COLOR CODE DECODERS FROM TORIC CODE DECODERS

Aleksander Kubica

PERIMETER INSTITUTE

Institute for Quantum Computing

work w/ N. Delfosse

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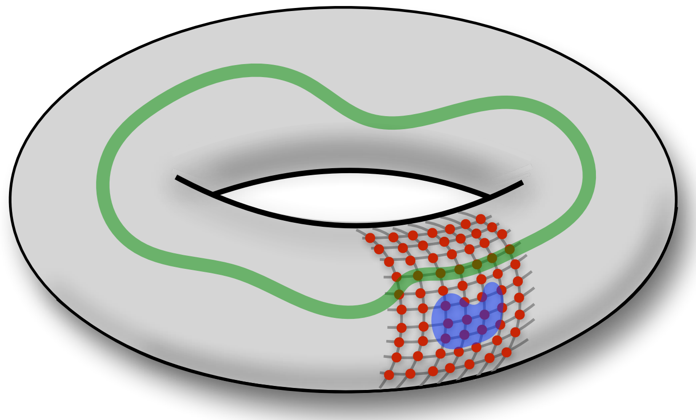
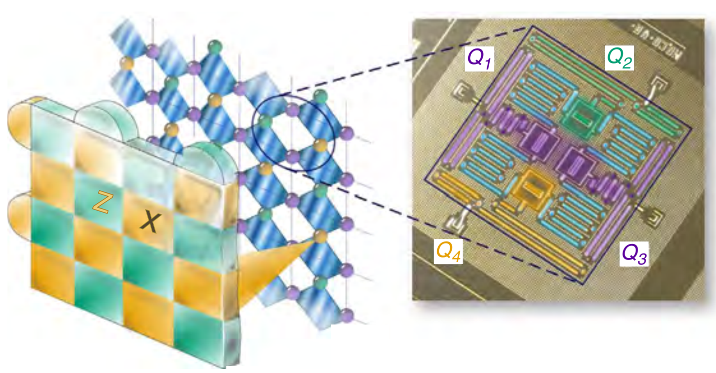
TOPOLOGICAL QUANTUM ERROR-CORRECTING CODES

- Want to reliably store & process q. information. Need QECCs!
- Topological codes = geometrically local generators, logical info encoded non-locally.



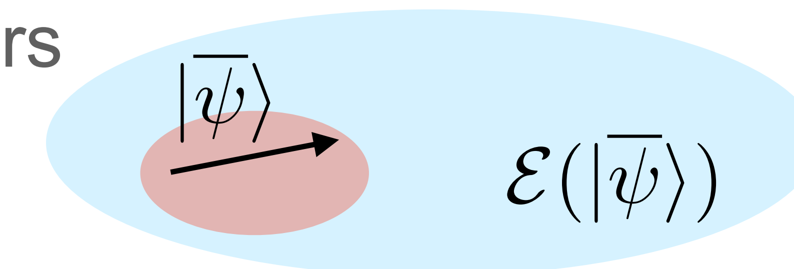
- Examples: toric & color codes.
- Desired properties:
  — can be built in the lab,
  — fault-tolerant logical gates,
  — efficient decoders
  — high thresholds.



Córcoles et al., Nat. Commun. 6 (2015)

DECODING PROBLEM FOR STABILIZER CODES

- **Stabilizer codes** *[G96]*: commuting Pauli operators
  code space = (+1)-eigenspace of stabilizers.



- **Quantum error-correction game:**

$$|\psi\rangle \xrightarrow{\text{encode}} |\overline{\psi}\rangle \xrightarrow{\text{noise}} \overbrace{\boxed{\mathcal{E}(|\overline{\psi}\rangle) \xrightarrow{\text{recovery}} \mathcal{R}\circ\mathcal{E}(|\overline{\psi}\rangle)}}^{\textbf{decoding}} \xrightarrow{\text{read off}} |\psi'\rangle$$

move outside
the code space

measure stabilizers to
discretize and diagnose errors

- **Decoding** = classical algorithm to find error correction from syndrome.

- **Threshold** p_{th} = max error rate tolerated by code (family).

Gottesman'96

WHY COLOR CODE?

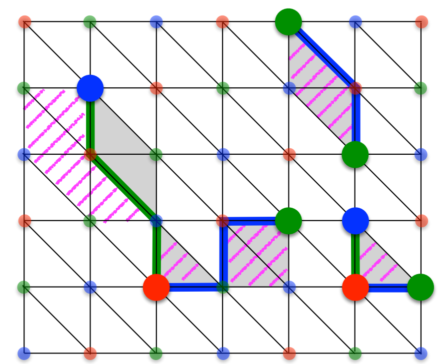
- Leading approach to scalable q. computing — **2D toric code** (surface).
- Difficulty: **fault-tolerant non-Clifford** gate (needed for universality).
- **Color code** as alternative to toric code
  - 😀 easier computation in 2D,
  - 😀😀 more qubit efficient,
  - 😀😀😀 code switching *[B15,BKS]* instead of magic state distillation.
- **Unfortunately**, color code
  - 🙁 seems difficult to decode,
  - 🙁🙁 seems to exhibit worse performance than toric code.

Bombin'15; Beverland et al. (in prep.)

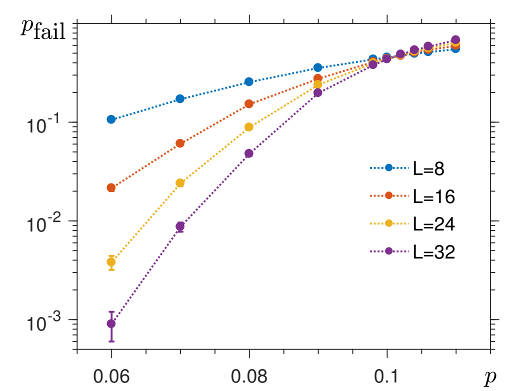
MAIN RESULTS & OUTLINE

Results: <u>efficient</u> decoders for color code in d ≥ 2 dim w/ <u>high thresholds</u>.

1. Toric & color codes in 2D.



2. **Restriction Decoder**: color code decoding by using toric code decoding.



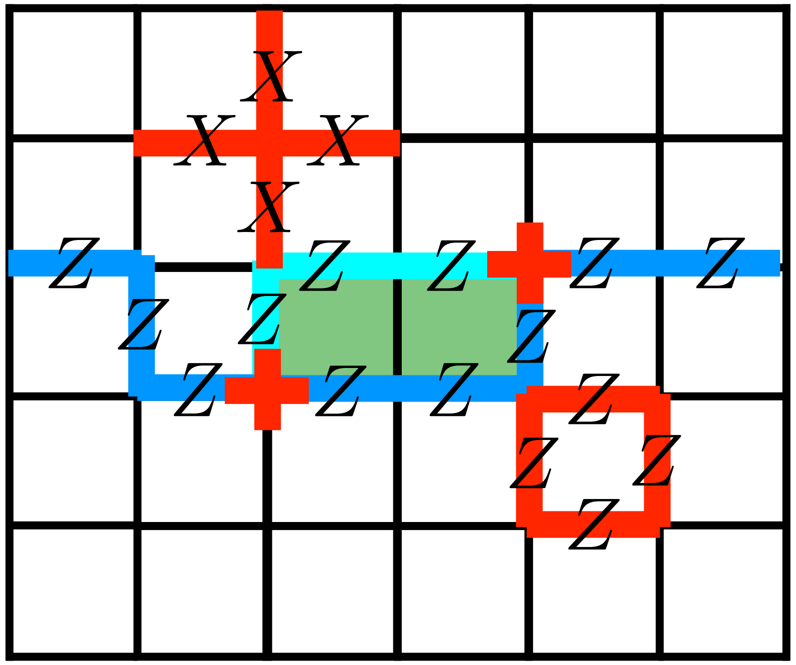
3. **High thresholds**: color code performance matches toric code.

$$
\begin{tikzcd}
C_{d-k-1}(\mathcal{L}) \arrow[r, "\partial_{d-k-1,d}"] \arrow[d, "\pi_C^{(2)}"] & C_d(\mathcal{L}) \arrow[r, "\partial_{d,k-1}"] \arrow[d, "\pi_C^{(1)}"] & C_{k-1}(\mathcal{L}) \arrow[d, "\pi_C^{(0)}"] \\
C_{k+1}(\mathcal{L}_C) \arrow[r, "\partial^C_{k+1}"] & C_k(\mathcal{L}_C) \arrow[r, "\partial^C_k"] & C_{k-1}(\mathcal{L}_C)
\end{tikzcd}
$$

4. Extra: going beyond 2D & neural network decoding.

2D TORIC CODE & DECODING

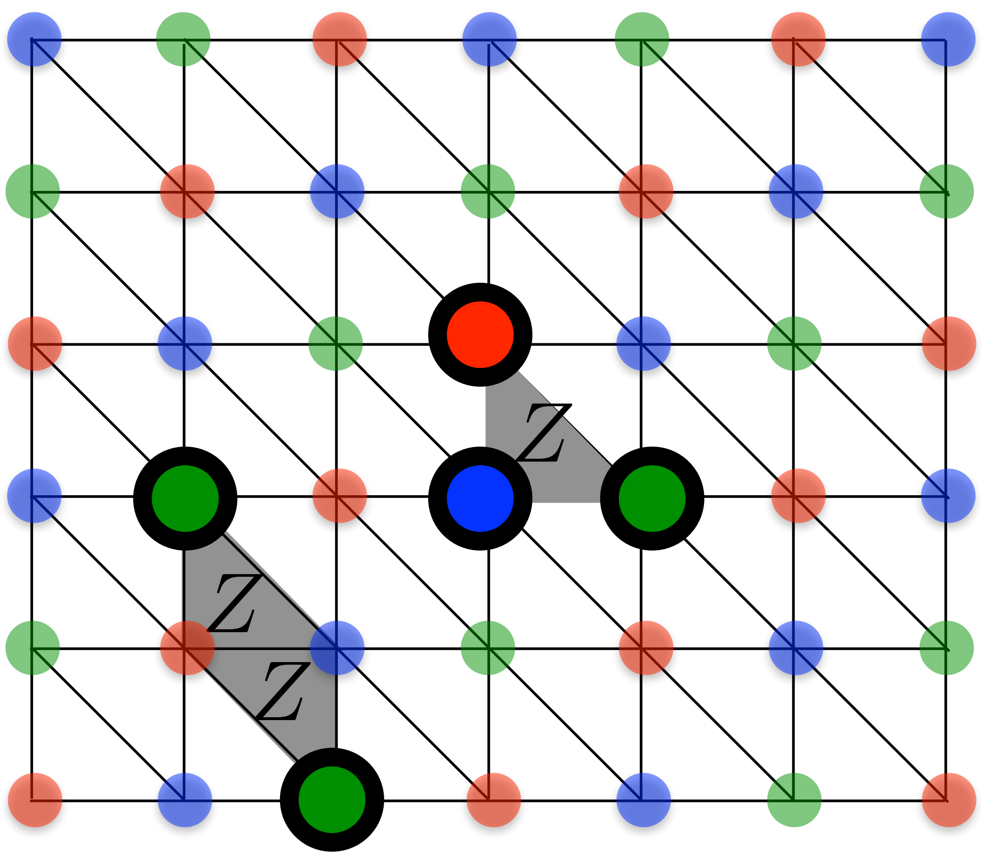
- **2D toric code** *[K97]*:
  — qubits = edges,
  — stabilizers = Z-faces & X-vertices,
  — Z-errors = edges,
  — excitations = vertices.

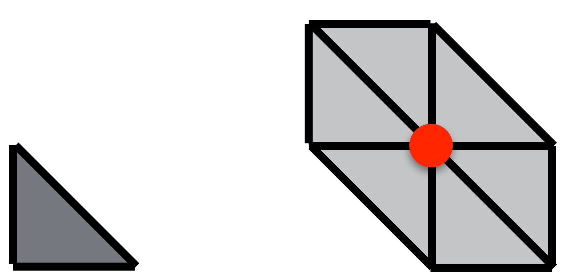


- **Decoding =** finding position of errors from violated stabilizers = pairing up excitations!
- **Successful decoding** iff error and correction differ by stabilizer.
- Toric code decoders *[DKLP02,H04,DP10,DN17,…]*: MWPM, RG, UF, …

Kitaev'97; Dennis et al.'02; Duclos-Cianci&Poulin'10; Harrington'04; Delfosse&Nickerson'17

2D COLOR CODE





qubit stabilizer

- **Lattice**: triangles, 3-colorable vertices.
- **2D color code** *[BM08]*:
  — qubits = triangles,
  — stabilizers = X- & Z-vertices.
- Color and toric codes related *[KYP15]…*
- …but decoding seems to be challenging as excitations created in pairs & triples!
- **Set-up:** error syndrome

  2D ← 0D

  local lift 1D TC decoder

Bombin&Martin-Delgado'06; Kubica et al.'15

COLOR CODE DECODER FROM TORIC CODE DECODER

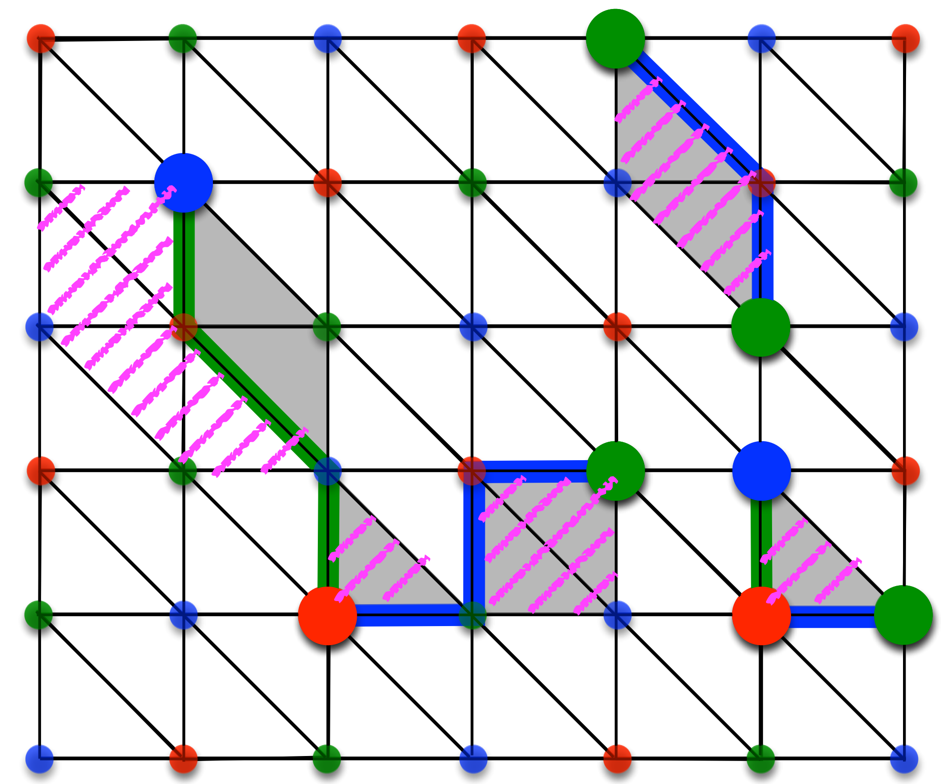
- **Restriction Decoder:** restricted lattice $\mathcal{L}_{RG}$, restricted syndrome s_{RG}.

    1. Use toric code decoder for $\mathcal{L}_{RG}$ and s_{RG}.

       Repeat for $\mathcal{L}_{RB}$ and s_{RB}.

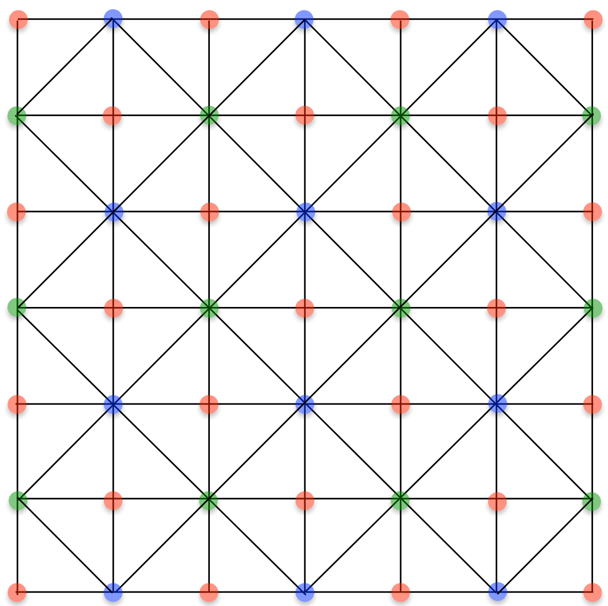
    2. For all R vertices v find some faces $f(v)$.

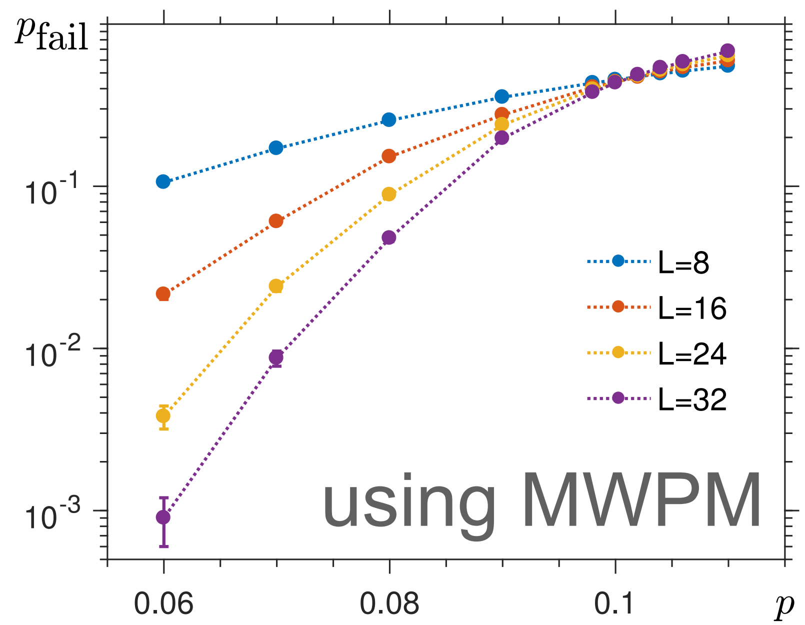
    3. Color code correction = $\sum f(v)$.

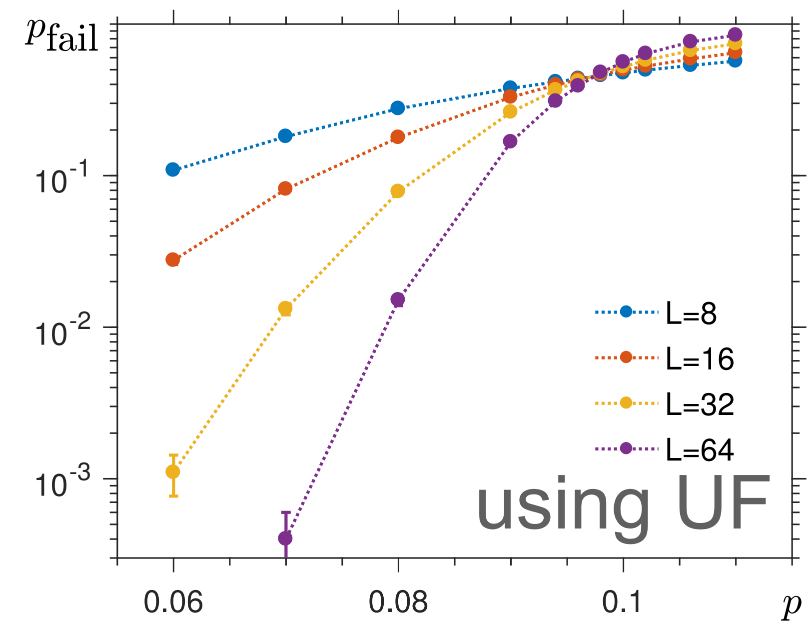


- Comments:
  — any toric code decoder can be used,
  — local lifting procedure to find $f(v)$,
  — similar for $d \geq 2$ dim.

NUMERICS







- Square-octagon lattice, phase-flip noise and ideal measurements.
- Color code threshold ~ **10.2%** on a par w/ toric code threshold ~ 10.3%.
- Previous highest thresholds 7.8% ~ 8.7% *[SR12,BDCP12,D14].*
- For **almost-linear time** decoder, use UF (instead of MWPM).

Sarvepalli&Raussendorf'12; Bombin et al.'12; Delfosse'14

GOING BEYOND 2D

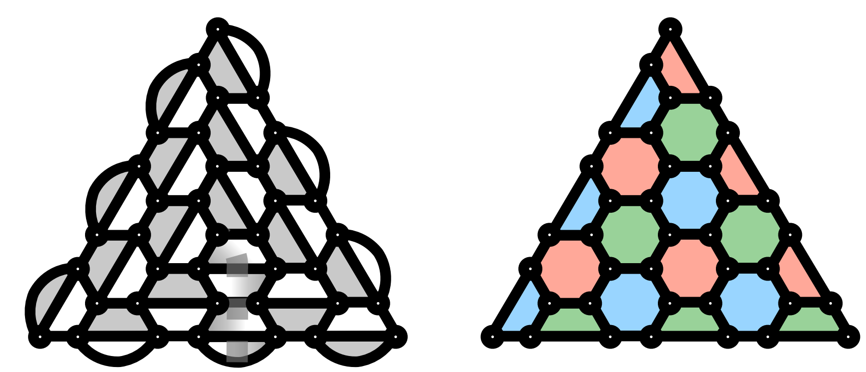
- **Restriction Decoder:** toric code decoding + local lifting procedure.
- **Theorem 1:** the k[th] homology groups of the color code lattice $\mathcal{L}$ and the restricted lattice $\mathcal{L}_C$ are isomorphic.
- **Lemma:** morphism between color and toric code chain complexes

$$\begin{tikzcd}
C_{d-k-1}(\mathcal{L}) \arrow[r, "\partial_{d-k-1,d}"] \arrow[d, "\pi_C^{(2)}"] & C_d(\mathcal{L}) \arrow[r, "\partial_{d,k-1}"] \arrow[d, "\pi_C^{(1)}"] & C_{k-1}(\mathcal{L}) \arrow[d, "\pi_C^{(0)}"] \\
C_{k+1}(\mathcal{L}_C) \arrow[r, "\partial_{k+1}^C"] & C_k(\mathcal{L}_C) \arrow[r, "\partial_k^C"] & C_{k-1}(\mathcal{L}_C)
\end{tikzcd}$$

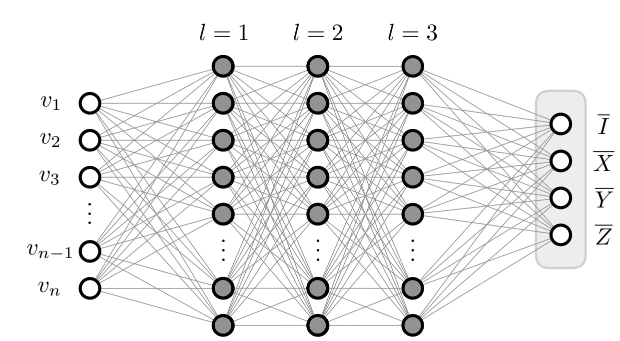
- **Theorem 2:** Restriction Decoder for the d-dim color code succeeds iff toric code decoding succeeds.

EXTRA: NEURAL-NETWORK DECODING [MKJ19]

- Decoders designed and analyzed for **simplistic noise models**. Dominant **sources of errors** not known/device-dependent.
- Generic stabilizer codes are hard to decode *[HL11,IP13]*.
- Desirable decoding methods should:
  — minimize human input,
  — be easily adaptable to different noise/code,
  — be efficient and have good performance.



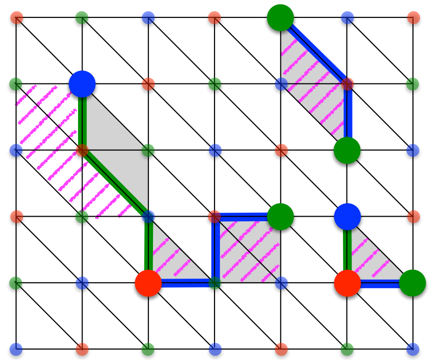
- **Idea:** decoding as a classification problem *[TM16]*.
- *[MKJ19]:* neural-network decoding is **versatile** and **outperforms** efficient decoders.



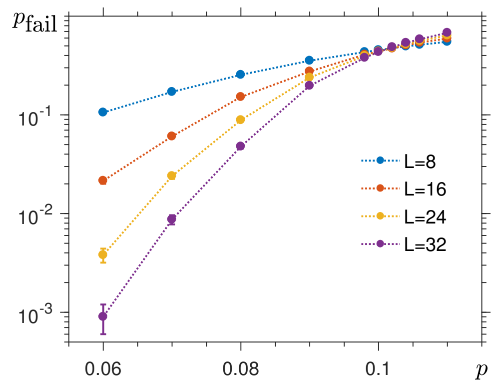
Maskara, K., Jochym-O'Connor'19; Hsieh&LeGall'11; Iyer&Poulin'13; Torlai&Melko'16

DISCUSSION

- **Restriction Decoder:** efficient decoder of color code in $d \geq 2$ dim by using toric code decoding.



- Restriction Decoder threshold ~ **10.2%**
  — better than all previous results for 2D color code,
  — on a par with 2D toric code ~ 10.3%.



- Things to explore: boundaries, circuit-level thresholds, …

- **Take-home**: q. computing based on 2D color code worth pursuing!

THANK YOU!
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