Fault-tolerant verification of quantum supremacy & Accreditation of NISQ devices

Animesh Datta

Department of Physics, University of Warwick, UK

Samuele Ferracin, Theodoros Kapourniotis

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Quantum Physics

Nonadaptive fault-tolerant verification of quantum supremacy with noise

Theodoros Kapourniotis, Animesh Datta

(Submitted on 28 Mar 2017 (v1), last revised 27 Feb 2019 (this version, v3))

arXiv.org > quant-ph > arXiv:1811.09709

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Quantum Physics

Accrediting outputs of noisy intermediate-scale quantum computing devices

Samuele Ferracin, Theodoros Kapourniotis, Animesh Datta

(Submitted on 23 Nov 2018 (v1), last revised 30 May 2019 (this version, v2))



- Dominic Branford
- Samuele Ferracin
- Jamie Friel
- Evangelia Bisketzi
- Aiman Khan

- Andrew Jackson
- Theodoros Kapourniotis
- Max Marcus
- Francesco Albarelli



Quantum supremacy





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Quantum supremacy



Why quantum supremacy? It used to be ...

- quantum simulator
- Q quantum computer
- guantum 'supreme' device

Manin/Feynman (1980/82)

Aaronson/Arkhipov (2013)

Shor (1994) 🚆



It may look like the promise of quantum information is shrinking

The slide down from computation to supremacy is because

- Experiments are hard!
- All of DiVincenzo's criteria need fulfilling





Figure: Experimental advances have been enormous (Google, UMD) We still don't have a big enough system with low enough noise If we had a universal QC, we wouldn't be talking about supremacy

Theoretical shortcomings

- Many examples of exponential improvement in QIP
- Simon's algorithm (oracle separation between BPP & BQP)
- Shor's algorithm (compared to best known classical algorithm)
- X Hofstadter butterfly @ Google (provably polynomial)

• ...

No theoretical impossibility of classical polynomial algorithms

Tang, 1807.04271



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So, we've settled for

Quantum supremacy

- Theoretical proof of exponential gaps (with conjectures)
- Sub-universal (typically sampling) problems
- The idea has been around for a long time

Knill/Laflamme, DQC1, 1998-

Terhal/DiVincenzo, Fermionic QC, 2002-

 Revived interest after complexity-theoretic hardness proofs (sampling problems with conjectures)

Bremner/Jozsa/Shepherd/Montanaro, IQP, 2010-Aaronson/Arkhipov, BosonSampling, 2013-

Morimae/Fujii/Fitzsimons, DQC1k 2014-

Fefferman/Umans, FourierSampling, 2015-

Farhi/Harrow, QAOA, 2016- Google, RandomSampling, 2016-

Gao/Wang/Duan, IsingSampling, 2016

Some performed/proposed experiments

Oxford, Vienna, Rome, Brisbane, Shanghai, Google, IBM, ...



Quantum supremacy experiments

What do quantum supremacy experiments prove?

Figure: Boson sampling (Oxford), Random sampling(Google)



Is quantum supremacy really easier than quantum computation?

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• Can imperfect/noisy experiments 'show' quantum supremacy?



All experiments are imperfect and noisy

- Can imperfect/noisy experiments 'show' quantum supremacy?
- Physical system must be quantum (non-classical)

Rahimi-Keshari/Ralph/Caves, PRX, 6, 021039, (2016)

[need low(er) noise/imperfection]

Computational task must be supreme (super-classical)

DWave

Neville et al. Nat. Phys. 13, 1153 (2017) Google/IBM

[need large(r) system]



All experiments are imperfect and noisy

- Can imperfect/noisy experiments 'show' quantum supremacy?
- But even with better and larger systems ...

Noise

- Is the problem still hard?
- Otherwise experiments useless (for quantum supremacy)

Imperfections

- Is the solution correct?
- Not solving decision problems

The two fundamental issues are

- Proofs of hardness of sampling (with noise)
- Verification of quantum supremacy (with imperfections)

Aaronson/Chen, 1612.05903

Harrow/Montanaro, Nature 549, 203 (2017)

• Is quantum supremacy easier than quantum simulation?

• Is quantum supremacy easier than quantum computation?

• If so, by how much?



Verification of quantum computation

- I. Direct certification, benchmarking (Hardware solution) Certify a small system, hope it stills holds for a big one
- II. Interactive proof system: verification (Software)

(Software solution)



Aharonov, Ben-Or, Broadbent, Fitzsimmons, Hayashi, Kashefi, Morimae, Vazirani, Vidick, ...

To verify, must trust

Our work: 'Prepare-and-send' protocol

Verification of quantum computation



Aharonov, Ben-Or, Broadbent, Fitzsimmons, Hayashi, Kashefi, Morimae, Vazirani, Vidick, ...

To verify, must trust

Our work: 'Prepare-and-send' protocol

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Verification scheme for quantum supremacy

New definition of verifiability over i.i.d. repetitions based on

$$\mathsf{var} \equiv rac{1}{2}\sum_{oldsymbol{x}} |q^{\mathsf{exc}}(oldsymbol{x}) - q^{\mathsf{nsy}}(oldsymbol{x})|,$$

Fitzsimmons/Kashefi, PRA 96, 012303 (2017)

- (1) Takes as input a verification protocol, $M \in \mathbb{N}, l \in [0, 1]$
- (2) Outputs a string and a bit.
- (3) The bit determines if the string is accepted or rejected.
- (4) After running M i.i.d repetitions of (1) it outputs one of the M output strings at random. Accept if at least a fraction I of the protocols accept and reject otherwise.

Verifiability

Definition (Verifiability)

A scheme is verifiable if its output is

• (δ', δ) -complete: For an honest prover having only bounded noise, the scheme accepts at least with probability δ' , and

$$\operatorname{var} \leq 1 - \delta$$

for the output string.

• $(\varepsilon', \varepsilon)$ -sound: For any, including adversarial, prover if the scheme accepts then

$$\operatorname{var} \leq \varepsilon$$

with confidence ε' .

Kapourniotis/AD, arXiv:1703.09568

Blindness is a necessary ingredient in our verification scheme

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Kapourniotis/AD, arXiv:1703.09568

Blindness is a necessary ingredient in our verification scheme

Our work: Trap-based verification of Ising sampling problem

Ising sampler

• Translationally-invariant, nonadaptive, Ising spin model

$$\mathcal{H} = -\sum_{\langle i,j \rangle} J Z_i Z_j + \sum_i B_i Z_i$$

• The probability p_x of measuring bit string x from partition function \mathcal{Z}_x

$$p_{\mathbf{x}} = \frac{|\mathsf{Tr}(e^{-i(\mathcal{H} + \frac{\pi}{2}\sum_{i}x_{i}Z_{i})}|^{2}}{2^{2mn}} \equiv \frac{|\mathcal{Z}_{\mathbf{x}}|^{2}}{2^{2mn}}$$

Gao/Wang/Duan, PRL, 118, 40502 (2017)

• Partition function at imaginary temperatures insightful

Lee/Yang, Phys. Rev. 87, 410 (1952)

Fujii/Morimae, NJP 19, 033003 (2017) Goldberg/Guo, Computational Complexity 26, 765 (2017)



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Traps



Figure: Verifier chooses a random ordering of $2\kappa + 1$ graph states.Single qubit traps.Kapourniotis/AD, arXiv:1703.09568



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- 'Prepare-and-send' protocol
- Blindness (Quantum one-time pad)

$$\mathcal{N}_j = (1 - \epsilon_{V,P})\mathcal{I} + \mathcal{E}_j$$

where

•
$$\epsilon_V = ||\mathcal{E}_j||_\diamond$$
 for preparation noise [Verifier]

• $\epsilon_P = ||\mathcal{E}_j||_{\diamond}$ for entangling/measurement noise [Honest Proverse]

Our results

(Short term aim of experiments)

Theorem (Non-fault tolerance verification scheme)

There exists a verification scheme with

$$M = \frac{\log(1/\beta)}{2\kappa^2 N^2 (\epsilon_V + \epsilon_P)^2},$$
$$I = 1 - \kappa N (2\epsilon_V + 4\epsilon_P)$$

that is

$$\left(1-eta,1-\sqrt{\textit{N}(\epsilon_{V}+3\epsilon_{P})}
ight)-\textit{complete}$$

and

$$\left(1-eta,\sqrt{\kappa \mathcal{N}(3\epsilon_V+5\epsilon_P)+\Delta_\kappa}
ight)-\textit{sound},$$

where $\Delta_{\kappa} = \kappa! (\kappa + 1)! / (2\kappa + 1)! \sim 2^{-\kappa}$.

Kapourniotis/AD, arXiv:1703.09568

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Definition (Verifiability)

A scheme is verifiable if its output is

• (δ', δ) -complete: For an honest prover having only bounded noise, the scheme accepts at least with probability δ' , and

$$\operatorname{var} \leq 1 - \delta$$

for the output string.

• $(\varepsilon', \varepsilon)$ -sound: For any, including adversarial, prover if the scheme accepts then

 $\mathrm{var} \leq \varepsilon$

with confidence ε' .

Kapourniotis/AD, arXiv:1703.09568

For verifiable quantum supremacy, we need

 $N(\epsilon_V + 3\epsilon_P)$ const.

and

$$\kappa N(3\epsilon_V + 5\epsilon_P) + \Delta_{\kappa} \quad \downarrow$$

Impossible in large systems(N) with constant noise($\epsilon_{P,V}$)



Solution: Quantum fault tolerance

- Use FT (3D cluster state) encoding for universal QC
- X RHG encoding require adaptive operations (gate distillation)

Raussendor/Harrington/Goyal, NJP 9, 199 (2007)

• On target computation, use free postselection due to Fujii

1610.03632

• Trap computation is Clifford, so nonadaptive



Solution: Quantum fault tolerance

✗ RHG encoding require adaptive operations (gate distillation)

Raussendor/Harrington/Goyal, NJP 9, 199 (2007)

• On target computation, use free postselection due to Fujii

1610.03632

- Trap computation is Clifford, so nonadaptive
- FT thresholds



FT thresholds for verifiable quantum supremacy

Supremacy easier than universal QC

 $\epsilon_{\mathsf{thres}} = 1.97\%$

- Replace error correction with error detection
- Works as isolated trap qubits isolated can be retransmitted individually
- Same completeness & soundness with κ replaced by $\textit{M}\kappa$

ϵ	$\epsilon_{\sf thres}/20$	$\epsilon_{\sf thres}/50$	$\epsilon_{\sf thres}/100$
Μ	$3 imes 10^8$	2863	54

- M independent of problem size
- Improved by judicious braiding or other topological code
- \bullet Larger ϵ_{thres} with simpler problem specific code

Kapourniotis/AD, arXiv:1703.09568



X Leaking logical measurement angles in magic state distillation

 \pmb{x} For distillation, need to reveal information about state distilled

Ising Sampler and Trap Computations MBQC (Logical layer)

Protected topology using defects

Blind 3D cluster-state MBQC (Physical layer)

Kapourniotis/AD, arXiv:1703.09568

On target computation, use free postselection due to Fujii

$$\operatorname{var}^{\operatorname{Post}} \equiv \frac{1}{2} \sum_{\boldsymbol{x}} |q^{\operatorname{exc}}(\boldsymbol{x}|y=0) - q^{\operatorname{nsy}}(\boldsymbol{x}|y=0)|$$

Definition (Verifiability of a scheme for post-selected distribution)

A scheme is verifiable <u>conditioned on the post-selection register</u> being zero, if its output is

• (δ', δ) -complete: For an honest prover having only bounded noise, the scheme accepts at least with probability δ' , and

$$\operatorname{var}^{\operatorname{Post}} \leq 1 - \delta$$

for the the output string.

• $(\varepsilon', \varepsilon)$ -sound: For any, including adversarial, prover if the scheme accepts, then

$$\operatorname{var}^{\operatorname{Post}} \leq \epsilon$$

with confidence
$$\varepsilon'$$

Our results

Theorem (Fault-tolerant verification scheme)

There exists a verification scheme with

 $M = \log(1/eta)/(2\epsilon''^2)$

and

 $I=(1-2\epsilon''),$

that is

$$(1-eta,1-\sqrt{\epsilon''})-\mathit{complete}$$

and

$$(1-eta,\sqrt{3\epsilon''+\Delta_\kappa})-\mathit{sound}$$

where $\Delta_{\kappa} = \kappa!(\kappa+1)!/(2\kappa+1)!$.

Milestone towards FT QC

Kapourniotis/AD, arXiv:1703.09568



Conjecture (Average-case hardness)

For $0 \le \alpha_1, \beta_1 \le 1$, approximating the probability distribution of the Ising sampler by $p^{apx}(\mathbf{x}|y=0)$ up to multiplicative error

$$|m{
ho}^{ ext{apx}}(m{x}|y=0)-m{q}^{ ext{exc}}(m{x}|y=0)|\leq lpha_1m{q}^{ ext{exc}}(m{x}|y=0)$$

in time $poly(|\mathbf{x}|, 1/\alpha_1, 1/\beta_1)$ is #P-hard for at least a fraction β_1 of \mathbf{x} instances.

Conjecture (Anti-concentration)

There exist some $0 \le \alpha_2, \beta_2 \le 1, 1/\alpha_2 \in poly(1/\beta_2)$ such that for all x

$$\operatorname{prob}\left(\boldsymbol{q}^{\operatorname{exc}}(\boldsymbol{x}|\boldsymbol{y}=\boldsymbol{0}) \geq \frac{\alpha_2}{2^N}\right) \geq \beta_2$$

More general than Bremner/Montanaro/Shepherd, PRL 117, 080501 (2016)

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Theorem (Fault-tolerant hardness)

Assume that the two Conjectures hold. Then sampling from the output distribution of the experimental Ising sampler $q^{nsy}(\mathbf{x}, y)$ with a classical machine, assuming a $(\varepsilon', \varepsilon)$ -sound verification scheme accepts with

$$\varepsilon \leq \frac{(\beta_1 + \beta_2 - 1 - 2^{-N})\alpha_1\alpha_2}{2},$$

implies, with confidence ε' , a collapse in the polynomial hierarchy to the third level.

Kapourniotis/AD, arXiv:1703.09568

First FT verification of quantum supremacy

FT supremacy verification milestone for FT QC

Kapourniotis/AD, arXiv:1703.09568

We still need

- bespoke FT thresholds for verifying specific supremacy models
- bespoke error correcting codes for specific supremacy models
- verification schemes for specific architectures
- to use verification schemes in experiments
 - short term (Thm 1)
 - long term (Thm 2/3)
 - But we want everything now. NISQ devices...



If/when a NISQ device solves a hard problem (not in NP), how do we know its done so correctly?

- X NISQ devices are noisy and imperfect
- X Cannot check efficiently on a classical computer

"Quantum Accreditation"



To build big (intermediate) systems, start with small ones

- State tomography
- Process tomography
- Measurement (Detector) tomography

Too many parameters for NISQ devices

Good gate fidelities are not enough

- Randomised benchmarking
- Gate set tomography

Knill et. al., PRA 77, 012307 (2008)

Blume-Kohout et. al., Nat. Comm. 8, 14485 (2017)

Makes unrealistic assumptions



Numerous references ...

PHYS: Statistical methods (e.g., cross entropy)

nadequate | Bouland et al., Nat. Phys. (2018)

TCS: Interactive proof system

Childs, Aharonov, Ben-Or, Broadbent, Eisert, Fitzsimmons, Hayashi, Kashefi, Mahajan

Morimae, Vazirani, Vidick, Zhu, Us

- Hide easy 'trap' computations within hard computation
- Check the correctness of the 'traps'
- Bound distance between ideal (p_{id}) and actual (p_{act}) output Exorbitant overheads (due to MBQC)
- Even constant overheads are impractical



CS meets experiments: Scalable vs practical



Figure: CS: Verifier, prover, and a shared register C.

Figure: Experiments: System and environment.





Our work

In the circuit model



Figure: A six-qubit example of target circuit.

- Several (v > 1) trap circuits
- Traps designed to capture all noise
- Trap and target circuits of same size

- In the circuit model
- Different trust assumptions (noise model)
- N1: Noise in state preparation, entangling gates, measurements is arbitrary CPTP map encompassing system & environment

$$\rho_{\mathsf{out}} = \mathsf{Tr}_{E} \big[\circ_{p=1}^{q} \mathcal{N}_{SE}^{(p)} \big(\mathcal{E}_{S}^{(p)} \otimes \mathcal{I}_{E} \big) (\rho_{S} \otimes \rho_{E} \big) \big]$$

and is unbounded in diamond norm;

N2: Single qubit gates are trusted



Accreditation Protocol - One run

$$\rho_{\mathsf{out}} = \mathsf{Tr}_{E} \big[\circ_{p=1}^{q} \mathcal{N}_{SE}^{(p)} \big(\mathcal{E}_{S}^{(p)} \otimes \mathcal{I}_{E} \big) (\rho_{S} \otimes \rho_{E}) \big]$$

Protocol $\{\mathcal{E}^{(p)}_S\}_{p=1}^q$ accredits outputs in presence of $\{\mathcal{N}^{(p)}_{SE}\}_{p=1}^q$ if

$$\begin{array}{ll} \rho_{\mathsf{out}} &=& b \; \tau_{\mathsf{out}}^{\prime\,\mathsf{tar}} \otimes |\mathsf{acc}\rangle\langle\mathsf{acc}| \\ &+& (1-b) \bigg(I \; \sigma_{\mathsf{out}}^{\mathsf{tar}} \otimes |\mathsf{acc}\rangle\langle\mathsf{acc}| + (1-I) \tau_{\mathsf{out}}^{\mathsf{tar}} \otimes |\mathsf{rej}\rangle\langle\mathsf{rej}| \bigg), \end{array}$$

where

 $\begin{array}{l} \sigma_{\rm out}^{\rm tar} \left(\tau_{\rm out}^{\prime \ \rm tar} \right) \text{ is target circuit state after noiseless (noisy) protocol,} \\ \tau_{\rm out}^{\rm tar} \text{ is an arbitrary state for the target circuit,} \\ |{\rm acc}\rangle \text{ is the state of the flag indicating acceptance,} \\ |{\rm rej}\rangle = |{\rm acc} \oplus 1\rangle, \\ 0 \leq l \leq 1, \ 0 \leq b \leq \varepsilon \text{ and } \varepsilon \in [0,1]. \end{array}$

 $1-\varepsilon$ is the *credibility* of the accreditation protocol.





Figure: One target computation and v trap computations.

Correlated noise across all v + 1 circuits - in space and time.

• Use
$$U_{i,j}'=X_i^{lpha_{i,j}'}Z_i^{lpha_{i,j}}U_{i,j},\,lpha_{i,j},lpha_{i,j}'\in\{0,1\}$$
 are random bits

- Pauli twirl decomposes noise into combination of local Pauli
- Traps designed to capture all local Pauli noise

- After *d* protocol runs (with same target and *v* different traps),
- If all runs are affected by i.i.d. noise,
- then, with confidence $1 e^{-2d\theta^2}$, $heta \in (0, N_{\sf acc}/d)$

$$rac{1}{2}\sum_{ar{s}}\left|p_{\mathsf{noiseless}}(ar{s})-p_{\mathsf{noisy}}(ar{s})
ight|\leq rac{arepsilon}{N_{\mathsf{acc}}/d- heta}$$
 ,

for all $N_{\sf acc} \in [0, d]$ protocol runs ending with $|{\sf acc}\rangle$ flag bit.



Theorem

Suppose that all single-qubit gates are <u>noiseless</u>. For any number $v \ge 3$ of trap circuits, our protocol can accredit the outputs of a noisy quantum computer affected by noise of the form N1 with

$$arepsilon = rac{\kappa}{\mathsf{v}+1}$$
 ,

where $\kappa = 3(3/4)^2 \approx 1.7$.

Ferracin/Kapourniotis/AD, 1811.09709



Noisy single qubit gates

- Different trust assumptions (noise model)
- N1: Noise in state preparation, entangling gates, measurements is arbitrary CPTP map encompassing system & environment

$$\rho_{\mathsf{out}} = \mathsf{Tr}_{\mathsf{E}} \big[\circ_{p=1}^{q} \mathcal{N}_{SE}^{(p)} \big(\mathcal{E}_{S}^{(p)} \otimes \mathcal{I}_{E} \big) (\rho_{S} \otimes \rho_{E}) \big]$$

and is unbounded in diamond norm;

N2: Noise in single-qubit gates is arbitrary (inc. gate-dependent) CPTP map encompassing system & environment

$$\widetilde{\mathcal{U}}_j = \mathcal{N}_j^{(k)} ig(\mathcal{U}_j \otimes \mathcal{I}_E ig) \; \; ext{with} \; \; ||\mathcal{N}_j^{(k)} - \mathcal{I}_{SE}||_\diamond \leq r_j^{(k)}$$

and $0 \le r_j^{(k)} < 1$ (bounded in diamond norm).



Theorem

Our protocol with $v \ge 3$ of trap circuits can accredit the outputs of a noisy quantum computer affected by noise of the form N1 and N2 with

$$\varepsilon = g \frac{\kappa}{\nu + 1} + 1 - g , \qquad (1)$$

where
$$\kappa = 3(3/4)^2 \approx 1.7$$
 and $g = \prod_{j,k} (1 - r_{\max, j}^{(k)})$.

Ferracin/Kapourniotis/AD, 1811.09709



Experimental use

$$rac{1}{2}\sum_{\overline{s}}\left|p_{\mathsf{noiseless}}(\overline{s})-p_{\mathsf{noisy}}(\overline{s})
ight|\leq rac{arepsilon}{N_{\mathsf{acc}}/d- heta}$$
 ,

Since $N_{\rm acc}/d$ is an estimate of prob(acc) (and if prob(acc) $\geq \delta$.)



Verification & Accreditation

So, Accreditation

- is practical (and scalable)
- is inspired by trap-based verification schemes
- is different from verification
- combines best features from physics & CS
- inspires new mesothetic (verifier-in-the-middle) verification scheme



Ferracin/Kapourniotis/AD, 1811.09709



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Thank you!

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