# Black hole interiors, state dependence, and modular inclusions

1811.08900

Ro Jefferson

Gravity, Quantum Fields & Information
Albert Einstein Institute
www.aei.mpg.de/GQFI

It from Qubit, YITP Kyoto
May 31st, 2019

# Mirror operators as probes of black hole interior

Black hole information / firewall paradox: do black holes have smooth horizons? (AMPS 1207.3123)

Papadodimas-Raju: do there exist CFT operators that satisfy certain constraints? (1211.6767, 1310.6334, 1310.6335)

$$\langle\psi|\mathcal{O}_n(t,x)\tilde{\mathcal{O}}_m(t',x')|\psi\rangle = Z_\beta^{-1}\,\mathrm{tr}\left[e^{-\beta H}\mathcal{O}_m(t,x)\mathcal{O}_n(t'+i\beta/2,x')\right]$$

Explicit construction of operators behind the horizon
$\longrightarrow$ **state-dependent** *mirror operators*:

$$\tilde{\mathcal{O}}_n|\psi\rangle = e^{-\beta H/2}\mathcal{O}_n^\dagger|\psi\rangle\ , \qquad \tilde{\mathcal{O}}_n\mathcal{O}_m|\psi\rangle = \mathcal{O}_m\tilde{\mathcal{O}}_n|\psi\rangle\ .$$

**TL;DR:** state dependence is a natural & inevitable feature of representing information behind horizons.

# Traversable wormholes via double trace deformation

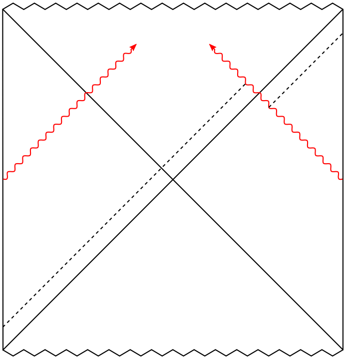
Consider thermofield double state dual to eternal AdS black hole:

$$|TFD\rangle = \frac{1}{\sqrt{Z_\beta}} \sum_i e^{-\beta E_i/2} |i\rangle_L |i\rangle_R$$

Gao, Jafferis, Wall (1608.05687) perturb the TFD by a relevant double-trace deformation:

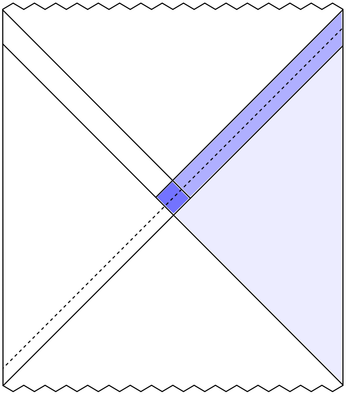
$$\delta S = \int \mathrm{d}^d x \, h \, \mathcal{O}_L \mathcal{O}_R$$

Decreases the energy of the TFD $\implies$ negative-energy shockwave in the bulk.

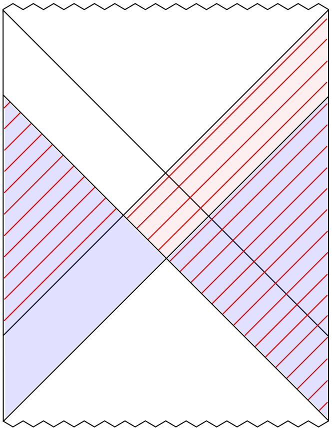


# A more physical picture

- Future horizons shrink, overlap allows null observer to cross.
- Preserves causality: observer is never "inside" the black hole; passage through wormhole is instantaneous.
- Left and right algebras are no longer independent due to bulk overlap.
- Relation between these two sets of operators is a *modular inclusion*.



# Modular inclusions $\longrightarrow$ state-dependent interiors



Modular inclusion of right (left) exterior algebras:

$$\mathcal{N}_R \subset \mathcal{M}_R \ , \quad \mathcal{M}'_R \subset \mathcal{N}'_R \ .$$

Interior state:

$$|\psi\rangle = D|\Omega\rangle \ , \quad D \in \mathcal{D}_R \equiv \mathcal{M}_R - \mathcal{N}_R \ .$$

How to represent $|\psi\rangle$ in exterior $\mathcal{N}_R$?

Find $N \in \mathcal{N}_R$ such that $N|\Omega\rangle = D|\Omega\rangle$

*State-dependent*! $N \neq D$

Information behind horizon does not admit local representation in either CFT $\longrightarrow$ no state-independent operators!

# Tomita-Takesaki in a nutshell

- Given a von Neumann algebra $\mathcal{A}$, TT theory provides canonical construction of commutant $\mathcal{A}'$.
- Consider Hilbert space $\mathcal{H}$ with cyclic & separating vacuum state $\Omega$.

  cyclic States spanned by $\mathcal{O} \in \mathcal{A}$ are dense in $\mathcal{H}$.
  separating $\mathcal{O}|\Omega\rangle = 0$ if and only if $\mathcal{O} = 0$.
- Starting point: antilinear map $S : \mathcal{H} \to \mathcal{H}$, $S\mathcal{O}|\Omega\rangle = \mathcal{O}^\dagger|\Omega\rangle$.
- Note that $S$ is a *state dependent* operator!
- Admits a unique polar decomposition $S = J\Delta^{1/2}$

  $J$ modular conjugation, $J^2 = 1$, $J^{-1} = J$
  $\Delta$ modular operator, $\Delta = S^\dagger S = e^{-K}$.
  $K$ modular hamiltonian $K \equiv -\log(S^\dagger S)$.
- Invariance of the vacuum: $S|\Omega\rangle = J|\Omega\rangle = \Delta|\Omega\rangle = |\Omega\rangle$.

# (ok, two nutshells...)

Fundamental result of TT theory comprised of two facts:

1. Modular operator $\Delta$ defines a 1-parameter family of modular automorphisms

$$\Delta^{it}\mathcal{A}\Delta^{-it} = \mathcal{A}\ , \qquad \forall t \in \mathbb{R}$$

$\implies$ $\mathcal{A}$ is invariant under *modular flow*.

E.g., subregion-subregion duality, $S_{\text{blk}}(\rho|\sigma) = S_{\text{bdy}}(\rho|\sigma)$ (1512.06431).

2. Modular conjugation induces isomorphism between $\mathcal{A}$ and $\mathcal{A}'$

$$J\mathcal{A}J = \mathcal{A}'$$

$\implies$ $\forall\mathcal{O} \in \mathcal{A},\ \exists\mathcal{O}' = J\mathcal{O}J$ such that $[\mathcal{O},\mathcal{O}'] = 0.$

Map between left and right Rindler wedges, or across black hole horizon!

# Mirror operators from TT theory (1708.06328)

Let $\mathcal{O} \in \mathcal{A}$ be a unitary operator; state $|\phi\rangle = \mathcal{O}|\Omega\rangle$ is indistinguishable from vacuum for observers $\mathcal{O}' \in \mathcal{A}'$:

$$\langle\phi|\mathcal{O}'|\phi\rangle = \langle\Omega|\mathcal{O}^\dagger\mathcal{O}'\mathcal{O}|\Omega\rangle = \langle\Omega|\mathcal{O}'|\Omega\rangle$$

But state $|\psi\rangle = \Delta^{1/2}\mathcal{O}|\Omega\rangle$ indistinguishable from vacuum for observers in $\mathcal{A}$!

$$|\psi\rangle = J^2\Delta^{1/2}\mathcal{O}|\Omega\rangle = JS\mathcal{O}|\Omega\rangle = J\mathcal{O}^\dagger|\Omega\rangle = J\mathcal{O}^\dagger J|\Omega\rangle = \mathcal{O}'|\Omega\rangle$$

where $\mathcal{O}' \equiv J\mathcal{O}^\dagger J \in \mathcal{A}'$.

State $|\psi\rangle$ is localized in $\mathcal{A}'$, but operator $\Delta^{1/2}\mathcal{O}$ is not!

$$\mathcal{O}' \neq \Delta^{1/2}\mathcal{O} \qquad \text{but} \qquad \mathcal{O}'|\Omega\rangle = \Delta^{1/2}\mathcal{O}|\Omega\rangle$$

$\longrightarrow$ Excitations behind horizon represented as *state-dependent* mirror operators.

# Reeh-Schlieder $\implies$ state dependence

Inability to encode information behind horizon in terms of state-independent operators localized to exterior is a natural consequence of the Reeh-Schlieder theorem.

*State-dependence reflects interplay between locality and unitarity.*

Witten's example (1803.04993): suppose $|\phi\rangle$ represents excitation in $\mathcal{D}_R \subset \mathcal{M}_R$. Define $D \in \mathcal{D}_R$ such that

$$\langle\phi|D|\phi\rangle = 1 \qquad \text{and} \qquad \langle\Omega|D|\Omega\rangle = 0$$

Reeh-Schlieder ($\Omega$ cyclic) $\implies$ can reproduce $|\phi\rangle$ arbitrarily well using operators localized entirely outside $\mathcal{D}_R$:

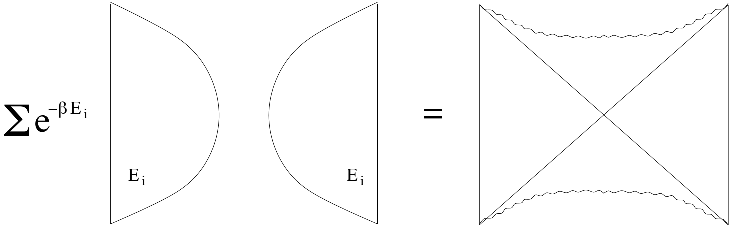
$$\exists N \in \mathcal{N}_R \;\; \text{s.t.} \;\; \langle\phi|D|\phi\rangle \approx \langle\Omega|N^\dagger D N|\Omega\rangle = \langle\Omega|N^\dagger N D|\Omega\rangle$$

$N$ unitary $\implies$ contradiction!

# Spacetime from quantum entanglement

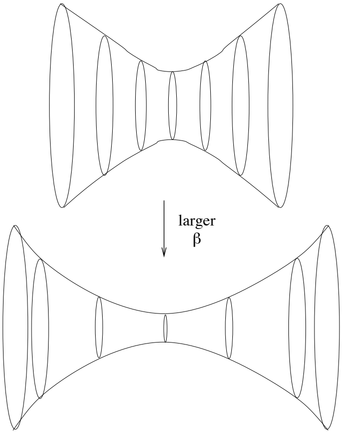
Product of CFTs: $|\Psi\rangle = |\Psi_1\rangle \otimes |\Psi_2\rangle$ dual to two disconnected spacetimes.

Entangled state: $|TFD\rangle \simeq \sum_i e^{-\beta E_i/2} |i\rangle_L |i\rangle_R$ superposition of disconnected pairs.




*Classical connectivity arises by entangling the dofs in the two components.* – van Raamsdonk (1005.3035)

# Disentangling the TFD



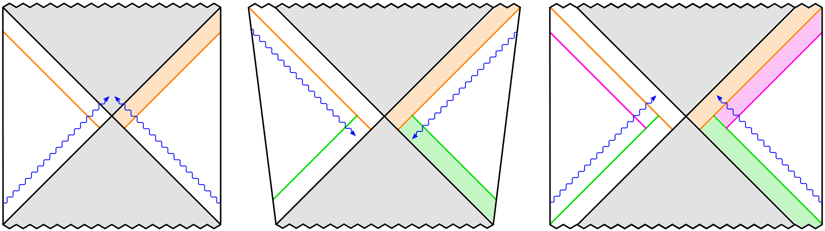

$$I(A,B) = S(A) + S(B) - S(A \cup B)$$

$$I(A,B) \geq \frac{\left(\langle \mathcal{O}_A \mathcal{O}_B \rangle - \langle \mathcal{O}_A \rangle \langle \mathcal{O}_B \rangle\right)^2}{2|\mathcal{O}_A|^2 |\mathcal{O}_B|^2}$$

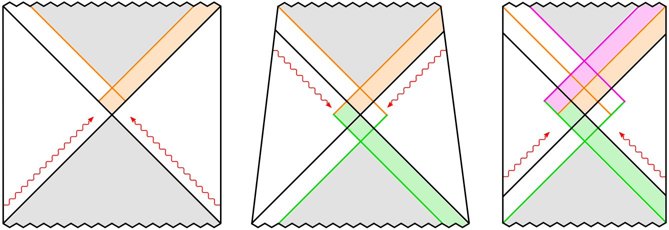
$$\langle \mathcal{O}_A(x) \mathcal{O}_B(x) \rangle \sim e^{-mL}$$

Length of wormhole $\overset{?}{\longleftrightarrow}$ amount of entanglement

# Modular theory $\longrightarrow$ It from Qubit?



$$\ldots \subset \mathcal{N}_{-3} \subset \mathcal{N}_{-2} \subset \mathcal{N}_{-1} \subset \mathcal{N}_0$$



$$\mathcal{N}_0 \subset \mathcal{N}_1 \subset \mathcal{N}_2 \subset \mathcal{N}_3 \subset \ldots$$

# Future connections (1811.08900)

- Why Ryu-Takayanagi: deeper relationship between entanglement and spacetime geometry?
- It-from-Qubit, ER=EPR: spacetime emergence consistent with boundary Hilbert space factorization?
- Black hole complementarity: global Hilbert space, but with state-dependent interior.
- Ontological foundation for QEC in holography: bulk algebra cannot hold at level of operators in CFT (1411.7041).
- Precursors: preservation of unitarity à la Reeh-Schlieder underlies holographic non-locality?
- Complexity: probing beyond horizons, holographic shadows?

*Can we make these ideas more precise?!*