Computational power of dual-unitary quantum circuits

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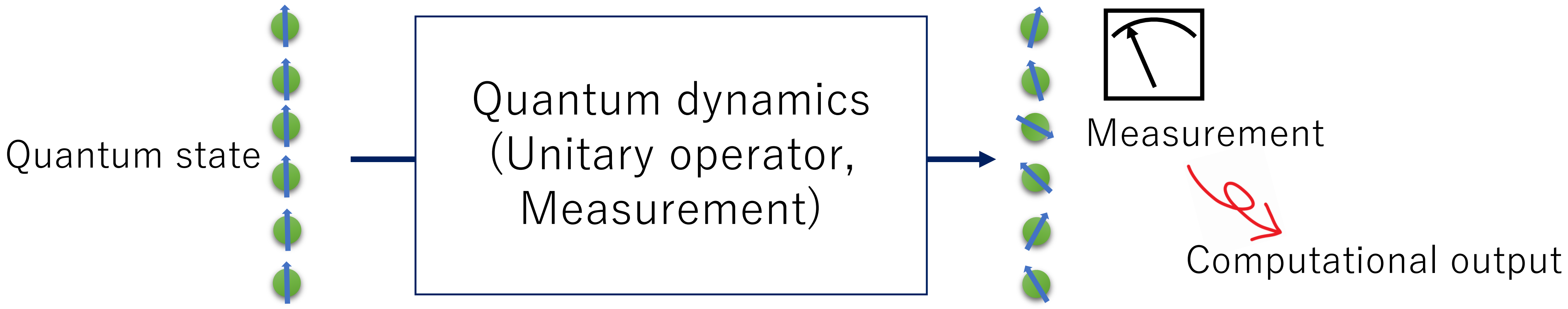
What is Quantum Computation?

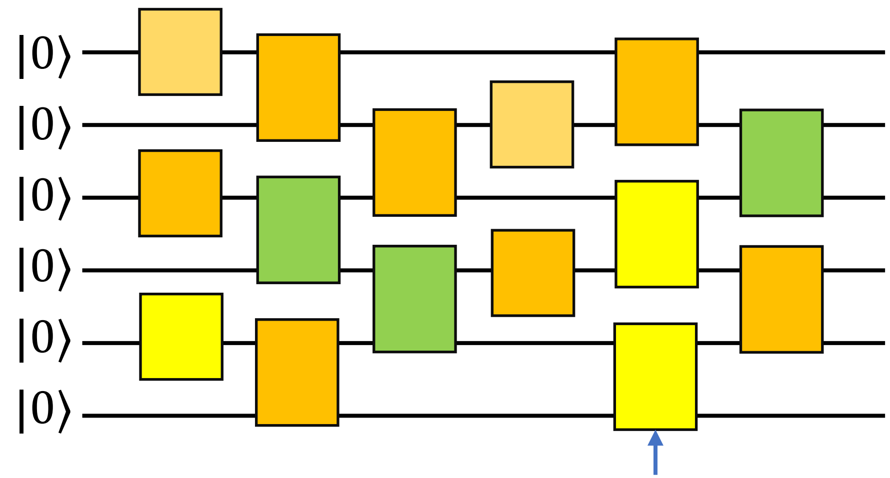
Quantum computer: a computer which makes use of quantumness



- Outperforms classical computers for certain problems (E.g. FACTORING [Shor '94])

Quantum circuit model

Qubits: $|\psi\rangle = a|0\rangle + b|1\rangle \in \mathbb{C}^2$



t Unitary operator: $U = e^{-iHt}$

Output

- Measure local observables

  Decision problem such as Factoring

- Sampling from a wave-function distribution

  Showing quantum spremacy

  F. Arute, et al., *Nature* 574.7779 (2019)

Universal gates set:

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix} \quad CZ = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Approximate arbitrary unitary operaots

Boykin, et. al., '99

CZ gate can be replaced by arbitrary entangling 2-qubit gates M. Bremner, et. al., '02

Classically simulatable quantum circuits

What makes quantum computation **different** from classical one?

When is quantum computation **tractable** for classical one?

General quantum circuits

Classical computer

Classically simulatable quantum circuits

E.g.

Low-entangled circuit
G. Vidal '03

$$|\Psi\rangle = \sum_{\alpha=1}^{\chi_A} \lambda_\alpha |\psi_\alpha^A\rangle \otimes |\phi_\alpha^{\bar{A}}\rangle$$

$$\chi_A = poly(n)$$

Clifford circuit
D. Gottesman '97

$$U_C = \{H, T^2, CZ\}$$

$$U_C{}^{\dagger} P U_C = P'$$

P, P':Pauli operators

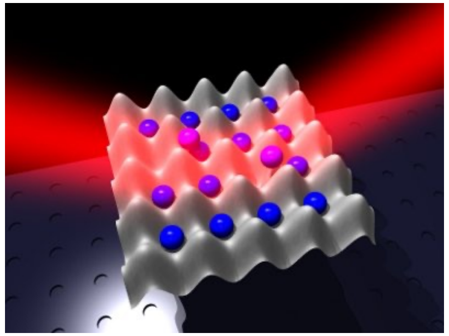
Matchgates circuit
L. Valiant '01

$$U_M = \begin{pmatrix} A_{11} & 0 & 0 & A_{12} \\ 0 & B_{11} & B_{12} & 0 \\ 0 & B_{21} & B_{22} & 0 \\ A_{21} & 0 & 0 & A_{22} \end{pmatrix}$$

$$A, B \in SU(2)$$

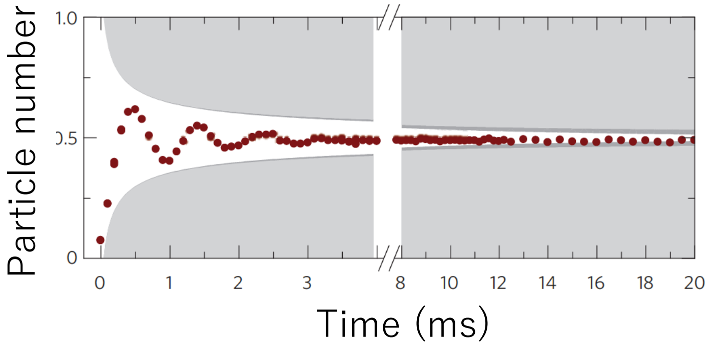
Non-equilibrium systems and Computation

Isolated quantum systems



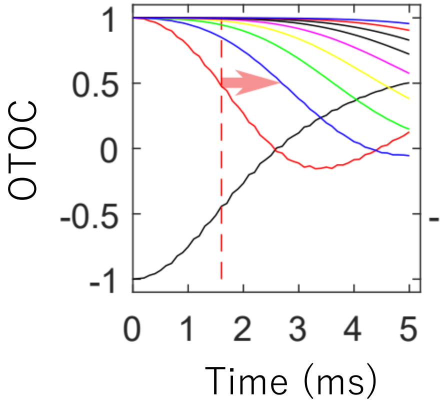
(https://dep.ftmc.uam.es/quantum-information-for-molecular-physics/)

Control of cold atoms



Thermalization

Trotzky, Stefan, et al., *Nature physics* (2012)



Quantum Chaos

Joshi, Manoj K., et al. *PRL* 124.24 (2020)

Solvable model

Free-fermion

Do not thermalize
Not chaotic

dual-unitary QC

Do thermalize
Chaotic

Classically simulatable QC

Matchgate L. Valiant (2001)

Independent interest of computer science

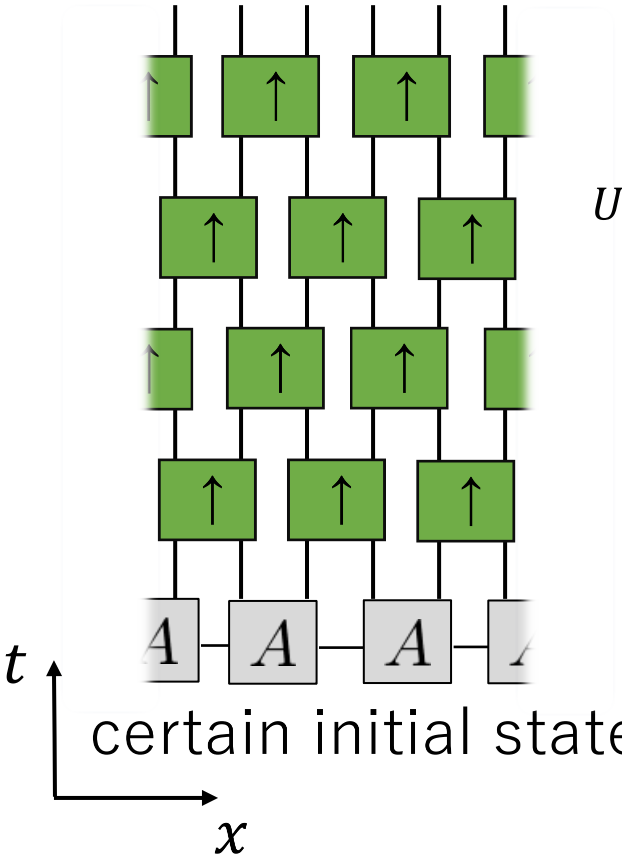
This is related to free fermion.

B. Terhal, D. Divinvenzo (2001)

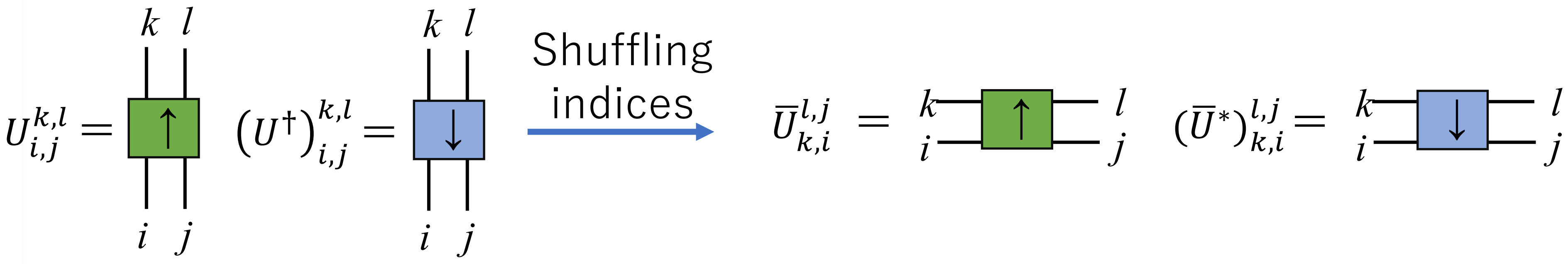
?

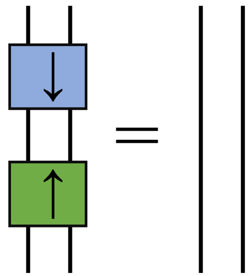
Definition of 1D dual-unitary quantum circuits

Bertini, Kos, Prosen ‘19 Piroli, Bertini, Cirac, Prosen ‘20

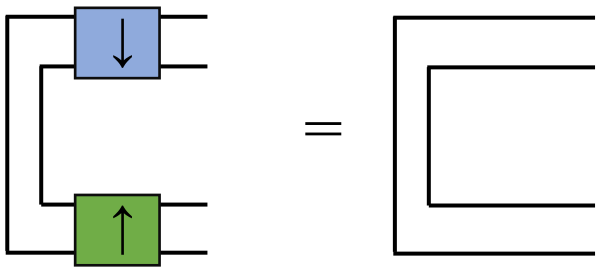


certain initial states



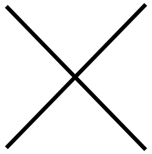


Unitarity



Unitarity in space direction

E.g. $\mathrm{SWAP}|i\rangle|j\rangle = |j\rangle|i\rangle$

SWAP = 

<u>General form</u>

$$U = u_1 \otimes u_2 e^{-i\left(\frac{\pi}{4}X\otimes X + \frac{\pi}{4}Y\otimes Y + JZ\otimes Z\right)} v_1 \otimes v_2$$

Solvable initial states

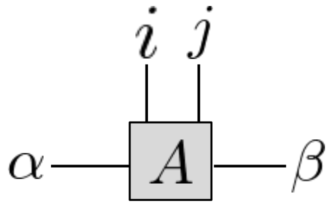
Piroli, Bertini, Cirac, Prosen ‘20

MPS

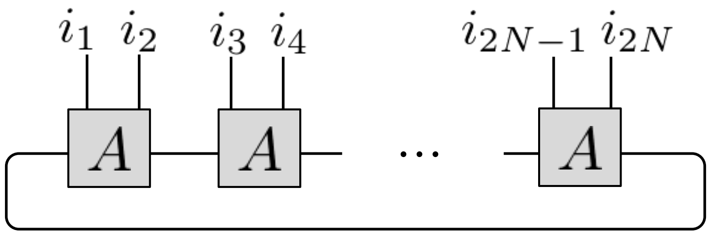
$A^{(i,j)}$:$\chi\times\chi$matrix

$$|\Psi^N(A)\rangle = \sum_{\{i_j\}}^{2} \mathrm{Tr}\left(A^{(i_1,i_2)}A^{(i_3,i_4)}\dots A^{(i_{2N-1},i_{2N})}\right)|i_1 i_2 \dots i_{2N}\rangle$$

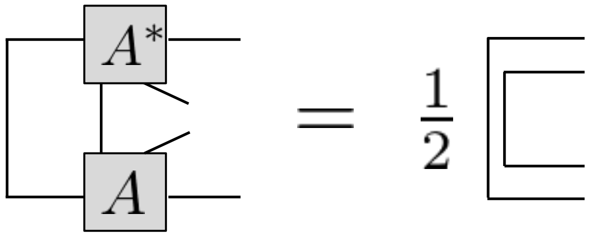
$(A^{(i,j)})_{\beta,\alpha} =$



$= \sum_{\{i_j\}}^{2}$

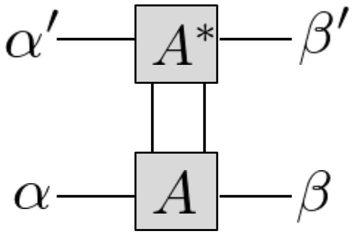


Condition ①



Condition ②

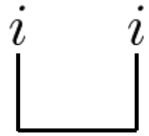
$E_{\beta'\beta,\alpha'\alpha} =$



Transfer matrix E has unique maximum eigenvalue

The simplest case：EPR pair chain

$$|\mathrm{EPR}\rangle^{\otimes N}, \quad |\mathrm{EPR}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = \frac{1}{\sqrt{2}}\sum_{i=1}^{2}$$

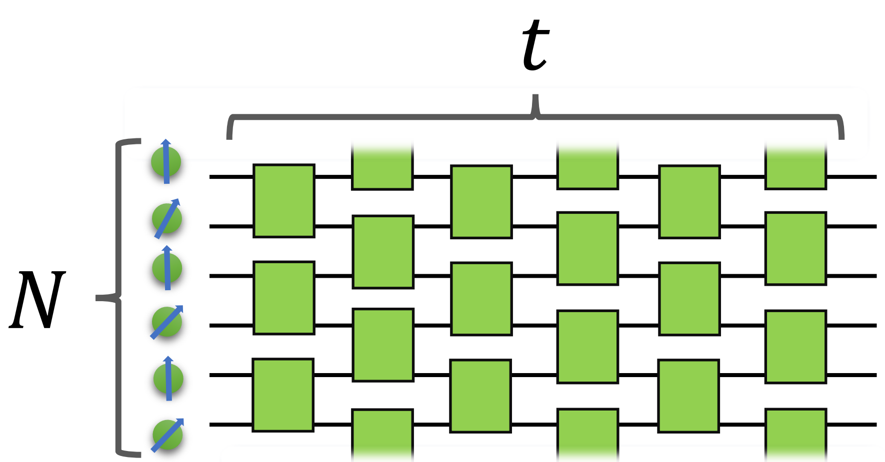


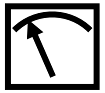
Computational power of 1D DUQC

Computational output: local measurement outcome $\langle\psi(t)|O|\psi(t)\rangle$

Problem

Fixing the number of qubit N, determine $\langle\psi(t)|O|\psi(t)\rangle$ of DUQC in time t.





Result : local observables (1D))

Early time : $t \leq N$

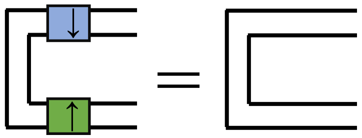
Local expectation value
$= \mathrm{Tr}(O) + O(c^N)$,
$c < 1$: real number

Classically simulatable

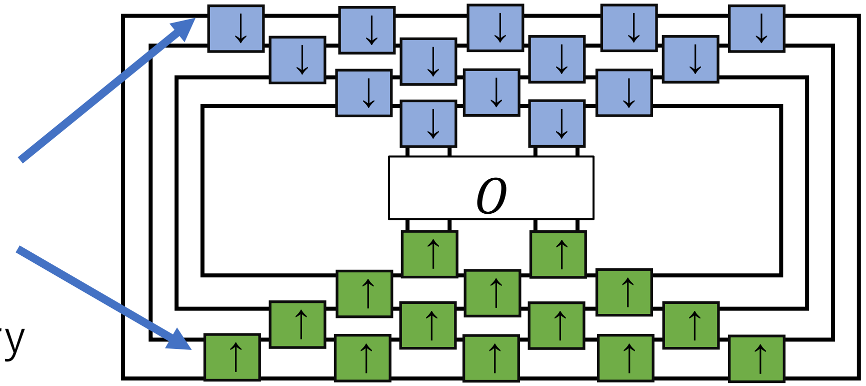
Late time: $t = \mathrm{poly}(N)$

Universal quantum computation

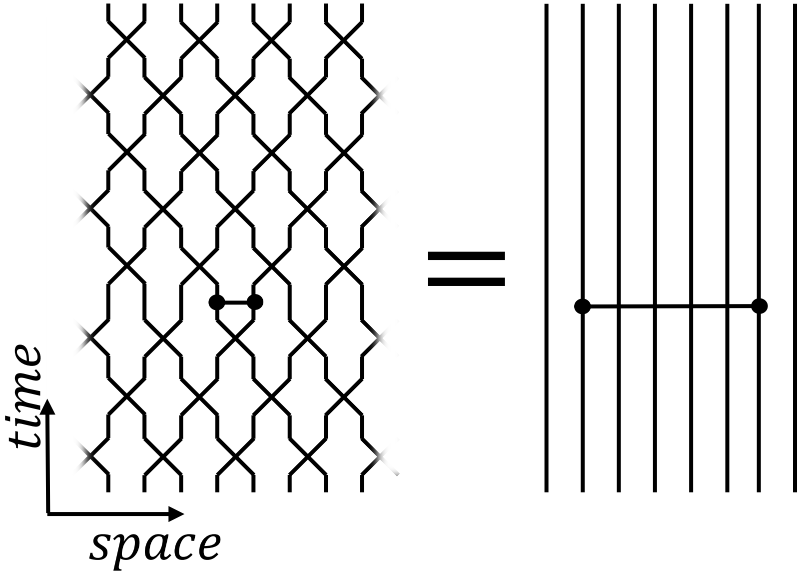
Outline of proof



Def of dual-unitary



Contraction of tensor-network



Construct 2-qubit gate

Universal gate set

M. J. Bremner, et al., *PRL* (2002)

Result : local observables (1D))

Early time：$t \leq N$

Local expectation value

$$= \mathrm{Tr}(O) + O(c^N),$$

$c < 1$: real number

Classically simulatable

Computational complexity makes a transition!

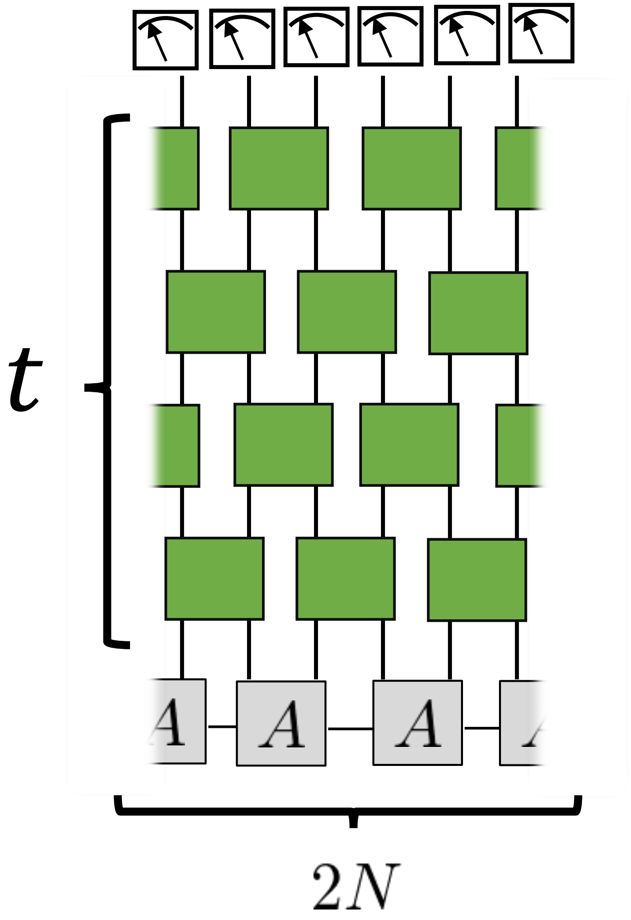
Late time: $t = \mathrm{poly}(N)$

Universal quantum computation

Sampling complexity of 1D dual-unitary QC

$\boxed{\text{problem}}$ Sample output of dual-unitary QC up to multiplicative error.

Measure in $\{|0\rangle, |1\rangle\}$ basis



$z \in \{0,1\}^{2N}$ with probability p_z

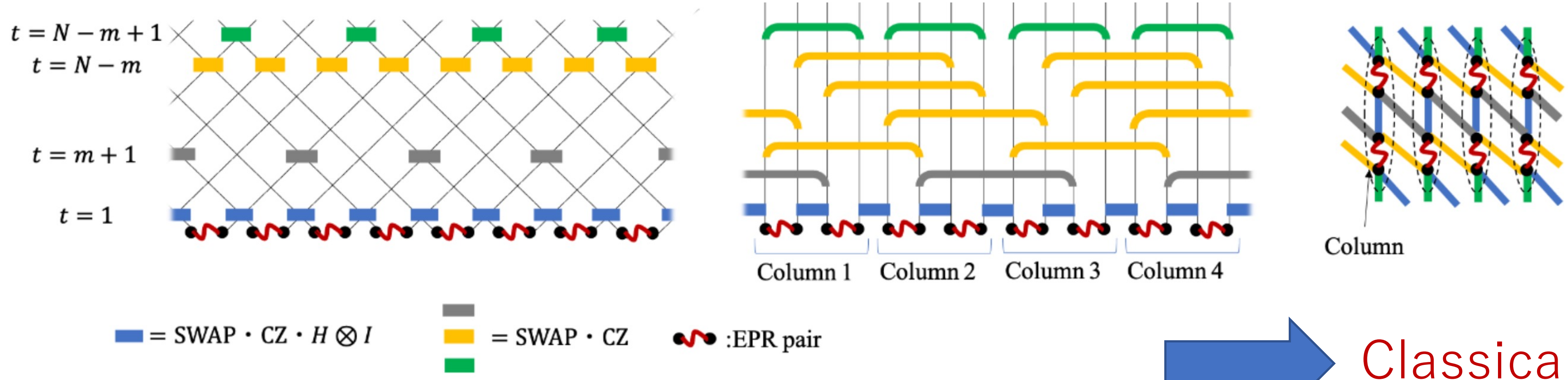
Is there a classical computer whose output $\{q_z\}$ satisfies $|p_z - q_z| \leq c p_z$? (for some constant c)

Result : sampling complexity (1D)

・Sampling from measurement output distribution of 2D cluster states is inefficient by classical computers under a computational complexity assumption.

Raussendorf, Briegel ‘01　Bremner, *et al.*, ‘11

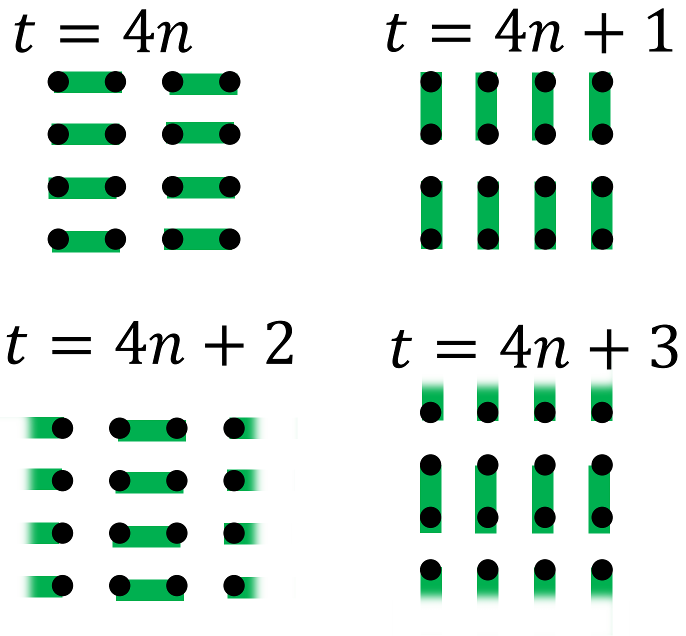
・1D DUQC can generate a state equivalent to a 2D cluster state.



Classical sampling is hard.

2D dual-unitary QC

Quantum circuits:



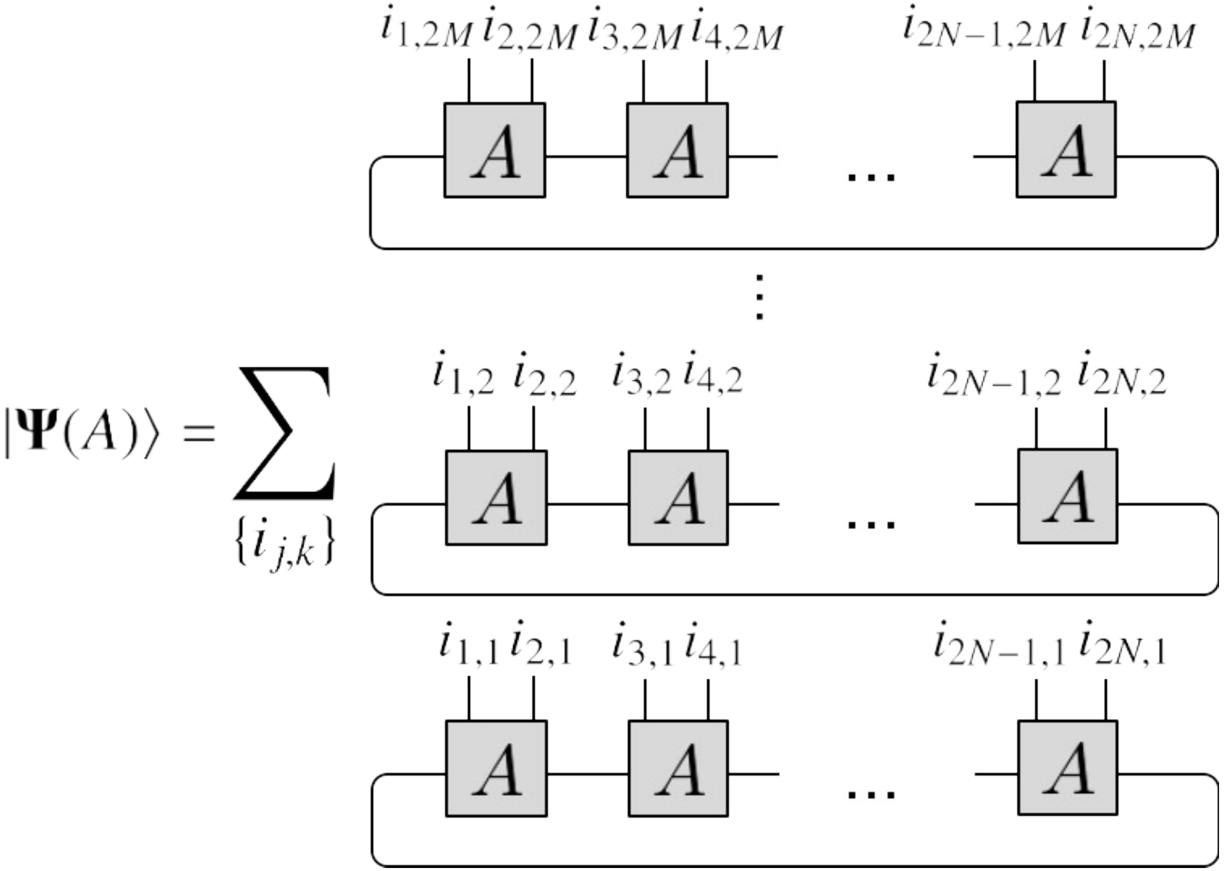
• :qubit

: unitary gate

dual-unitary gate (even time)

arbitrary two-qubit unitary gate (odd time)

Initial state: rows of initial states in 1D case



Result : Local observables (2D)

Computational power of 2D DUQC where output is obtained by local measurement.

Result

Early time : $t \leq 2N$

$$\left\langle \Psi^N(A) \left| \boldsymbol{U}^\dagger O \boldsymbol{U} \right| \Psi^N(A) \right\rangle = \frac{1}{2^{l^2}} \mathrm{Tr}(O) + O(c^N),$$

$c < 1$: real number

Classically simulatable

Late time: $t = poly(N)$

Arbitrary quantum algorithms can be implemented.

universal quantum computation

Classically simulatable | Universal quantum computation

$t \sim 2N$ $\quad$ $t = poly(N)$ $\quad$ → t

Sampling complexity (2D)

problem Sample from output distribution of 2D dual-unitary QC up to multiplicative error.

Depth-four 2D dual-unitary QC can generate 2D cluster states.

Sampling problem of depth-four 2D dual-unitary quantum circuits

|| hardness

Sampling problem of depth-four 2D quantum circuits

Conclusion

DUQCs is a new type of classically simulatable quantum circuits,
where computational complexity highly depends on computational time.

(might be an example for “computational phase transition”)

Deshpande, Abhinav, et al. PRL 121.3 (2018)   Napp, John, et al. *arXiv:2001.00021* (2019).

<u>Future work</u>

・ computational power of DUQCs with non-solvable initial states

・relation between other exactly solvable models and quantum computation