Exceptional dynamical quantum phase transitions in periodically driven systems

Ryusuke Hamazaki

RIKEN CPR Hakubi/iTHEMS

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Ref: R. Hamazaki, Nature Communications 12, 5108 (2021).

Outline

Part I: Introduction

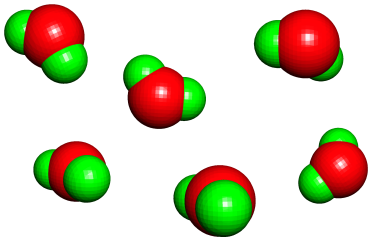
- Motivation
- Dynamical quantum phase transitions (DQPT)

Part II: Exceptional DQPT in periodically driven systems

R. Hamazaki, Nature Communications 12, 5108 (2021).

Nonequilibrium quantum statistical mechanics

Statistical mechanics
at equilibrium



$$\xrightarrow{\frac{e^{-\beta H}}{Z}}$$

Quantum statistical mechanics out of equilibrium?

Artificial quantum systems (e.g. ultra cold atoms)

Control of microscopic parameters

→ Observation of macroscopic dynamics

For long times, generic many-body systems thermalize

via eigenstate thermalization hypothesis (ETH)

$$\langle \psi(0) | \hat{O} | \psi(0) \rangle \xrightarrow[t \to \infty]{} \mathrm{Tr}[\hat{\rho}_{\mathrm{can}} \hat{O}]$$

Nevertheless, interesting physics can appear at finite times

e.g.) Dynamical quantum phase transition (DQPT)

Quantum return probability as "dynamical free energy"

Quantum overlap concerning nonequilibrium processes

$$\langle \psi_f | \psi(t) \rangle = \langle \psi_f | e^{-iHt} | \psi_i \rangle$$

Dynamical free energy density

[M. Heyl et al. Phys. Rev. Lett. 110, 135704 (2013)]

$$F = -\frac{1}{L} \log | \langle \psi_f | e^{-iHt} | \psi_i \rangle |$$

*Similar to equilibrium free energy

$$F_{\mathrm{eq}} = -\frac{1}{L} \log \mathrm{Tr}[e^{-\beta H}]$$

*Analytic continuation $\beta \to it$

Generalized expectation value

[E. Canovi et al., Phys. Rev. Lett. 113, 265702 (2014)]

$$\langle A \rangle_{\mathrm{gexp}} = \frac{\langle \psi_f | A e^{-iHt} | \psi_i \rangle}{\langle \psi_f | e^{-iHt} | \psi_i \rangle}$$

cf) $\langle A \rangle = \dfrac{\mathrm{Tr}[A e^{-\beta H}]}{\mathrm{Tr}[e^{-\beta H}]}$

Not standard (local) observables, but in principle experimentally accessible using, e.g., interferometry

Dynamical quantum phase transition (DQPT)

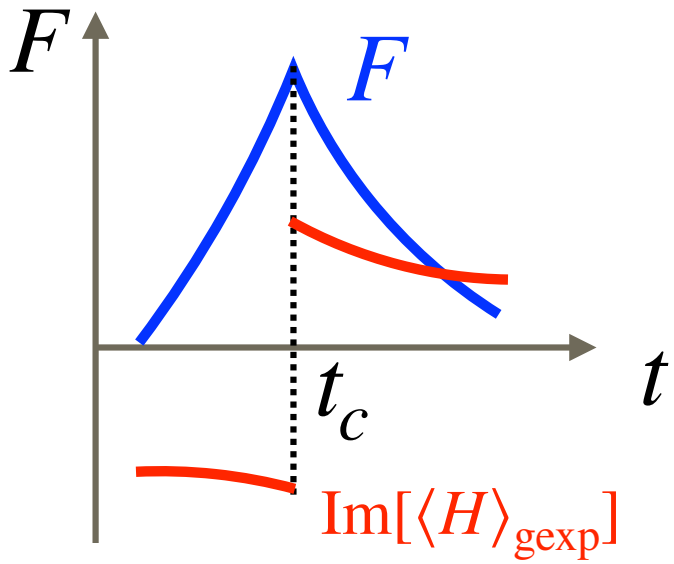
DQPT: Singularity of dynamical free energy at finite times in the thermodynamic limit

[M. Heyl et al. Phys. Rev. Lett. 110, 135704 (2013)]

Typical example:

F has a cusp as a function of time

cf) $\mathrm{Im}[\langle H \rangle_{\mathrm{gexp}}] = -\dfrac{dF}{dt}$ has a jump



Experiments

*Trapped ions

Motivation of our work

DQPT is regarded as a finite-time QPT,
but not much is known yet

Is there a notion of symmetry breaking or universality?

Result

Yes.
Using periodically driven (*unitary*) quantum Ising model,
we find DQPT whose universality originates from hidden *nonunitarity*,
i.e., an **exceptional point** under **antiunitary symmetry breaking**

[R. Hamazaki, Nature Communications 12, 5108 (2021).]

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"Space-time duality" elucidates hidden nonunitary structure!

[R. Hamazaki, Nature Communications 12, 5108 (2021).]

Model and setup

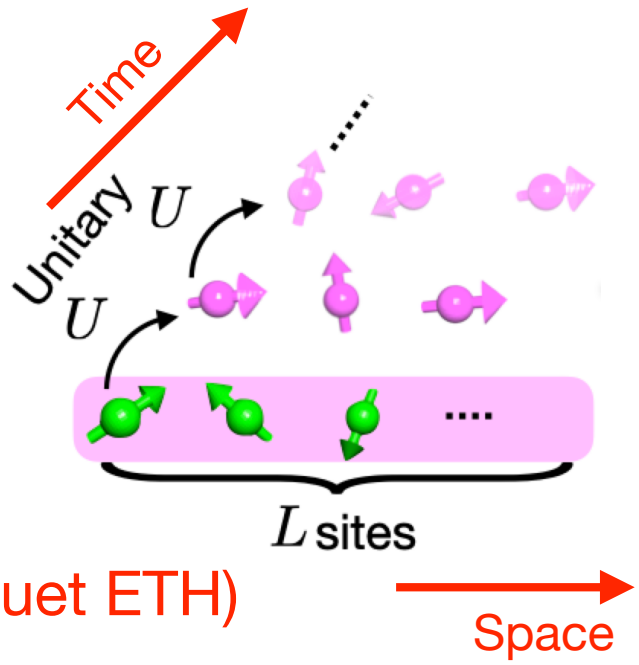
Periodically driven 1D Ising chain

$$|\psi(T)\rangle = U^T|\psi\rangle$$

$$U = e^{-i\sum_{j=1}^{L} b\sigma_j^x} e^{-i\sum_{j=1}^{L} J\sigma_j^z\sigma_{j+1}^z - i\sum_{j=1}^{L} h\sigma_j^z}$$

spin rotation

Ising interaction
+ longitudinal field



Trivial infinite-temperature stationary state (Floquet ETH)

Two types of dynamical free energy

$$F_{L,T}^{\downarrow\uparrow} = -\frac{1}{L}\log|\langle\downarrow\cdots\downarrow|U^T|\uparrow\cdots\uparrow\rangle|$$

Generalized expectation value

$$\langle A\rangle_{\mathrm{gexp}} = \frac{\mathrm{Tr}[AU^T]}{\mathrm{Tr}[U^T]}$$

$$F_{L,T}^{s} = -\frac{1}{L}\log|\overline{\langle\psi|U^T|\psi\rangle}| = -\frac{1}{L}\log|\mathrm{Tr}[U^T]| + \log 2$$

(average over initial states)

[R. Hamazaki, Nature Communications 12, 5108 (2021).]

DQPT of the stroboscopic Ising model

Change b with $J = -\dfrac{\pi}{4}$ in the thermodynamic limit

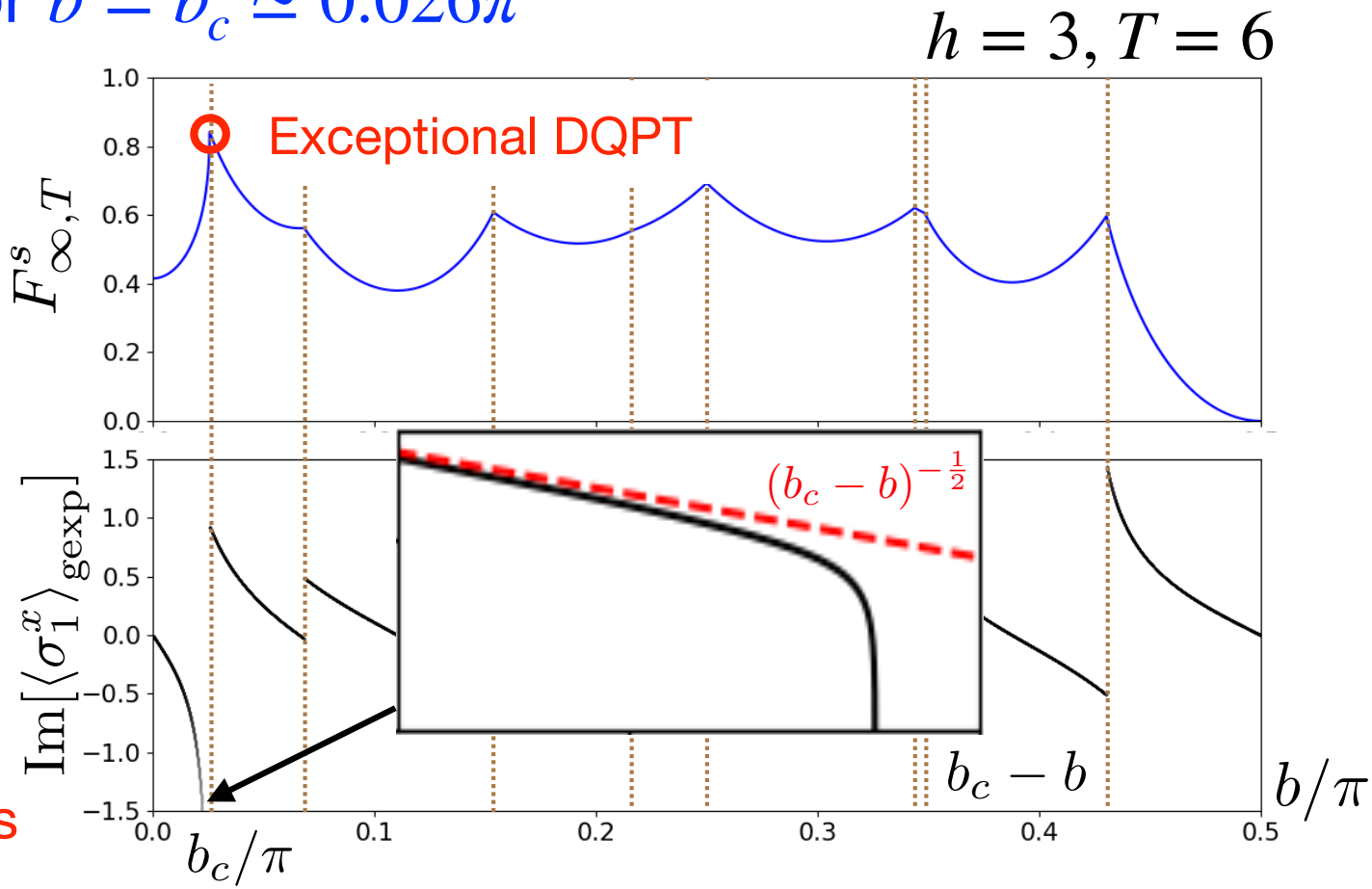
Many singularities for $F^s_{\infty,T}$, but its derivative ($\propto \mathrm{Im}[\langle\sigma_1^x\rangle_{\mathrm{gexp}}]$) is divergent only for $b = b_c \simeq 0.026\pi$

Exceptional DQPT:

$$\frac{\partial F^s_{\infty,T}}{\partial b} \sim (b_c - b)^{-\frac{1}{2}}$$

near criticality (only for $b < b_c$)

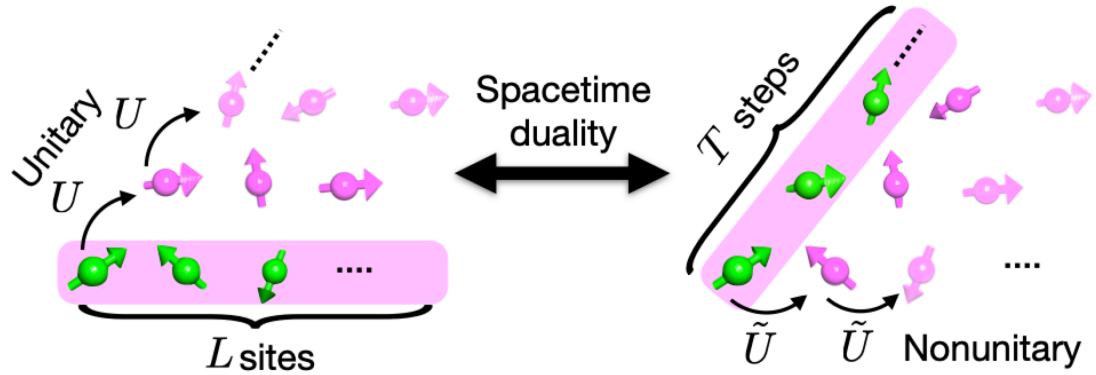
$\dfrac{\partial F^s_{\infty,T}}{\partial b}$ only exhibits a jump for ordinary DQPTs



[R. Hamazaki, Nature Communications 12, 5108 (2021).]

Spacetime duality

Exact mapping using many-body transfer matrix



[M. Akila et al. J Phys. A (2016)]

[B. Bertini et al. PRL (2018)]

cf) Suzuki-san's talk

$$F^s_{L,T} = -\frac{1}{L}\log\left|\frac{\mathrm{Tr}[U^T]}{2^L}\right|$$

$$\mathrm{Tr}\left[U^T\right] = \left(\frac{\sin 2b}{2i}\right)^{LT/2} \sum_{\{s_{\tau,j}\}} e^{-i\sum_{\tau=1}^{T}\sum_{j=1}^{L}\left(Js_{\tau,j}s_{\tau,j+1} + J's_{\tau,j}s_{\tau+1,j} + hs_{\tau,j}\right)}$$

$$= -\frac{1}{L}\log|\mathrm{Tr}[\tilde{U}^L]|$$

cf) Classical partition function in 2D Ising model

$$\tilde{U} = Ce^{-i\sum_{\tau=1}^{T}\tilde{b}\sigma^x_\tau}e^{-i\sum_{\tau=1}^{T}\tilde{J}\sigma^z_\tau\sigma^z_{\tau+1} - i\sum_{\tau=1}^{T}h\sigma^z_\tau}$$

$$\tilde{b} = -\pi/4 - i\log(\tan J)/2,\ \tilde{J} = -\pi/4 - i\log(\tan b)/2 \in \mathbb{C}$$

Nonunitary T-spin operator propagating through space

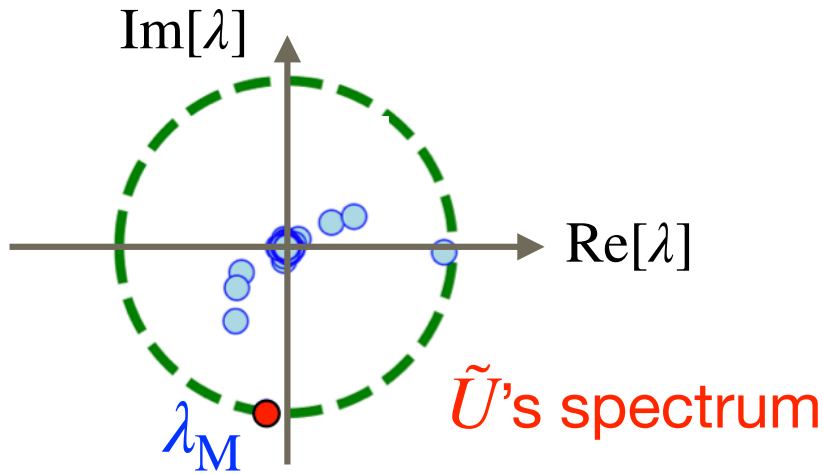
Dual operator and dynamical free energy

$$F^s_{L,T} = -\frac{1}{L}\log\left|\frac{\mathrm{Tr}[U^T]}{2^L}\right| = -\frac{1}{L}\log|\mathrm{Tr}[\tilde{U}^L]|$$

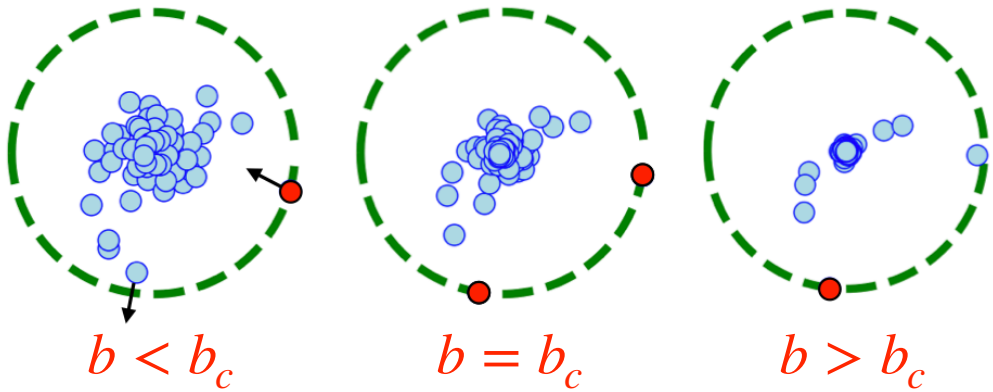
Thermodynamic limit

$$F^s_{\infty,T} = -\log|\lambda_{\mathrm{M}}|$$

λ_{M}: $\tilde{U}$'s eigenvalue whose absolute value is maximal



Ordinary DQPT: switch of the largest eigenvalues



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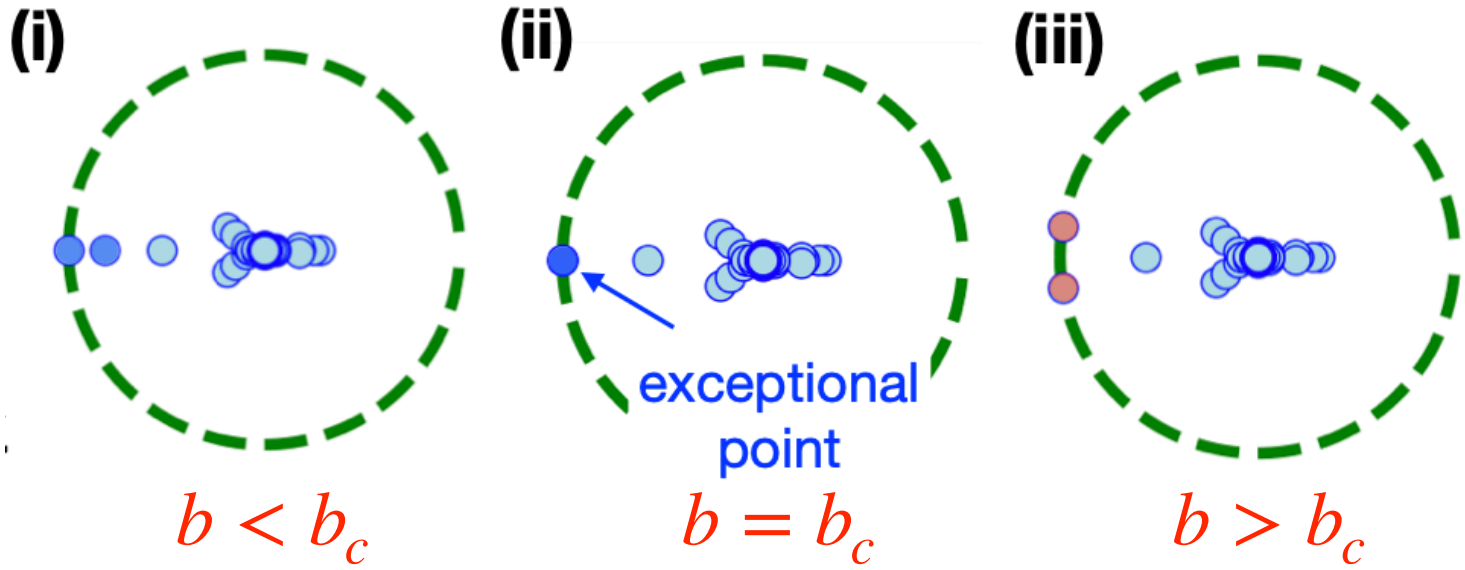
Mechanism of exceptional DQPT

Exceptional DQPT: coalescence of the largest eigenvalues



This eigenvalue coalescence (exceptional point):
mathematics of nonunitary matrices with antiunitary symmetry

(Review) Mathematics of nonunitary matrices: antiunitary symmetry and exceptional points

Nonunitary matrix $X(g)$

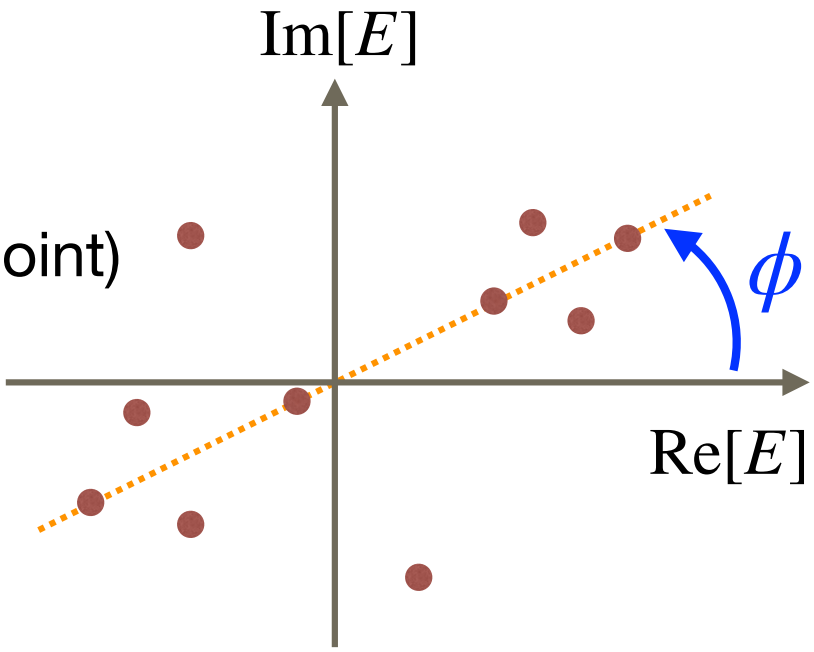
If $VX^*V^\dagger = e^{i\phi}X$ for some unitary operator V with $\phi \in \mathbb{R}$, we say that X possesses antiunitary symmetry (AUS)

→ Spectrum of X on the complex plane becomes symmetric around a line with slope ϕ

Spontaneous AUS breaking

By changing parameter g, two eigenvalues on the line coincide at $g = g_c$ (exceptional point) and then form a pair

Eigenvalue difference typically exhibits universal singularity $\sim \sqrt{|g - g_c|}$



Mechanism of exceptional DQPT

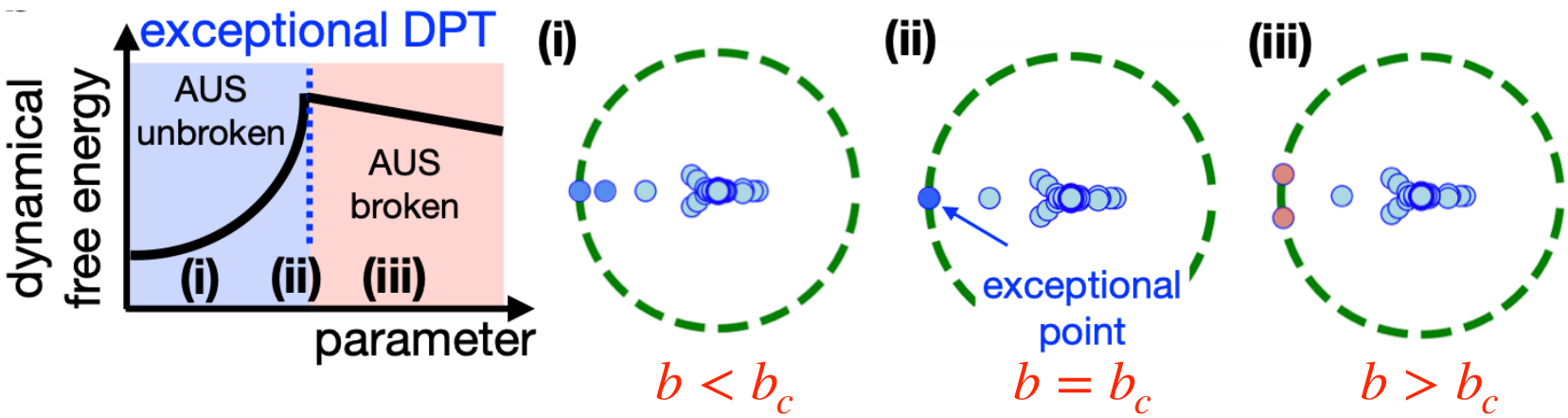
Exceptional DQPT: spontaneous AUS breaking occurs

Necessary condition: AUS (class AI) is hidden in $\tilde{U}$

$\leftarrow J = -\frac{\pi}{4} + \frac{n\pi}{2}, T = 2m\ (n \in \mathbb{Z}, m \in \mathbb{N})$ for original U (case for $F^{s}_{\infty,T}$)

$$\left(\prod_{\tau=1}^{T} e^{i\frac{\pi}{2}\sigma^y_\tau}\right)\tilde{U}^*\left(\prod_{\tau=1}^{T} e^{i\frac{\pi}{2}\sigma^y_\tau}\right)^\dagger = e^{i\phi}\tilde{U} \qquad \left(\prod_{\tau=1}^{T} e^{i\frac{\pi}{2}\sigma^y_\tau}\right)\left(\prod_{\tau=1}^{T} e^{i\frac{\pi}{2}\sigma^y_\tau}\right)^* = \mathbb{I}$$

We can also discuss the condition for $F^{\downarrow\uparrow}_{\infty,T}$ to exhibit exceptional DQPT



[R. Hamazaki, Nature Communications 12, 5108 (2021).]

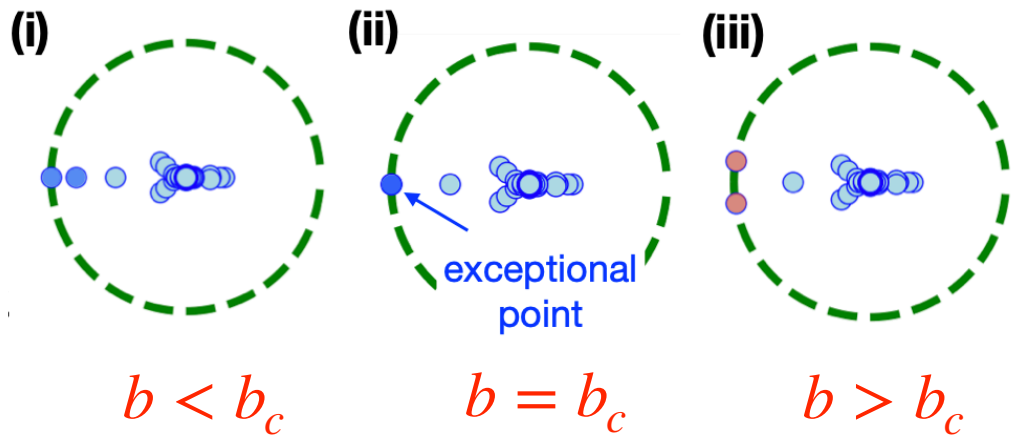
Divergence of generalized expectation value

Typically, the eigenvalue difference becomes $|b - b_c|^{1/2}$

*Generalized expectation value

$$\mathrm{Im}[\langle \sigma_1^x \rangle_{\mathrm{gexp}}] \propto \frac{dF^s_{\infty,\mathrm{T}}}{db}$$

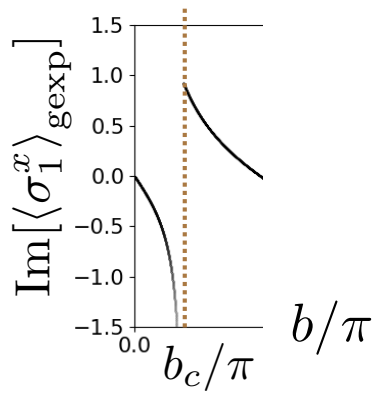
$$= -\frac{d \log |\lambda_{\mathrm{M}}|}{db}$$



Change in radial direction: significant for symmetry-unbroken case ($b < b_c$)

$$\mathrm{Im}[\langle \sigma_1^x \rangle_{\mathrm{gexp}}] \sim \begin{cases} (b_c - b)^{-1/2} & (b < b_c) \\ \text{finite} & (b > b_c) \end{cases}$$

Divergence only for AUS-unbroken phase



[R. Hamazaki, Nature Communications 12, 5108 (2021).]

Generalized correlation function

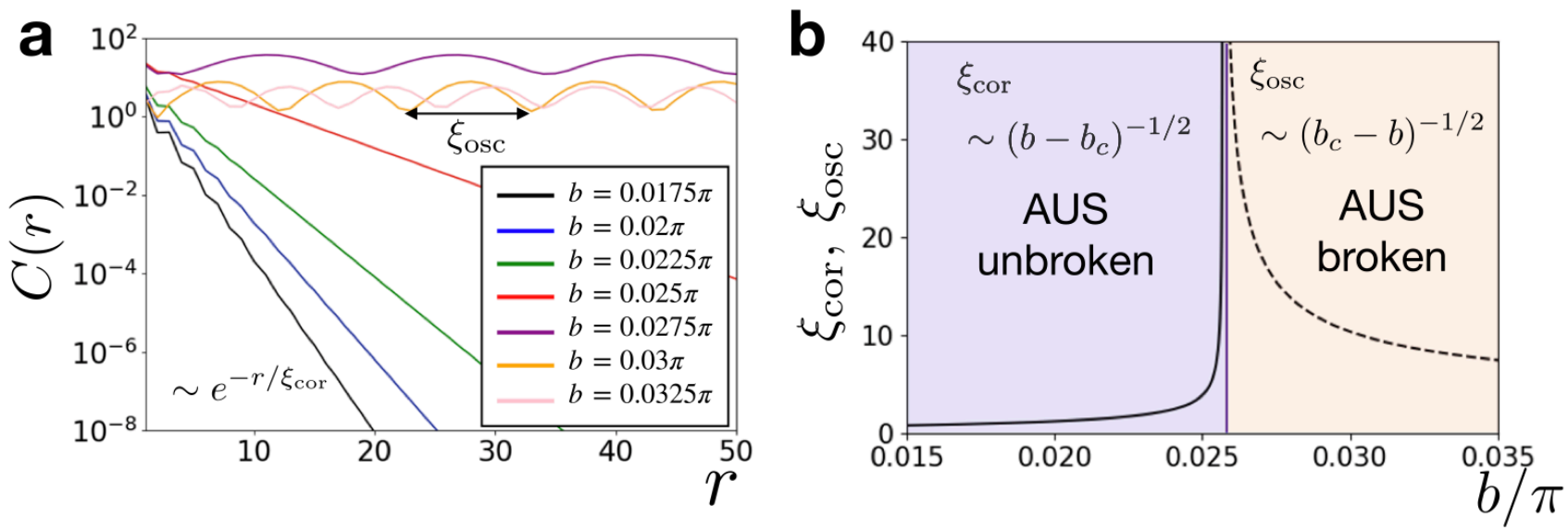
Typically, the eigenvalue difference becomes $|b - b_c|^{1/2}$

*Generalized correlations

$$C(r) = |\langle \sigma_1^z \sigma_{r+1}^z \rangle_{\mathrm{gexp}} - \langle \sigma_1^z \rangle_{\mathrm{gexp}} \langle \sigma_{r+1}^z \rangle_{\mathrm{gexp}}|$$

$$= \begin{cases} c_1 e^{-r/\xi_{\mathrm{cor}}}, & \xi_{\mathrm{cor}} \sim (b_c - b)^{-1/2} \quad (b < b_c) \\ a + c_2 \cos\left(\frac{r}{\xi_{\mathrm{osc}}} + \theta_0\right) & \xi_{\mathrm{osc}} \sim (b - b_c)^{-1/2} \quad (b > b_c) \end{cases}$$

Oscillatory long-range order for AUS-breaking phase



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Signature of exceptional DQPT from (quasi)local observables

$$F_{L,T} = -\frac{1}{2L}\log|\langle\psi_f|U^T|\psi_i\rangle|^2 = -\frac{1}{2L}\log\langle\psi_i(T)|P_f|\psi_i(T)\rangle \qquad P_f = |\psi_f\rangle\langle\psi_f|$$

For $F^{\downarrow\uparrow}_{L,T} = -\frac{1}{2L}\log|\langle\downarrow\cdots\downarrow|U^T|\uparrow\cdots\uparrow\rangle|^2 = -\frac{1}{2L}\log\langle\uparrow\cdots\uparrow(T)|P_f|\uparrow\cdots\uparrow(T)\rangle$

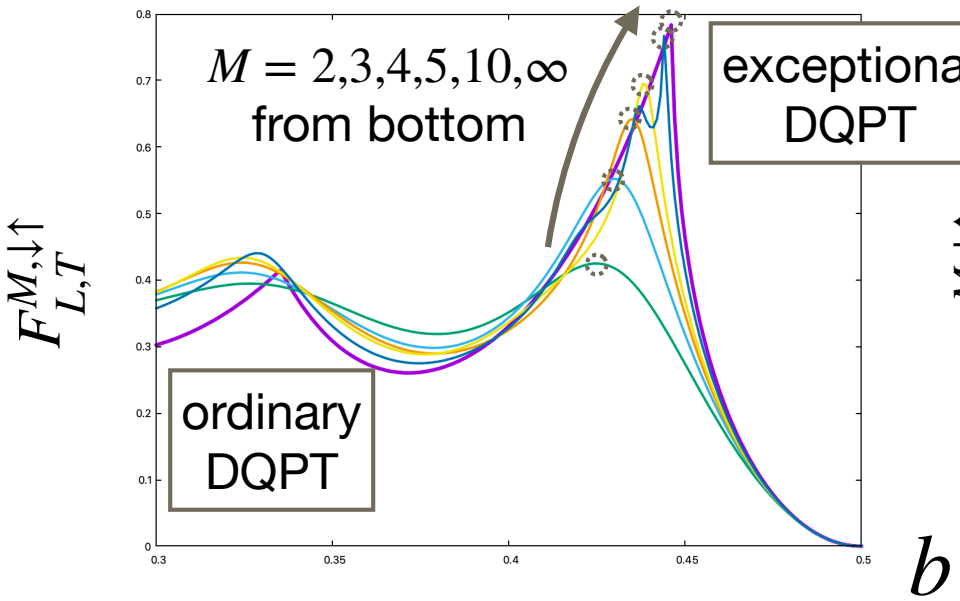
$P_f = |\downarrow\cdots\downarrow\rangle\langle\downarrow\cdots\downarrow|$

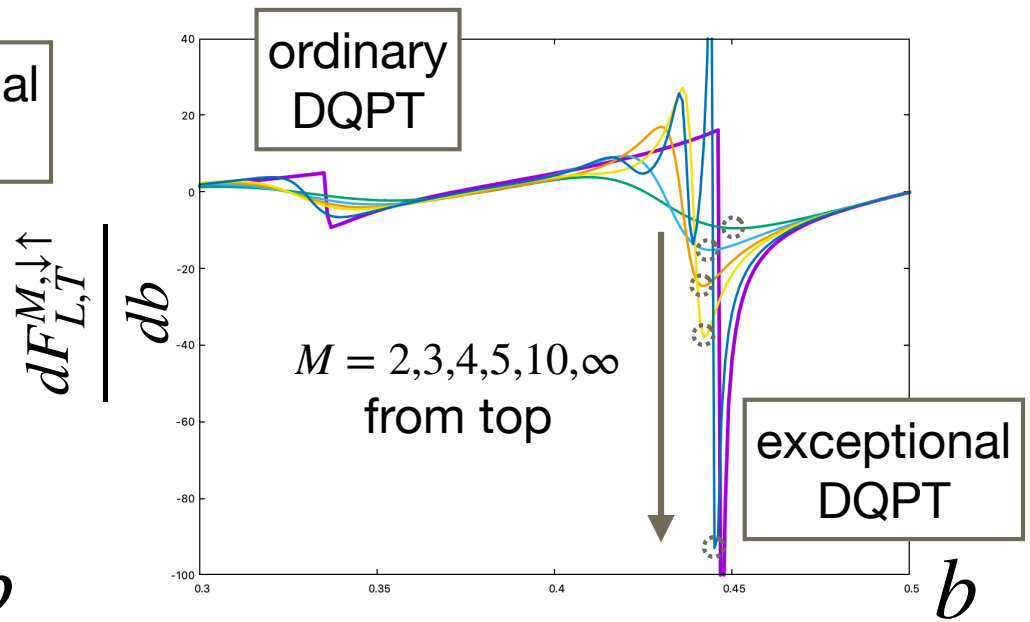
P_f is a nonlocal projection→ Alternatives with local observables?

We introduce

$$F^{M,\downarrow\uparrow}_{L,T} = -\frac{1}{2M}\log|\langle\psi_i(T)|P^M_f|\psi_i(T)\rangle| \qquad P^M_f = \prod_{i=1}^{M}|\downarrow_i\rangle\langle\downarrow_i| \qquad M \ll L$$

(quasi)local





Signature of exceptional DQPT from (quasi)local observables

A sharp peak for the derivative appears when M is increased
→ Signature of exceptional DQPT (with strong singularity)

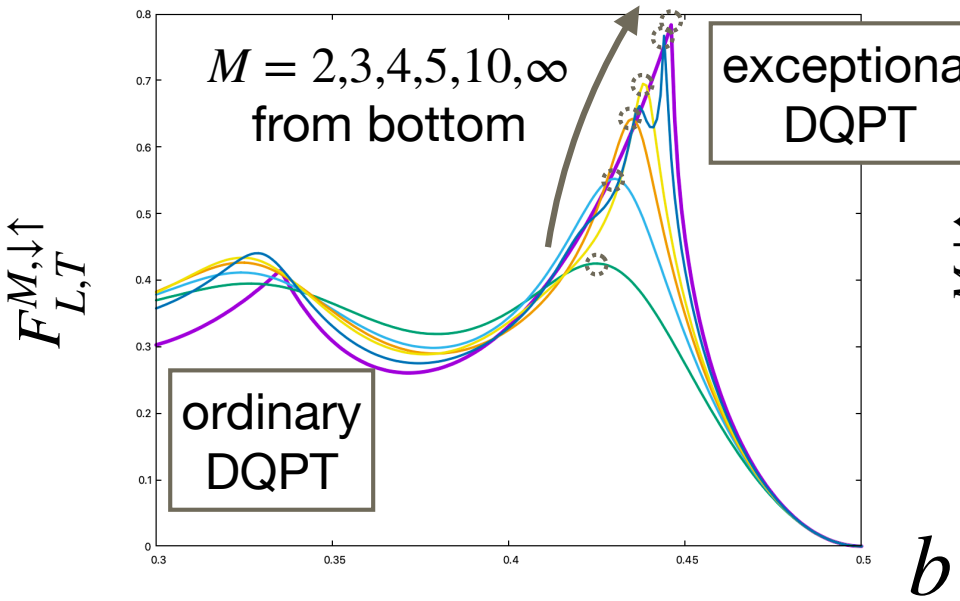
P_f is a nonlocal projection → Alternatives with local observables?

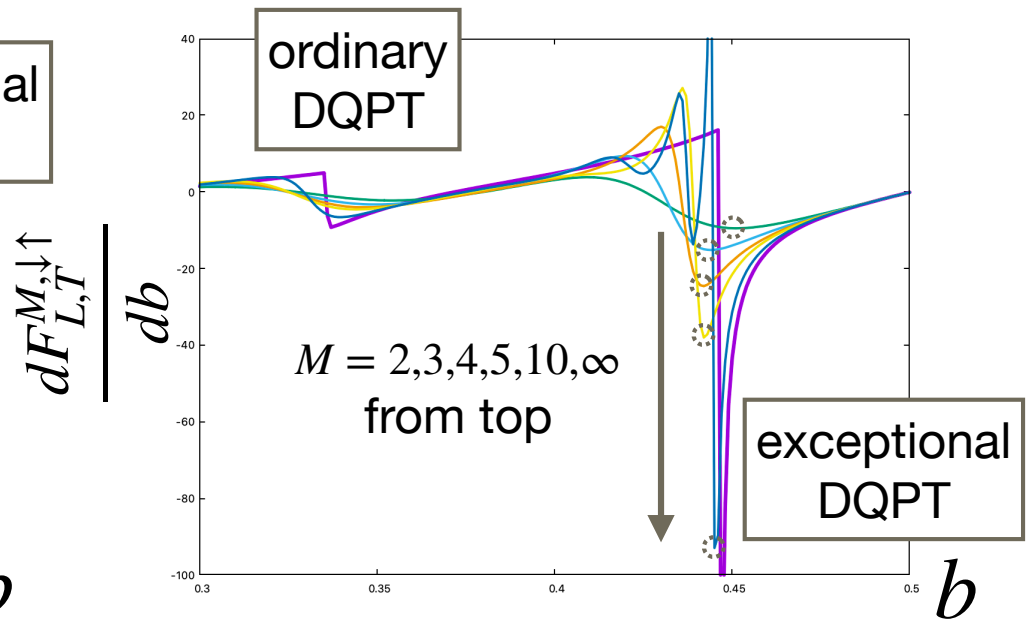
We introduce

$$F_{L,T}^{M,\downarrow\uparrow} = -\frac{1}{2M}\log|\langle\psi_i(T)|P_f^M|\psi_i(T)\rangle|$$

$$P_f^M = \prod_{i=1}^{M} |\downarrow_i\rangle\langle\downarrow_i|$$

$M \ll L$
(quasi)local





Conclusion

We discover a new type of universal DQPT
in periodically driven quantum many-body systems

Strong singularity is attributed to the AUS breaking
through the exceptional point, which is hidden in
the spacetime-dual nonunitary operator
of the original unitary dynamics

[R. Hamazaki, Nature Communications 12, 5108 (2021).]