Information leakage from black holes with symmetry

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Outline of the talk

Outline

1. **Black hole information paradox**
2. **Review of the Hayden-Preskill toy model**
   - Q.I. approach to the paradox
3. **Summary of our results**
   - Information leakage from a rotating black hole
4. **Technical contribution**
   - Partial decoupling theorem
5. **Summary and Discussions**

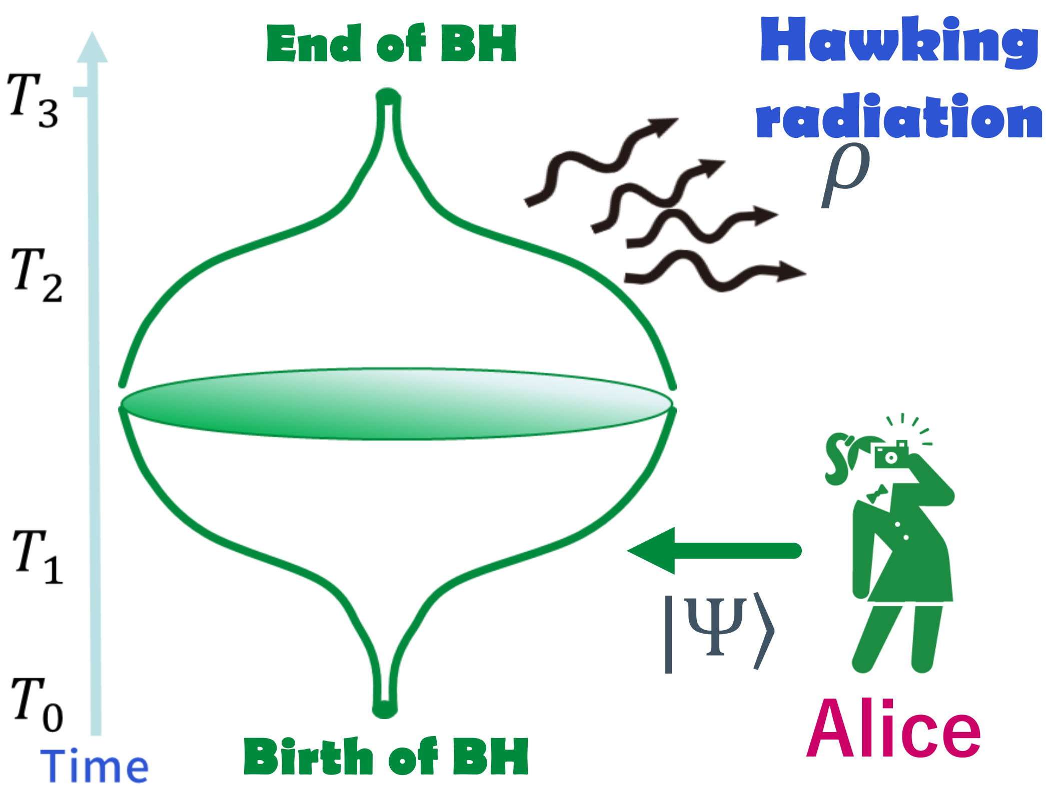
Black hole information paradox 1

Information paradox of black holes

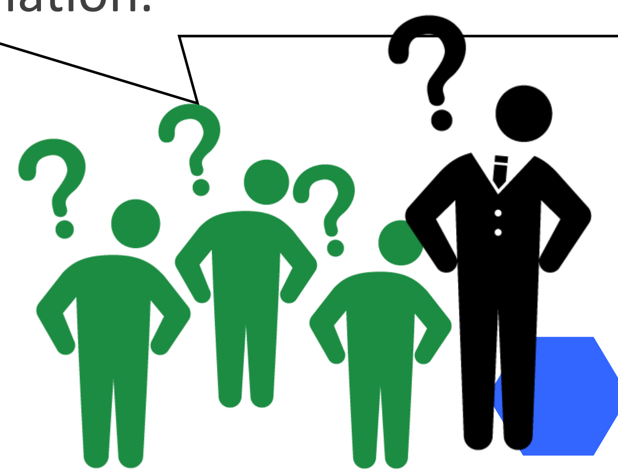
Does Hawking radiation carry away information from black holes?

Quantum theory → YES, since the dynamics is unitary & reversible.

Contradiction??



Hawking radiation is **thermal** and does not seem to carry any information.



Black hole information paradox 2

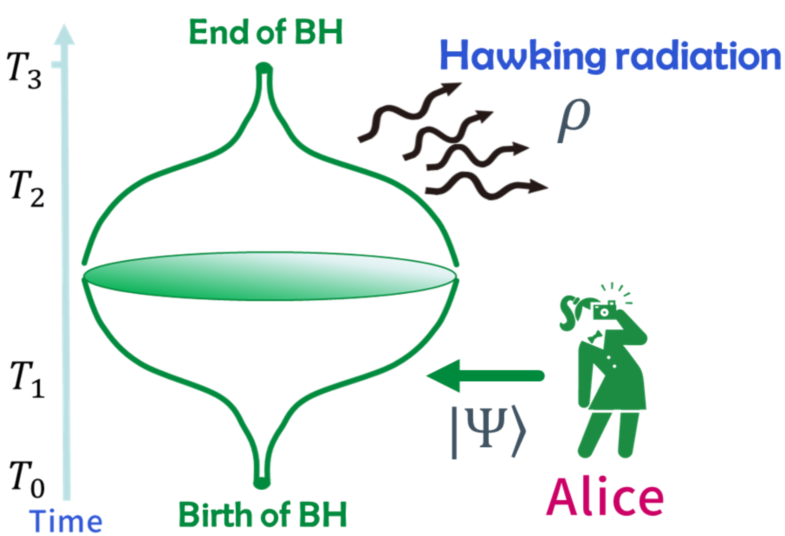
Information paradox of black holes

Does Hawking radiation carry away information from black holes?

Quantum theory → YES, since the dynamics is unitary & reversible.

- The holographic principle indicates that
  → the whole dynamics should be unitary.
  → the information is preserved = radiation should carry info.

Thermal?



How does radiation carry the info. away from black holes? **How quickly?**

Hayden-Preskill toy model [‘07]

Quantum information theoretic proposal towards the resolution.

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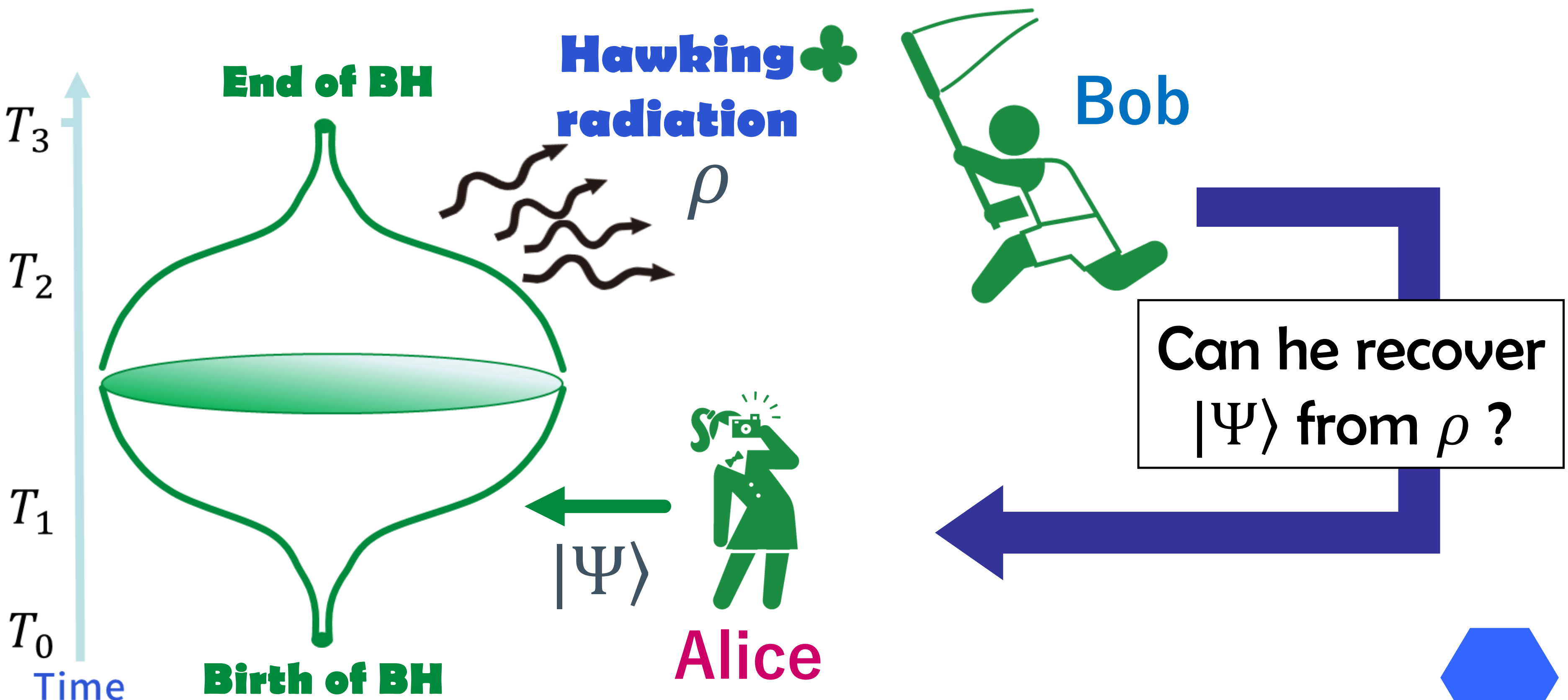
Hayden-Preskill toy model 1

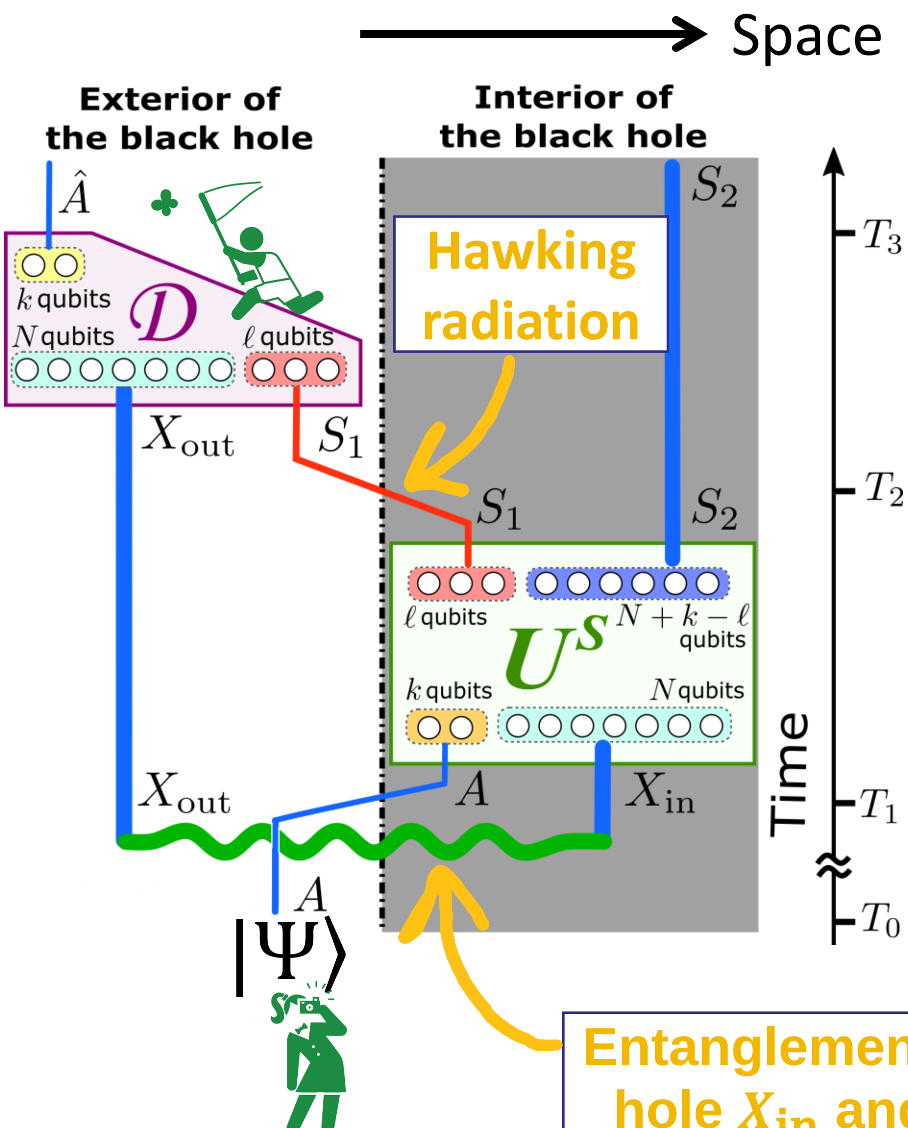
Hayden-Preskill toy model

Consider if $|\Psi\rangle$ is recoverable from the radiation ρ.

(Recovery $\Leftrightarrow$ the info. has been already leaked out)



Hayden-Preskill toy model 2



Entanglement between the initial black hole X_{in} and the early radiation X_{out}

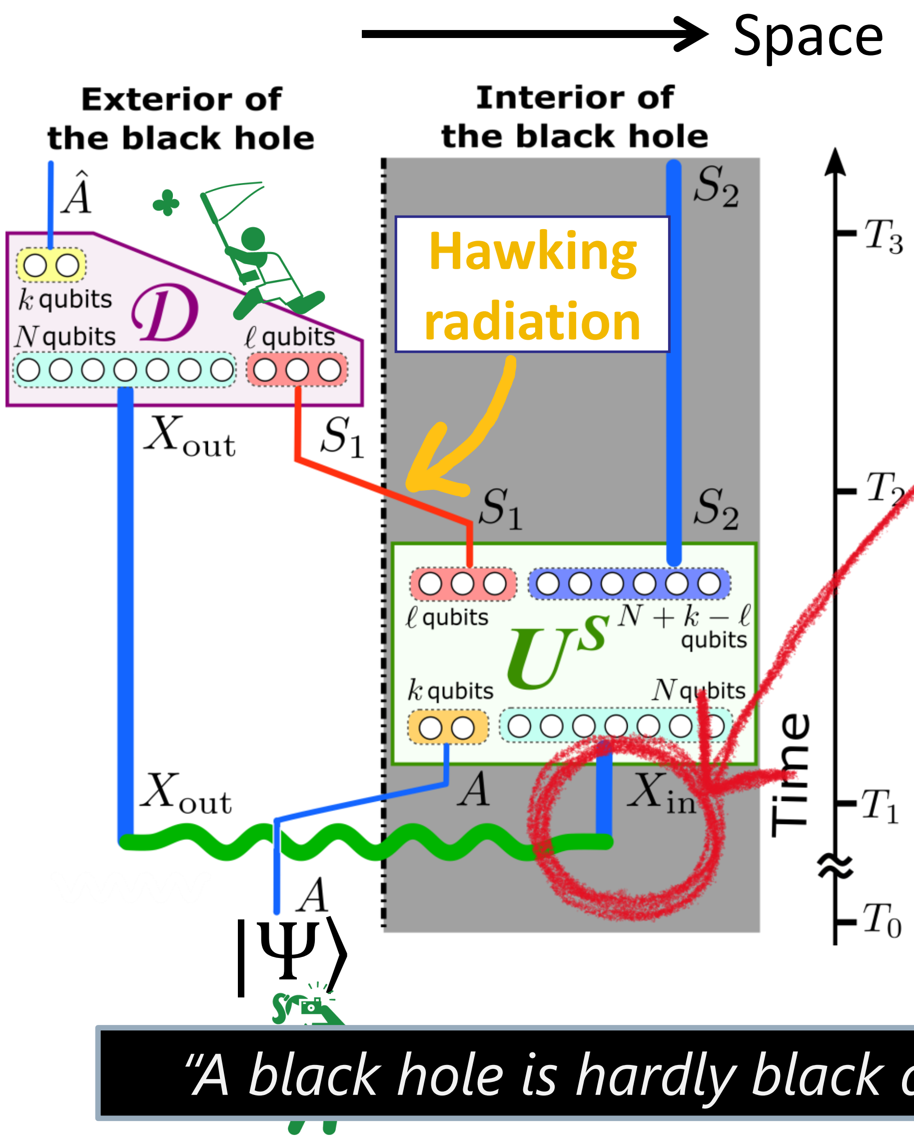
Setting:

1. Alice throws her quantum info. A (k qubits) into a black hole X_{in} (N qubits).
2. The whole black hole $S = AX_{\mathrm{in}}$ undergoes time evolution U^S.
3. A part S_1 (ℓ qubits) of S is evaporated.
4. Bob applies a recovery operation to S_1 and early radiation X_{out}.

Assumption:

- U^S is unitary and is sufficiently **Haar scrambling** (Haar random).

Hayden-Preskill toy model 3



Error Δ in recovering Alice's info.

$$\Delta \leq 2^{(N - H_{\min}(\xi))/2 + k - \ell}$$

- k: # of Alice's qubits
- ℓ: # of Hawking radiation
- N: Size of the initial BH

[HP '07] [Dupuis et al '14]

- For **young** BHs (no early radiation), $\Delta \leq 2^{k+N/2-\ell}$.
- For **old** BHs (early radiation is maximally entangled), $\Delta \leq 2^{k-\ell}$.

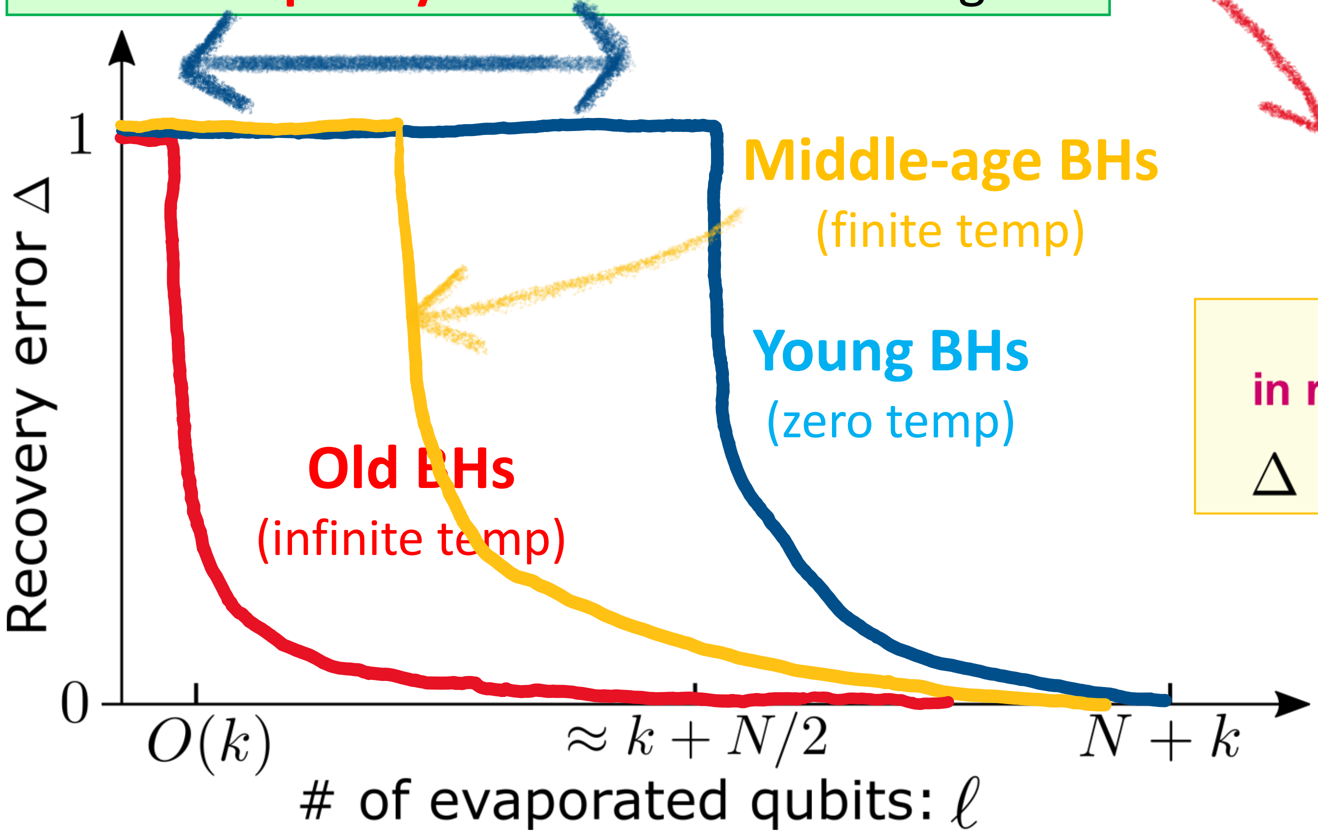
No matter how large the BH is, Alice's info leaks out quickly.

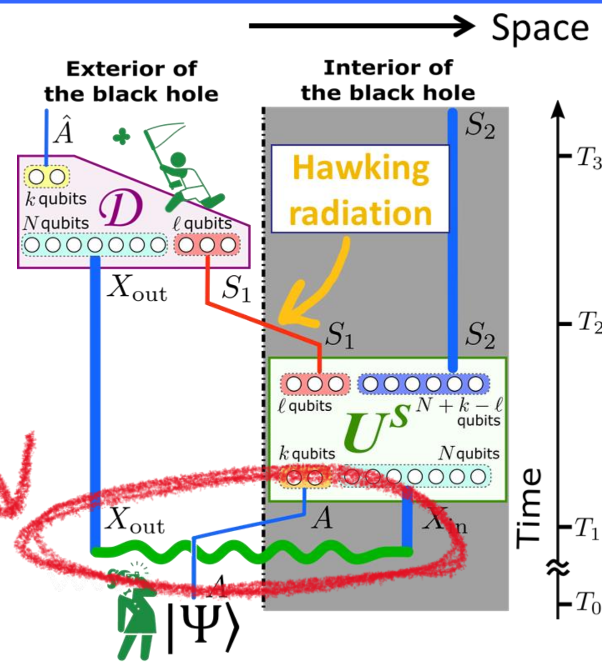
"A black hole is hardly black at all. It is an information mirror"

Hayden-Preskill toy model 4

Information leakage from black holes

More entanglement b/t X_{in} and X_{out},
→ **more quickly** the BH starts releasing info.





Error Δ in recovering Alice's info. is

$$\Delta \leq 2^{(N - H_{\min}(\xi))/2 + k - \ell}$$

Hayden-Preskill toy model 5

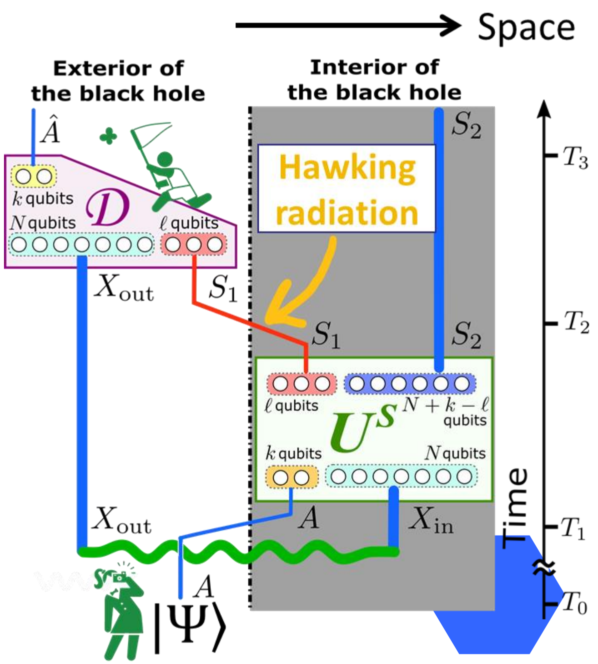
"A black hole is hardly black at all. It is an information mirror"

Far reaching consequences (incomprehensive)**:**

- Scrambling
  [Sekino & Susskind '08] [Lashkari et al '13] [Shenker & Stanford '15]…
- Out-of-Time-Ordered-Correlators (OTOCs)
  [Roberts & Stanford '15] [Hosur et al '16] …
- Firewalls
  [AMPS '13] [Yoshida '19]…
- Holographic principles…

To quantum information:

- Decoding algorithm of random encoder
  [Yoshida & Kitaev '17] [Landsman et al '19]
- Information theory is useful also in physics?



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Our motivation – symmetry of BHs -

What happens if we take the symmetry of BHs into account?

Immediate implication:

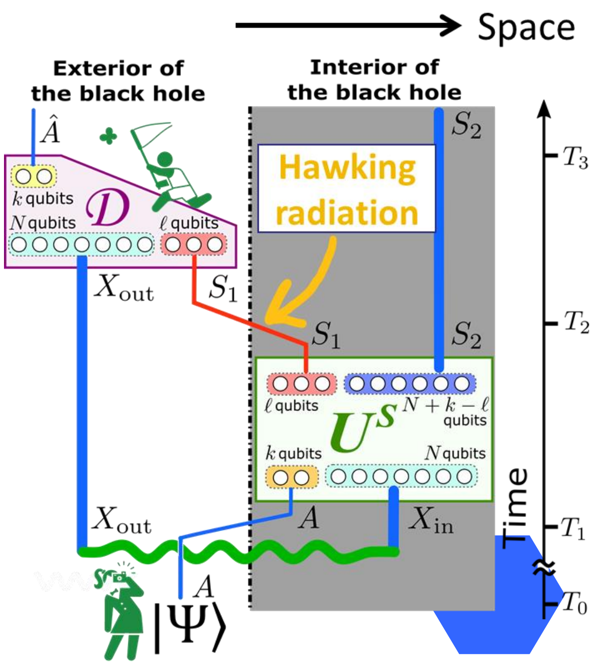
- ∃conservation quantities
  → U^S CANNOT be fully scrambling.

How does this affect the information leakage?

No exact symmetry in Q. gravity

- Harlow & Oguri '19, etc…
- ∃approximate symmetry to be consistent with classical BHs
- In early time, symmetry restricts U^S.

We start with an exact symmetry.



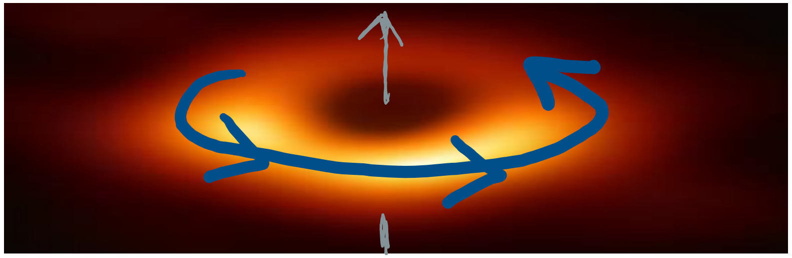
Information leakage from Kerr black holes 1

What happens

if we take the symmetry of BHs into account?

- **Kerr black holes** = BHs with an axial symmetry

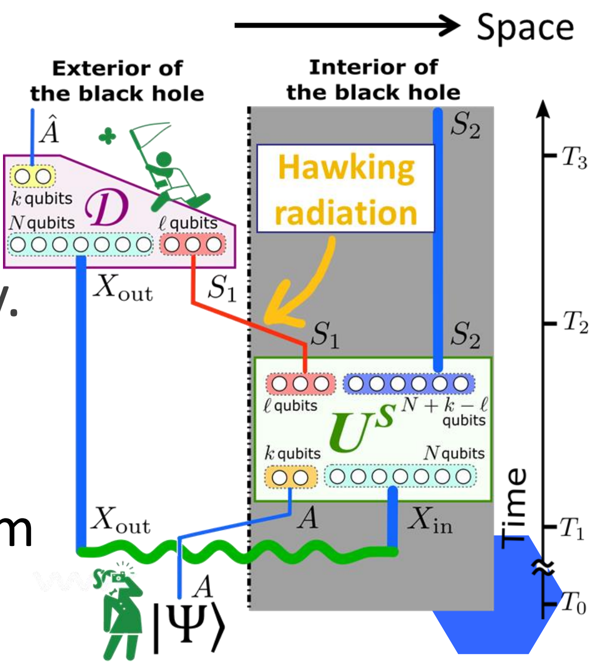
  → Z-component of angular momentum is conserved.



- The U^S should commute with the symmetry.

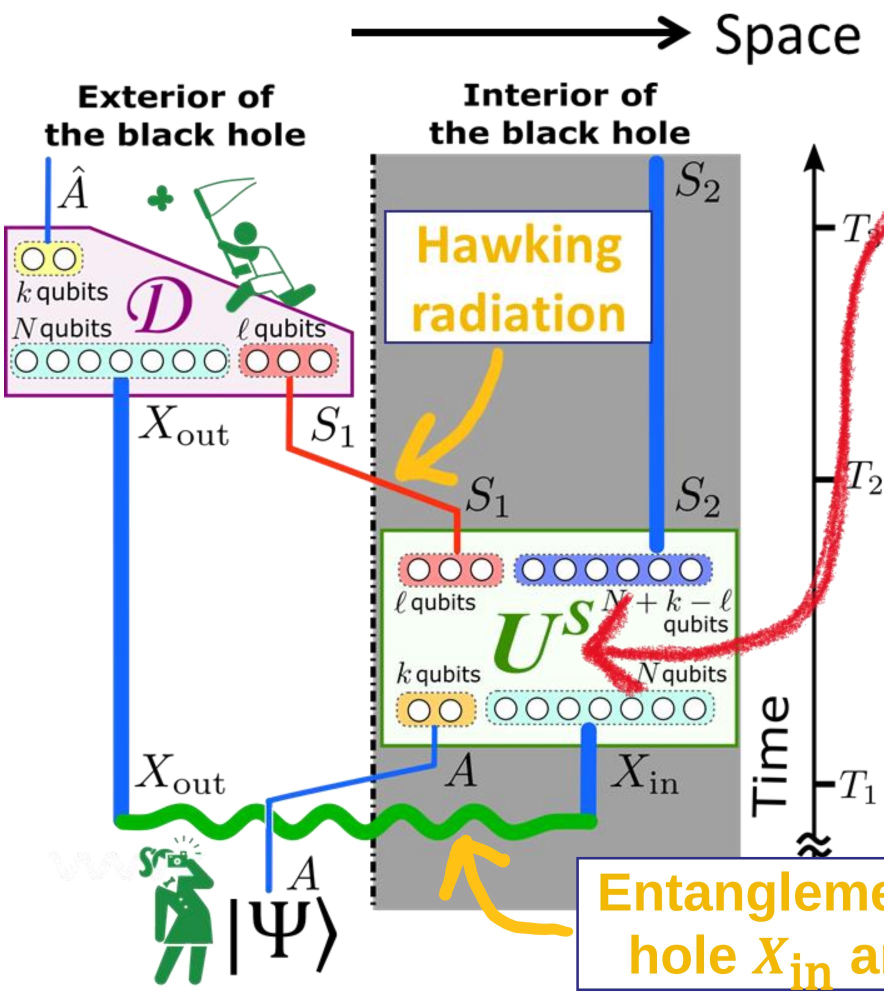
$$U^S = \bigoplus_{m=0}^{N+k} U_m^S$$

  ✓ m is the Z-component of angular momentum



Information leakage from Kerr black holes 2

Information leakage from Kerr black holes



$$U^S = \bigoplus_{m=0}^{N+k} U_m^S$$

Assumption:

U_m^S is Haar scrambling in each subspace

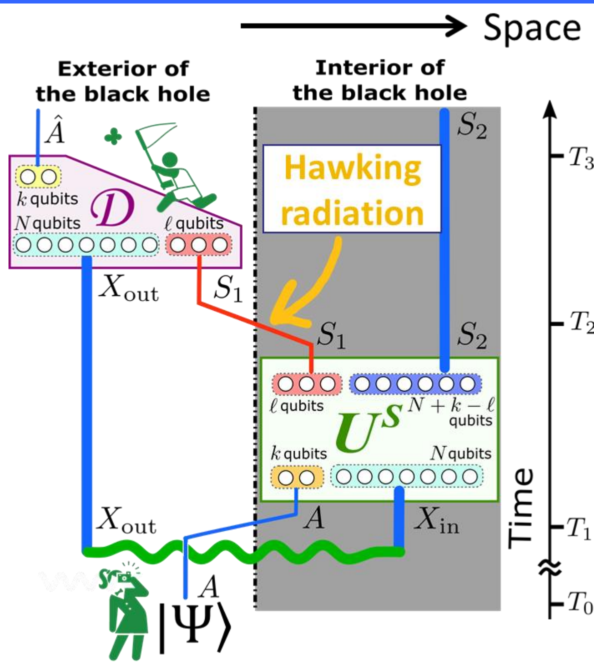
U^S: partial scrambling

Interplay b/t **entanglement** and **asymmetry** **of the initial black hole**

Entanglement between the initial black hole X_{in} and the early radiation X_{out}

Summary of our result 1

Information leakage from Kerr black holes



HP result without any symmetry

- ✓ Entanglement of the initial BH

$$\Delta \leq 2^{(N-H_{\min}(\xi))/2+k-\ell}$$

When BH has an axial symmetry...

- ✓ Entanglement of the initial BH, and its relation to symmetry
- ✓ Asymmetry of the state of the initial black hole

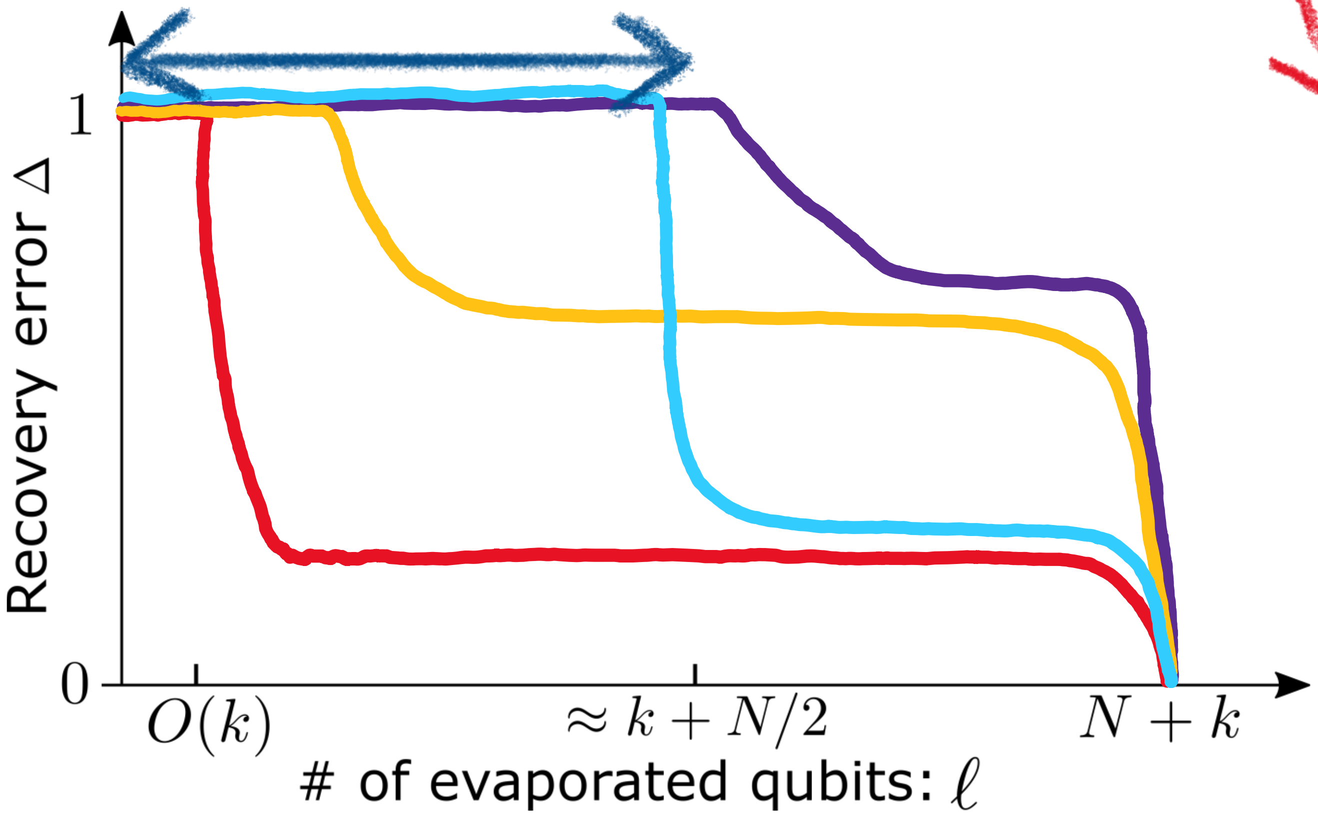
For symmetry-inv. Info of Alice: $\Delta_{inv} \leq 2^{-\frac{1}{2}H_{min}(SS|ER)_{\tau*\rho}}$

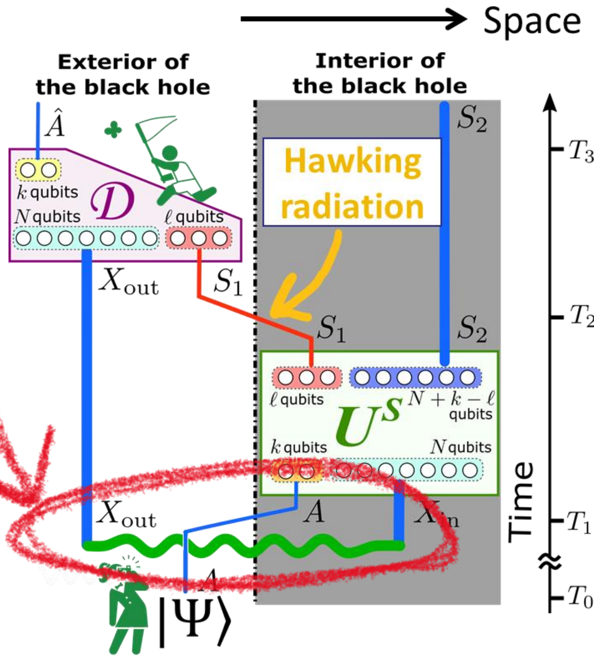
For the whole Info of Alice: $\Delta \leq 2^{-\frac{1}{2}H_{min}(SS|ER)_{\tau*\rho}} + \eta(\xi)$

Summary of our result 2

Information leakage from Kerr black holes

More entanglement, un-affected by symmetry, b/t X_{in} and X_{out}
→ **more quickly** the BH starts releasing info.

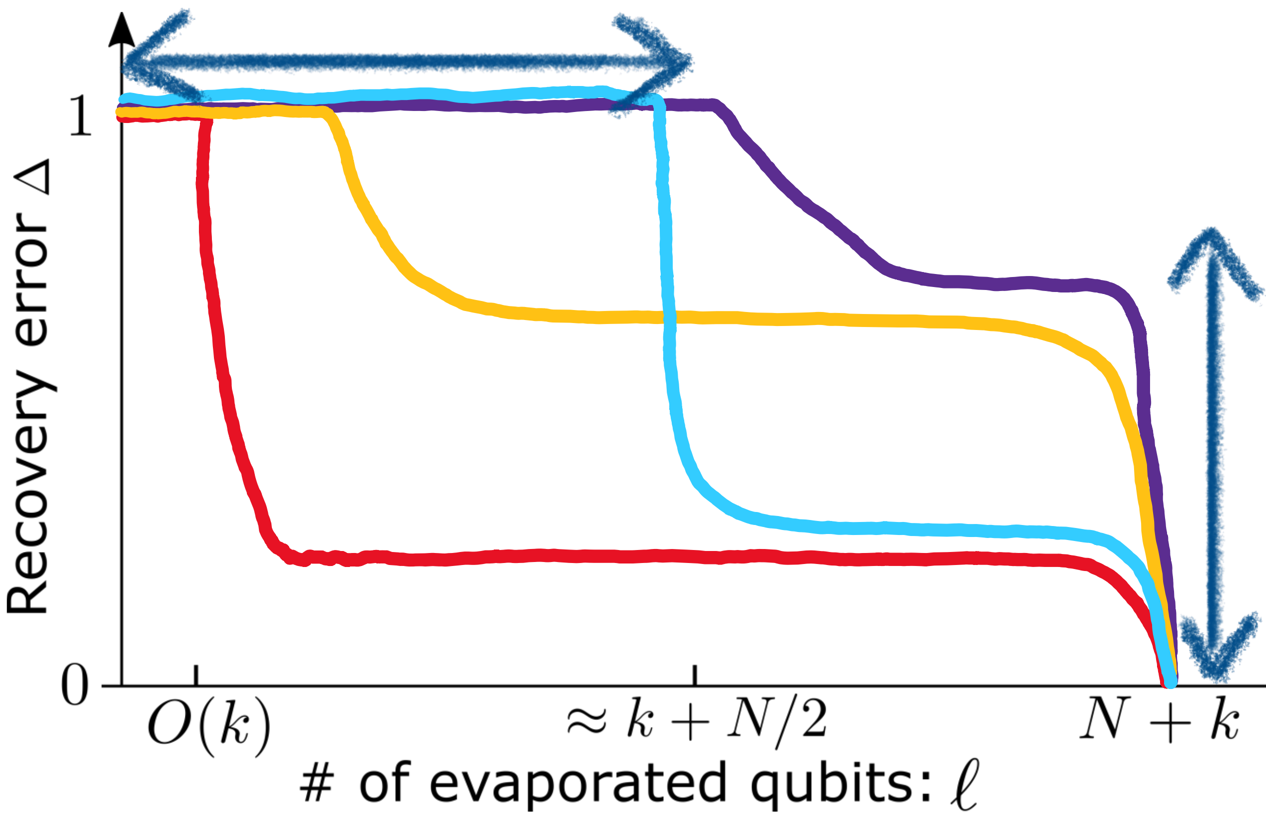


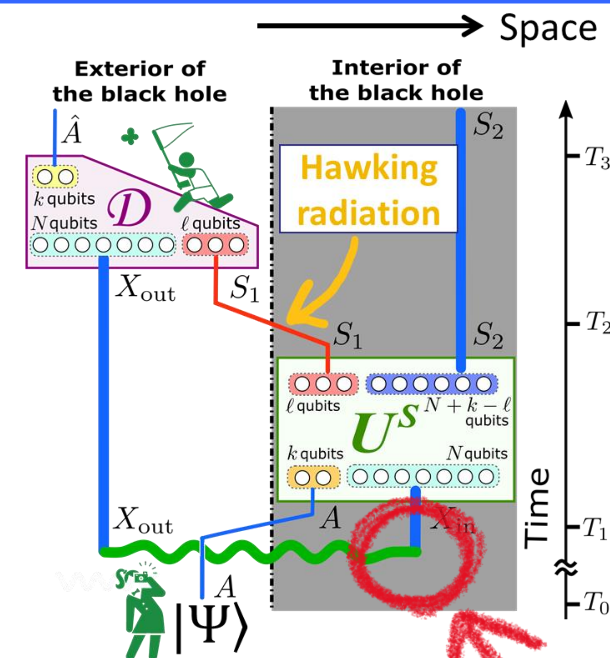


Summary of our result 2

Information leakage from Kerr black holes

More entanglement, un-affected by symmetry,
b/t $X_{\rm in}$ and $X_{\rm out}$
→ **more quickly** the BH starts releasing info.





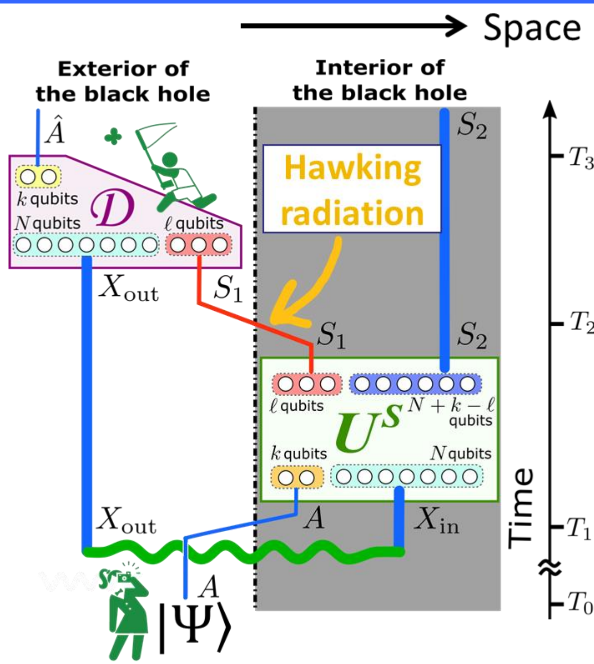
∃residual info.

(symmetry-variant info.)

More asymmetry in $X_{\rm in}$
→ **Less** residual info.
(numerical observation)

Summary of our result 3

Information leakage from Kerr black holes



- When the initial BH X_{in} is **maximally entangled** with the early radiation X_{out} (infinite temp.),

The recovery error: $\Delta \lesssim 2^{k-\ell} + O(N^{-0.5})$

- k: # of Alice's qubits
- ℓ: # of Hawking radiation
- N: Size of the initial BH

(If $\nexists$symmetry, $\Delta \leq 2^{k-\ell}$ [HP07])

- The info leaks out extremely quickly iff the initial Kerr BH is sufficiently large ($N \gg O(2^k)$).

A Kerr black hole is an information mirror iff it is sufficiently large.

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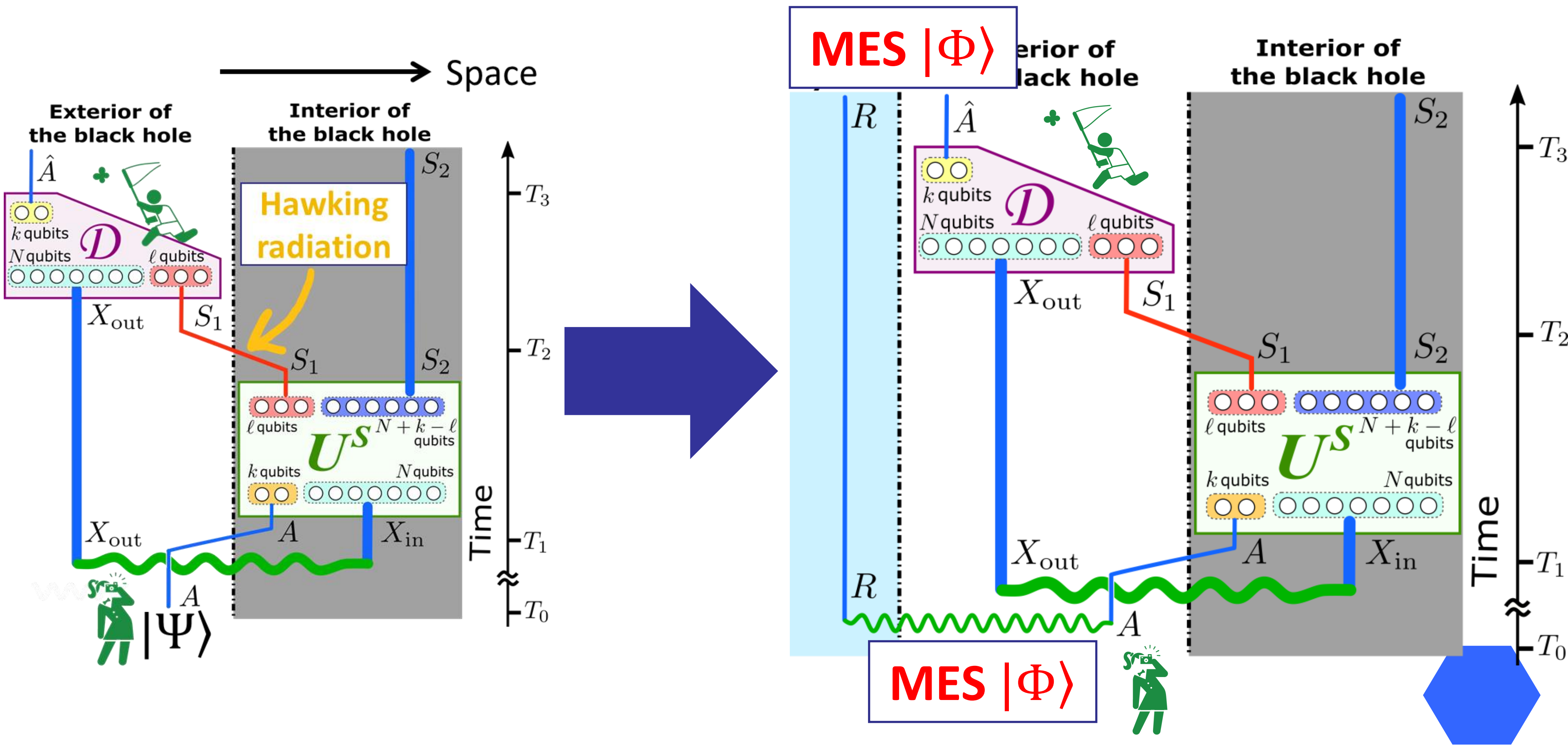
Symmetry-invariant and -variant info. 1

- Information of A is stored in the correlation b/t the reference R.
  - ✓ Under certain assumptions, MES $|\Phi\rangle^{AR}$ is sufficient.
- The information in $|\Phi\rangle^{AR}$ can be classified in terms of symmetry.



Symmetry-invariant and -variant info. 2

- The information in $|\Phi\rangle^{AR}$ can be classified in terms of symmetry
  - ✓ Hilbert space $\mathcal{H}^A = \bigoplus_{\kappa=0}^{k} \mathcal{H}^A_\kappa$ (Decomp. by the axial symmetry)
  - ✓ P^A_κ: projection onto $\mathcal{H}^A_\kappa$

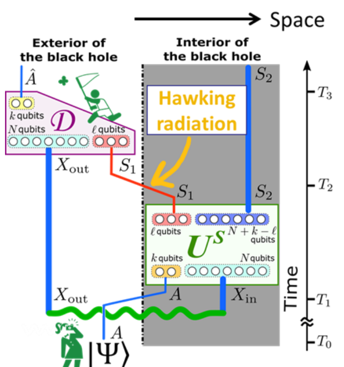
Invariant under rotation

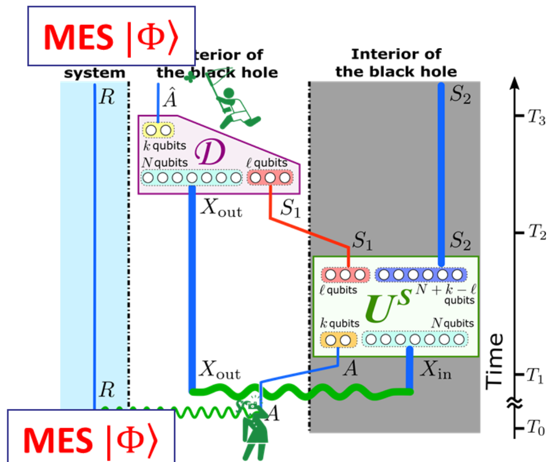
$$\Phi^{AR} = \Phi^{AR}_{\mathrm{diag}} + \Phi^{AR}_{\mathrm{off}}$$

where $\Phi^{AR}_{\mathrm{diag}} = \sum_{\kappa=0}^{k} \Phi^{AR}_{\kappa\kappa}$, and $\Phi^{AR}_{\kappa\kappa'} = (P^A_\kappa \otimes I^R)\Phi^{AR}(P^A_{\kappa'} \otimes I^R)$.

Information in $\Phi^{AR}_{\mathrm{diag}}$
= **symmetry-invariant** info.
e.g.) conserved quantity

Remaining
= **symmetry-variant** info.
e.g.) coherence b/t different conserved quantities

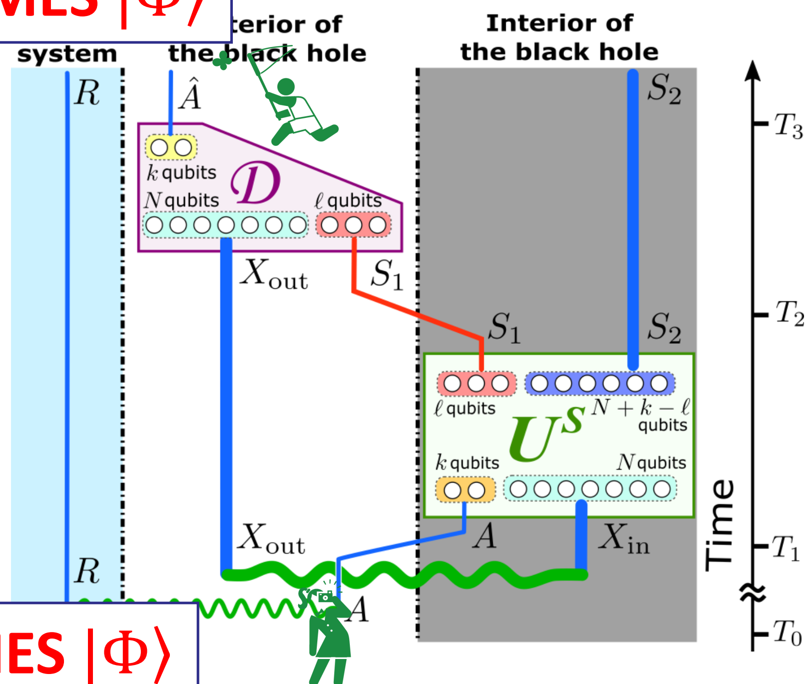




Symmetry-invariant and -variant info. 3

How quickly symmetry-invariant/-variant info. of Alice leaks out from a Kerr BH?

MES $|\Phi\rangle$



MES $|\Phi\rangle$

symmetry-invariant part

+ symmetry-variant part

Decoupling approach

The most elegant approach to quantum communicational tasks

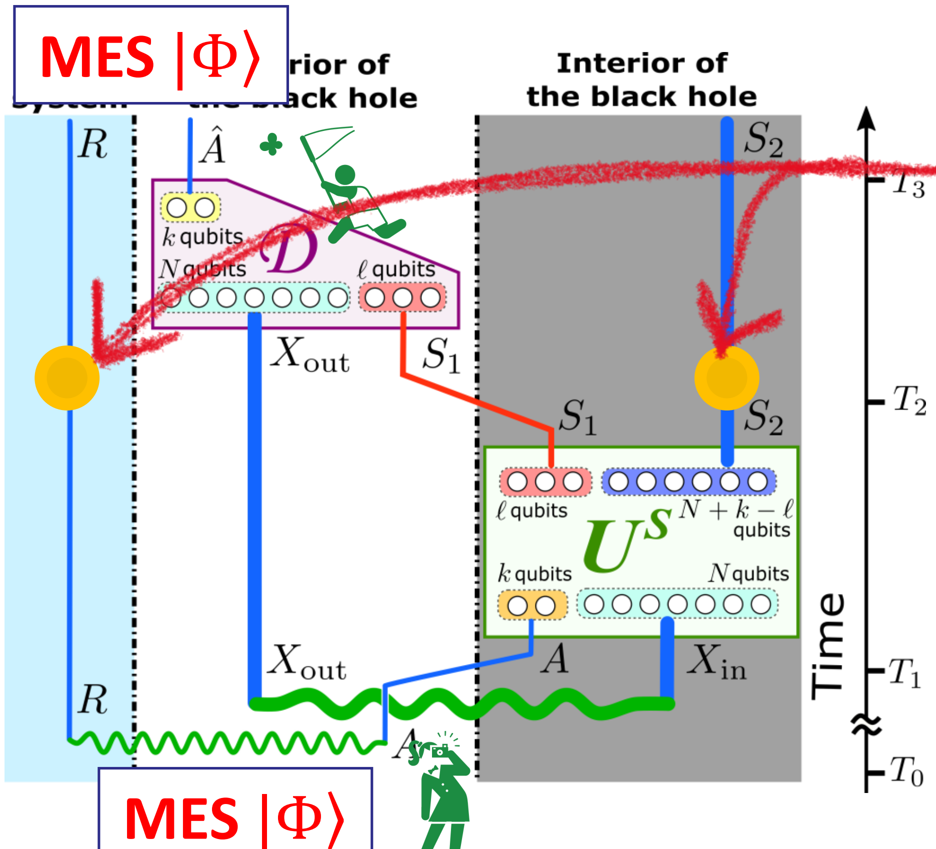
[Horodecki,Oppenheim&Winter '05]

[Abeyesinghe,Devetak,Hayden& Winter '09]

Decoupling approach 1

HP approach in detail:

1. Assume that U^S is Haar scrambling.
2. Use the one-shot decoupling.



"Decoupling"

$$\Psi_U^{RS_2} \approx \frac{I^R}{d_R} \otimes \sigma^{S_2}$$

σ: any state

$\exists$a good decoder $\mathcal{D}$ for Bob to recover $|\Phi\rangle$

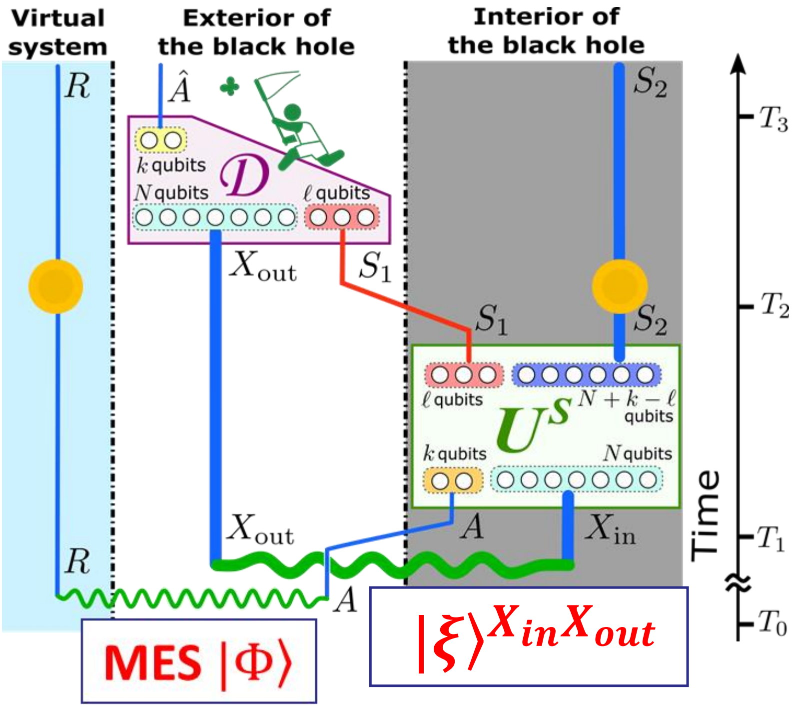
Decoupling approach

Decoupling approach 2

HP approach in detail:

1. Assume that U^S is Haar scrambling.
2. Use the one-shot decoupling.

$$\Psi_U^{RS_2} = \mathrm{Tr}_{S_1}\left[U^S(\Phi^{AR} \otimes \xi^{X_{\mathrm{in}}})U^{S\dagger}\right]$$



Direct consequence of **decoupling theorem** [Dupuis et al '14]

For Haar scrambling U^S,

$$\Psi_U^{RS_2} \approx \frac{I^R}{d_R} \otimes \frac{I^{S_2}}{d_{S_2}}$$

$$2^{(N - H_{\min}(\xi^{X_{\mathrm{in}}}))/2 + k - \ell}$$

with high probability.

Decoupling approach 3

HP approach in detail:

1. Assume that U^S is Haar scrambling.
2. Use the one-shot decoupling.

Decoupling theorem (simplified) [Dupuis et.al. 2014]

For a state ρ^{SR}, a CPTP map $\mathcal{T}^{S\to E}$, and a Haar scrambling U^S,

$$\|\mathcal{T}^{S\to E}(U^S \rho^{SR} U^{S\dagger}) - \tau^E \otimes \rho^R\|_1 \leq 2^{-\frac{1}{2}H_{\min}(S'S|ER)_{\tau\otimes\rho}}$$

with high probability, where τ^{SE}: state representation of $\mathcal{T}^{S\to E}$ and $H_{\min}(S'S|ER)_{\tau\otimes\rho}$ is the conditional min-entropy.

Our approach to the Kerr BH:

1. The U^S is a partial scrambling due to the symmetry.
2. Prove PARTIAL decoupling and use it.

$$U^S = \bigoplus_{m=0}^{N+\ell} U_m^S$$

Partial decoupling approach 1

Partial decoupling (simplified) [E. Wakakuwa and YN 2019]

For a state ρ^{SR}, a CPTP map $\mathcal{T}^{S\to E}$, and a partial scrambling $U^S = \bigoplus U_m^S$,

$$\|\mathcal{T}^{S\to E}\left((\oplus_m U_m^S)\rho^{SR}(\oplus_m U_m^S)^\dagger\right) - \sum_m \tau_{mm}^E \otimes \rho_{mm}^R\|_1 \leq 2^{-\frac{1}{2}H_{\min}(S'S|ER)_{\tau*\rho}}$$

with high probability.

Separable state

Katrhi-Rao product $*$

("block-wise" tensor product)

- One-shot & converse, and a generalization are shown.
- Useful in QIT
  - ✓ Interpolating decoupling thm and dequantization thm [Dupuis '12]
  - ✓ Classical&quantum hybrid channel coding [Devetak and Shor '03]
  - ✓ Relative thermalization, area law, with symmetry?

Decoupling theorem (simplified) [Dupuis et.al. 2014]

$$\|\mathcal{T}^{S\to E}(U^S\rho^{SR}U^{S\dagger}) - \tau^E \otimes \rho^R\|_1 \leq 2^{-\frac{1}{2}H_{\min}(S'S|ER)_{\tau\otimes\rho}}$$

Product state

Tensor product

Partial decoupling approach 2

Partial decoupling (simplified) [E. Wakakuwa and YN 2019]

For a state ρ^{SR}, a CPTP map $\mathcal{T}^{S\to E}$, and a partial scrambling $U^S = \bigoplus U_m^S$,

$$\left\|\mathcal{T}^{S\to E}\left((\oplus_m U_m^S)\rho^{SR}(\oplus_m U_m^S)^\dagger\right) - \sum_m \tau_{mm}^E \otimes \rho_{mm}^R\right\|_1 \leq 2^{-\frac{1}{2}H_{\min}(S'S|ER)_{\tau*\rho}}$$

with high probability.

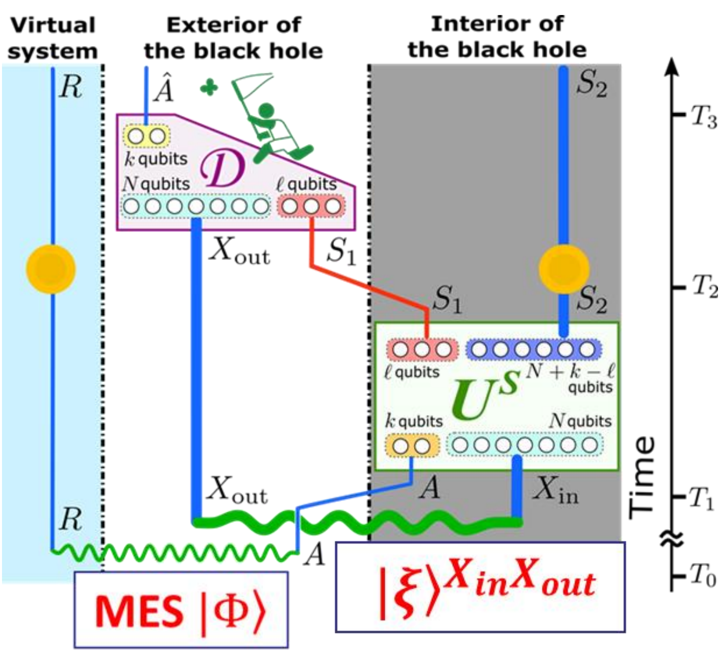
Separable state

→ Leakage of sym-inv info.

Katrhi-Rao product $*$

("block-wise" tensor product)

→ speed of leakage



Error in recovering symmetry-inv. Info of Alice:

$$\Delta_{inv} \leq 2^{-\frac{1}{2}H_{min}(SS|ER)_{\tau*\rho}}$$

Partial decoupling approach 3

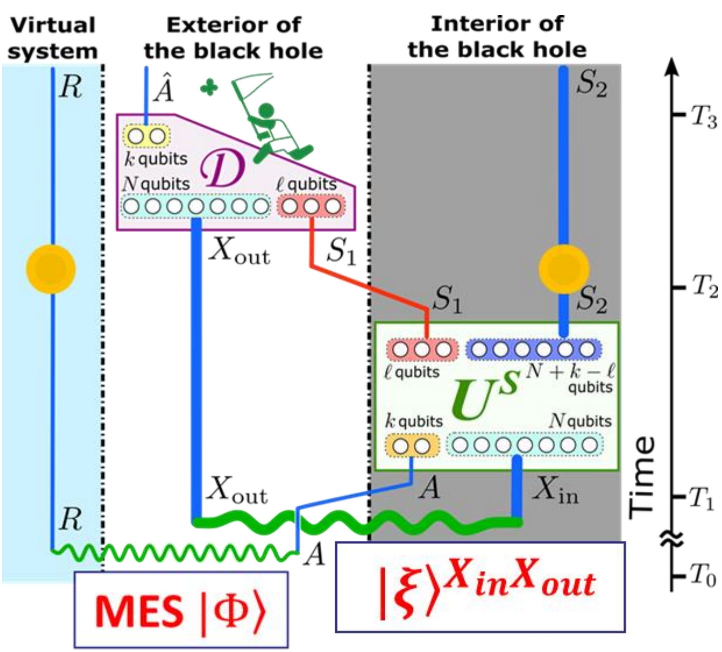
Partial decoupling (simplified) [E. Wakakuwa and YN 2019]

For a state ρ^{SR}, a CPTP map $\mathcal{T}^{S\to E}$, and a partial scrambling $U^S = \bigoplus U_m^S$,

$$\left\|\mathcal{T}^{S\to E}\left((\oplus_m U_m^S)\rho^{SR}(\oplus_m U_m^S)^\dagger\right) - \sum_m \tau_{mm}^E \otimes \rho_{mm}^R\right\|_1 \le 2^{-\frac{1}{2}H_{\min}(S'S|ER)_{\tau*\rho}}$$

with high probability.

What about the whole information, including **symmetry-variant** one?



From the difference b/t partial decoupling and full decoupling…

Error in recovering the whole Info of Alice:

$$\Delta \le 2^{-\frac{1}{2}H_{min}(SS|ER)_{\tau*\rho}} + \eta(\xi)$$

ξ: state of the initial BH

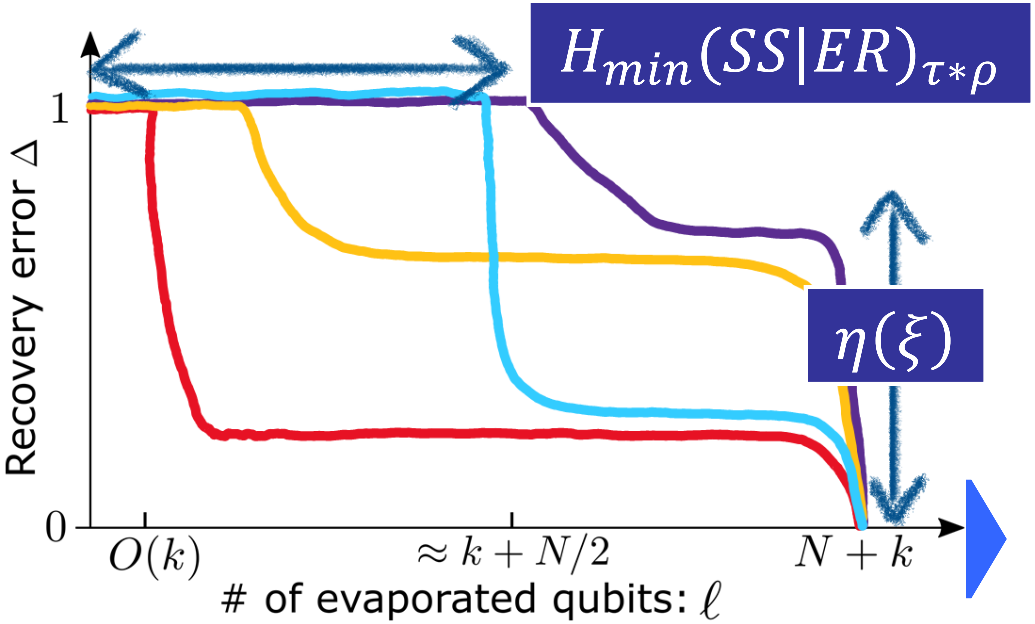
Information leakage from Kerr BHs 1

For **symmetry-inv.** Info of Alice: $\Delta_{inv} \leq 2^{-\frac{1}{2}H_{min}(SS|ER)_{\tau*\rho}}$

For **the whole** Info of Alice: $\Delta \leq 2^{-\frac{1}{2}H_{min}(SS|ER)_{\tau*\rho}} + \eta(\xi)$

- $H_{min}(SS|ER)_{\tau*\rho}$
  - ✓ $\tau * \rho$ is constructed from
    - Alice's source A
    - Initial black hole ξ
    - Symmetry
    - The evaporation process.
  - ✓ generally increases when ℓ increases.

- $\eta(\xi)$ (ξ is a state of the initial BH.)
  - ✓ Fluctuation of S_z.
  - ✓ depends on ℓ only weakly.



Information leakage from Kerr BHs 2

For **symmetry-inv.** Info of Alice: $\Delta_{inv} \leq 2^{-\frac{1}{2}H_{min}(SS|ER)_{\tau * \rho}}$

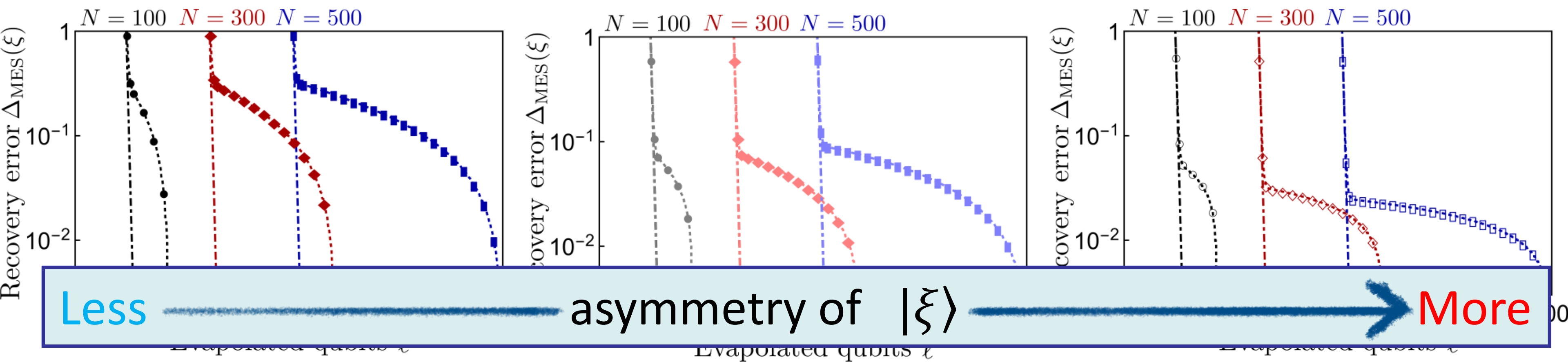
For **the whole** Info of Alice: $\Delta \leq 2^{-\frac{1}{2}H_{min}(SS|ER)_{\tau * \rho}} + \eta(\xi)$

- Pure initial BH ($\xi^{X_{in}}$= pure) for $\langle S_Z \rangle = 0$.

Case 1: Support ony on one subspace.

Case 2: Supp. is $O(\sqrt{N})$ spread.

Case 3: Support is $O(N)$ spread.



$$\Delta_{inv} \lesssim 2^{k+N/2-\ell} \text{ (dashed-dotted lines)}$$

$$\Delta \lesssim 2^{k+N/2-\ell} + O(1)$$

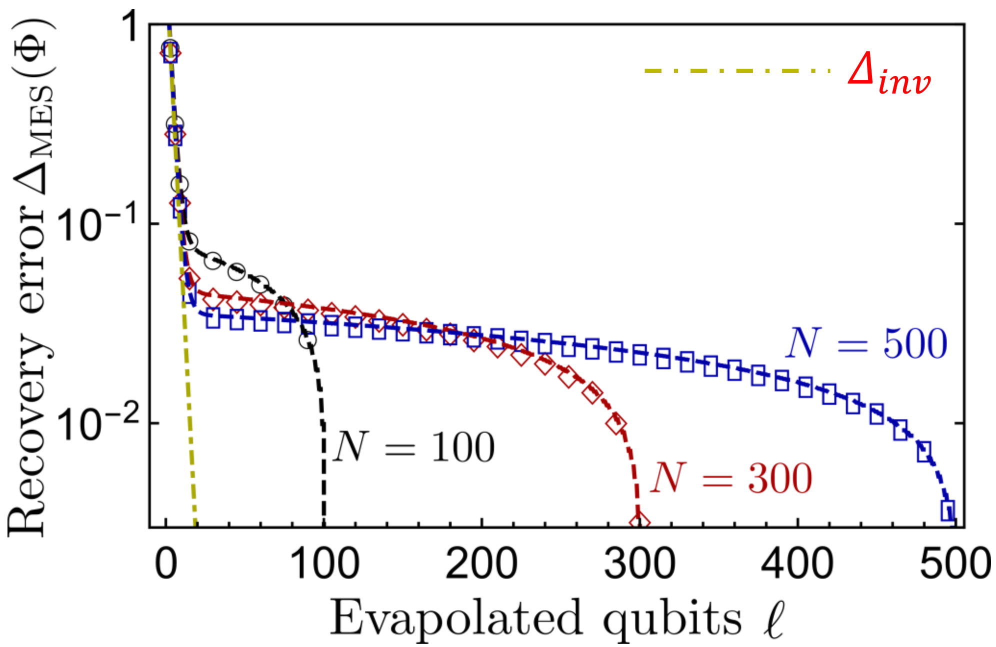
$$\Delta \lesssim 2^{k+N/2-\ell} + O(N^{-0.5})$$

Information leakage from Kerr BHs 3

For **symmetry-inv.** Info of Alice: $\Delta_{inv} \leq 2^{-\frac{1}{2}H_{min}(SS|ER)_{\tau*\rho}}$

For **the whole** Info of Alice: $\Delta \leq 2^{-\frac{1}{2}H_{min}(SS|ER)_{\tau*\rho}} + \eta(\xi)$

- Initial BH max. entangled with the early radiation ($\xi^{X_{in}X_{out}} = \Phi^{X_{in}X_{out}}$).



$$\Delta_{inv} \lesssim 2^{k-\ell},$$
$$\Delta \lesssim 2^{k-\ell} + O(N^{-0.5})$$

- This is likely to be optimal.
  - ✓ $O(N^{-1/2})$ amount of sym-var. info necessarily remains in the BH.

The information leaks out quickly

if and only if the initial BH is sufficiently large ($N \gg O(2^k)$).

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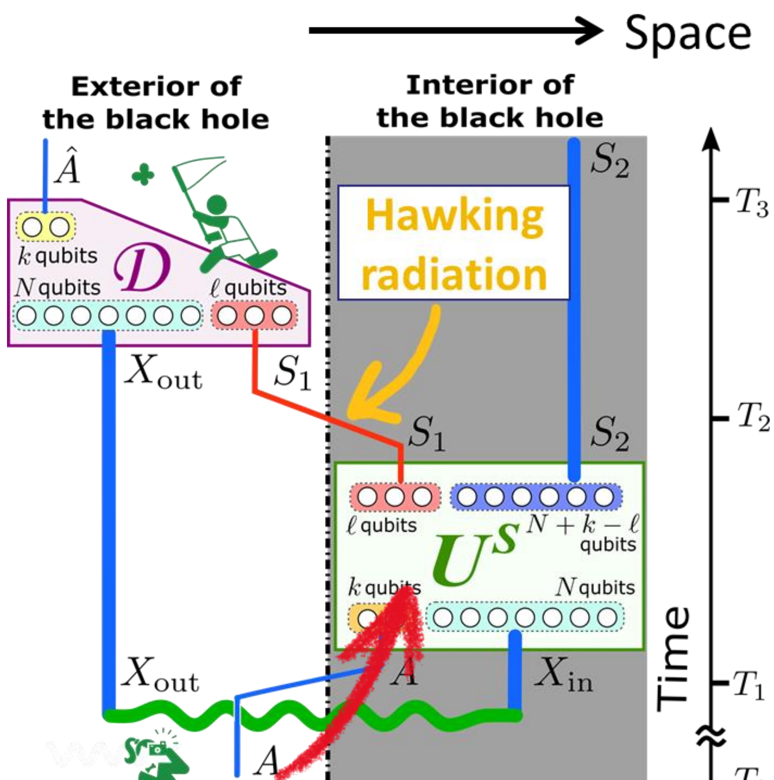
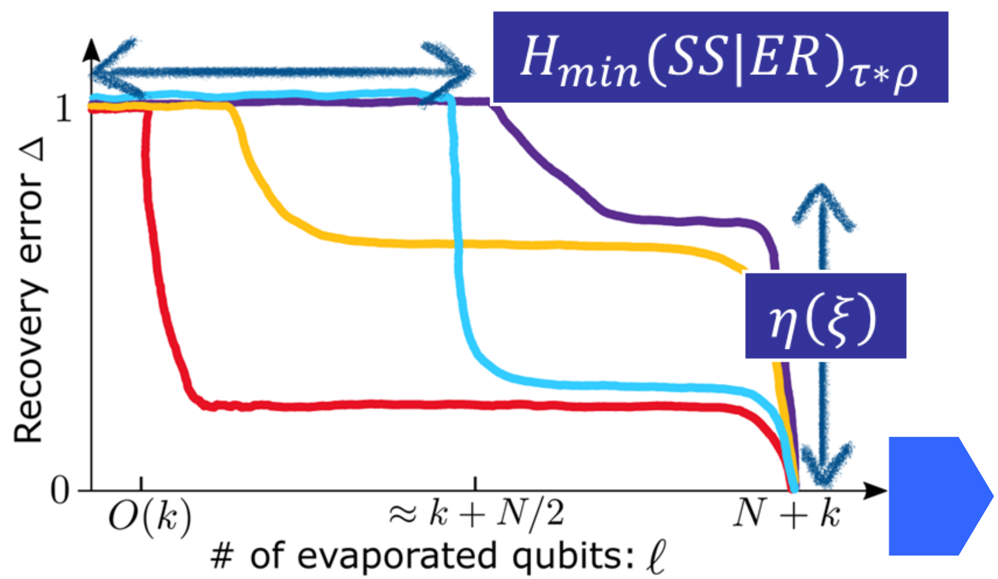
Summary



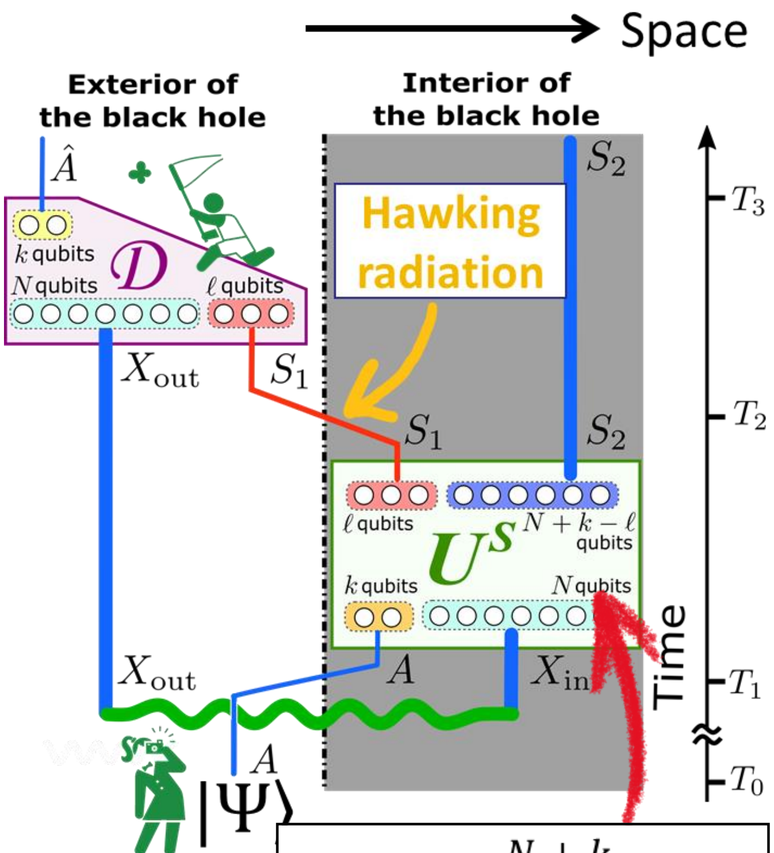
$$U^S = \bigoplus_{m=0}^{N+k} U_m^S$$

Information leakage problem of Kerr black holes

1. **Partial decoupling approach**
   - ✓ General tool and useful when ∃symmetry
   - ✓ E.g. energy, SO(3), charge, etc…
2. **Info leakage from Kerr BHs**
   - ✓ Symmetry-invariant/-variant info.
   - ✓ Two factors: entanglement & asymmetry

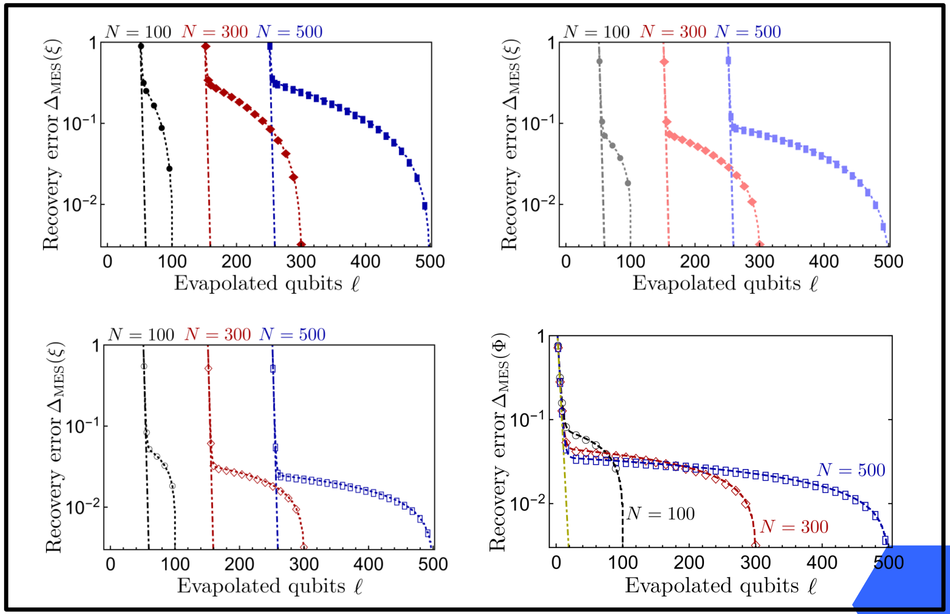


Discussion 1

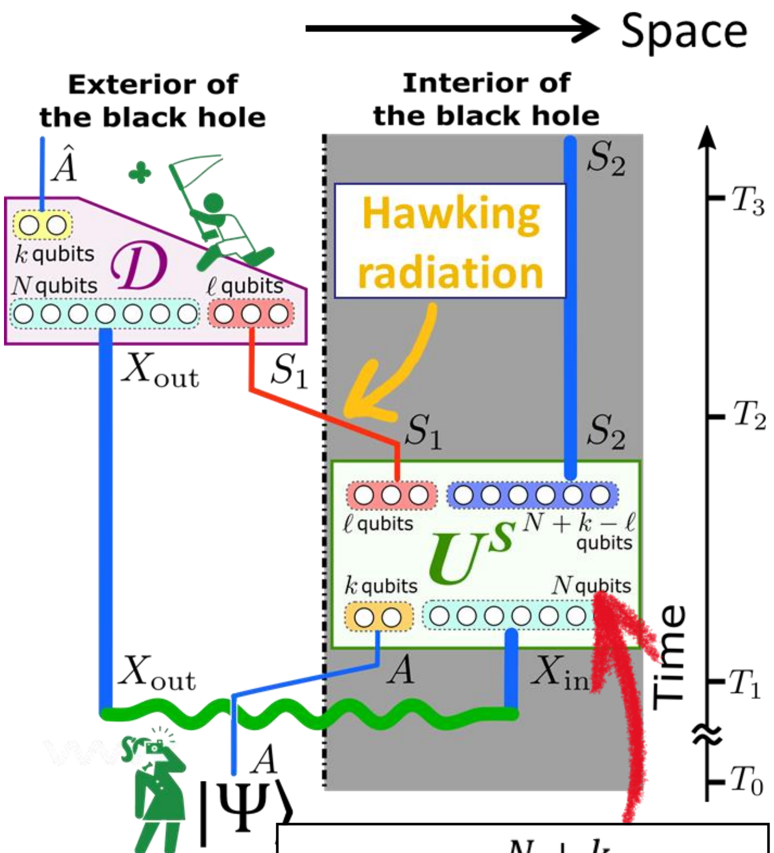


$$U^S = \bigoplus_{m=0}^{N+k} U_m^S$$

1. **Reasonable initial state ξ?**
   - ✓ We tried pure states and MES.
   - ✓ Reasonable assumptions on ξ incorporating with Penrose process?



Discussion 1

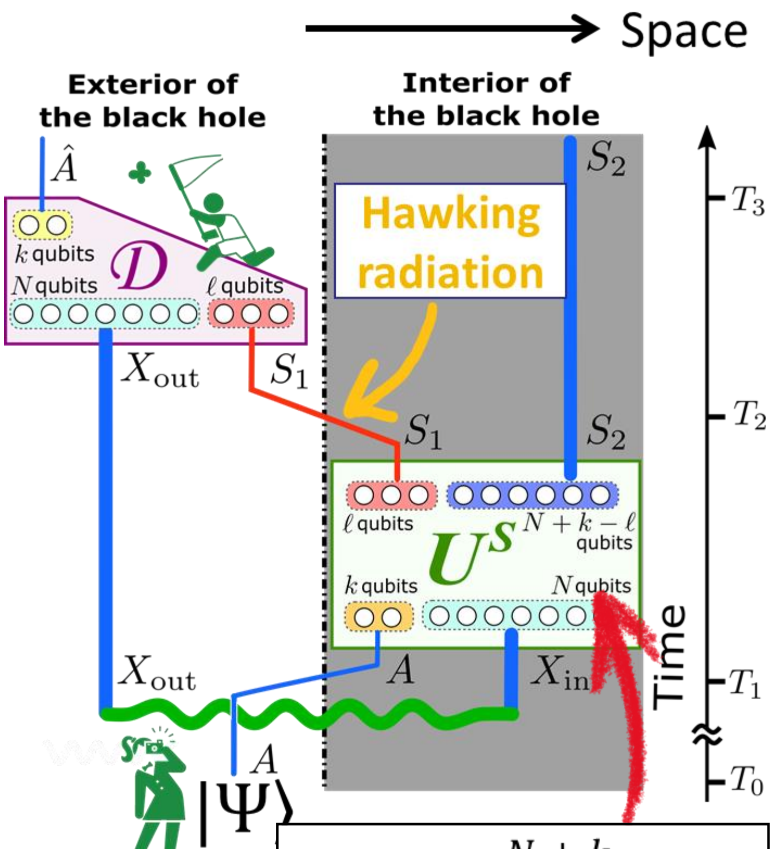


$$U^S = \bigoplus_{m=0}^{N+k} U_m^S$$

1. **Reasonable initial state ξ?**
   - ✓ We tried pure states and MES.
   - ✓ Reasonable assumptions on ξ incorporating with Penrose process?
2. **Weak violation of symmetry?**
   - ✓ Violation will be amplified during the time-evolution.
   - ✓ In the long-time limit, there should be a deviation from our results.

Operational approach to the symmetry violation in Q. gravity?

Discussion 2



$$U^S = \bigoplus_{m=0}^{N+k} U_m^S$$

Assumption:

U_m^S is Haar scrambling in each subspace

3. **Replacing Haar?**
   - ✓ Haar is normally replaced with unitary 2-designs.
   - ✓ Symmetry-preserving unitary design?
   - ✓ Implementation [Khemani et al '18]
4. **OTOC with symmetry?**
   - ✓ Argued that a decay of OTOC implies info recover.
   - ✓ How symmetry affects it?
5. **Non-unitary case?**
   - ✓ Time-evolution of BHs is not unitary.
   - ✓ Technically feasible, but what is the dynamics?

E. Wakakuwa, and YN (arXiv:1903.05796)
YN, E. Wakakuwa, and M. Koashi (arXiv:19xx.xxxxx)