AdS$_5$ Black Hole Entropy Without SUSY

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Microscopics of Black Hole Entropy

- The Bekenstein-Hawking area law for black hole entropy:

$$S = \frac{A}{4G_N} \, .$$

- In ***favorable cases*** string theory offers a ***statistical*** interpretation of the entropy $S = \ln \Omega$: specific constituents, ...

- Precise agreements were found ***in the classical limit*** but also beyond: ***higher derivative*** corrections, ***quantum*** corrections, ...

- These developments are among the ***most prominent successes*** of string theory as a ***theory of quantum gravity***.

AdS$_5$ Holography

- The best studied example of holography:
  String theory on AdS$_5 \times S^5$ is dual to ***N=4 SYM*** in $D = 4$.

- ***Microsopic details*** well understood (Quantum Field Theory!)

- The ***classical*** entropy of black holes in AdS$_5$ is a ***crude target***: just the asymptotic density of states.

- Yet: ***no quantitative agreements*** have been established in this context.

Recent Progress?

Several groups claimed ***precise agreements*** between entropy of ***supersymmetric*** AdS_5 black holes and the spectrum of N=4 SYM:

- Cabo-Bizet, Cassani, Martelli, and Murthy 1810.11442.
- Choi, Kim, Kim, and Nahmgoong 1810.12067.
- Benini and Milan 1811.04017.

But: they ***do not agree with each other*** and they are ***unclear about relation to previous negative results***.

This Talk (Draft Plan)

One option:

- Review recent (and not so recent) work ***authoritatively***.
- Also ***add generalizations*** and nuanced insights.
- Bonus: ***jokes*** about errors and misunderstandings (by others).

Drawbacks:

- ***Technicalities*** of subject not central to this workshop.
- Disclosure: many ***aspects remain confusing to me***.

Actual Talk

Goals:

- Study AdS_5 black holes ***away from the supersymmetric*** limit.
- Connect formal developments in string theory to physical regime ***central to this workshop***.
- Simple model for ***microscopic description of*** $\boldsymbol{AdS_5}$ black holes.
- Along the way: ***critical review*** of some work in the area.

Drawback:

- Legitimate questions about foundations.

FL+ Jun Nian, Yangwenxiao Zeng (work supported by DoE).

Quantum Numbers

- Geometry: AdS$_5 \times S^5$ has a (SUSY extension of) $SO(2,4) \times SO(6)$ symmetry.

- Fields in $SO(2,4)$ representations:
  conformal weight E, angular momenta $J_{a,b}$.

- Fields in $SO(6)$ representations: R-charges Q_I with $I = 1, 2, 3$.

- So asymptotic data of black holes in AdS$_5$:
  Mass M, Angular momenta $J_{a,b}$ and $3\, U(1)$ charges Q_I.

Classical Black Holes

- General solution (Wu 2011) .

  Independent mass M, angular momenta $J_{a,b}$, $U(1)$ charges Q_I.

  Not widely known (and exceptionally complicated).

- BPS mass (***"ground state energy"***): $M = \sum_I Q_I + g(J_a + J_b)$.

  Notation: coupling of gauged supergravity is $g = \ell_5^{-1}$.

- General BPS ***supersymmetric*** solution: Gutowski+Reall 2005.

- Feature: quantum numbers Q_I, J_a, J_b are related by ***a nonlinear constraint*** so ***rotation is mandatory***.

- Another feature: Only 2 SUSY's preserved $\frac{1}{16}$ ***of maximal***.

The Constraint on Charges

$$\frac{1}{2}N^2 J_a J_b + Q_1 Q_2 Q_3 = \left((Q_1 Q_2 + Q_2 Q_3 + Q_1 Q_3) - \frac{1}{2}N^2 (J_a + J_b) \right) \times \left(\frac{1}{2}N^2 + (Q_1 + Q_2 + Q_3) \right)$$

- Literature: black holes must have ***no closed timelike curves***
- Better:

$$M - M_{\rm BPS} = M - \sum_I Q_I - g(J_a + J_b) = (\ldots)^2 + (\ldots)^2$$

  BPS saturation gives $(\ldots)^2 = 0 \Rightarrow$ ***conditions give constraint***.
- But physics origin? null state condition from SUSY algebra??

The Entropy

$$S = 2\pi \sqrt{Q_1 Q_2 + Q_2 Q_3 + Q_1 Q_3 - \frac{1}{2} N^2 (J_a + J_b)}$$

- Q_I and $J_{a,b}$ are ***integral*** charges.
- ***Classical charges*** are $\sim N^2$ so the entropy is also $\sim N^2$.
- Flat space limit is nontrivial (bizarre) and not instructive.

Deconfinement

- There are two scales: $g = \ell_5^{-1}$ and G_5 in the problem.

- They are ***related as*** $\frac{\pi}{4G_5}\ell_5^3 = \frac{1}{2}N^2$
  (insert joke and/or cranky comment about practice in literature).

- The classical limit is $Q_I, J_{a,b}, M \sim N^2 \gg 1$.

- This is the ***deconfinement phase***.

- Physics question: is the ***low temperature phase*** deconfined?

(Suspense)

Beyond Supersymmetry

- ***Two perturbative paths break supersymmetry***.
- Recall: ***extremality*** = lowest mass given the conserved charges.
- The obvious path to break extremality: ***add energy*** (keeping charges fixed).

  Description: ***raise the temperature*** T beyond $T = 0$.
- An ***alternative path***: violate ***constraint*** by ***adjusting charges*** while preserving $M = M_{\rm ext}$.
- Description: ***"raise" potentials*** (for R-charges and angular momentum) from the values required by BPS.

Path I: Heat Capacity

- Black hole mass above BPS bound

$$M = M_{\text{BPS}} + \frac{1}{2} C_T T^2 \ .$$

- C_T is the ***heat capacity*** (divided by temperature) of the black hole. (The region of SYK,....).

- Gravity computations give

$$\frac{C_T}{T} = \frac{8Q^3 + \frac{1}{4}N^4(J_1 + J_2)}{\frac{1}{4}N^4 + \frac{1}{2}N^2(6Q - J_1 - J_2) + 12Q^2}$$

- Physics of this quantity: (essentially) the central charge.

  A measure of the ***number of degrees of freedom*** in ***low energy excitions***.

Path II: Capacitance

- BPS saturation implies the constraint so it is violated if the ***constraint is not enforced***.
- Then the ***extremal black hole mass exceeds the BPS bound***:

$$M_{\rm ext} = M_{\rm BPS} + \frac{1}{2}C_\varphi \varphi^2 \ .$$

- C_φ is the ***capacitance*** of the black hole. (The potential φ is defined precisely later)
- Gravity computations give

$$C_\varphi = \frac{8Q^3 + \frac{1}{4}N^4(J_1+J_2)}{\frac{1}{4}N^4 + \frac{1}{2}N^2(6Q^2+J_1+J_2)+12Q^2}$$

- ***Key observation***: $C_\varphi = \frac{C_T}{T}$.
- So: excitations violating the constraint "cost" the same as those violating the extremality bound!

Upshot: Gravity Computations

- The gold standard of ground states: ***supersymmetric*** $\equiv$ BPS.
- Somewhat mysteriously, BPS states must also satisfy a certain constraint.
- Excitations above the ground state "cost" energy $\frac{C_T}{T}$ that depends on BH parameters.
- Violations of the constraint "cost" energy C_φ that depends on BH parameters.
- These two types of excitations "cost" the same energy ***even though they are not obviously related***.

Effective Field Theory: UV vs. IR

- All low energy (IR) parameters are ultimately due to UV (microscopic) considerations.

- However, the precise relation between UV and IR is inscrutable in most cases.

- Current setting: enough structure that it may be realistic to compute IR parameters from UV.

  Encouragement: IR parameters relative simple functions of UV parameters.

- Moreover: IR theory suggests a symmetry that may have a UV origin.

A Supersymmetric Index

- The gravity regime corresponds to the strongly coupled regime of the dual gauge theory.

- Main idea for reliable analysis: ***protected states***.

- ***Preserved*** supersymmetry allows construction of the ***supersymmetric index***:

$$I = \mathrm{Tr}[(-)^F e^{-\Phi_I Q_I + \Omega_a J_a + \Omega_b J_b}]$$

- The grading $(-)^F$ computes (bosons - fermions) such that certain ***protected states*** will remain independent of coupling.

- Kinney, Maldacena, Minwalla, Raju (2005):

  All versions of the index is order ~ 1 (not N^2).

  Not sensitive to black hole phase (confined phase).

Recent Claims

Claim: ***protected*** versions of partition functions increase as $\sim N^2$.

Methodology:

- Localization.
- Enumeration of Free Fields.
- Integrable Systems/localization.

There are similarities and differences between the reported results and several known errors.

Central Point: Boundary Condition

- Euclidean path integral: ***rotation becomes imaginary***.

- Boundary conditions are twisted:

$$(\tau, \phi, \psi) \equiv (\tau + \beta, \phi - i\Omega_a \beta, \psi - i\Omega_a \beta)$$

- The preserved spinor has ***anti***periodic boundary conditions.

- SUSY requires ***complex potentials*** $\Phi_I, \Omega_{a,b}$

$$\Phi_1 + \Phi_2 + \Phi_3 - \Omega_a - \Omega_b = 2\pi i$$

- This was overlooked/not stressed by Kinney et.al.
  (but considered in an appendix)

- This point is ***technical but important***.

SUSY Localization

- Upshot: ***exploit SUSY to compute path integral exactly***.
- Strategy: deform integrand (without changing integral).
  Pick deformation so saddle point “approximation” becomes exact.
- Result of SUSY localization:

$$\ln Z = \frac{N^2}{2} \frac{\Phi_1 \Phi_2 \Phi_3}{\Omega_a \Omega_b}$$

Pro and con of SUSY localization:

- Pro: ***principled and very powerful***.
- Con: dominant saddle typically ***unphysical***.
  So computation is “magic”

Alternative: Free Field Theory

- The theory: 2 gauge d.o.f. + 6 scalars + fermion superpartners. All of them with $U(N)$ gauge indices.

- Single particle index (just $U(1)$):

$$1 - \frac{\prod_I (1 - e^{-\tilde{\Phi}_I})}{(1 - e^{-\tilde{\Omega}_1})(1 - e^{-\tilde{\Omega}_2})} \, .$$

- Challenges: ***multiple particle states*** and $U(N)$ indices.

Analysis

Special Korean maneuver:

- ***First*** assume that the rotation is slow $\Omega_a \ll \Phi_I$ ("Cardy Limit")
- Argue (***assume***) that $U(N)$ gauge indices just give a factor N^2.
- ***Then*** sum over multiparticle states
- Apply result for any Ω_a.

Result of free field computation:

$$\ln Z = \frac{N^2}{2} \frac{\Phi_1 \Phi_2 \Phi_3}{\Omega_a \Omega_b}$$

A “Miracle”

- Compute the entropy as the ***Legendre transform of the free energy*** (partition function $\ln Z$ as function of the potentials).
- ***Reality condition*** on the resulting entropy gives the constraint.
- Moreover, the ***real part*** of the Legendre transform ***gives the correct black hole entropy***.
- The justification of these steps is dubious but ***they suggest a free field representation of the strongly coupled limit***.

Historical Comments

- The ***joint*** representation of the black hole entropy and the constraint as the free energy

$$F = \frac{N^2}{2}\frac{\Phi_1\Phi_2\Phi_3}{\Omega_1\Omega_2}$$

  was known since '17 (Hosseini, Hristov, Zaffaroni).

- Recent derivations derive (find) the same answer *.

- A more general formula for any $\mathcal{N}=1$ theory (the "***generalized SUSY Casimir***")

$$F = \frac{16}{27}(3c-2a)\frac{\Phi_1\Phi_2\Phi_3}{\Omega_a\Omega_b}$$

- Outlook: the free field representation of the entropy may be justified for some purposes.

Beyond Supersymmetry

- Assume result for SUSY partition function.

- Apply when constraint

$$\Phi_1 + \Phi_2 + \Phi_3 - \Omega_a - \Omega_b = 2\pi i$$

  is violated (by a little bit).

- Apply away from extremality $T \neq 0$ (by a little bit)

- Result: leading order gives ***correct specific heat*** and ***capacitance***

Protection With No SUSY

- Model: a ***family of parameters*** where free gas description applies.
- Each is protected by BPS, but “which” BPS varies over parameter space.
- Slow motion on parameter space also protected (at first order away from BPS.)
- Disclosure: work in progress.

Final Comment

- Leading order away from BPS: nearAdS$_2$ limit.
- Much recent study (SYK,...) in the IR.
- My agenda: connect IR parameters to UV.