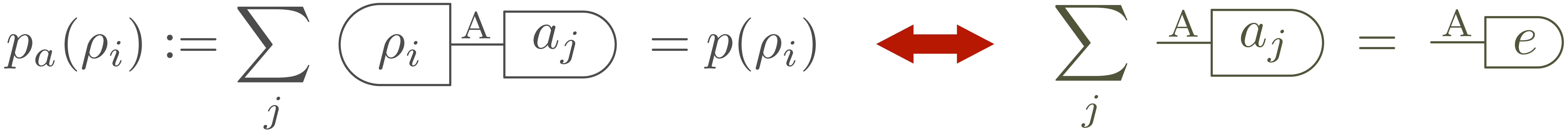
- $\sigma_y \otimes \sigma_y \in \mathsf{St}(\mathrm{AC})_{\mathbb{R}}$ $\quad (\mathscr{T} \otimes \mathscr{I}_{\mathrm{C}})(\sigma_y \otimes \sigma_y) = -\sigma_y \otimes \sigma_y$

Strongly causal theories

- Possibility of arbitrary conditional tests



- Causality implies no "backward" signalling



PP, Quantum **4**, 294 (2020)
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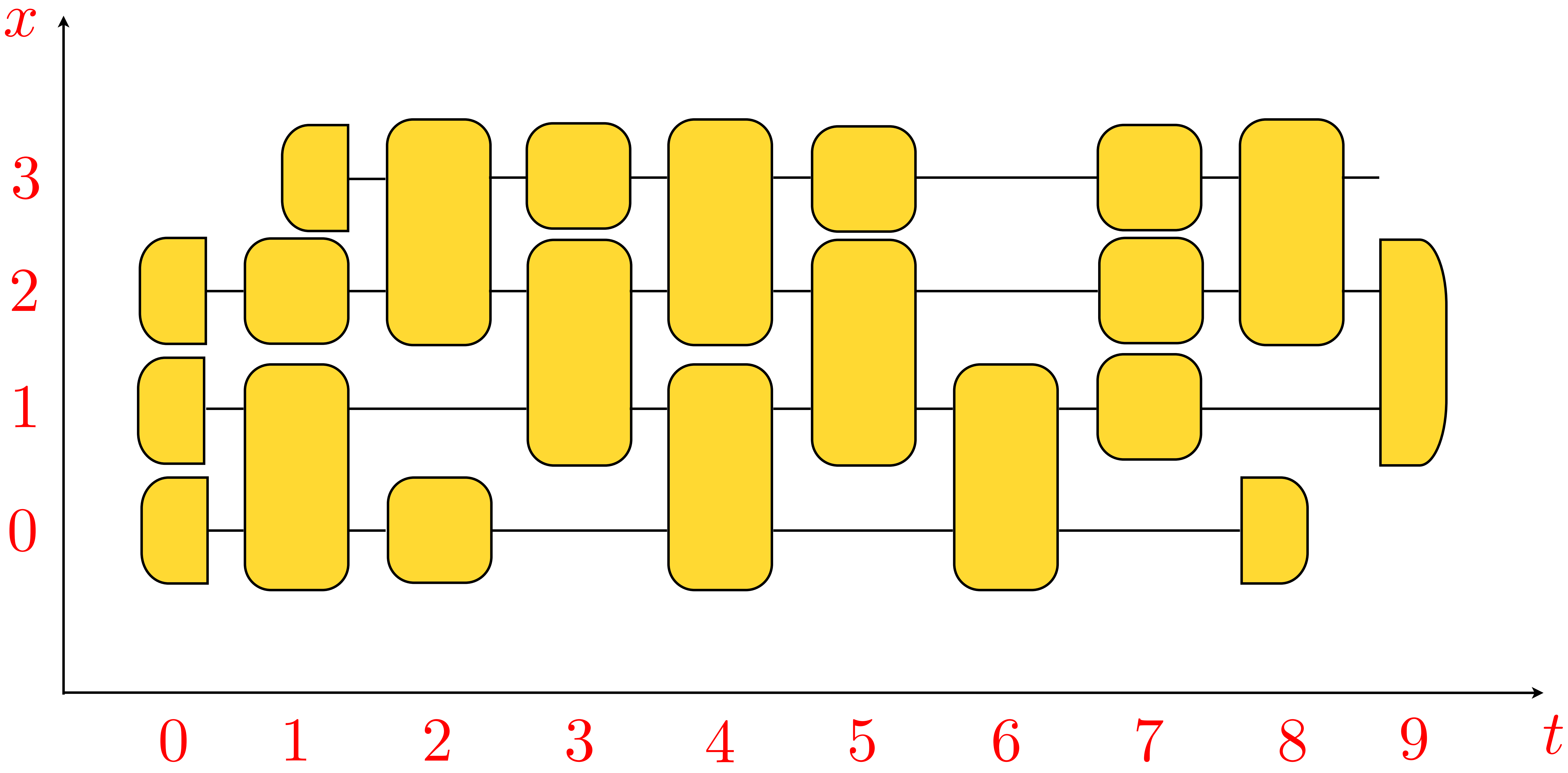
Circuits and causal chains

Logical space-time



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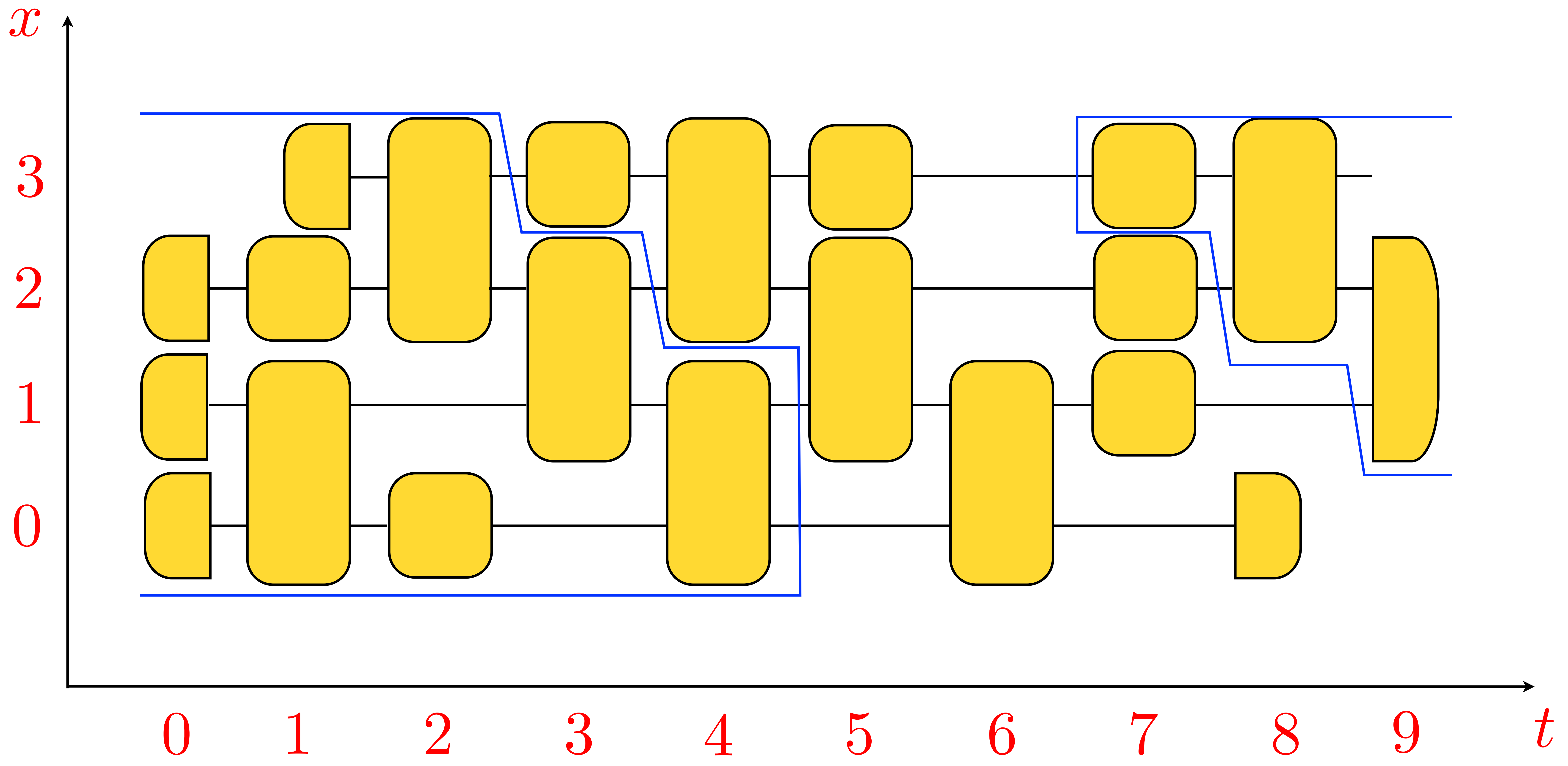

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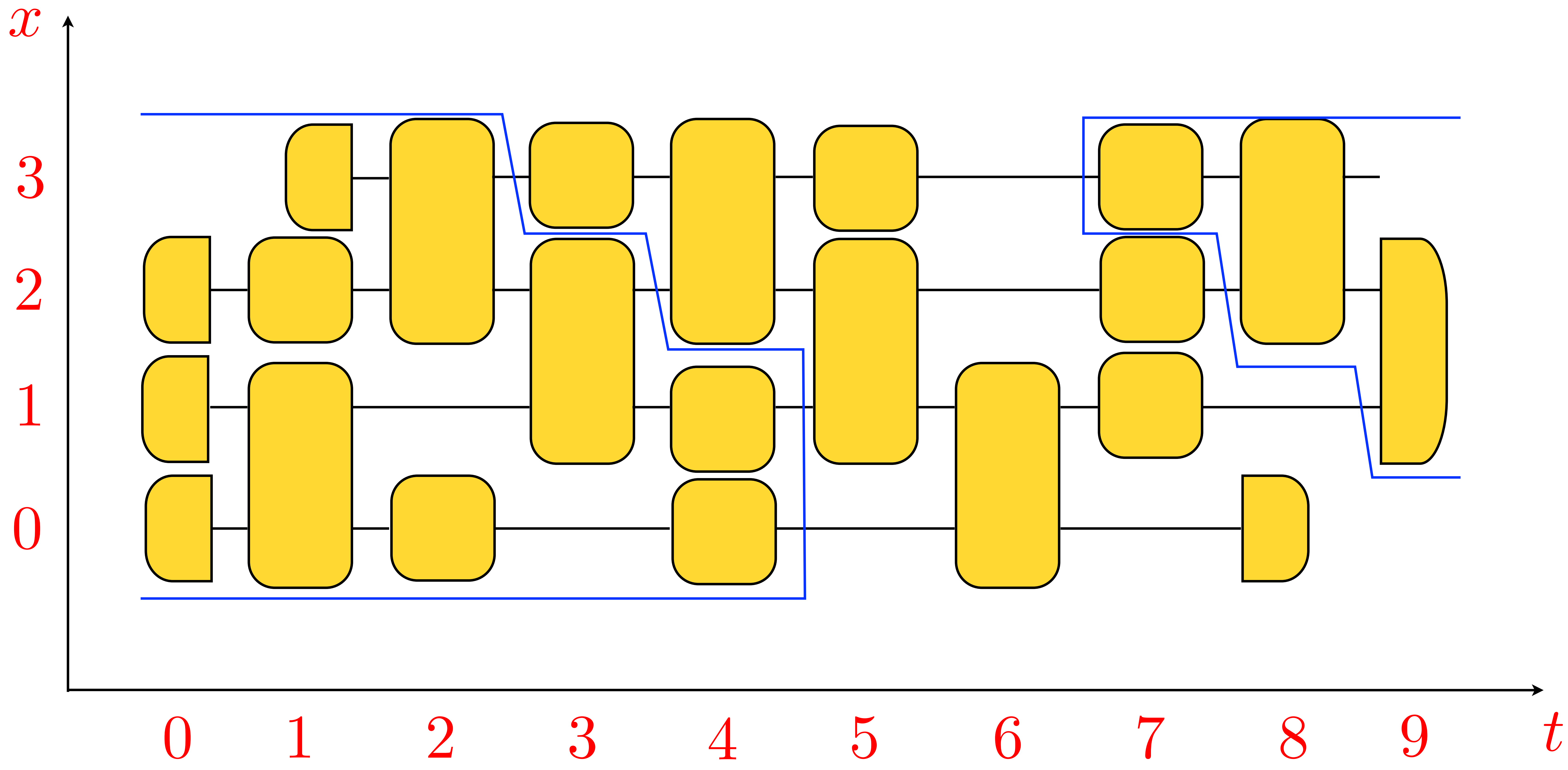

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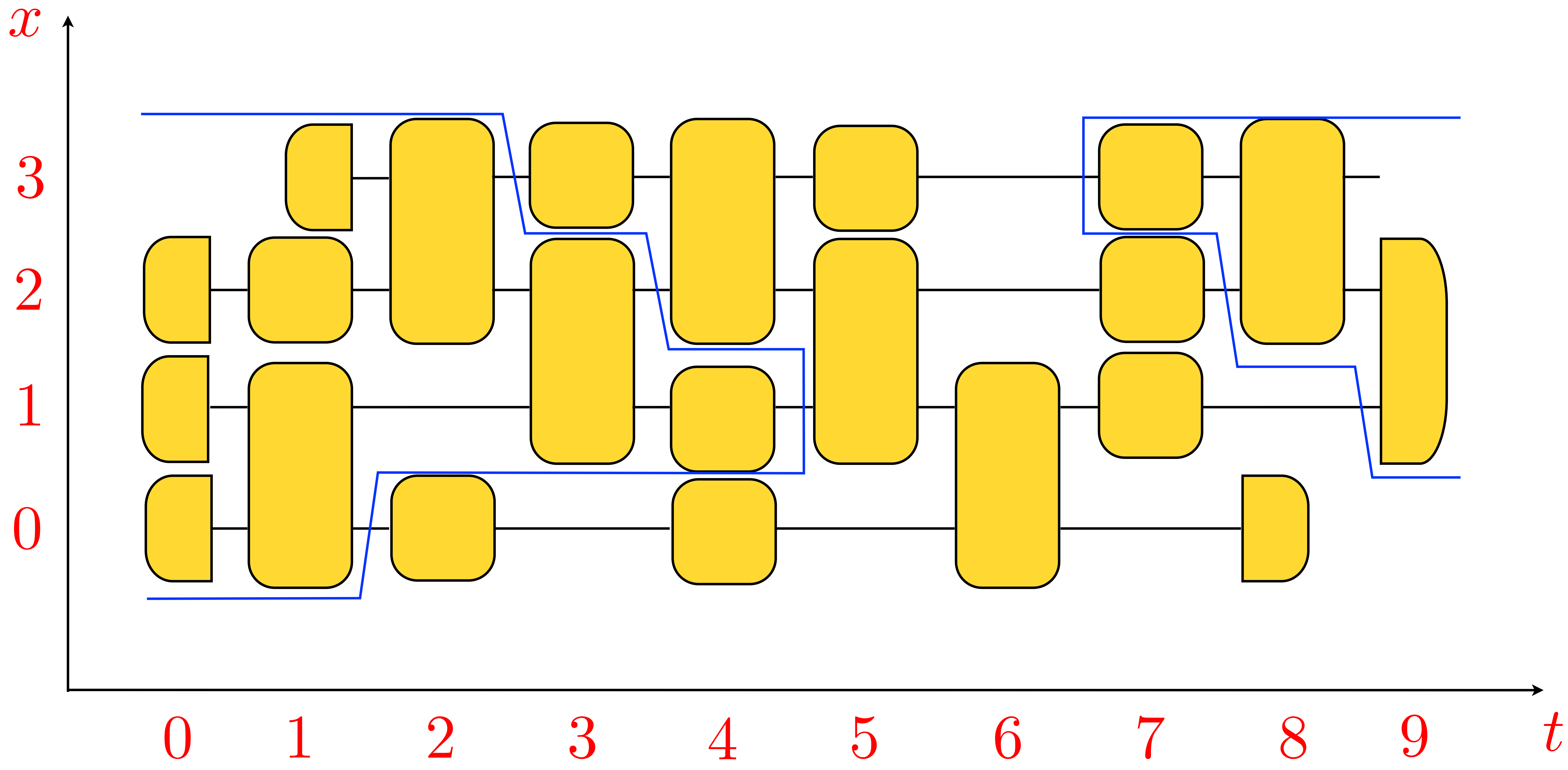

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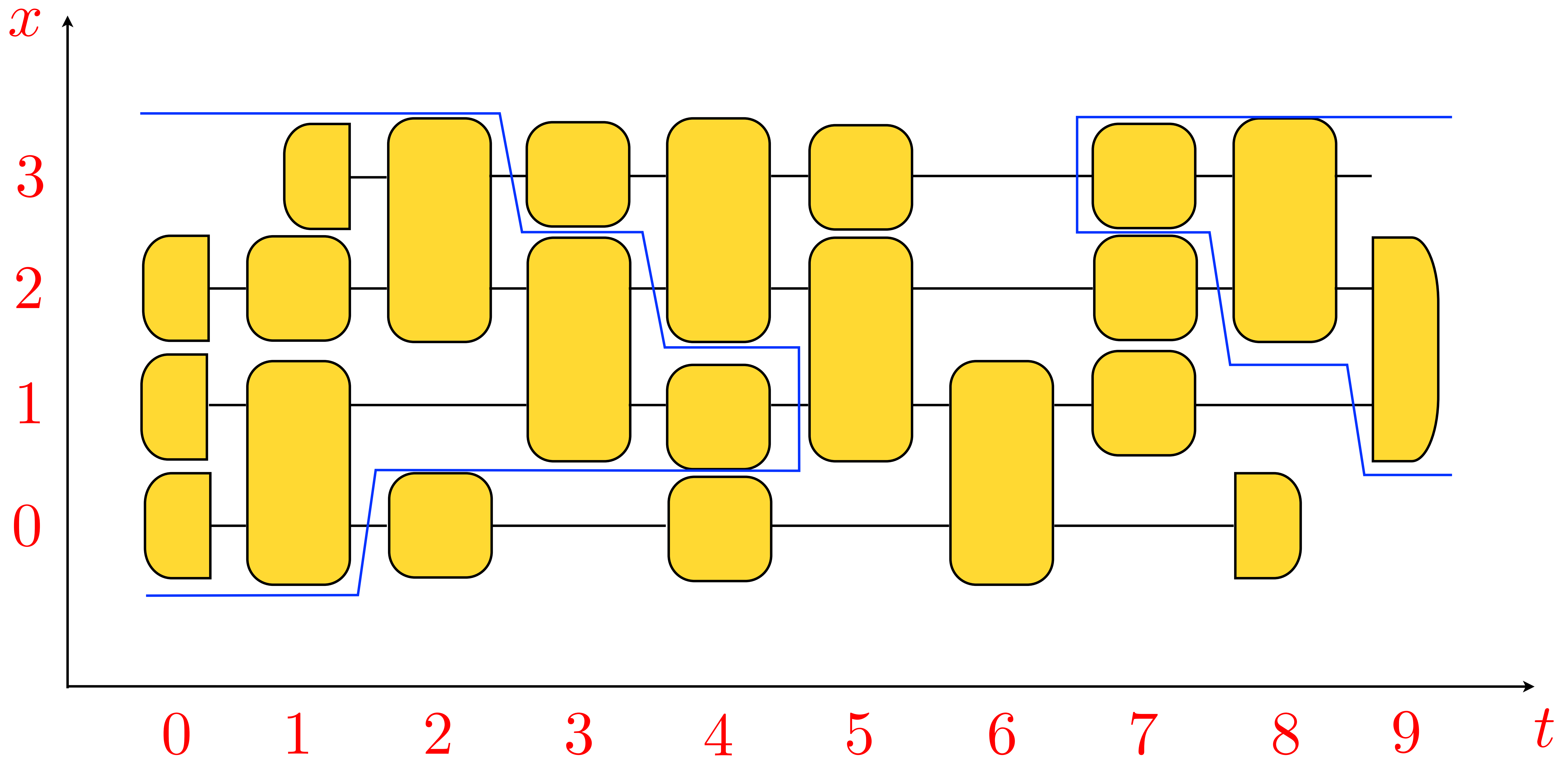
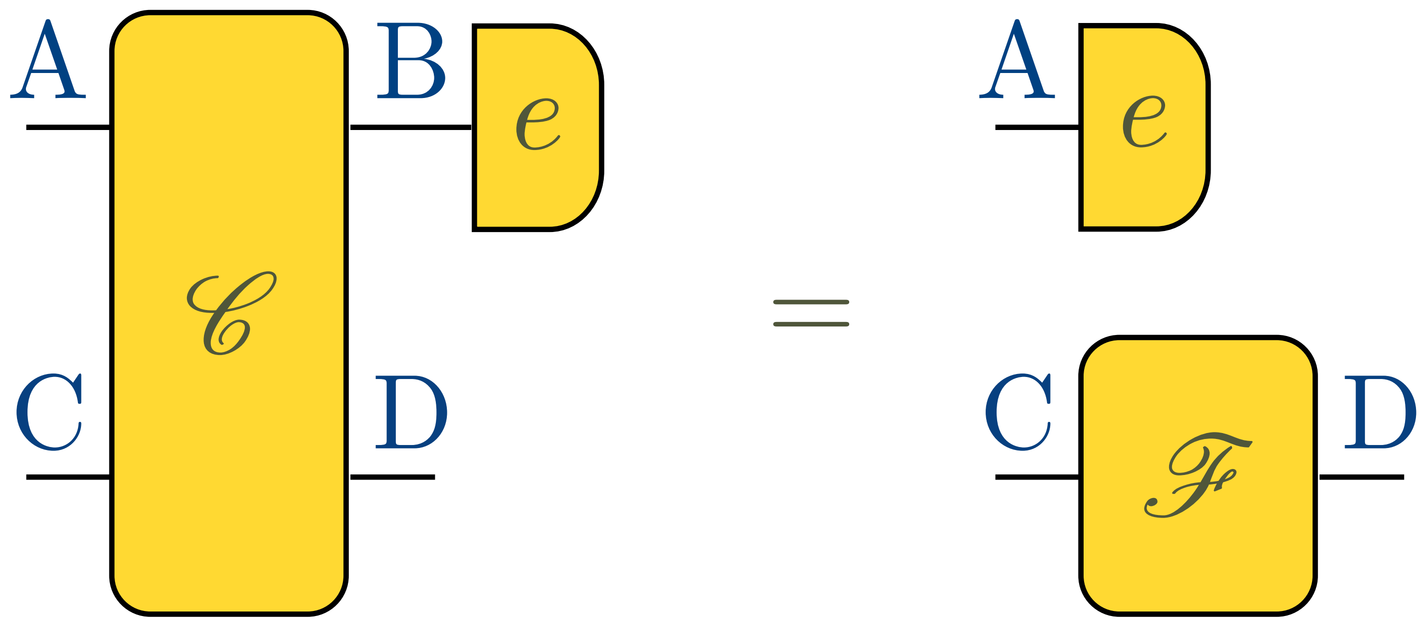


Can we make the causal order relation sharper?

G. M. D'Ariano, G. Chiribella, and PP, *Quantum Theory from first principles. An informational approach.* (CUP 2017)

Non-signalling

The usual approach



$A \not\rightarrow D$

No intervention on the state of A can influence the state of C

In quantum theory

$$\mathrm{Tr}_{\mathrm{B}}[R_{\mathscr{C}}] = I_{\mathrm{A}} \otimes R_{\mathscr{F}}$$

$R_{\mathscr{E}}$ denoting the Choi operator corresponding to $\mathscr{E}$

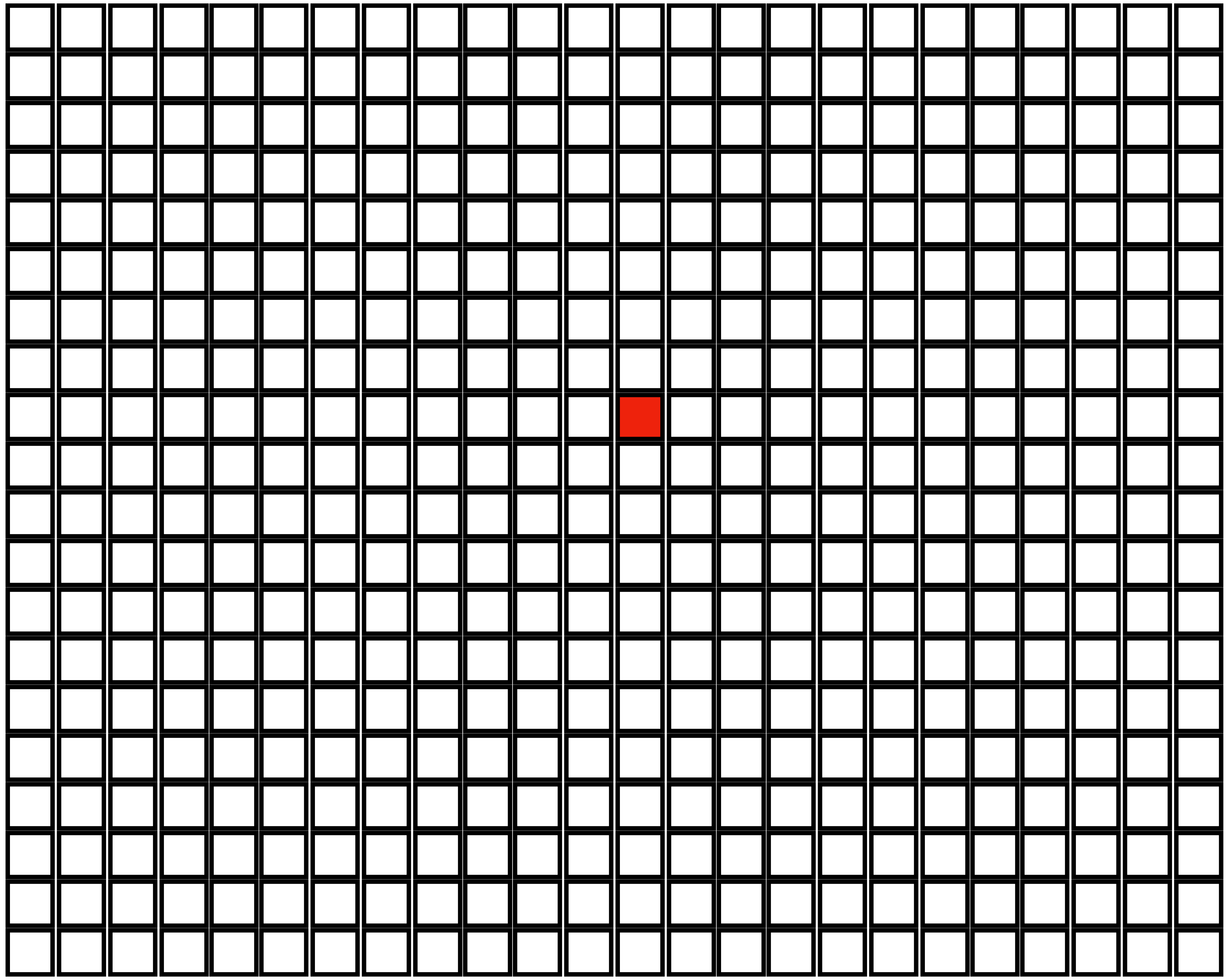
D. Beckman, D. Gottesman, M. A. Nielsen, and J. Preskill, Phys Rev. A **64**, 052309 (2001)
T. Eggeling, D. Schlingemann, and R.F. Werner, Europhys. Lett. **57**, pp. 782–788 (2002)

Quantum cellular automata

Neighbourhood of a cell

$\square = \mathcal{H}_x \leftrightarrow \mathsf{A}_x$

C.A.: $U : \bigotimes_x \mathcal{H}_x \rightarrow \bigotimes_x \mathcal{H}_x$



B. Schumacher and R.F. Werner, arXiv:quant-ph/040517

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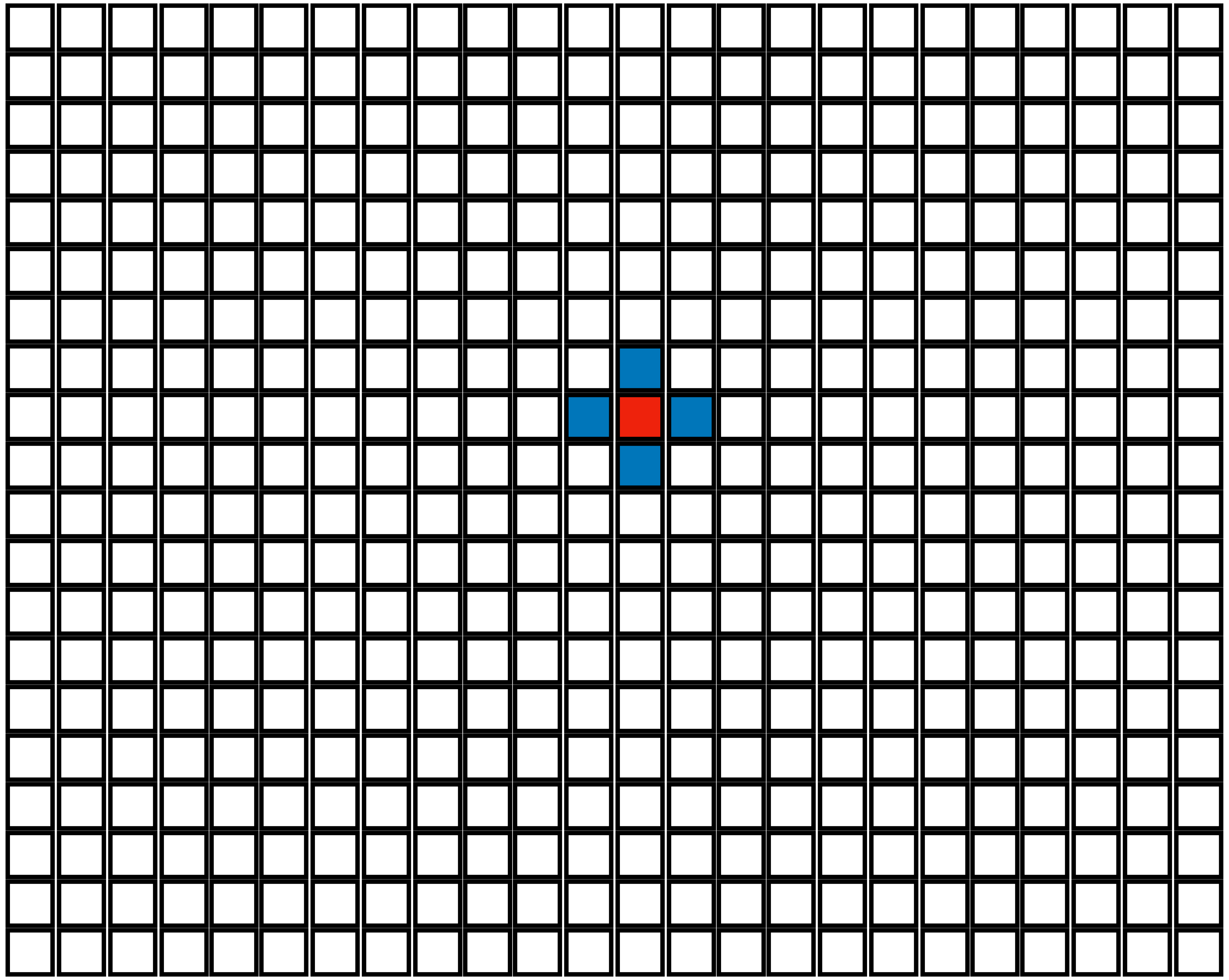
Neighbourhood of the cell x_0

$$U^{-1} \mathsf{A}_{x_0} U = \mathsf{A}_{\mathsf{N}(x_0)} \otimes I_{\bar{x}_0}$$



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Defining (no) causal influence in OPTs

The raw idea

- The definition is inspired by the notion of neighbourhood
- It holds for reversible transformations

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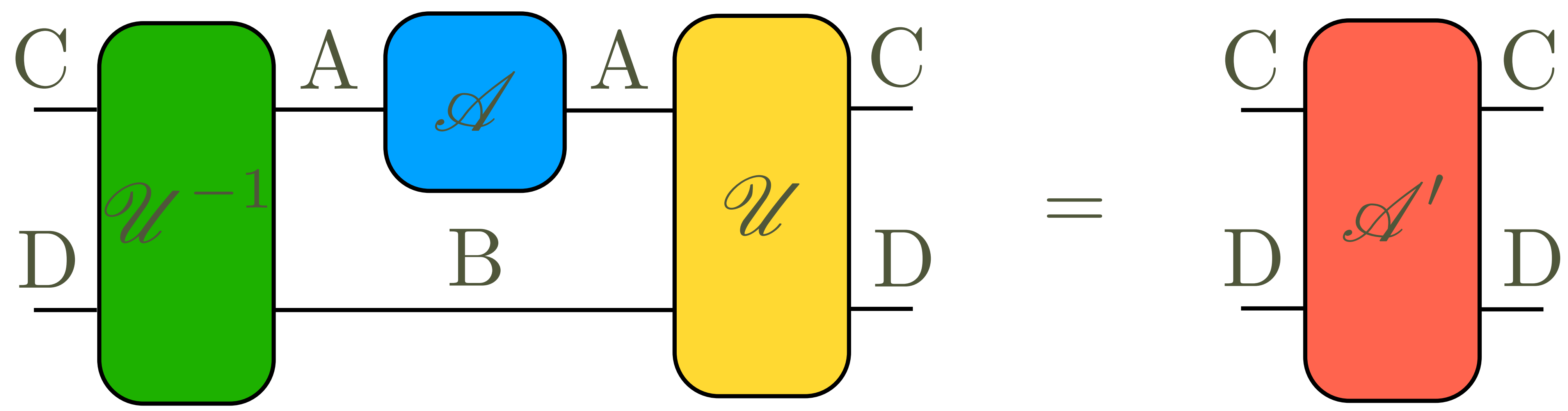

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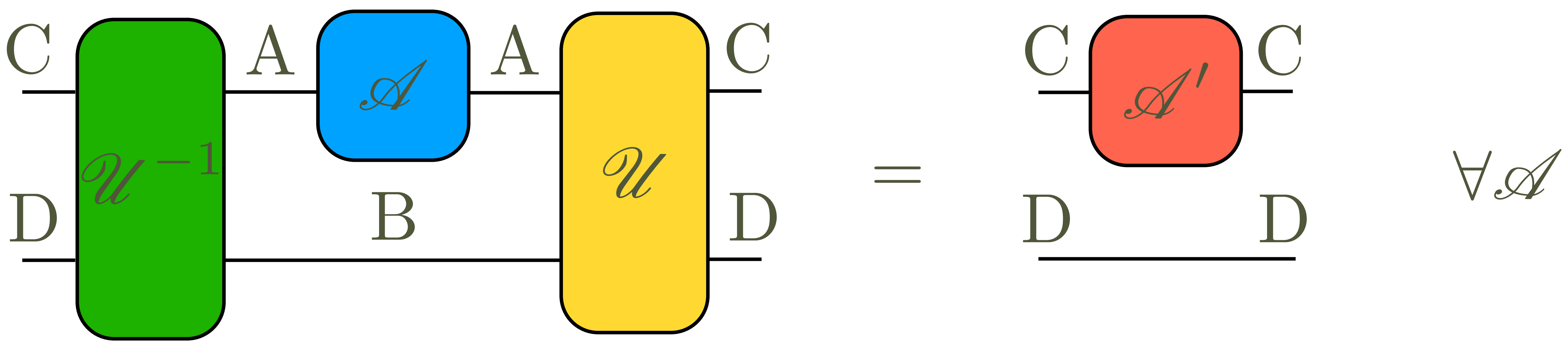

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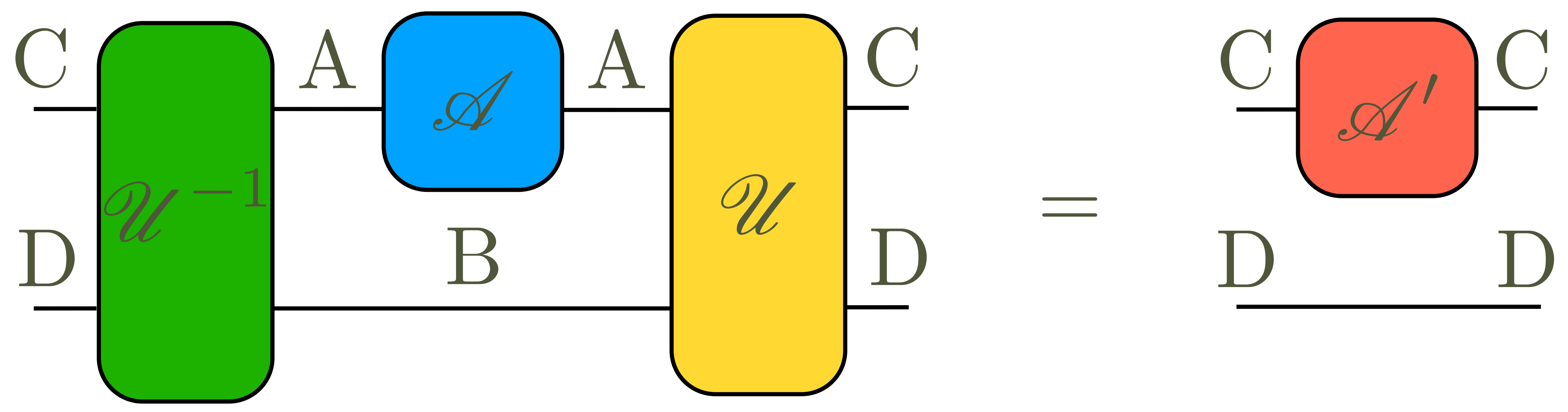

$\forall \mathscr{A} \qquad A \not\to D$

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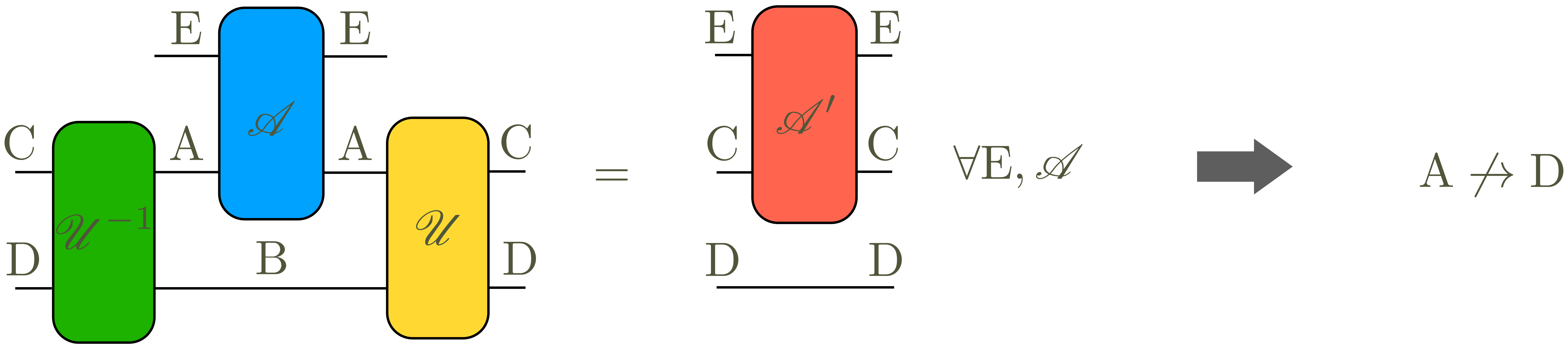
Defining (no) causal influence in OPTs

The precise notion

- Without local discriminability (local tomography/tomographic locality) we need to take into account interventions involving ancillary systems



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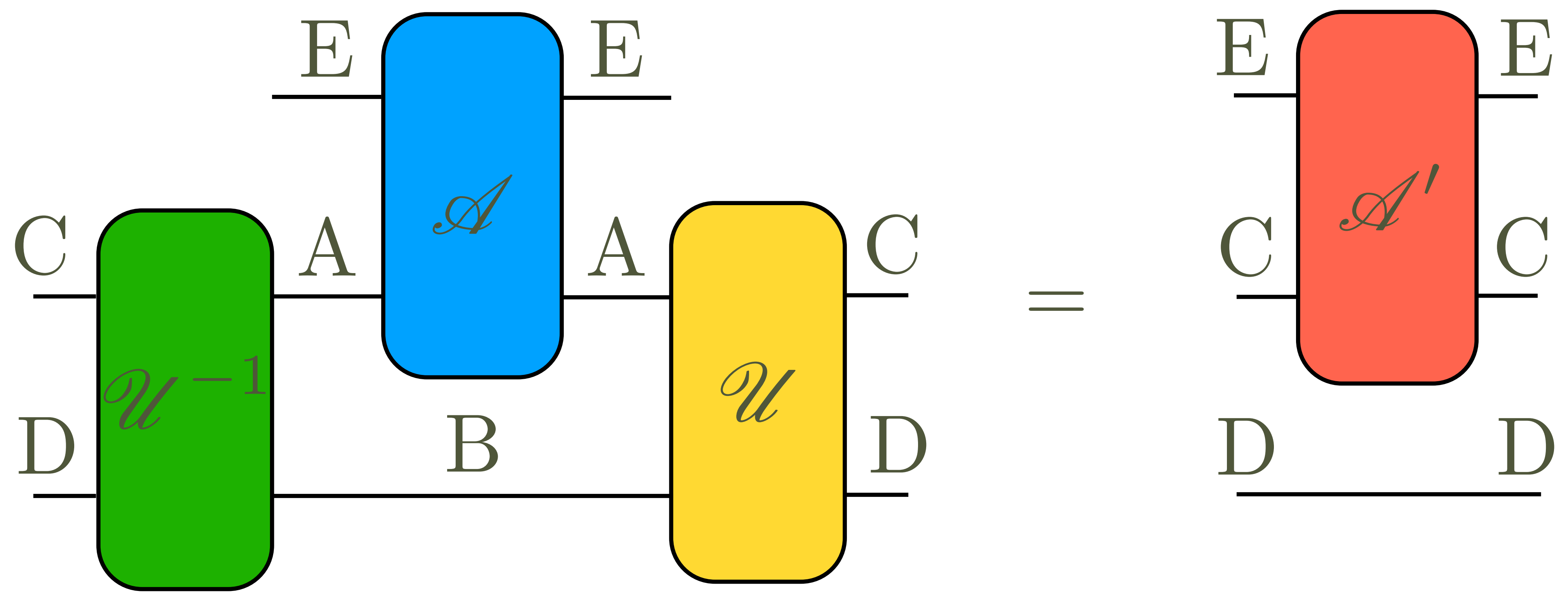
Explanation of the definition

Suppose that under $\mathscr{U}$ one has $\mathrm{A} \not\to \mathrm{D}$



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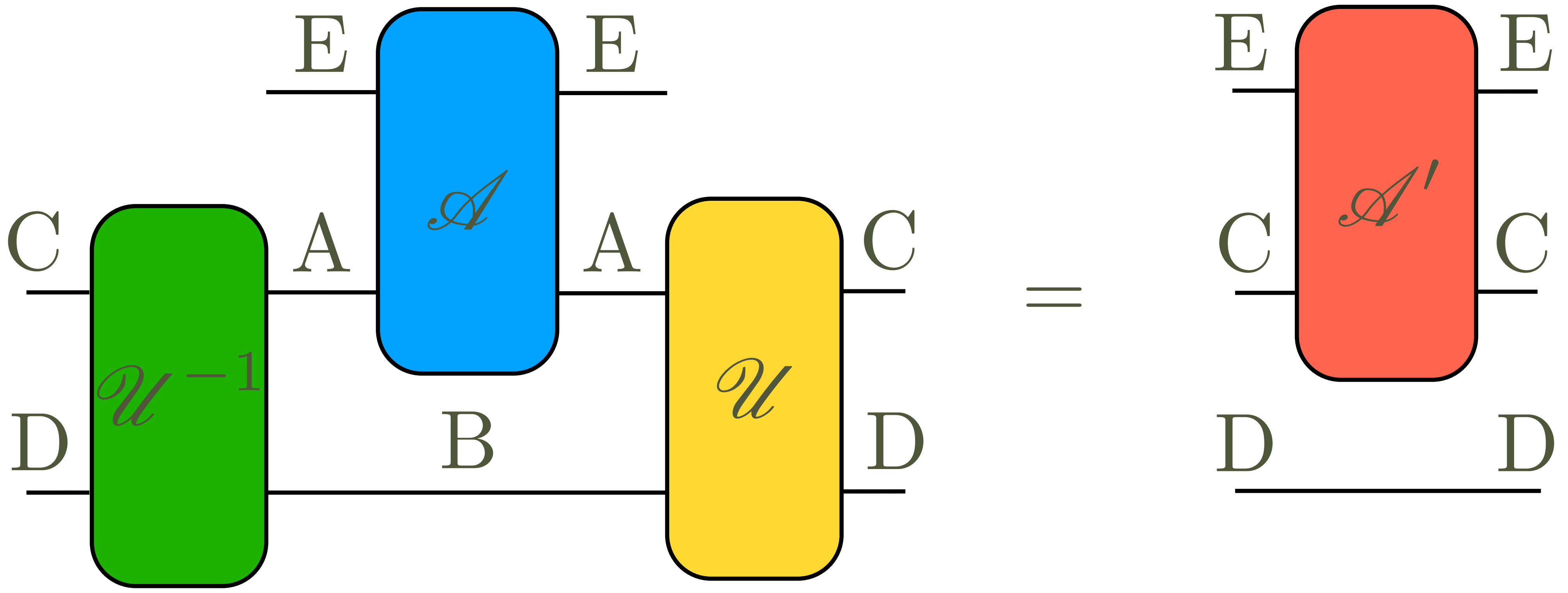
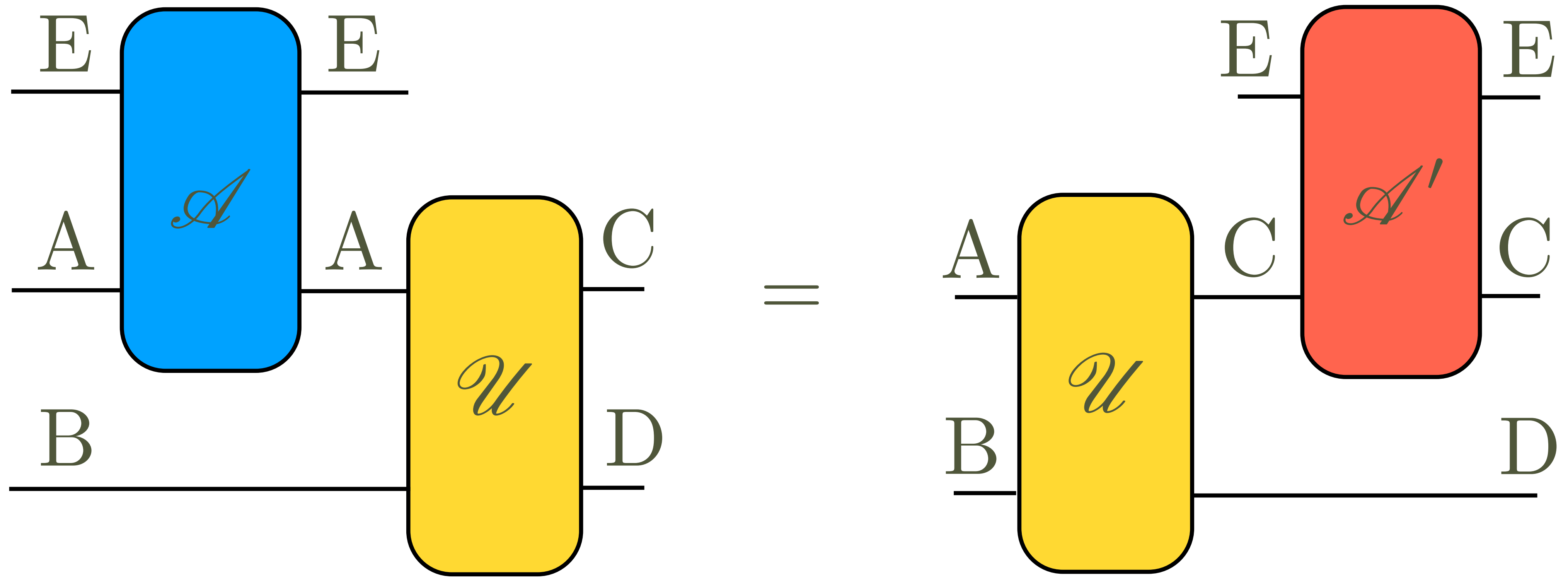
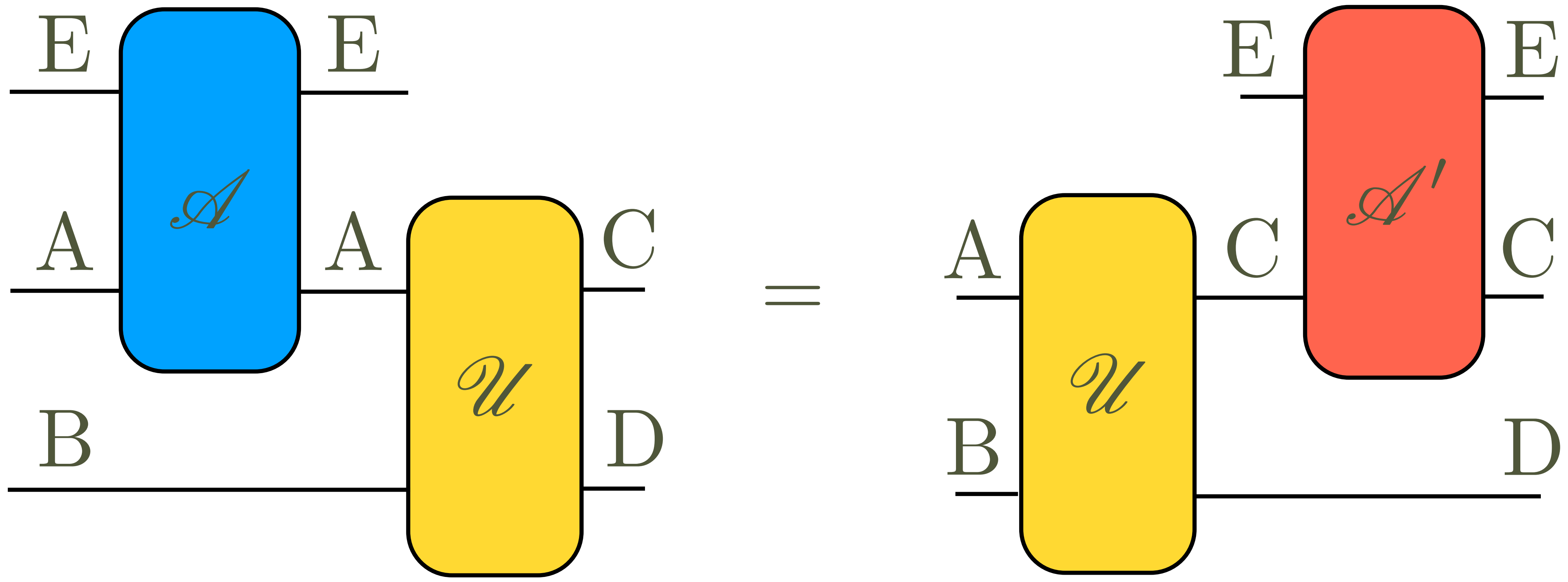


Equivalently:



Necessary condition for no C.I.: no-signalling

Let $\mathrm{A} \not\to \mathrm{D}$:



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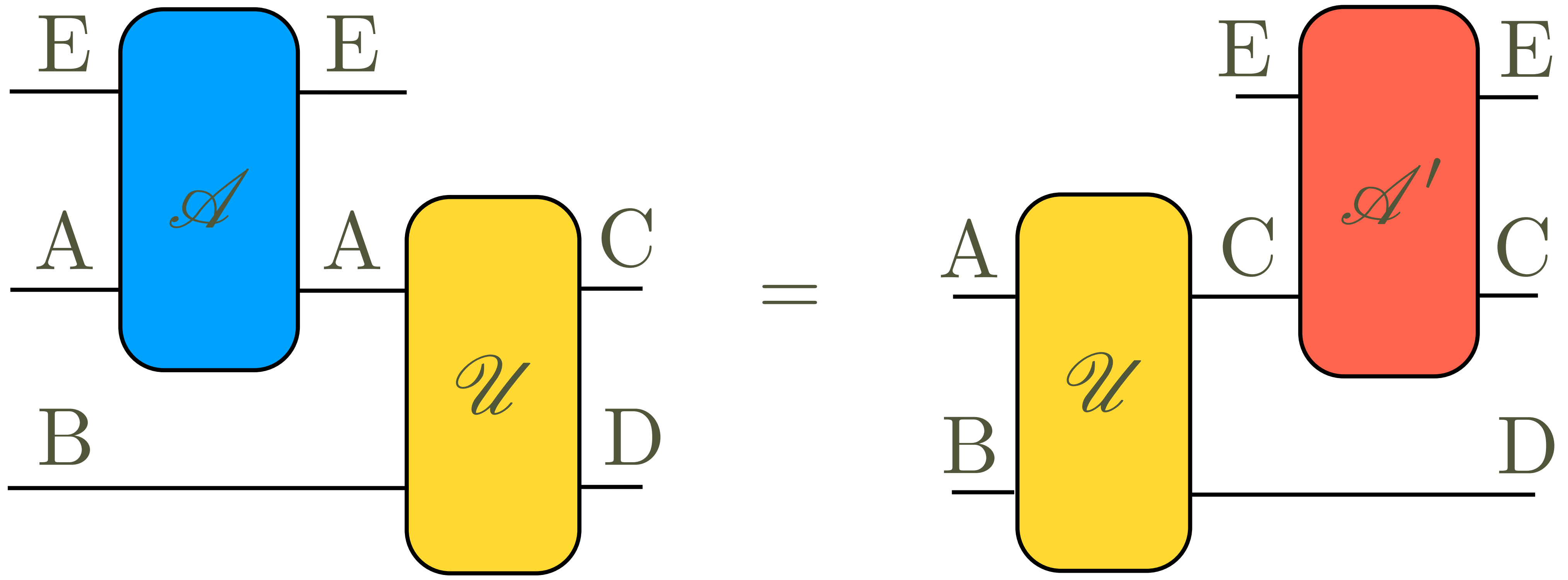
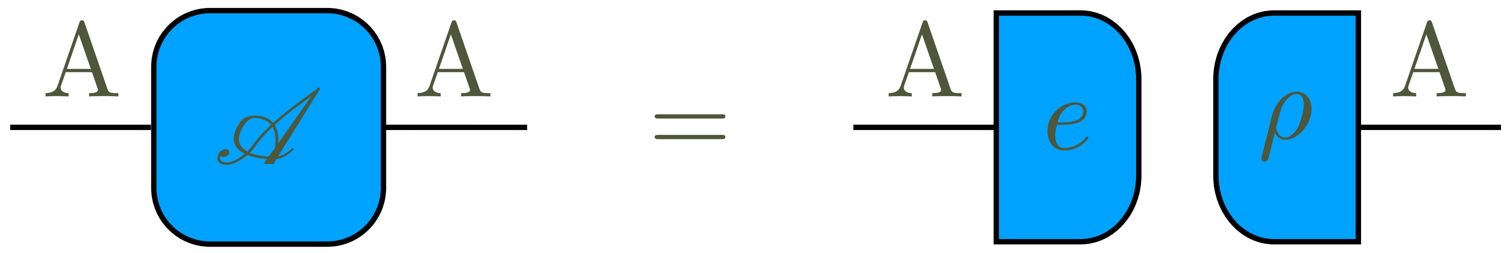
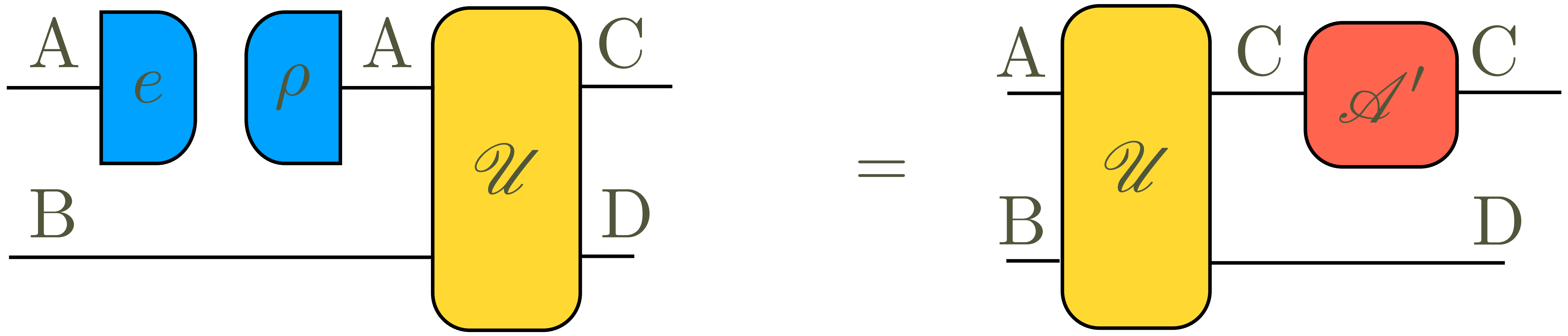


Consider $\mathrm{E} = \mathrm{I}$,





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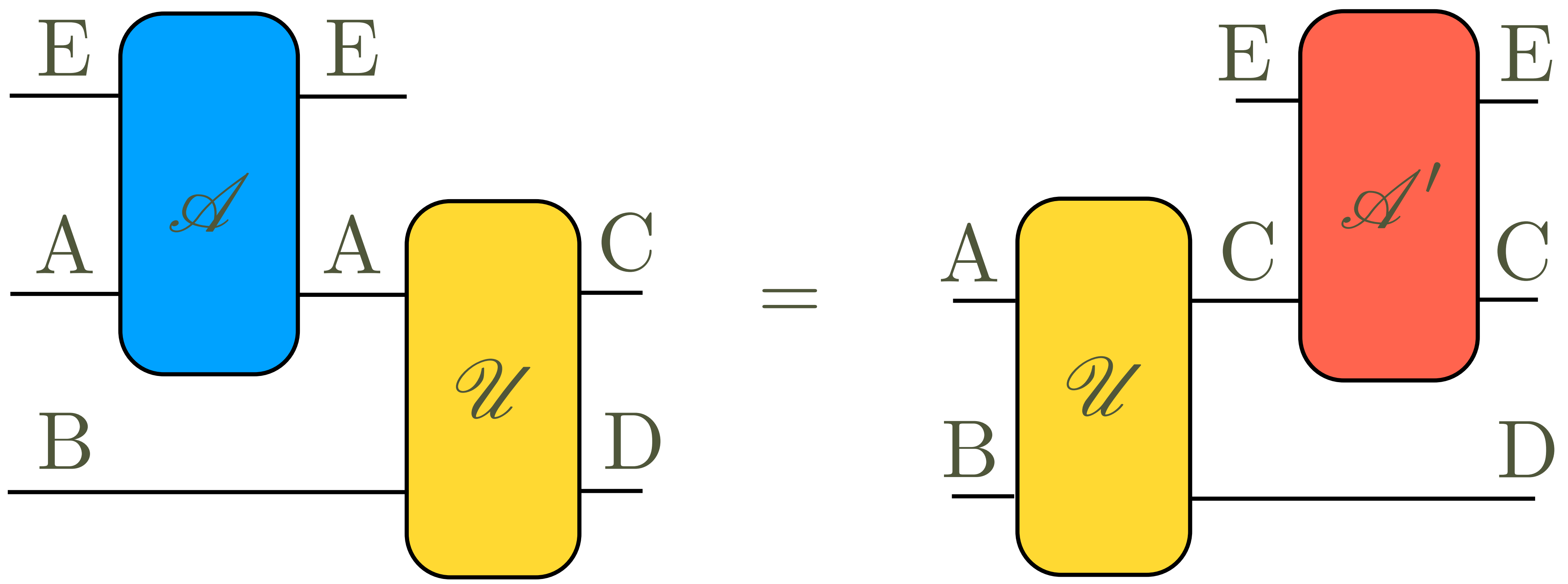

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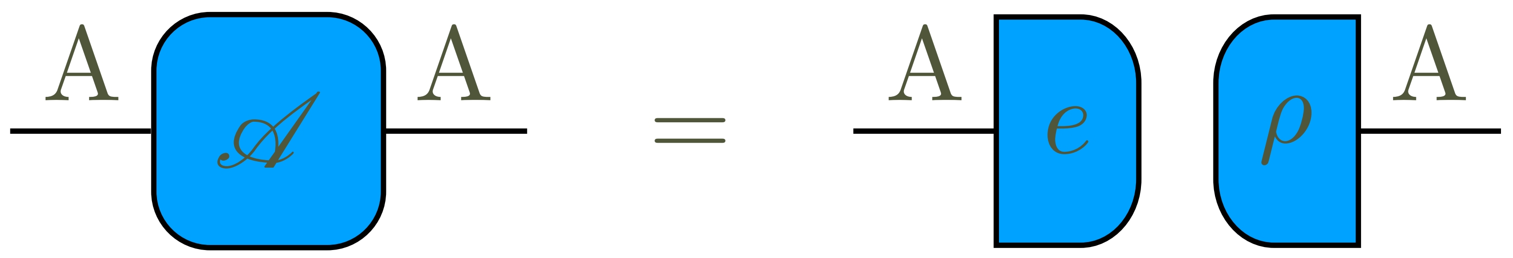
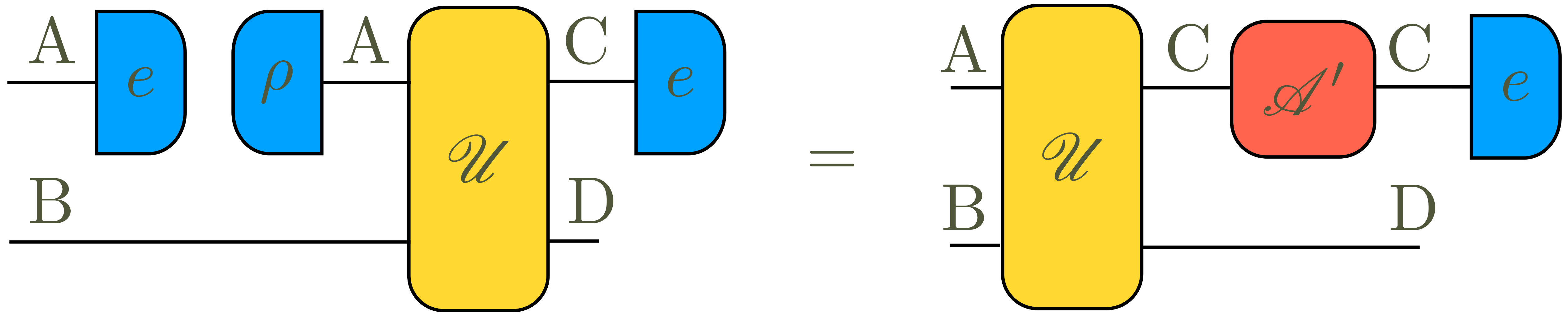


then discard C



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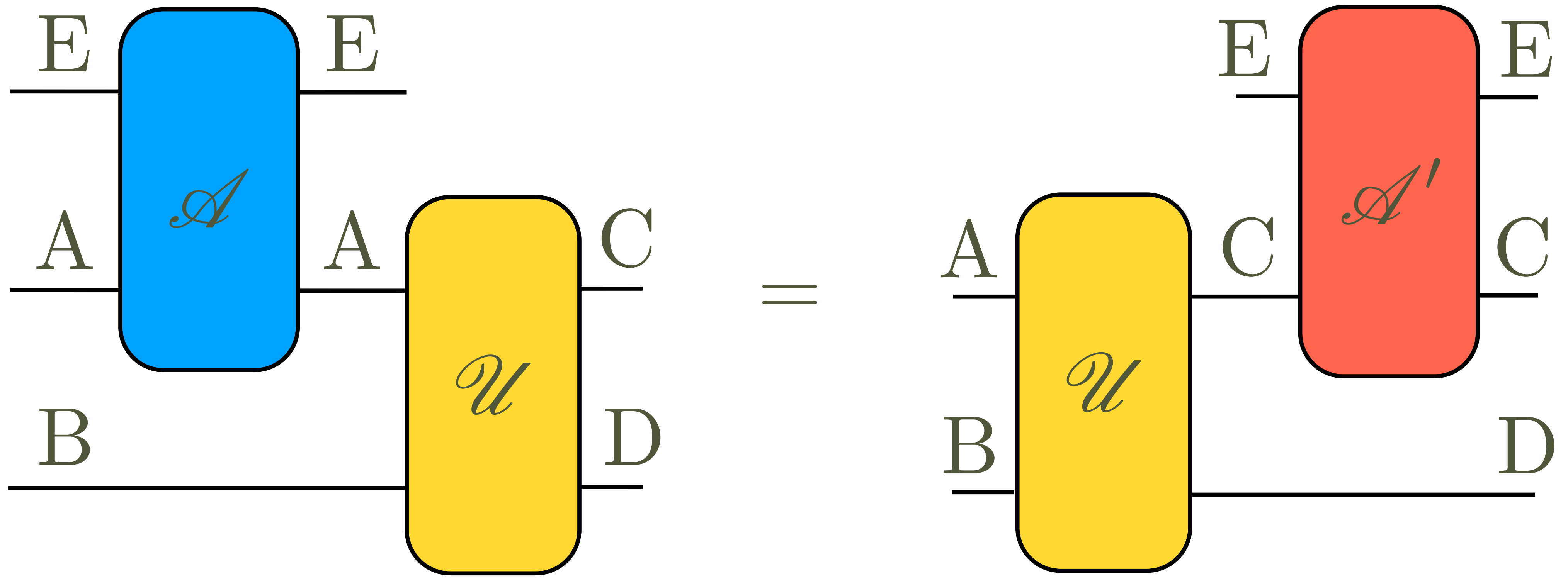

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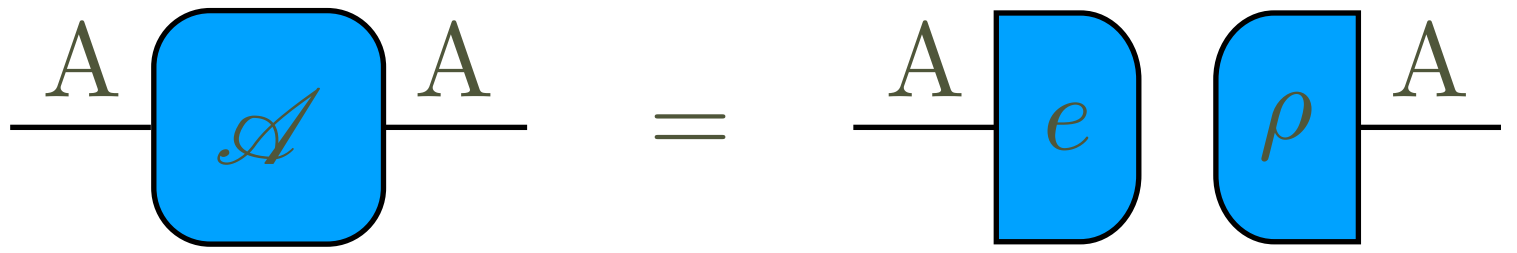

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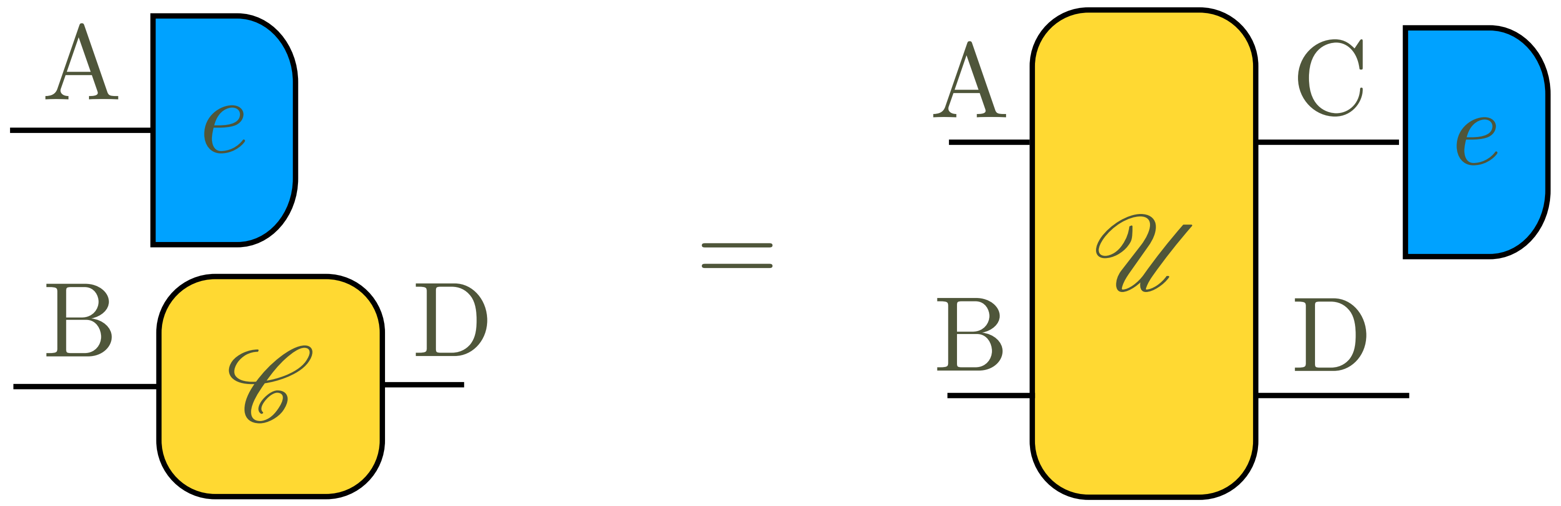

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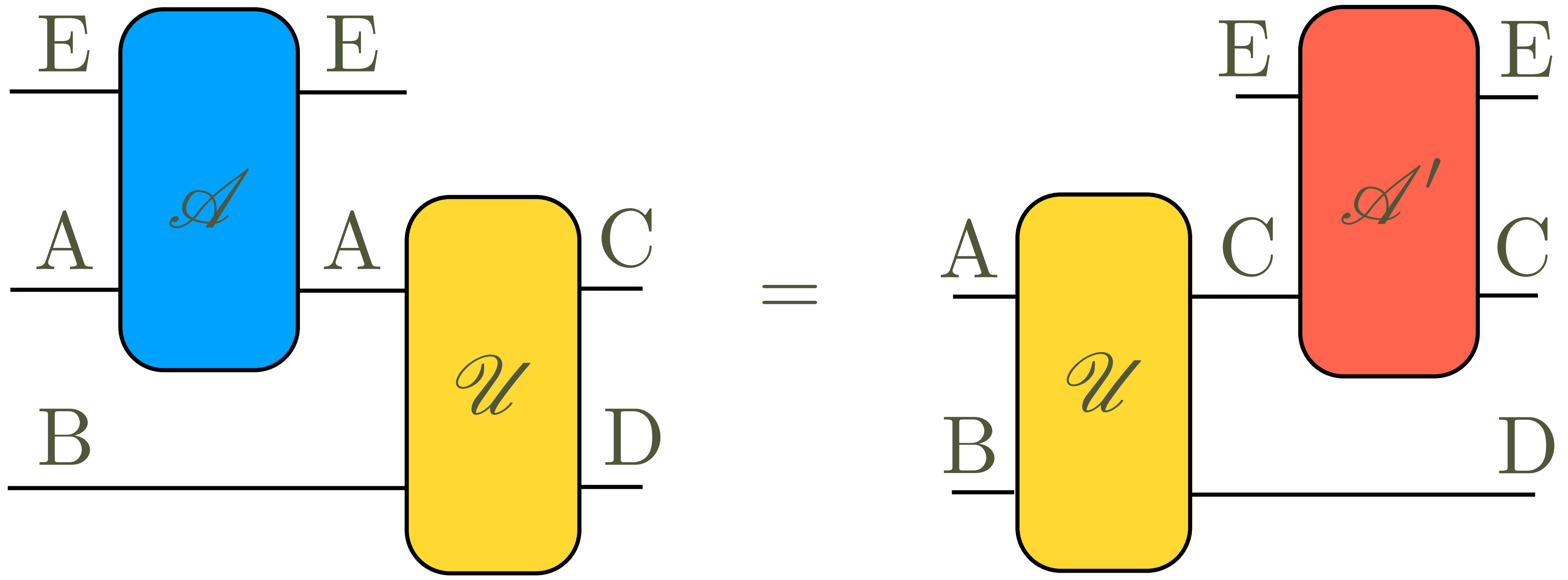

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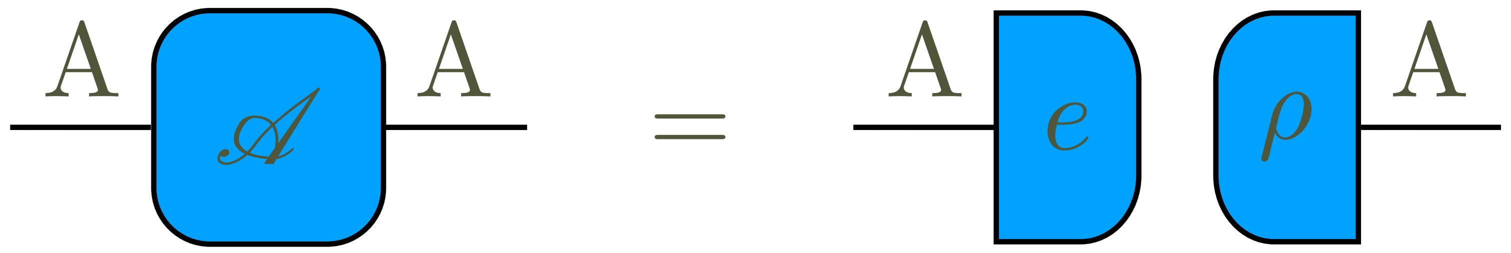

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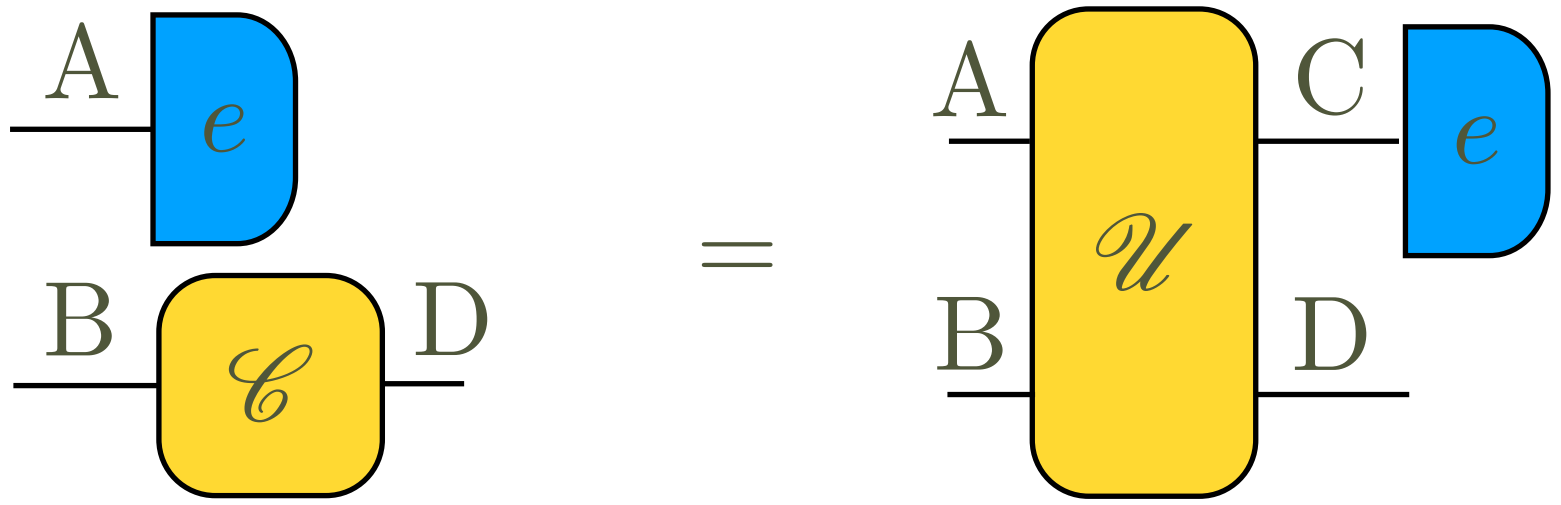

$\mathrm{A} \not\to \mathrm{D}$

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Necessary condition for no C.I.: no-signalling

Let $\mathrm{A} \not\to \mathrm{D}$:

$\Rightarrow$ $\mathrm{A} \not\rightsquigarrow \mathrm{D}$

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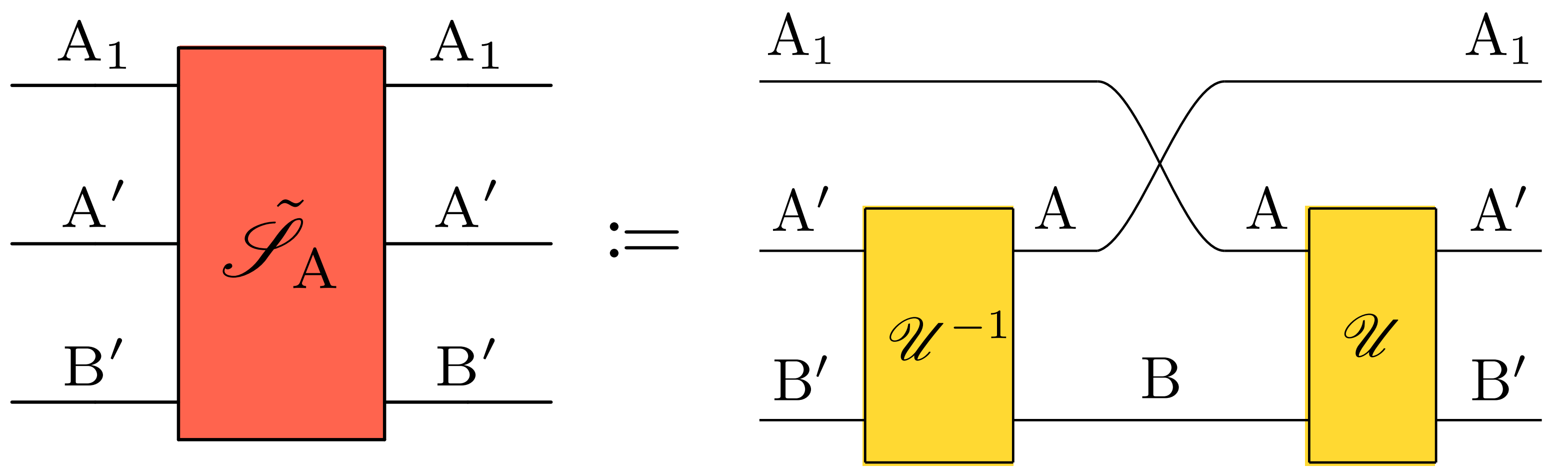
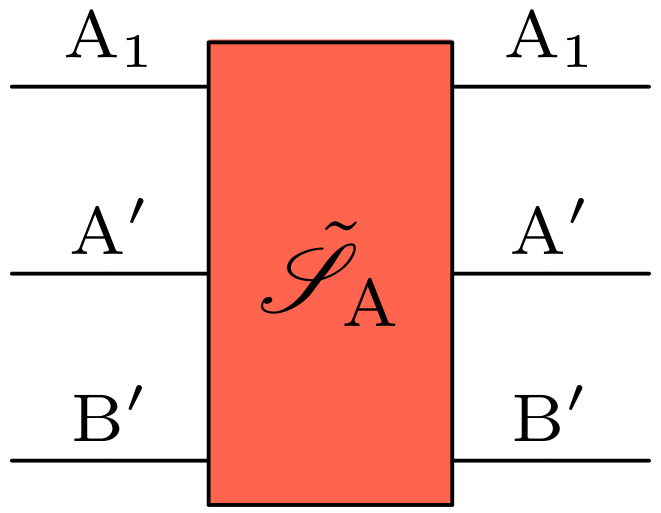
Necessary and sufficient condition

- Definition:



- Condition:

$\mathrm{A} \not\to \mathrm{B}'$ iff



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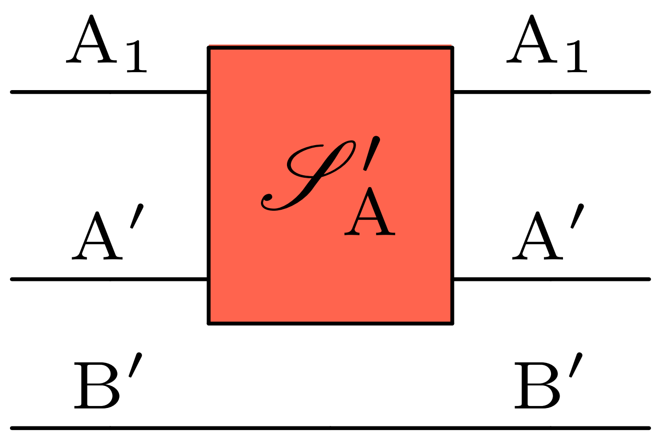
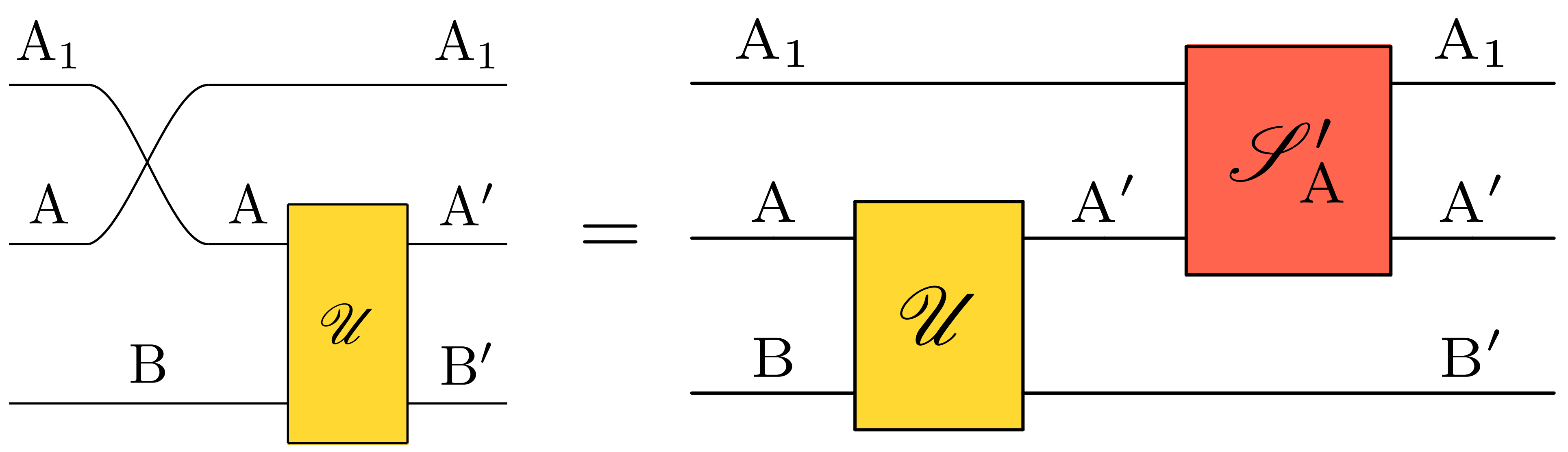


PP, Quantum **4**, 294 (2020)
PP, Quantum **5**, 515 (2021)

Necessary condition: comb structure

- Suppose that $\mathrm{A} \not\to \mathrm{B}'$. Then it must be



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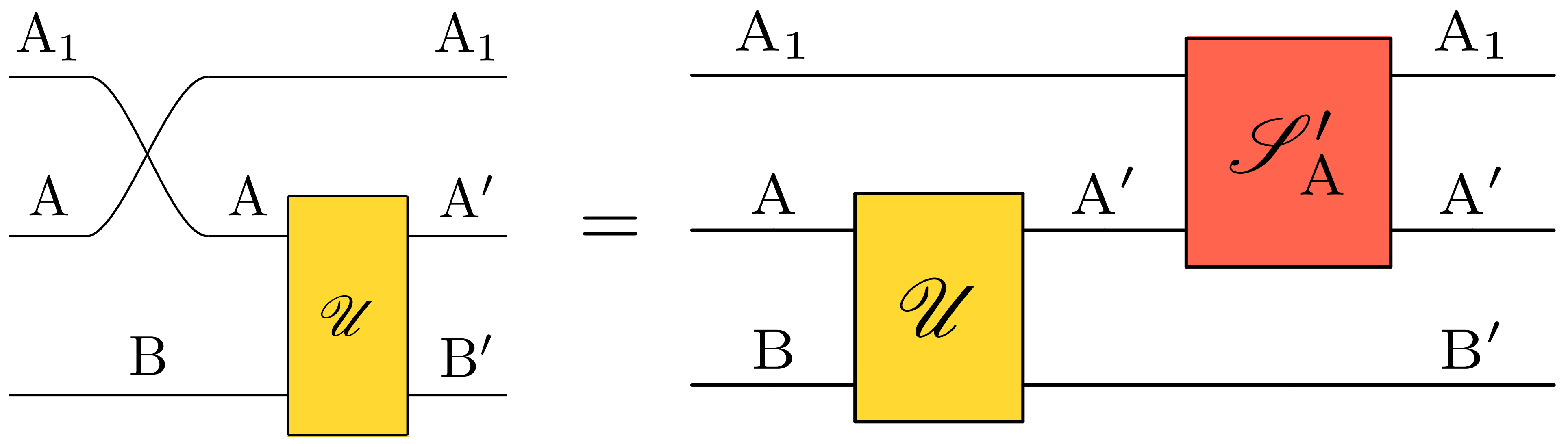
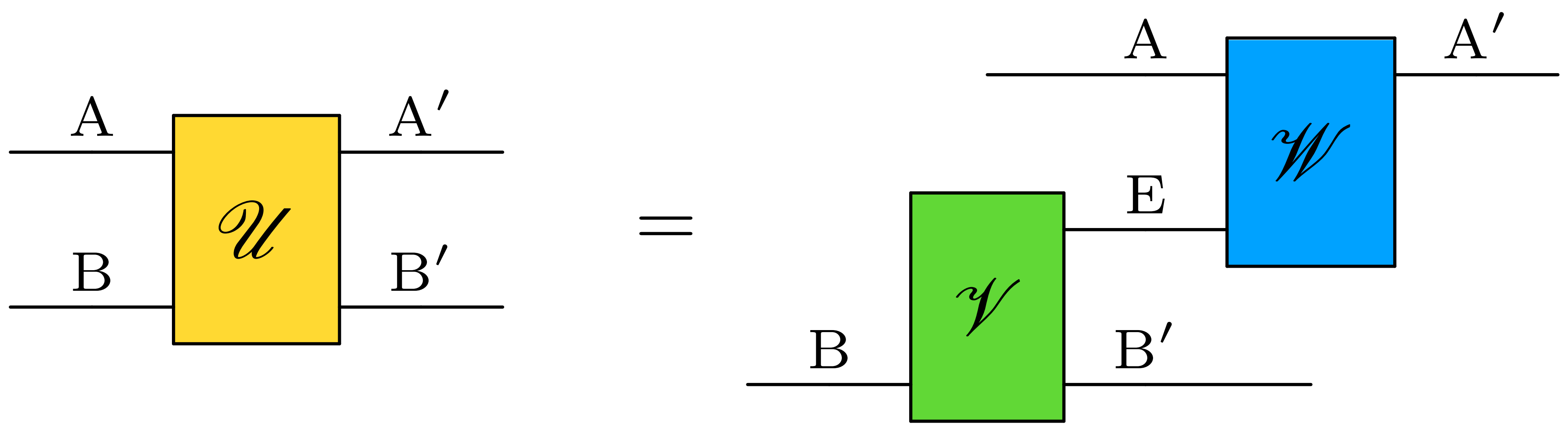
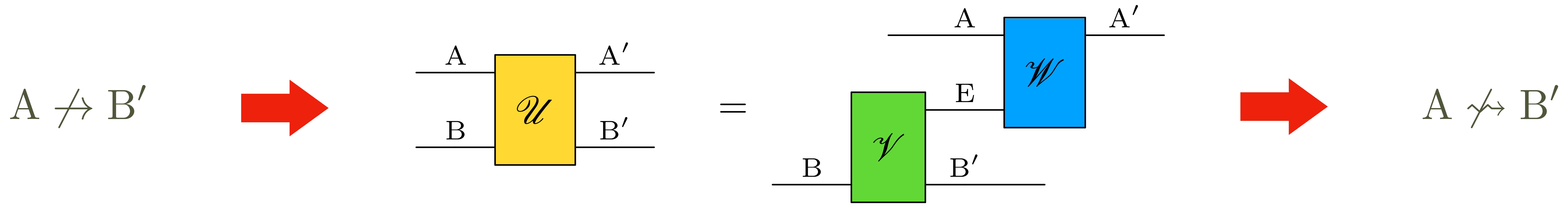


- Preparing a state of A and discarding A_1 we obtain that



PP, Quantum **5**, 515 (2021)

Chain of conditions



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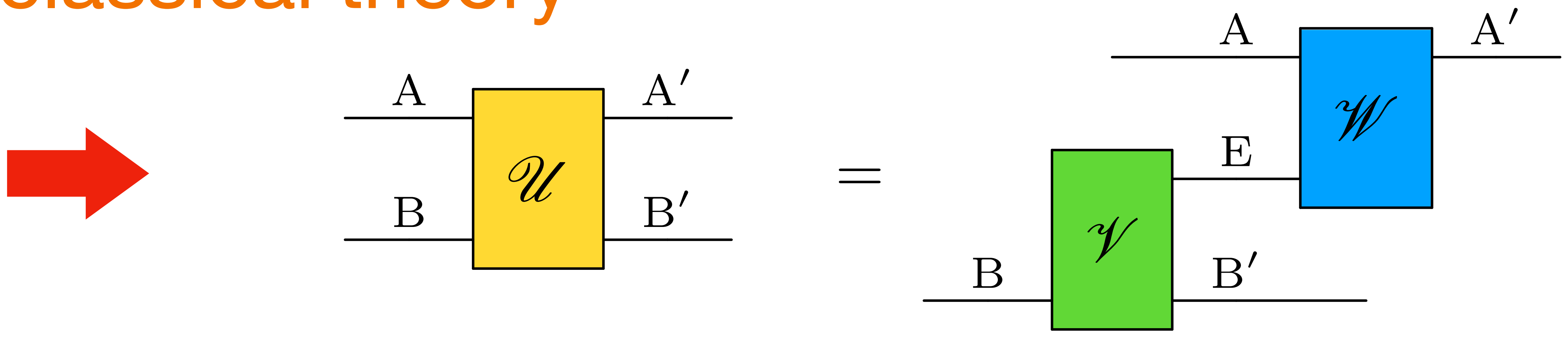
Classical theory

Example 1

- One can prove that in classical theory

$$A \not\rightsquigarrow B'$$



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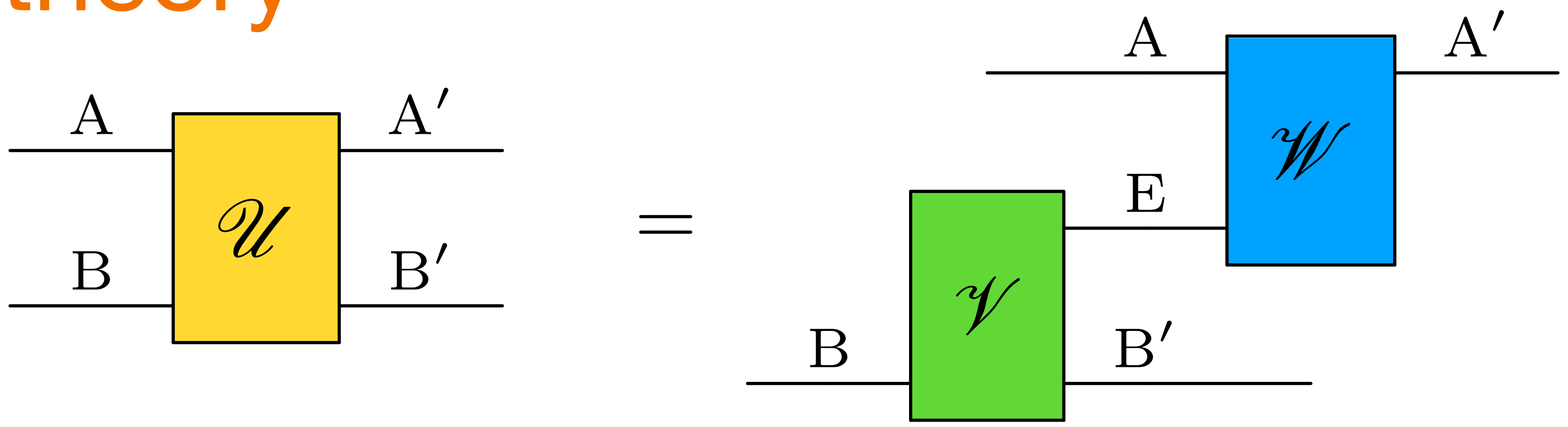
- One can prove that in classical theory

$A \not\rightsquigarrow B'$



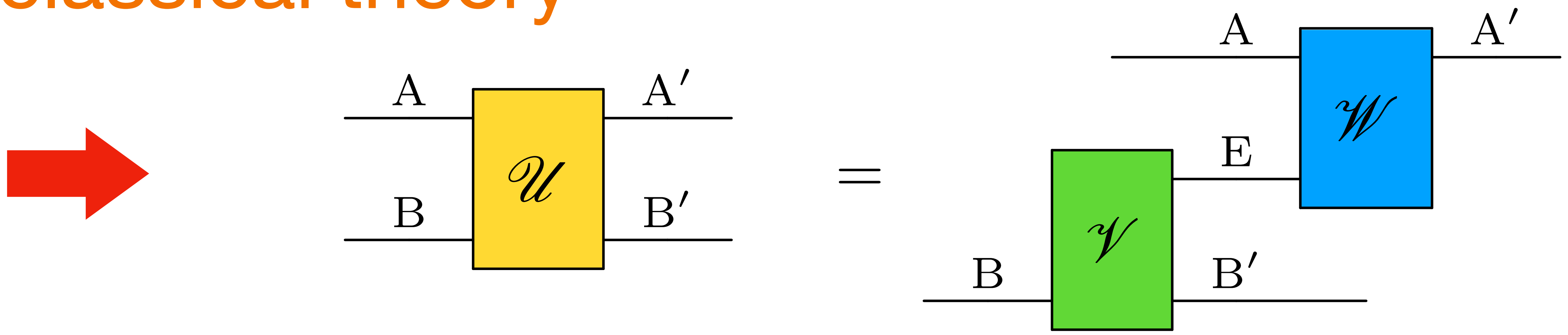
- Classical C-not

Classical theory

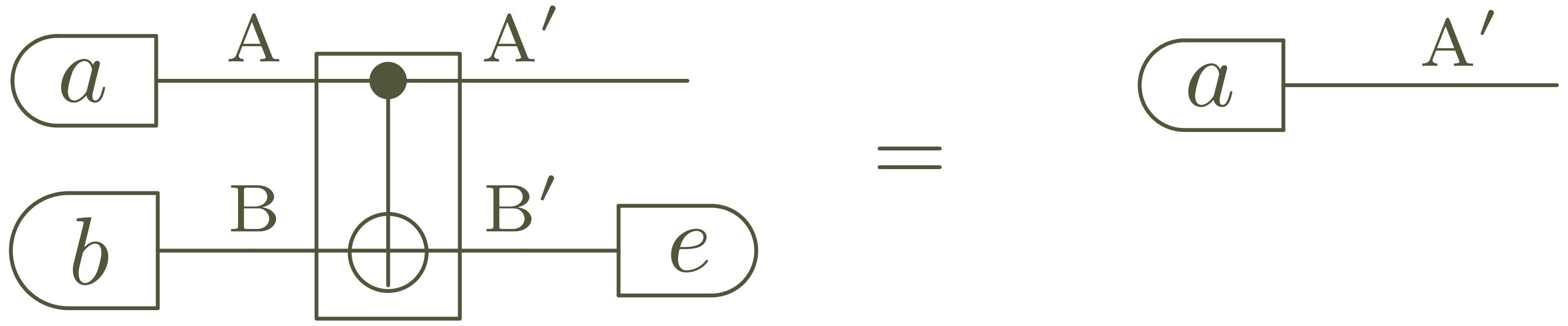
Example 1

- One can prove that in classical theory

$$A \not\rightsquigarrow B'$$



- Classical C-not



Classical theory

Example 1

- One can prove that in classical theory

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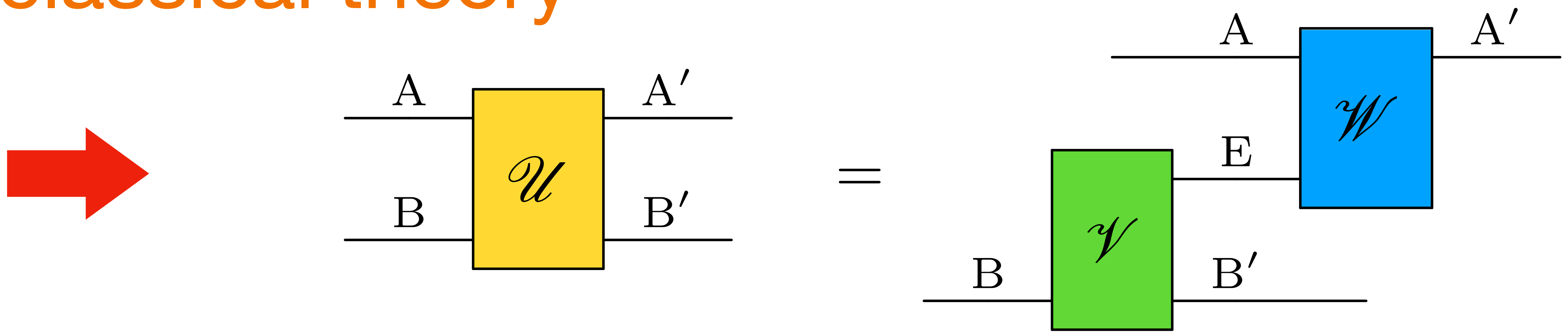
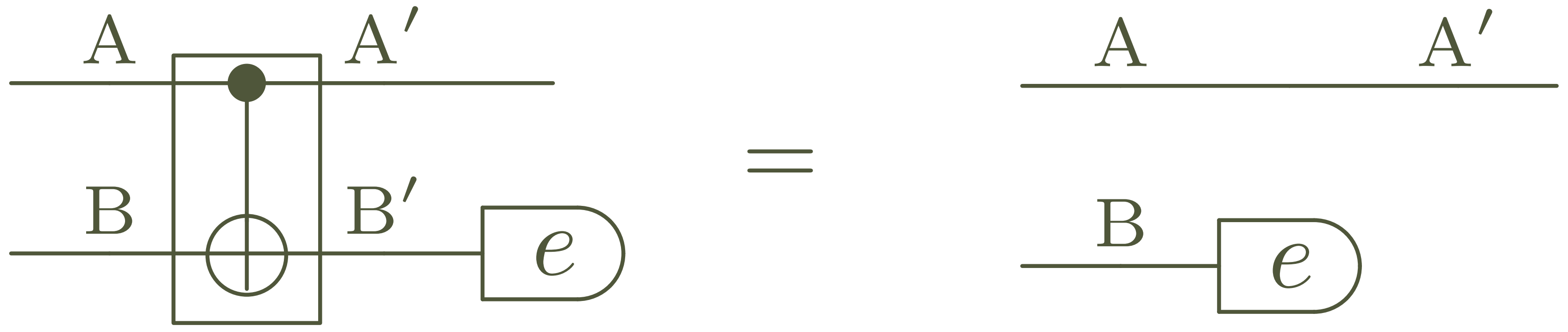


- Classical C-not

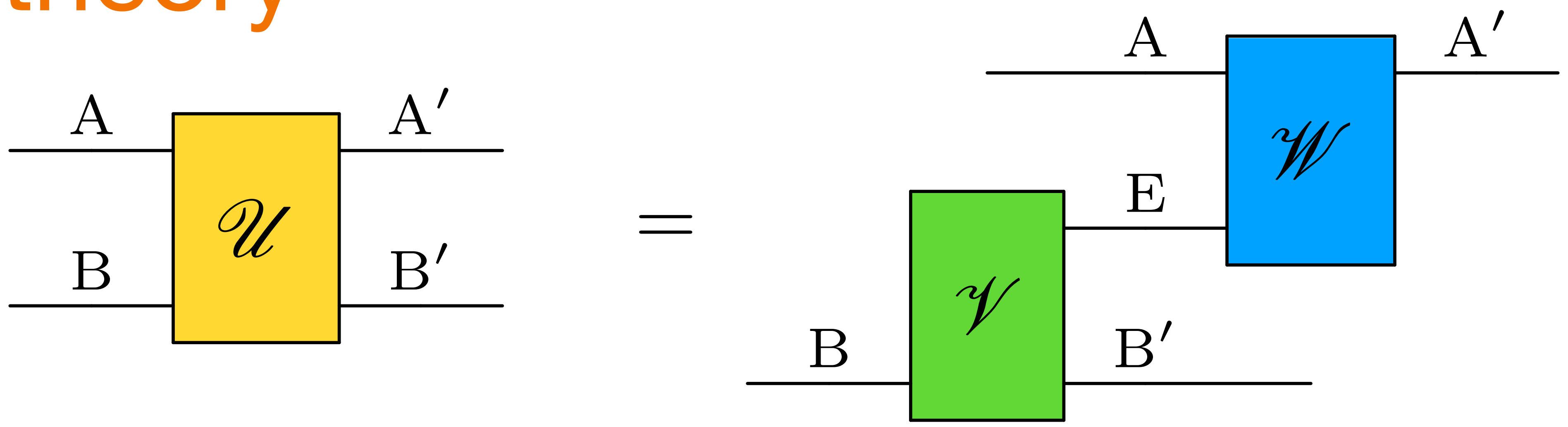


Classical theory

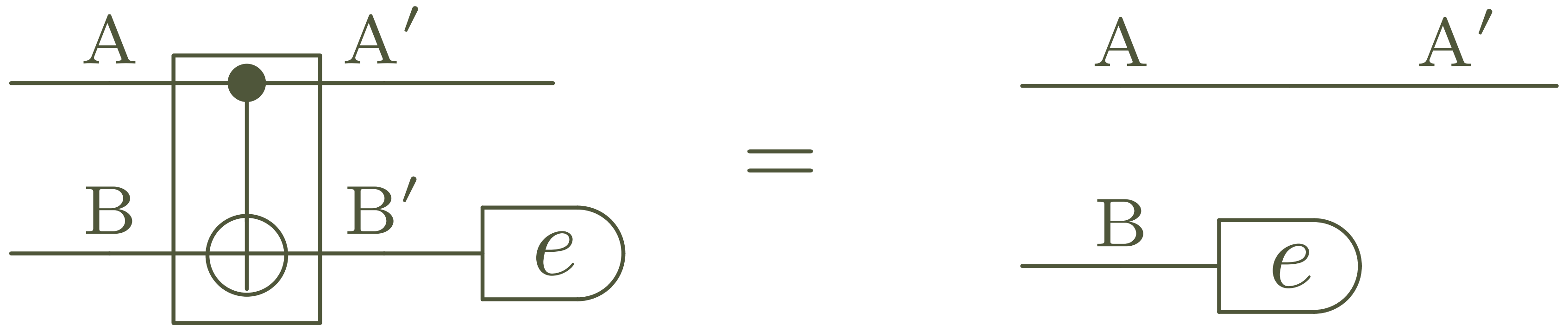
Example 1

- One can prove that in classical theory

$\mathrm{A} \not\rightsquigarrow \mathrm{B}'$



- Classical C-not



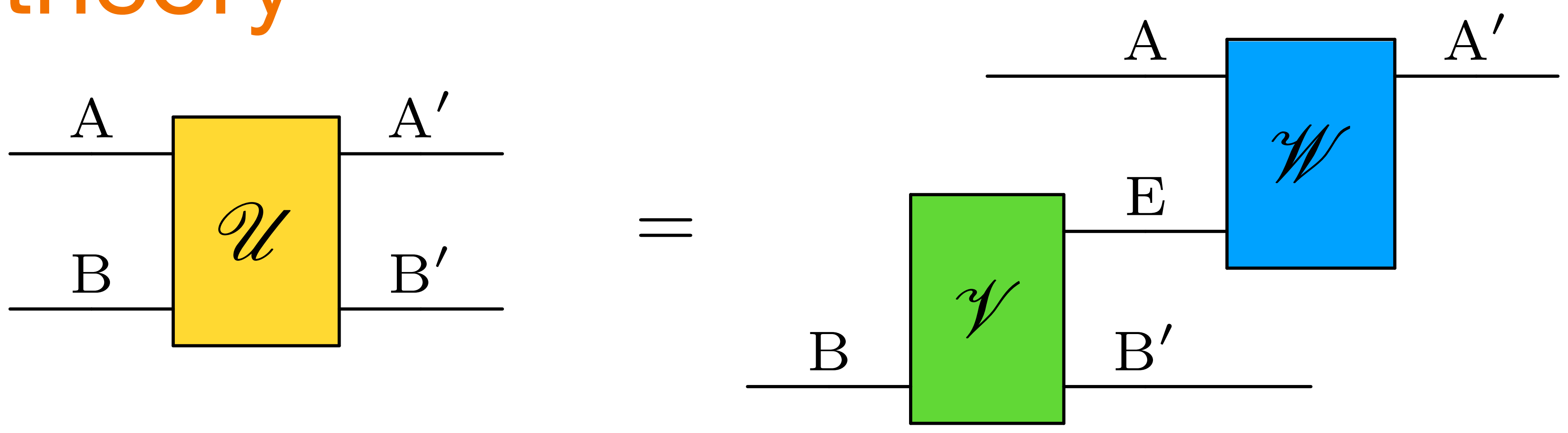
$\mathrm{B} \not\rightsquigarrow \mathrm{A}'$

Classical theory

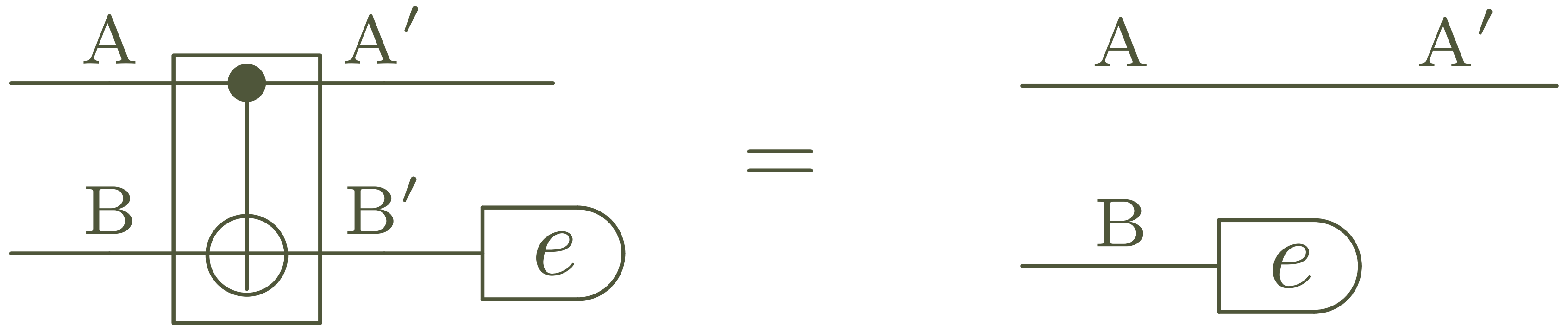
Example 1

- One can prove that in classical theory

$\mathrm{A} \not\rightsquigarrow \mathrm{B}'$ $\Rightarrow$



- Classical C-not



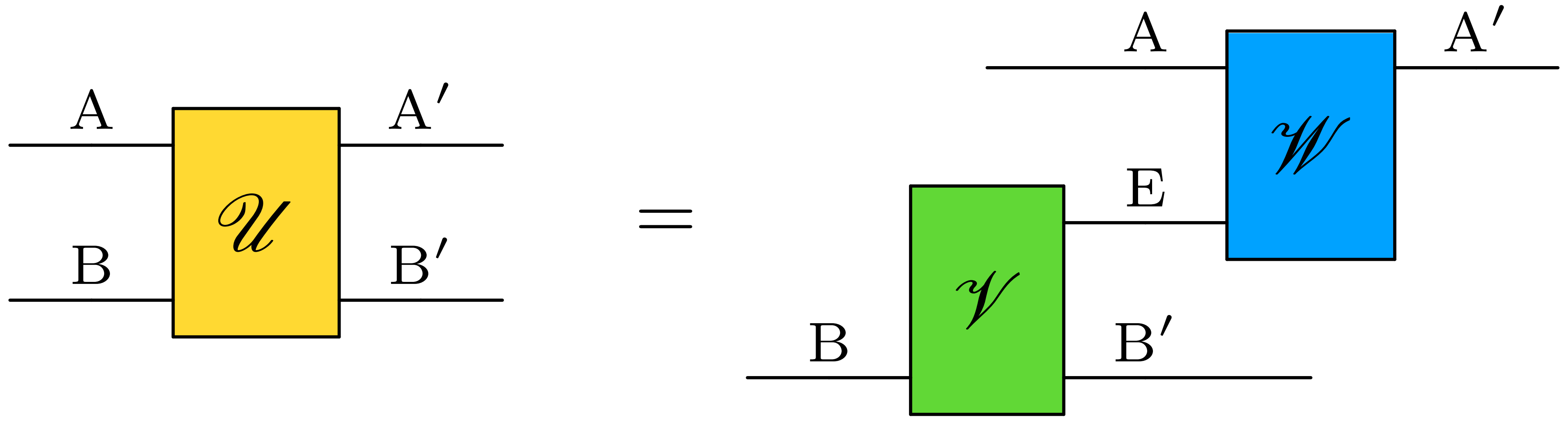
$\Rightarrow$ $\mathrm{B} \not\rightsquigarrow \mathrm{A}'$

  - However $\mathrm{B} \rightarrow \mathrm{A}'$

Chain of conditions

In classical theory

$$A \not\to B' \quad \Rightarrow$$



$$\Rightarrow \quad A \not\rightsquigarrow B'$$

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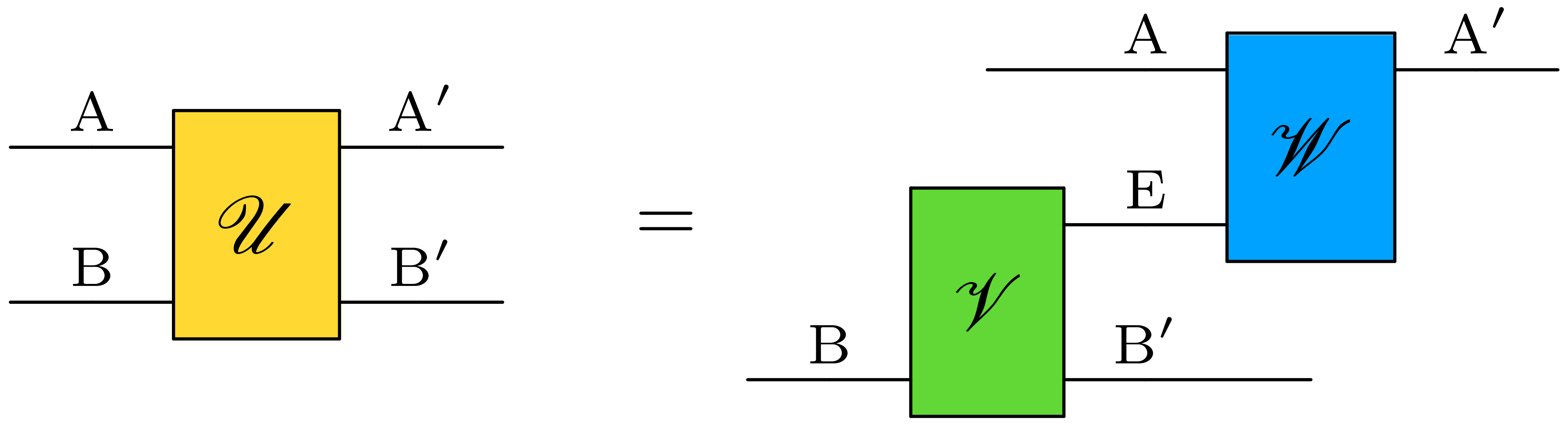
Chain of conditions

In classical theory

$A \not\to B'$



$A \not\rightsquigarrow B'$

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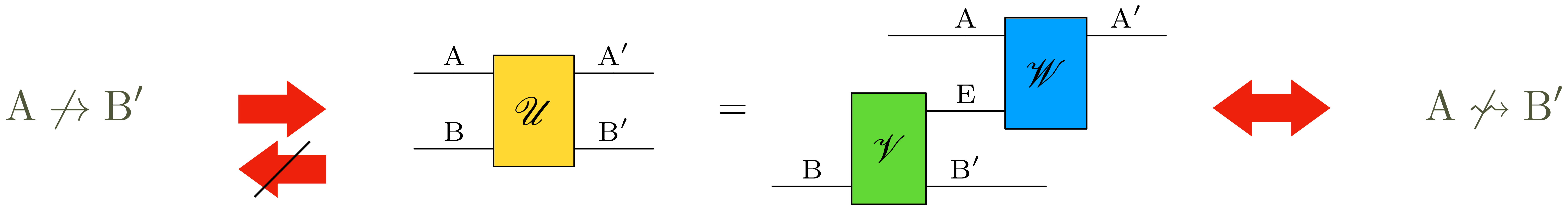
Chain of conditions

In classical theory



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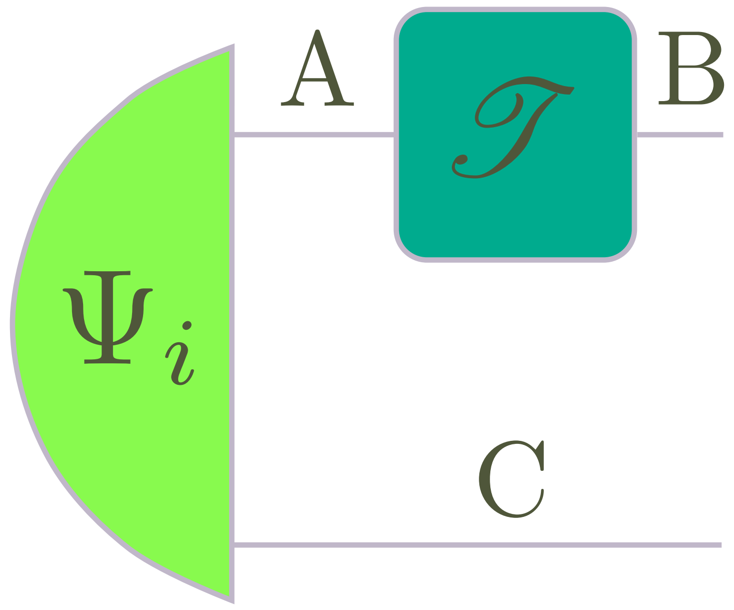
What is causal influence without signalling?

Classical C-NOT

The effects of a transformation must be evaluated also on correlations



Transformations like the C-NOT create correlations

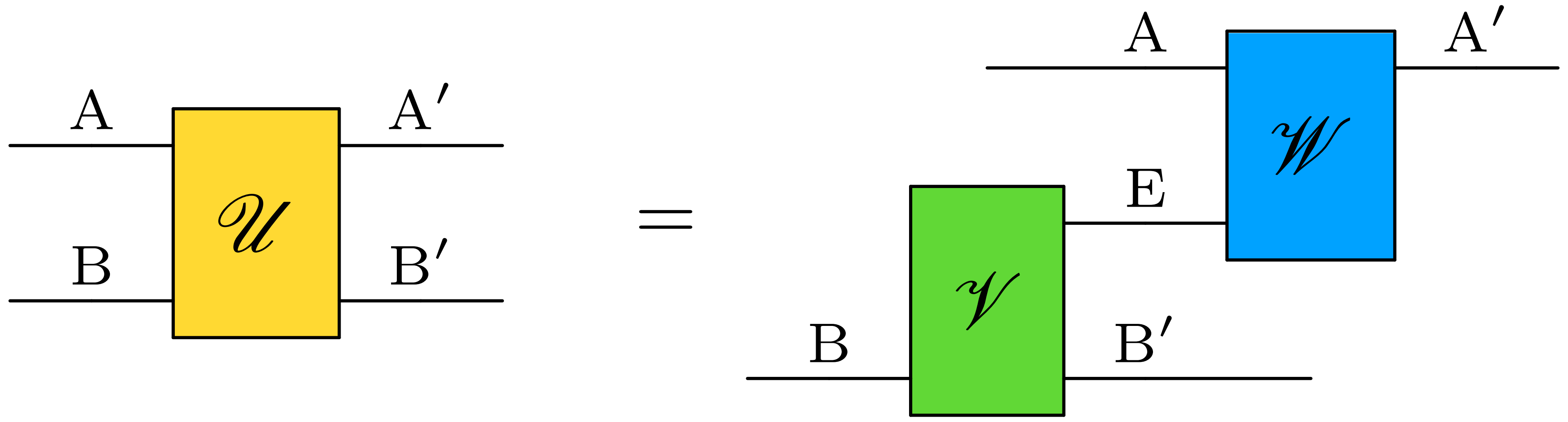
even if the local state is unchanged

Quantum theory

Example 2

- Also in quantum theory

$$\mathrm{A} \not\rightsquigarrow \mathrm{B}'$$



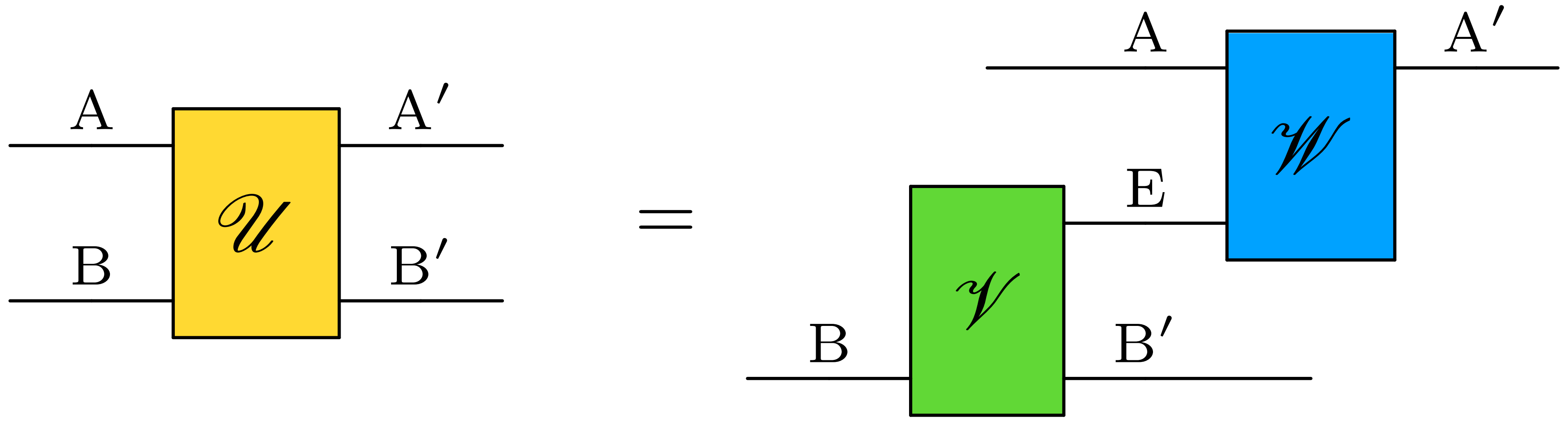
T. Eggeling, D. Schlingemann, and R.F. Werner, Europhys. Lett. **57**, pp. 782–788 (2002)

Quantum theory

Example 2

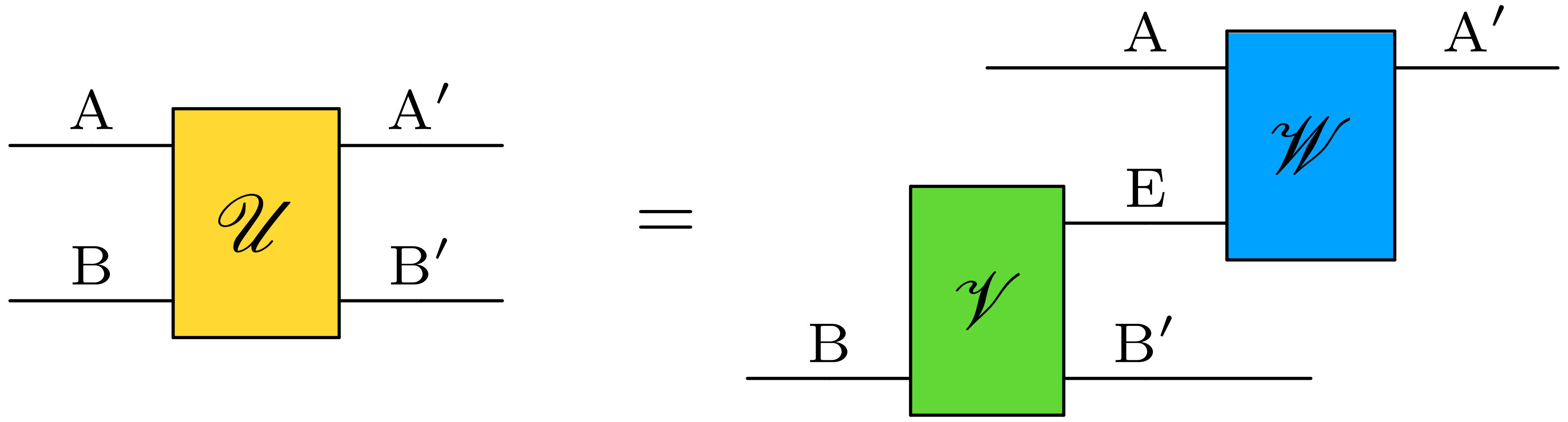
- Also in quantum theory

$A \not\rightsquigarrow B'$ ⇒



- Thus

$A \not\rightarrow B'$ ⇒



⇔ $A \not\rightsquigarrow B'$

T. Eggeling, D. Schlingemann, and R.F. Werner, Europhys. Lett. **57**, pp. 782–788 (2002)

Quantum theory

Example 2

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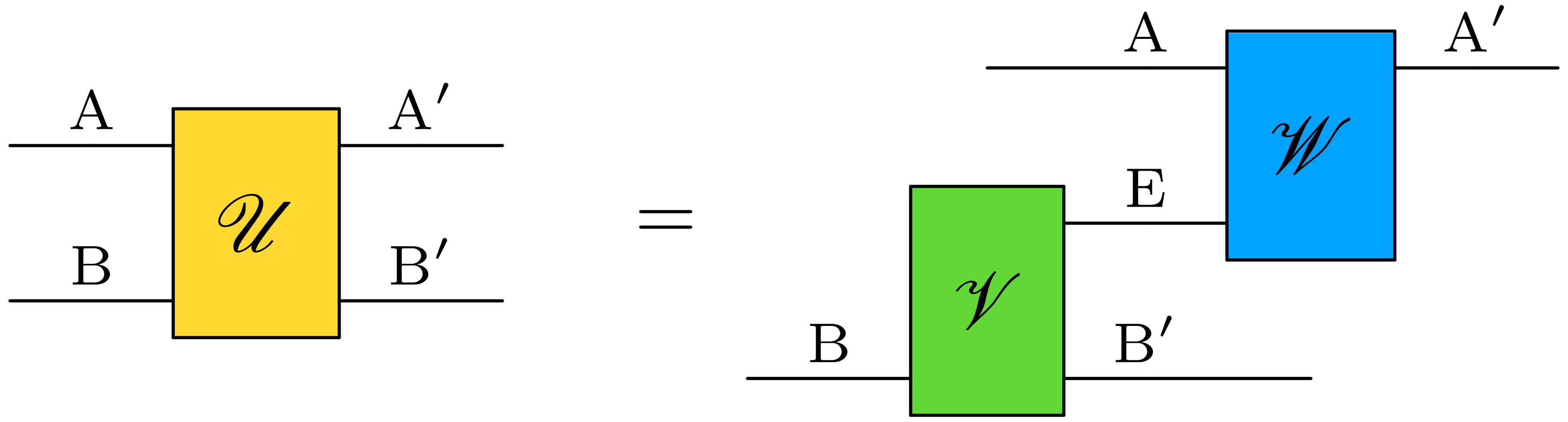
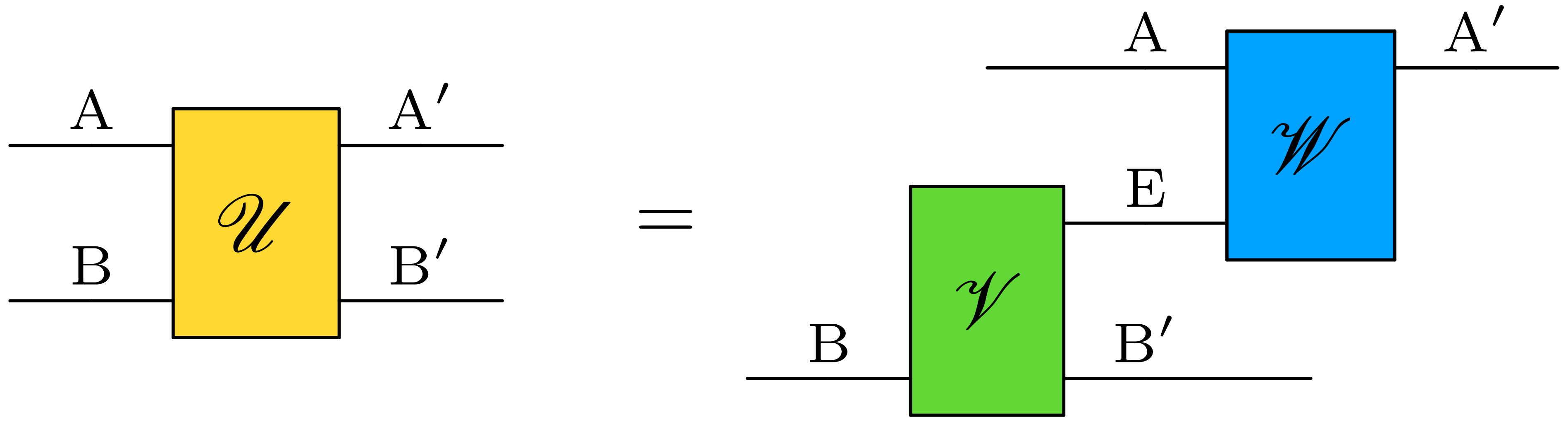


- Thus

$A \not\to B'$ ⇒



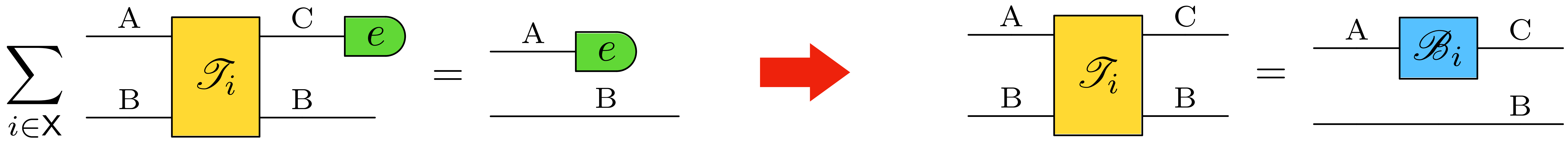
⇔ $A \not\rightsquigarrow B'$

- What about the first implication?

T. Eggeling, D. Schlingemann, and R.F. Werner, Europhys. Lett. **57**, pp. 782–788 (2002)

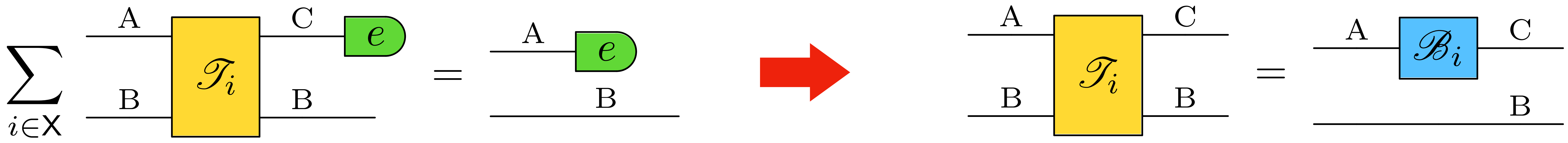
Quantum theory

- From the characterisation of Kraus decompositions of a given channel



Quantum theory

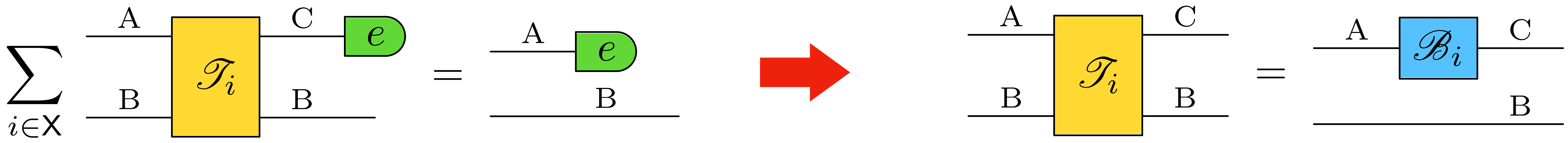
- From the characterisation of Kraus decompositions of a given channel



- Also from purification

Quantum theory

- From the characterisation of Kraus decompositions of a given channel



- Also from purification

➡ The above result holds also in Fermionic theory and Real Quantum theory

Quantum theory

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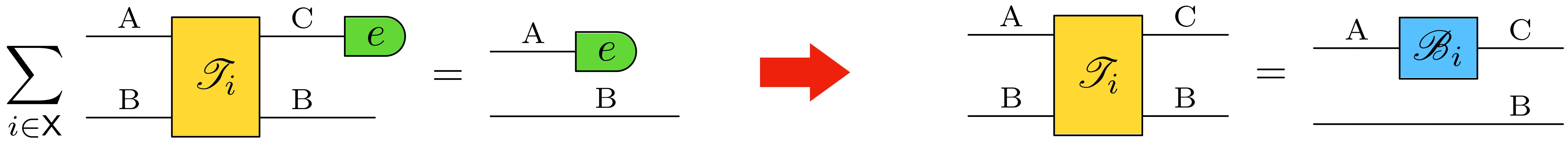

- Also from purification

➡ The above result holds also in Fermionic theory and Real Quantum theory

No interaction without disturbance

Quantum theory

- From no interaction without disturbance one has $\mathrm{A} \not\rightsquigarrow \mathrm{B} \Rightarrow \mathrm{A} \nrightarrow \mathrm{B}$

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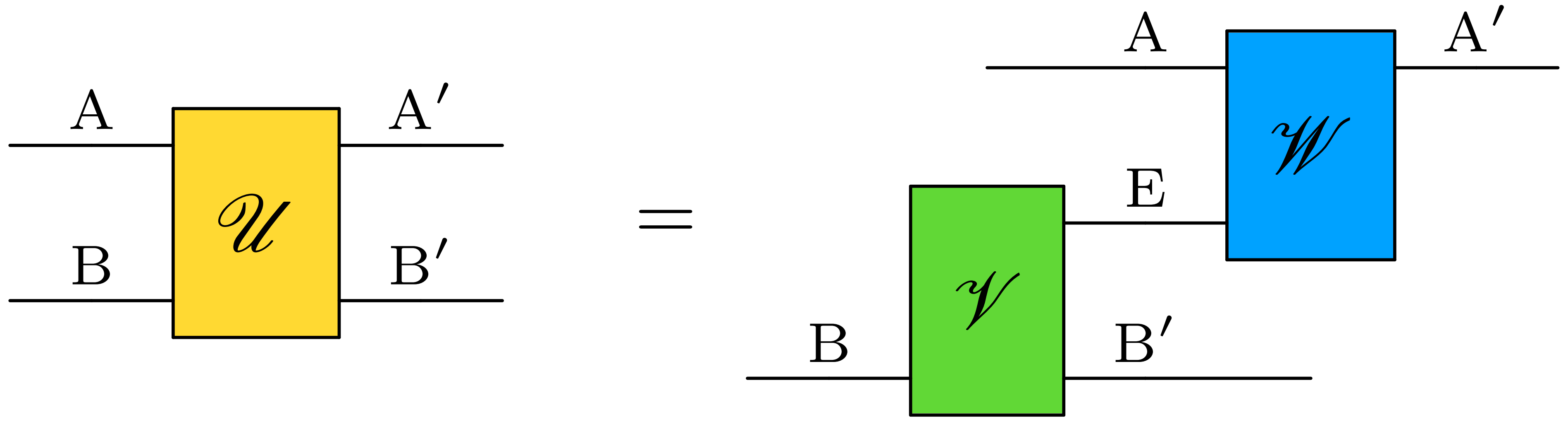
Quantum theory

- From no interaction without disturbance one has $A \not\rightsquigarrow B \Rightarrow A \nrightarrow B$
- Thus

$A \nrightarrow B' \Leftrightarrow$



$\Leftrightarrow A \not\rightsquigarrow B'$

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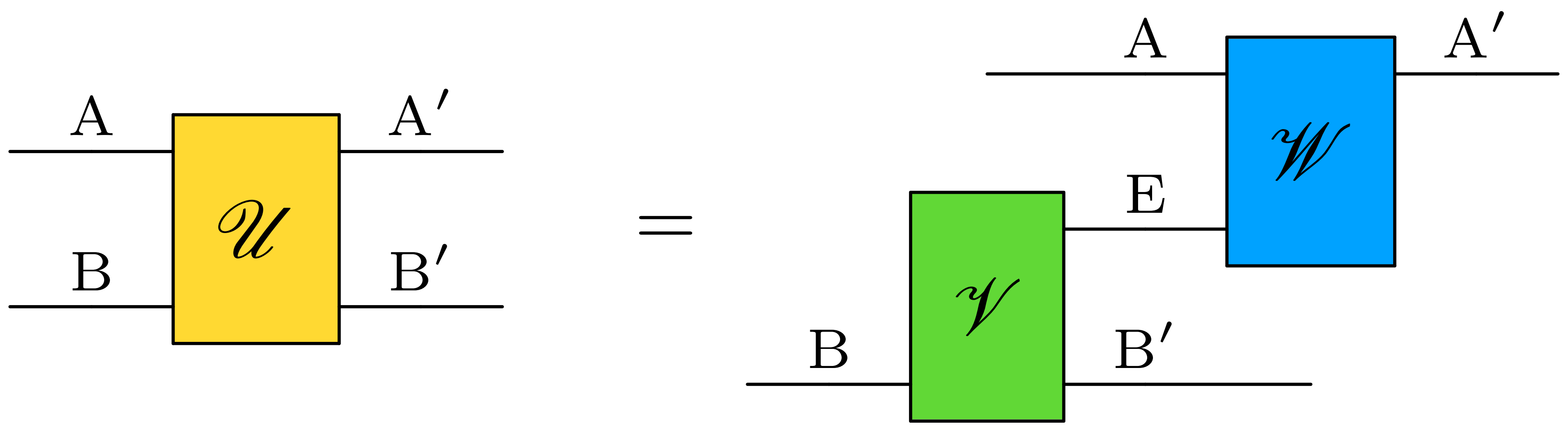
Quantum theory

- From no interaction without disturbance one has $\mathrm{A} \not\rightsquigarrow \mathrm{B} \Rightarrow \mathrm{A} \not\rightarrow \mathrm{B}$
- Thus

$\mathrm{A} \not\rightarrow \mathrm{B}' \Leftrightarrow$



$\Leftrightarrow \mathrm{A} \not\rightsquigarrow \mathrm{B}'$

- True in every theory with purification or just no interaction without disturbance

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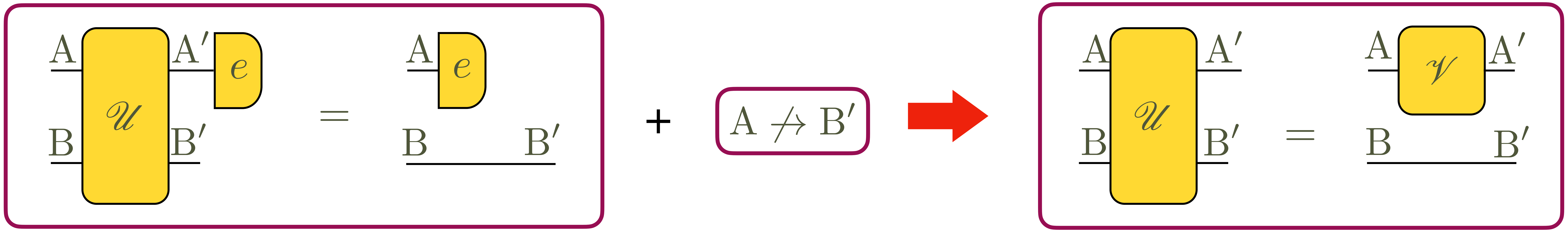
Interaction without disturbance

- What about a theory featuring interactions without disturbance?



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Interaction without disturbance

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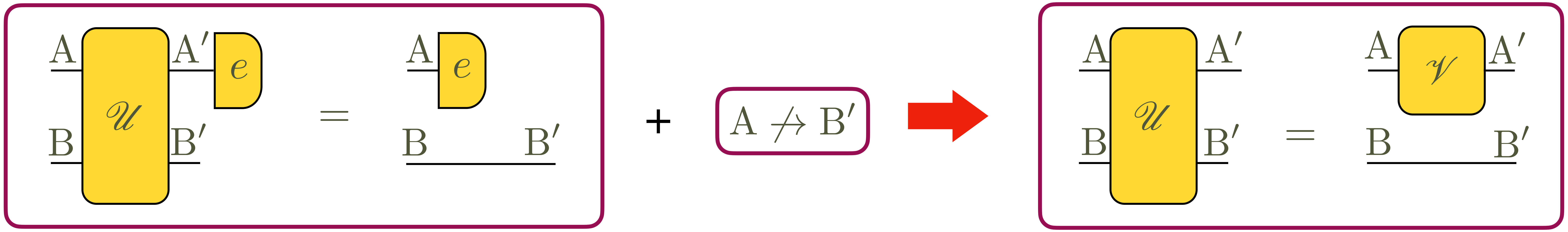
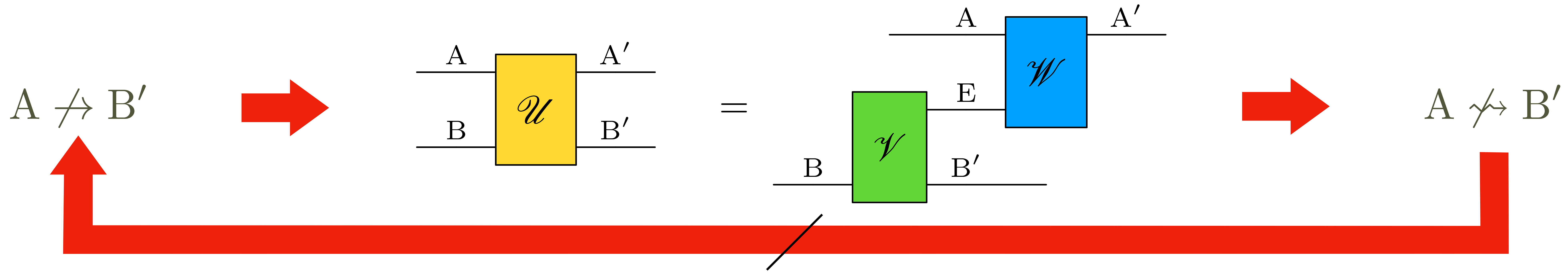


- Thus, if the special interaction without disturbance is reversible, one has



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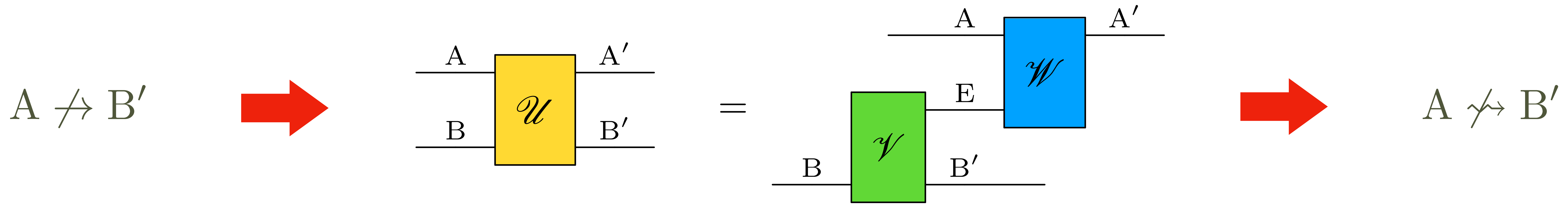
Open question

The quest for counterexamples

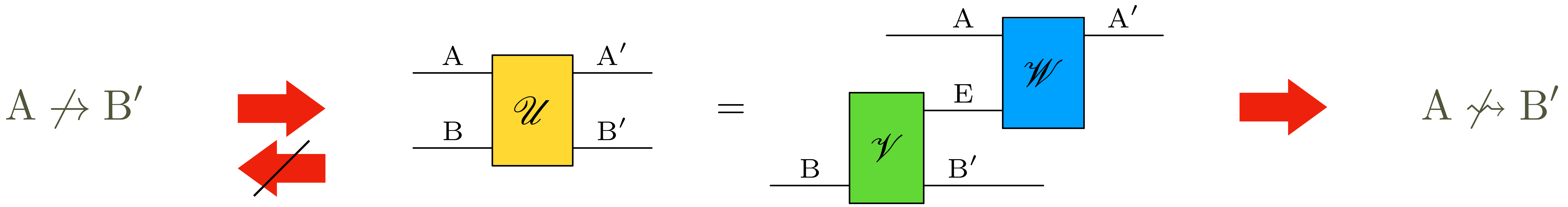
Open question

The quest for counterexamples



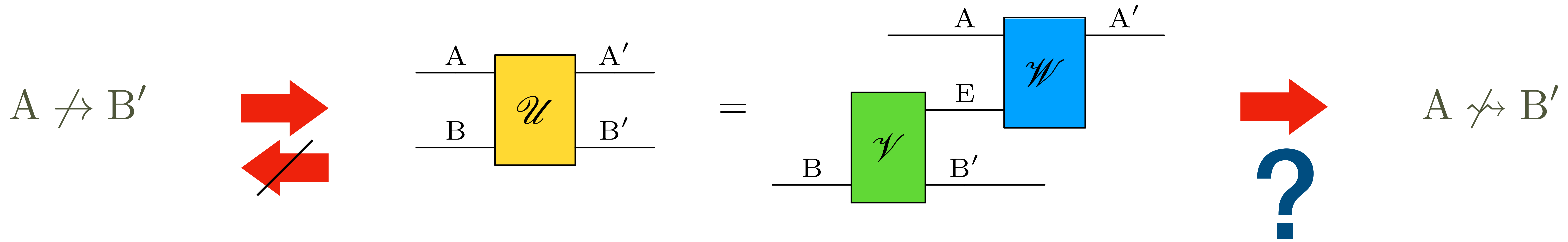
Open question

The quest for counterexamples



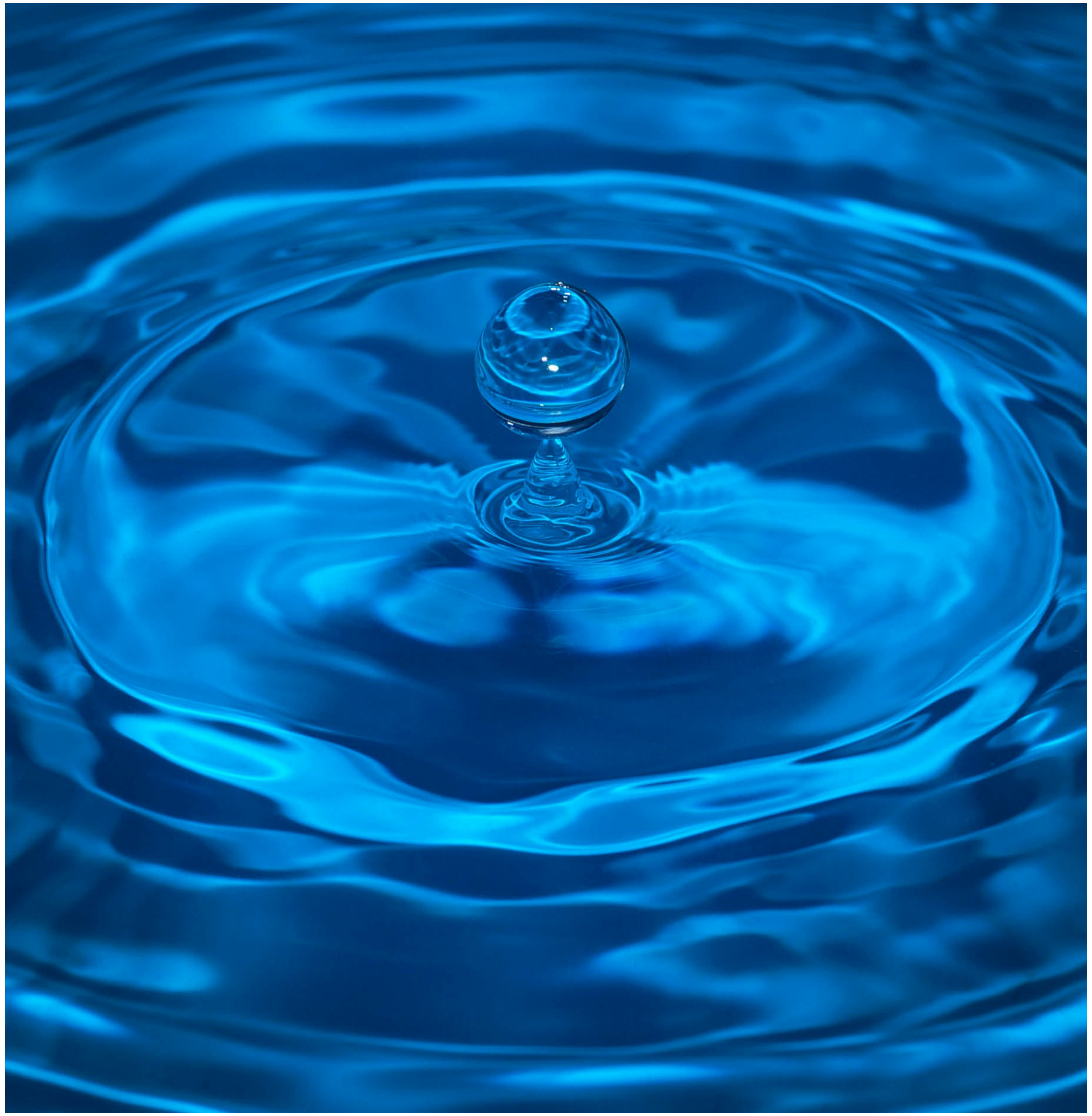
Open question

The quest for counterexamples



Conclusion

- Proposal: (no) causal influence
- Relation with comb structure and (no) signalling
- Classical and Quantum theory
- No interaction without disturbance
- Cellular automata and conservation principles



Again on the classical C-not

Comb structure