LIGHT CONE BOOTSTRAP AND UNIVERSALITY

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Based on

YK, JHEP 1901 (2019) 025

YK, Masamichi Miyaji, arXiv:1905.02191

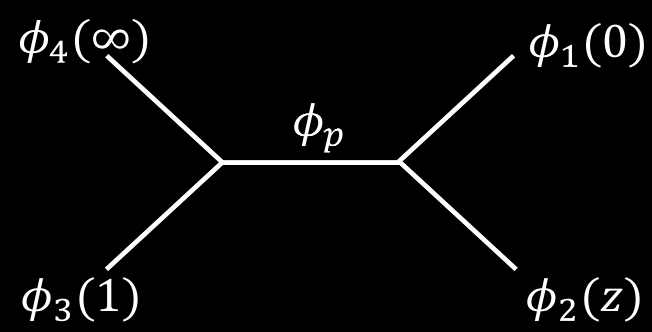
(Developments of JHEP01(2018)115, JHEP 1807 (2018) 010, JHEP 1808 (2018) 161)

Main Interest

$$\langle O_4 O_3 O_2 O_1 \rangle = \sum_p C_{12p} C_{p34}\, F_{34}^{21}(h_p|z)\bar{F}_{34}^{21}(\bar{h}_p|\bar{z})$$

Insert complete set
$1 = |\phi_p\rangle\langle\phi_p|$

Virasoro block,
perfectly determined by Virasoro algebra.



Contribution from single irrep.

Block is useful?

$$\langle O_B O_B O_A O_A \rangle \xrightarrow{z,\bar{z}\to 0} F^{AA}_{BB}(0|z)\bar{F}^{AA}_{BB}(0|\bar{z})$$

$$\xrightarrow{z \ll 1-\bar{z} \ll 1}$$

$$\xrightarrow{Regge\ limit}$$

$$\xrightarrow{c\ \to \infty}$$

Universality comes from **single block** !!

e.g., entanglement, OTOC, …

Block is useful again

$$\langle O_B O_B O_A O_A \rangle \longrightarrow F^{AA}_{BB}(0|z)\bar{F}^{AA}_{BB}(0|\bar{z})$$

Another decomposition
(OPE associativity)

$$\| \qquad\qquad \|$$

$$\langle O_B O_A O_A O_B \rangle = \sum_p C^2_{ABp} F^{AB}_{AB}(\bar{h}_p|1-z)\bar{F}^{AB}_{AB}(\bar{h}_p|1-\bar{z})$$

Universal coupling constant & spectrum
also come from single block !
Look very easy !!

e.g., spectrum at high energy (Cardy formula)
Eigenstate Thermalization Hypothesis

Serious problem

No simple closed form of block
(even in special limits, except for $z \to 0$)

To understand interesting universality,
we need

$$F^{AA}_{BB}(0|z) \xrightarrow[z \to 1]{} ?$$

so-called light cone singularity.

Strategy

Remind box:

$$h_i = \alpha_i(Q-\alpha_i),\; c = 1 + 6Q^2,\; Q = b + \frac{1}{b}$$

The key is an integral transformation (unitary $c > 1$ CFTs)

$$F_{BB}^{AA}(0|z) = \sum_n \# \times F_{AB}^{AB}(\alpha_A + \alpha_B + nb|1-z) + \int_{\frac{Q}{2}+0}^{\frac{Q}{2}+i\infty} \mathrm{d}\alpha_t\, \# \times F_{AB}^{AB}(\alpha_t|1-z)$$

where # is a kernel function.

The kernel is very complicated, but not important here.

By the trivial asymptotics (NOT $z \to 1$)

$$F_{AB}^{AB}(\alpha|z) \xrightarrow[z\to 0]{} z^{h_\alpha - h_A - h_B}$$

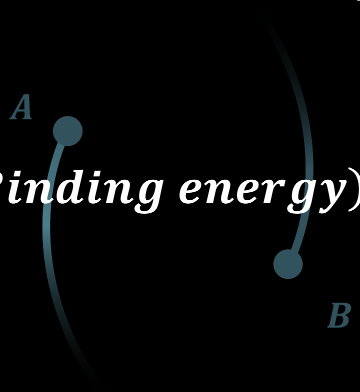
We can immediately obtain the light cone singularity!

Application I: Light Cone Bootstrap

[1810.01335], [1905.02191]

Assumption: unitary $c > 1$ CFT without extra currents (including holographic CFT)

In the bulk, for a spinning particle with O_A and O_B in AdS_3,



$$(\boldsymbol{Binding\ energy}) = \begin{cases} \bar{h}_{\bar{\alpha}_A+\bar{\alpha}_B+nb} - \bar{h}_A - \bar{h}_B, & \text{if } \bar{h}_{\bar{\alpha}_A+\bar{\alpha}_B+nb} < \dfrac{c-1}{24} \\ \dfrac{c-1}{24} - \bar{h}_A - \bar{h}_B, & \text{otherwise} \end{cases}$$

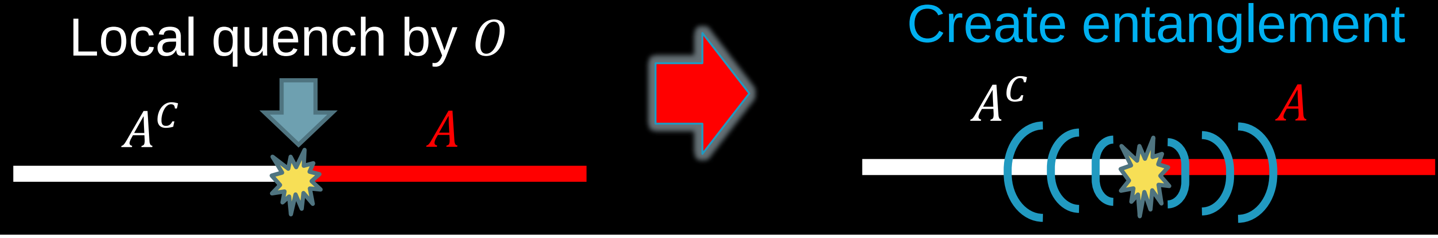
⇒We can detect the gravity interaction from CFT side!

The point is our result is non-perturbative! It would be interesting to reproduce the binding energy from AdS_3 quantum gravity.

Application II: Dynamics of Entanglement

[1905.02191], improvement of [1711.09913]

Setup: probe the growth of entanglement between A and A^C after a local quench



The growth of Renyi entanglement entropy after a local quench is given by the light cone singularity.

From our result, we find in holographic CFT,

- Logarithmic growth $S_A^{(n)}(t) \sim \log t$, non-perturbatively.
- Replica Transition at $n = 2$ (conjectured in [1711.09913] & [1804.06171])

Both of them cannot be found in RCFTs.
⇒Characteristics of the holographic CFT?

Comments

- Regge limit is also given by our approach

[1905.02191]

Future directions

- EE as Probe of Gravity Force

[Miyaji-YK, in preparation]

- Dynamics of EWCS & Reflected Entropy and Generalized EE (⇒Tamaoka's talk?)

[Tamaoka-YK, in preparation]

- Bootstrap in other limits
- OTOC, Dynamics of Negativity, multi-interval EE, …many applications!!

Thank you very much

Appendix

Kernel

Remind box:

$h_i = \alpha_i(Q - \alpha_i)$, $c = 1 + 6Q^2$, $Q = b + \frac{1}{b}$

Let's consider an integral transformation

$$F_{34}^{21}(h_{\alpha_s}|z) = \int_{\mathbb{S}} \mathrm{d}\alpha_t \, \mathbb{F}_{\alpha_s,\alpha_t}\begin{bmatrix} \alpha_2 & \alpha_1 \\ \alpha_3 & \alpha_4 \end{bmatrix} F_{14}^{23}(h_{\alpha_t}|1-z)$$

where $\mathbb{S} = [\frac{Q}{2} + 0, \frac{Q}{2} + i\infty]$.

The kernel function $\mathbb{F}$ is universal, determined only by Virasoro algebra.

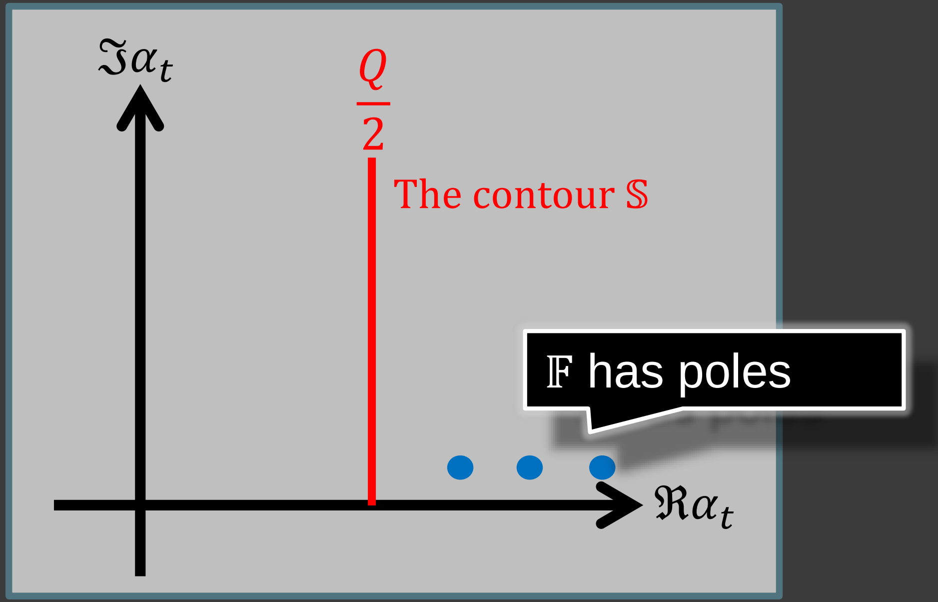
We particularly focus on the pole structure of $\mathbb{F}$.

Kernel

Remind box:

$$h_i = \alpha_i(Q-\alpha_i),\ c = 1+6Q^2,\ Q = b+\frac{1}{b}$$

$$F^{21}_{34}(h_{\alpha_s}|z) = \int_{\mathbb{S}} \mathrm{d}\alpha_t\, \mathbb{F}_{\alpha_s,\alpha_t}\begin{bmatrix}\alpha_2 & \alpha_1\\ \alpha_3 & \alpha_4\end{bmatrix} F^{23}_{14}(h_{\alpha_t}|1-z)$$

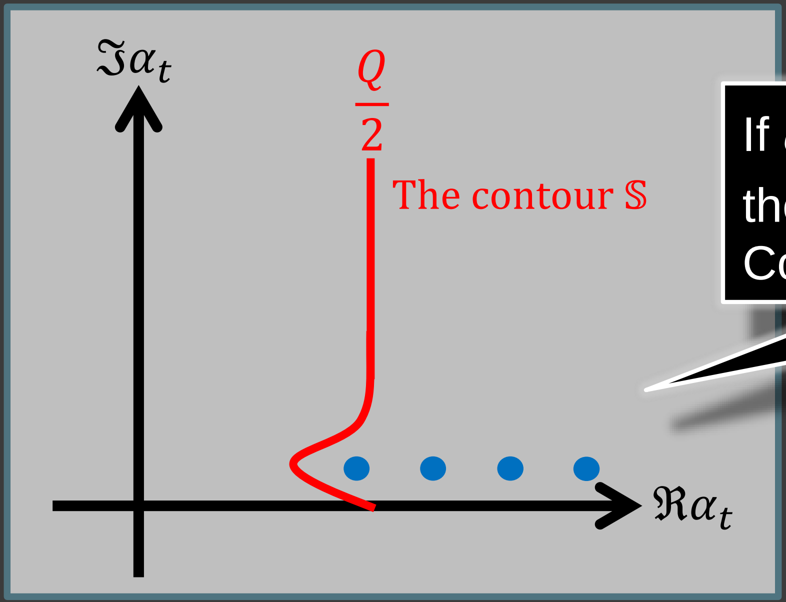


Kernel

<u>Remind box:</u>

$$h_i = \alpha_i(Q - \alpha_i),\ c = 1 + 6Q^2,\ Q = b + \frac{1}{b}$$

$$F_{34}^{21}(h_{\alpha_s}|z) = \int_{\mathbb{S}} \mathrm{d}\alpha_t\, \mathbb{F}_{\alpha_s,\alpha_t}\begin{bmatrix}\alpha_2 & \alpha_1\\ \alpha_3 & \alpha_4\end{bmatrix} F_{14}^{23}(h_{\alpha_t}|1-z)$$

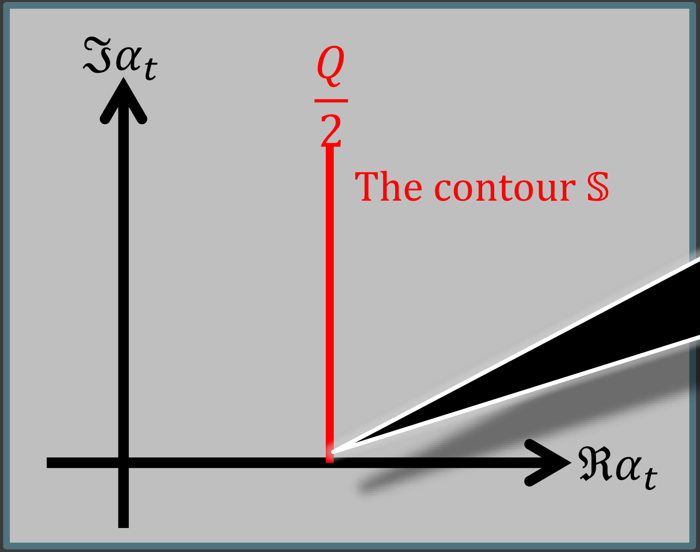


If $\alpha_1 + \alpha_4 < \frac{Q}{2}$ (or $\alpha_2 + \alpha_3 < \frac{Q}{2}$),
the poles cross the contour.
Consequently, the contour is deformed.

Kernel

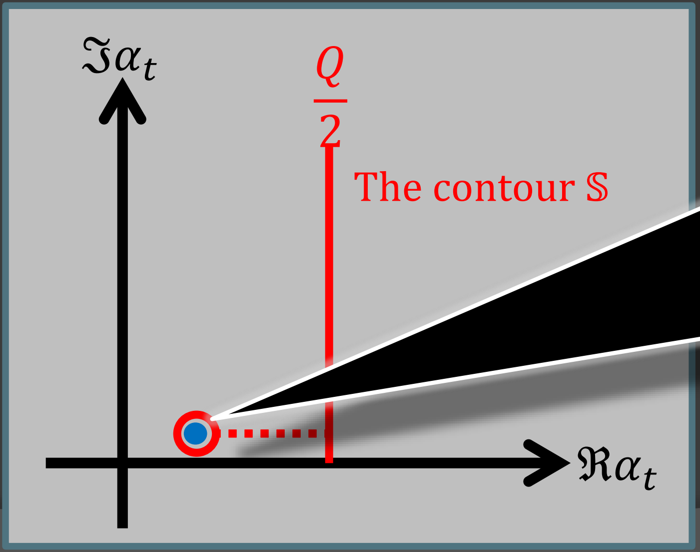
Remind box:

$$h_i = \alpha_i(Q-\alpha_i),\ c = 1+6Q^2,\ Q = b+\frac{1}{b}$$



if $\alpha_1+\alpha_4 > \frac{Q}{2}$ (and $\alpha_2+\alpha_3 > \frac{Q}{2}$),
the dominant contribution in the light cone limit of the integral is given by the root of $\mathbb{S}$.

$$(1-z)^{\frac{c-1}{24}-h_2-h_3}$$



if $\alpha_1+\alpha_4 < \frac{Q}{2}$ (or $\alpha_2+\alpha_3 < \frac{Q}{2}$),
the dominant contribution is given by the pole.

$$\begin{cases} (1-z)^{h_{\alpha_1+\alpha_4}-h_2-h_3} & \text{if } \alpha_1+\alpha_4 < \dfrac{Q}{2} \\ (1-z)^{h_{\alpha_2+\alpha_3}-h_2-h_3} & \text{if } \alpha_2+\alpha_3 < \dfrac{Q}{2} \end{cases}$$

which is more singular than $(1-z)^{\frac{c-1}{24}-h_2-h_3}$

Bootstrap

Remind box:

$h_i = \alpha_i(Q - \alpha_i)$, $c = 1 + 6Q^2$, $Q = b + \frac{1}{b}$

- The integral trans. is

$$F_{BB}^{AA}(0|z) = \sum_n \# \times F_{AB}^{AB}(\alpha_A + \alpha_B + nb|1 - z) + \int_{\frac{Q}{2}+0}^{\frac{Q}{2}+i\infty} \mathrm{d}\alpha_t \, \# \times F_{AB}^{AB}(\alpha_t|1 - z)$$

- The bootstrap eq. in the light cone limit is

$$\bar{F}_{BB}^{AA}(0|\bar{z}) = \int \mathrm{d}\bar{h}_p \, \rho_{AB}(\infty, \bar{h}_p) \bar{F}_{AB}^{AB}(\bar{h}_p|1 - \bar{z})$$

where we gather $\bar{z}$-independent terms into $\rho_{AB}(\infty, \bar{h})$, interpreted as the OPE coefficient density at large h_p.

Bootstrap

Remind box:

$$h_i = \alpha_i(Q - \alpha_i),\ c = 1 + 6Q^2,\ Q = b + \frac{1}{b}$$

- The integral trans. is

$$F_{BB}^{AA}(0|z) = \sum_n \# \times F_{AB}^{AB}(\alpha_A + \alpha_B + nb|1 - z) + \int_{\frac{Q}{2}+0}^{\frac{Q}{2}+i\infty} \mathrm{d}\alpha_t\, \# \times F_{AB}^{AB}(\alpha_t|1 - z)$$

We can compare them very easily!
That is, we can solve the bootstrap eq.!
(see [1905.02191])

- The bootstrap eq.

$$\bar{F}_{BB}^{AA}(0|\bar{z}) = \int \mathrm{d}\bar{h}_p\, \rho_{AB}\left(\infty, \bar{h}_p\right) \bar{F}_{AB}^{AB}\left(\bar{h}_p \middle| 1 - \bar{z}\right)$$

where we gather $\bar{z}$-independent terms into $\rho_{AB}\left(\infty, \bar{h}\right)$, interpreted as the OPE coefficient density at large h_p.

OPE Spectrum

Remind box:

$$h_i = \alpha_i(Q - \alpha_i),\ c = 1 + 6Q^2,\ Q = b + \frac{1}{b}$$

The spectrum of the OPE between V_A and V_B at large spin (i.e. $h \to \infty$) is

$$\bar{V}_A \times \bar{V}_B = \sum_{\substack{\bar{\alpha} = \bar{\alpha}_A + \bar{\alpha}_B + nb \\ n \in \mathbb{Z}_{\geq 0}}} \bar{V}_{\bar{\alpha}} + \int_{\frac{Q}{2}+0}^{\frac{Q}{2}+i\infty} \mathrm{d}\bar{\alpha}\, \bar{V}_{\bar{\alpha}}$$

Note that we can memorize this pattern by the Liouville fusion rule.

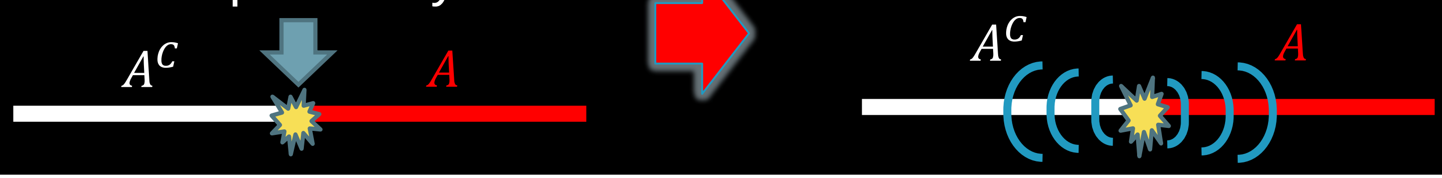
This is consistent with our previous numerical results [1711.09913] and [1804.06171].

Details for Dynamics of Entanglement

[1905.02191], improvement of [1711.09913]

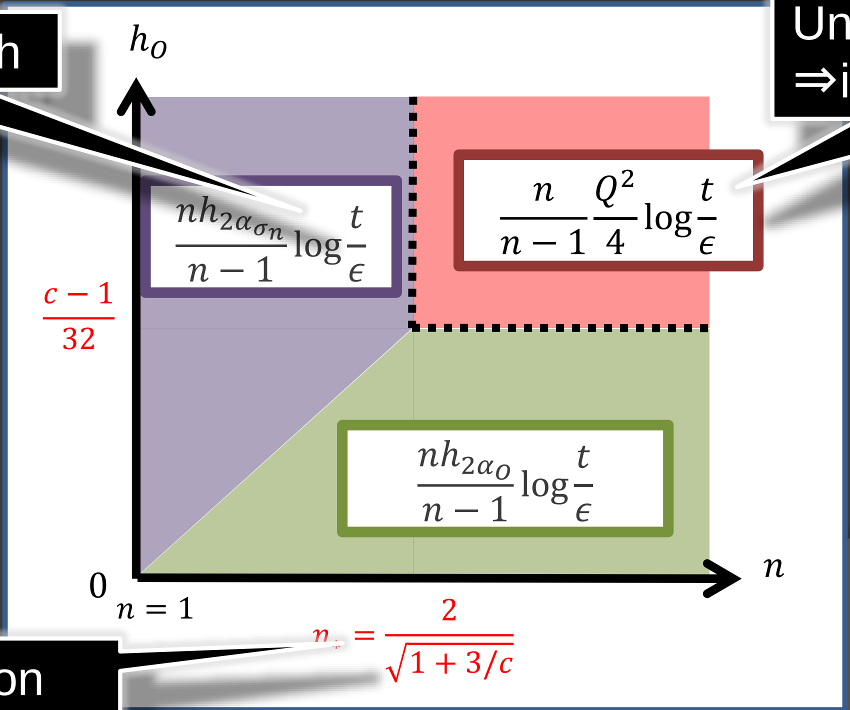
Setup: probe the growth of entanglement between A and A^C after a local quench

Local quench by O → Create entanglement



Logarithmic growth

Universality
⇒indicate scrambling?



Replica transition