Quantum Error Detection At Low Energies

arXiv:1902.02115 [quant-ph]

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IQIM
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It from Qubit
Simons Collaboration on
Quantum Fields, Gravity and Information

@KITP - September 2018

QINFO17 - Followon

@Santa Monica roller coaster ride: Losing everything we have done by killing our brain cells.

Outline

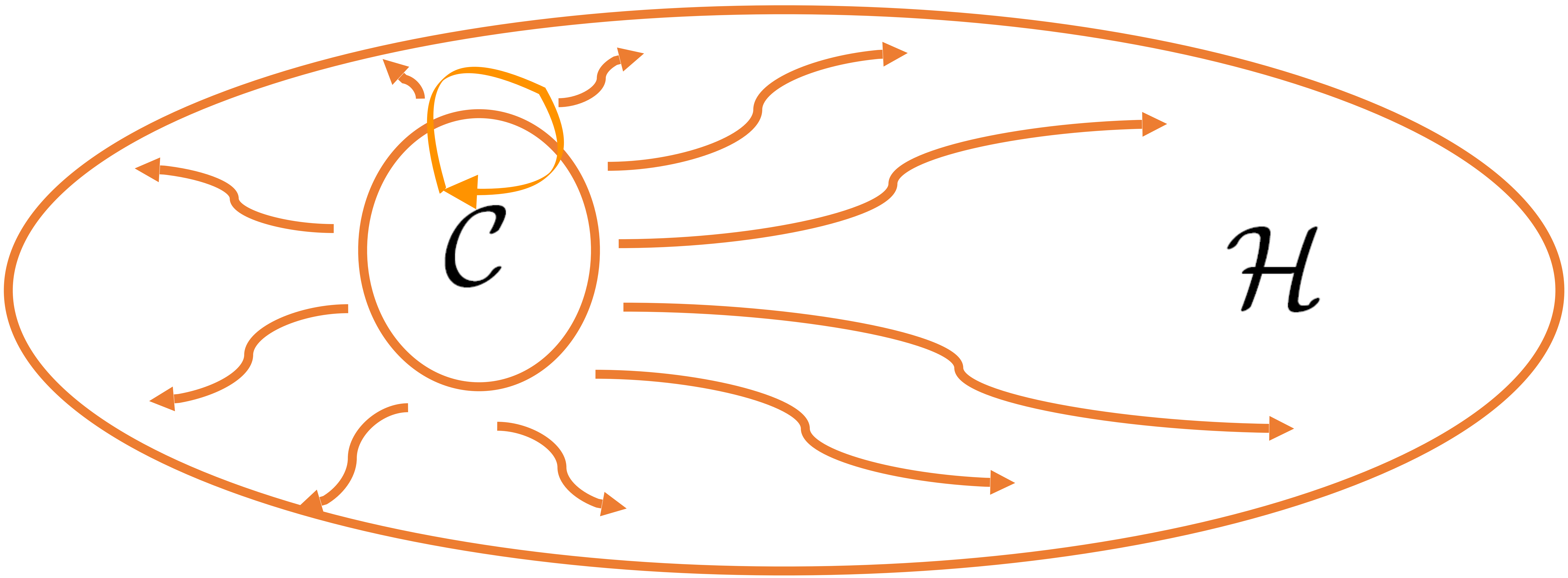
- Motivations
  - Error Correction & Detection
  - Physical Examples: Topological Order & Holography
- Approximate Quantum Error Correction & Detection
- A Matrix Product Encoding: No-Go Result: Trivial(=constant distance) ground space codes
- Getting around No-Go: Low energy space as codes:

A. Gapped excitations as codes: A general MPS formalism

B. Gapless excitations as codes: The Heisenberg XXX model

- Conclusions & Outlook

Motivations I - Quantum error detection/correction



$\mathscr{C}$: Code subspace $\quad\quad \log(\dim \mathscr{C}) = k$

$\mathscr{H}$: Physical Hilbert space $\quad\quad \log(\dim \mathscr{H}) = N$

Motivations I – Quantum Error Correction

$\mathscr{C} \subset \mathscr{H}$ is an $[[N, k, d]]$ quantum error correcting code against a noise channel $\mathscr{N}$ if there exists a recovery channel $\mathscr{R}$ such that

$$\mathscr{R}(\mathscr{N}(|\psi\rangle\langle\psi|)) = |\psi\rangle\langle\psi| \quad \forall |\psi\rangle \in \mathscr{C}$$

Motivations I – Quantum Error Correction

- Knill-Laflamme Conditions

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$$\Updownarrow$$

Let $\{|\psi_1\rangle, |\psi_2\rangle, \ldots, |\psi_{2^k}\rangle\} \in \mathscr{H}$ be an orthonormal basis for $\mathscr{C}$. They satisfy

$$\langle\psi_i|E_a^\dagger E_b|\psi_j\rangle = \lambda_{ab}\delta_{ij} \begin{cases} i = j : \text{Indistinguishability} \\ i \neq j : \text{No-collision in } \mathscr{H} \end{cases}$$

Motivations I – Quantum Error Detection

- Knill-Laflamme Conditions

$\mathscr{C} \subset \mathscr{H}$ is an $[[N, k, d]]$ quantum error detecting code against a noise channel $\mathscr{N}$ If

$$\langle\psi|\frac{P\mathscr{N}(|\psi\rangle\langle\psi|)P}{tr(P\mathscr{N}(|\psi\rangle\langle\psi|)P)}|\psi\rangle = 1.$$

$$\Updownarrow$$

Let $\{|\psi_1\rangle, |\psi_2\rangle, \ldots, |\psi_{2^k}\rangle\} \in \mathscr{H}$ be an orthonormal basis for $\mathscr{C}$. They satisfy

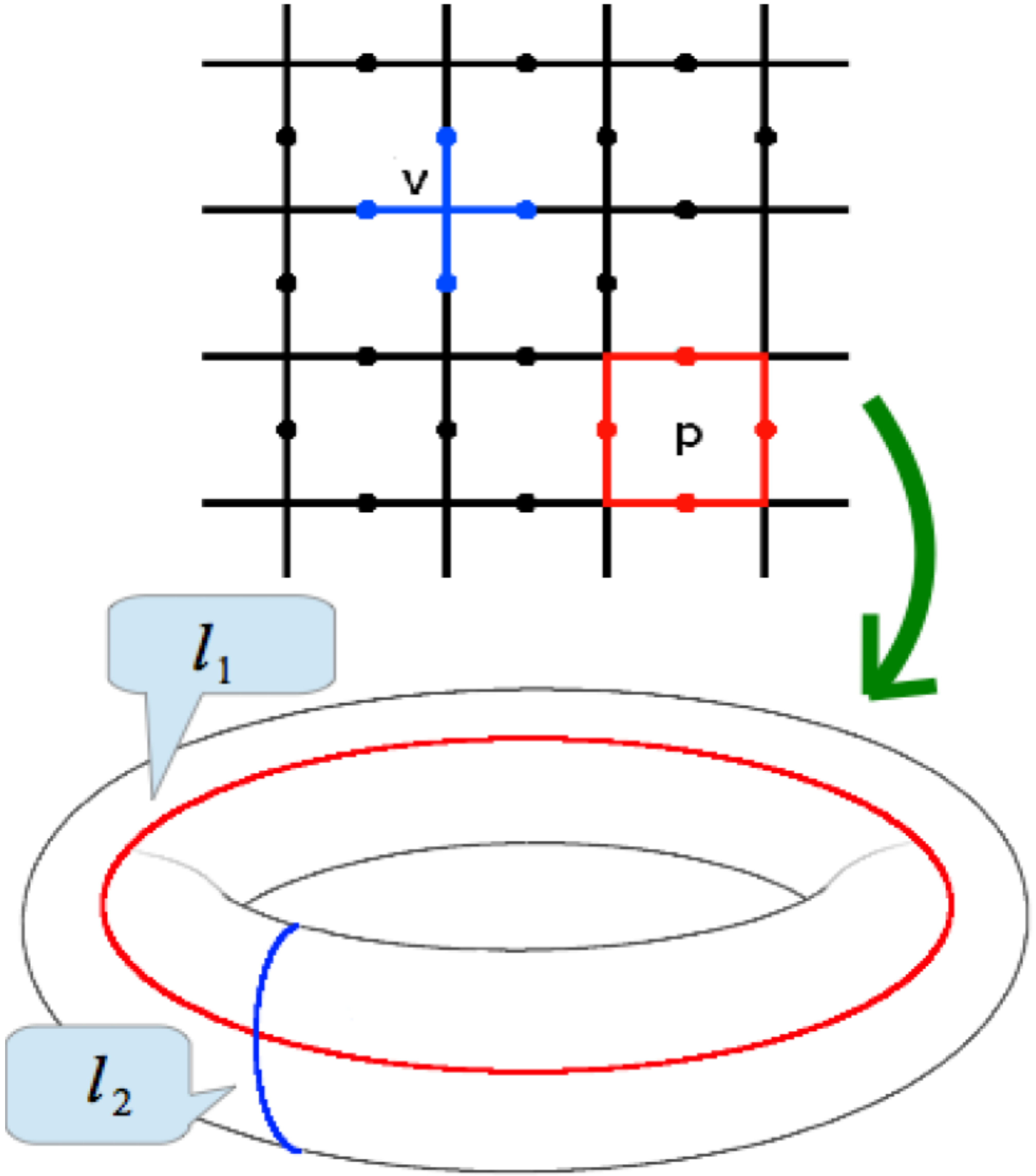
$$\langle\psi_i|E_a|\psi_j\rangle = \lambda_a\delta_{ij} \quad \begin{cases} i = j : \text{Indistinguishability} \\ i \neq j : \text{No-collision in } \mathscr{C} \end{cases}$$

“Wise” words from old men

- Richard Brower, Aspen, May 2019

“This is just a linear algebra problem!”

Motivations II – Topological Order (Kitaev)



$$A_v = \Pi_{i\in v} X_i, B_p = \Pi_{i\in p} Z_i$$

$$H = -\Sigma_v A_v - \Sigma_p B_p$$

Ground state space:

$$|\psi_1\rangle = \Sigma|even - l_1 \wedge even - l_2\rangle$$

$$|\psi_2\rangle = \Sigma|even - l_1 \wedge odd - l_2\rangle$$

$$|\psi_3\rangle = \Sigma|odd - l_1 \wedge even - l_2\rangle$$

$$|\psi_4\rangle = \Sigma|odd - l_1 \wedge odd - l_2\rangle$$

Locally Indistinguishable!

“Wise" words from old men

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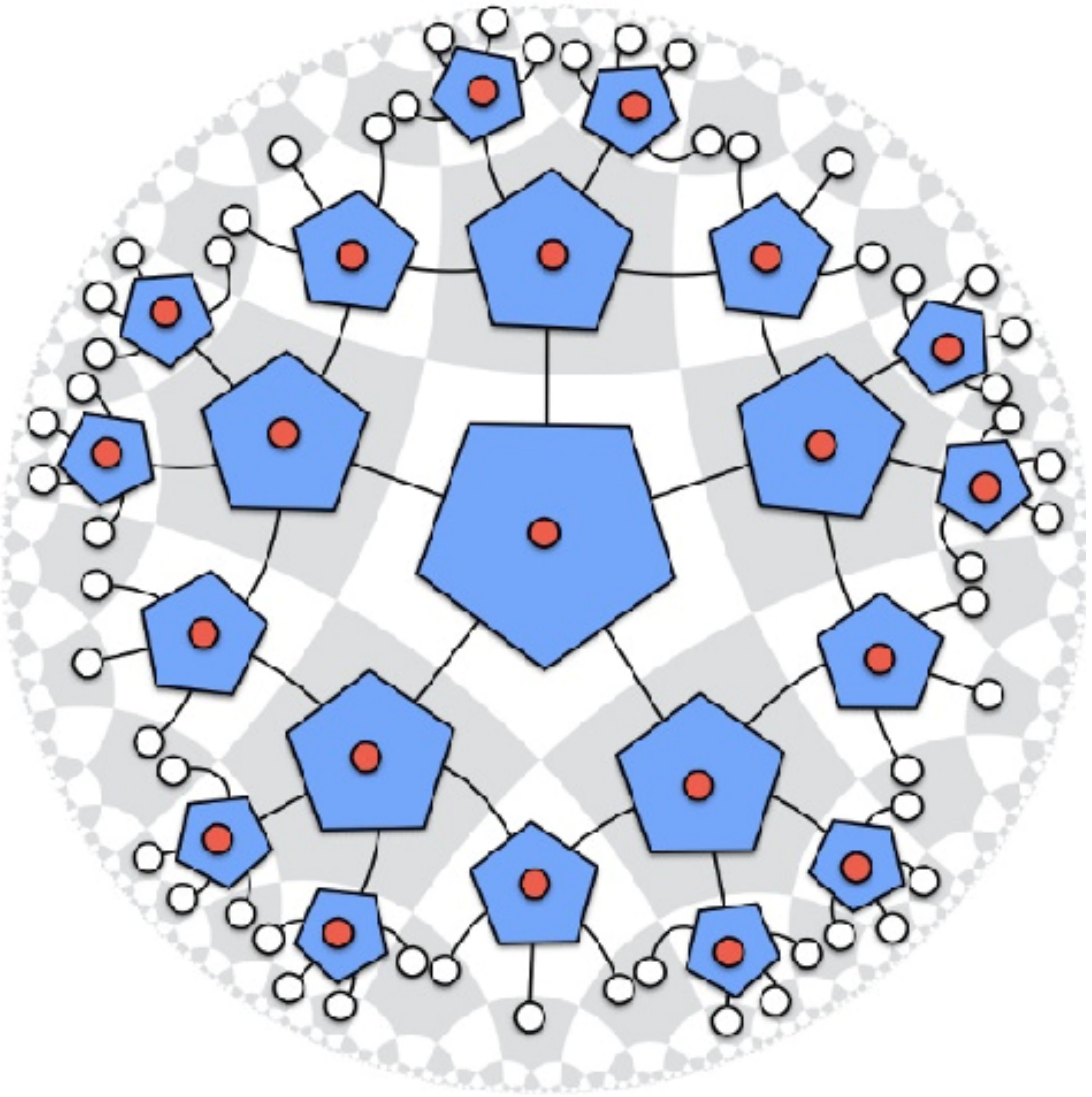
  “This is just a linear algebra problem!”

- Frank Verstraete, various places on Earth, 4-5 times between 2013 and 2016

  “Error-correction is not a problem of fundamental science anymore, it’s engineering.”

Motivations III – Holography (Almheiri, Dong, Harlow)



- Boundary: Physical Hilbert space
  Bulk: Code space
- Apparent puzzles like subregion duality and radial commutativity points out that

$$V : Bulk \to Boundary$$

is a quantum error correcting code against erasure channel.

- Conjecture: Low energy eigenspace of holographic CFTs are QECCs.


Topological order
EXACT
AdS/CFT holography
Kitaev
Toy model/idea:
Almheiri, Dong, Harlow
Quantum Error Correction
APPROXIMATE
Brandao, Crosson, MBS, Bowen arXiv:1710.04631 [quant-ph]
Chaos
Gapless models
High energies of 1D TI

What did we do at arXiv:1710.04631 [quant-ph]?

- A restrictive assumption of noise model (geometrically d-local)
- Don't address the error correction properties of generic gapped/gapless low energy subspace

What are we going to do now?

- No-go: Ground space of 1d local gapped Hamiltonians are trivial (=constant distance)
- Our cure for getting around the No-Go: Extend to low energy subspace

A. Low energy space of any local gapped Hamiltonian
B. Low energy space of the gapless Heisenberg XXX-model

Approximate Quantum Error Detection

$\mathscr{C} \subset \mathscr{H}$ is a $(\delta, \epsilon)[[N, k, d]]$ quantum error detecting code against noise channel $\mathscr{N}$ if the following holds for all $|\psi\rangle \in \mathscr{C}$

If $tr(P\mathscr{N}(|\psi\rangle\langle\psi|)) \geq \delta$ then $\langle\psi| \dfrac{P\mathscr{N}(|\psi\rangle\langle\psi|)P}{tr(P\mathscr{N}(|\psi\rangle\langle\psi|)P)} |\psi\rangle \geq 1 - \epsilon$.

What do we mean above?

- Remind that, we detect an error only when we go out of the code space.
- But there may be an overlap with code space, say of amount $\delta' \geq \delta$.
- In this case, we want to make sure that we make a logical error of amount at most ϵ . $\mathscr{C}$ is error-detecting if $\lim_{n\to\infty} \epsilon_n, \delta_n = 0.$

A sufficient condition for AQEDC

$\mathscr{C} \subset \mathscr{H}$ is a $(\delta, \epsilon)[[N, k, d]]$ quantum error detecting code against d-local noise $\mathscr{N}(\rho) = \sum_i p_i E_i \rho E_i^\dagger$ if the following holds for all $|\psi\rangle \in \mathscr{C}$

If $tr(P\mathscr{N}(|\psi\rangle\langle\psi|)) \geq \delta$ then $\langle\psi| \dfrac{P\mathscr{N}(|\psi\rangle\langle\psi|)P}{tr(P\mathscr{N}(|\psi\rangle\langle\psi|)P)} |\psi\rangle \geq 1 - \epsilon$.

$$\Uparrow \quad \epsilon = 2^{5k}\gamma^2\delta^{-1}$$

Let $\{|\psi_1\rangle, |\psi_2\rangle, \ldots, |\psi_{2^k}\rangle\} \in \mathscr{H}$ be an orthonormal basis for $\mathscr{C}$. They satisfy

$$\langle\psi_i|O_d|\psi_j\rangle \leq \lambda(O_d)\delta_{ij} + \gamma||O_d|| \quad \text{for some} \quad \gamma$$

A necessary condition for AQEDC

Let $\{|\psi_1\rangle, |\psi_2\rangle, \ldots, |\psi_{2^k}\rangle\} \in \mathscr{H}$ be an orthonormal basis for $\mathscr{C}$. Say

$$|\langle\psi_i|O|\psi_i\rangle - \langle\psi_j|O|\psi_j\rangle| = \eta$$

for some $i \neq j, \eta \in (0,1] : 1 - \eta \ll 1.$

$\Downarrow$

$\mathscr{C}$ is not an $(\delta, \epsilon)[[N, k, d]]$ quantum error detecting code for any

$\epsilon < 1 - 10(1-\eta)$ and $\delta < \eta^2$.

An MPS encoding

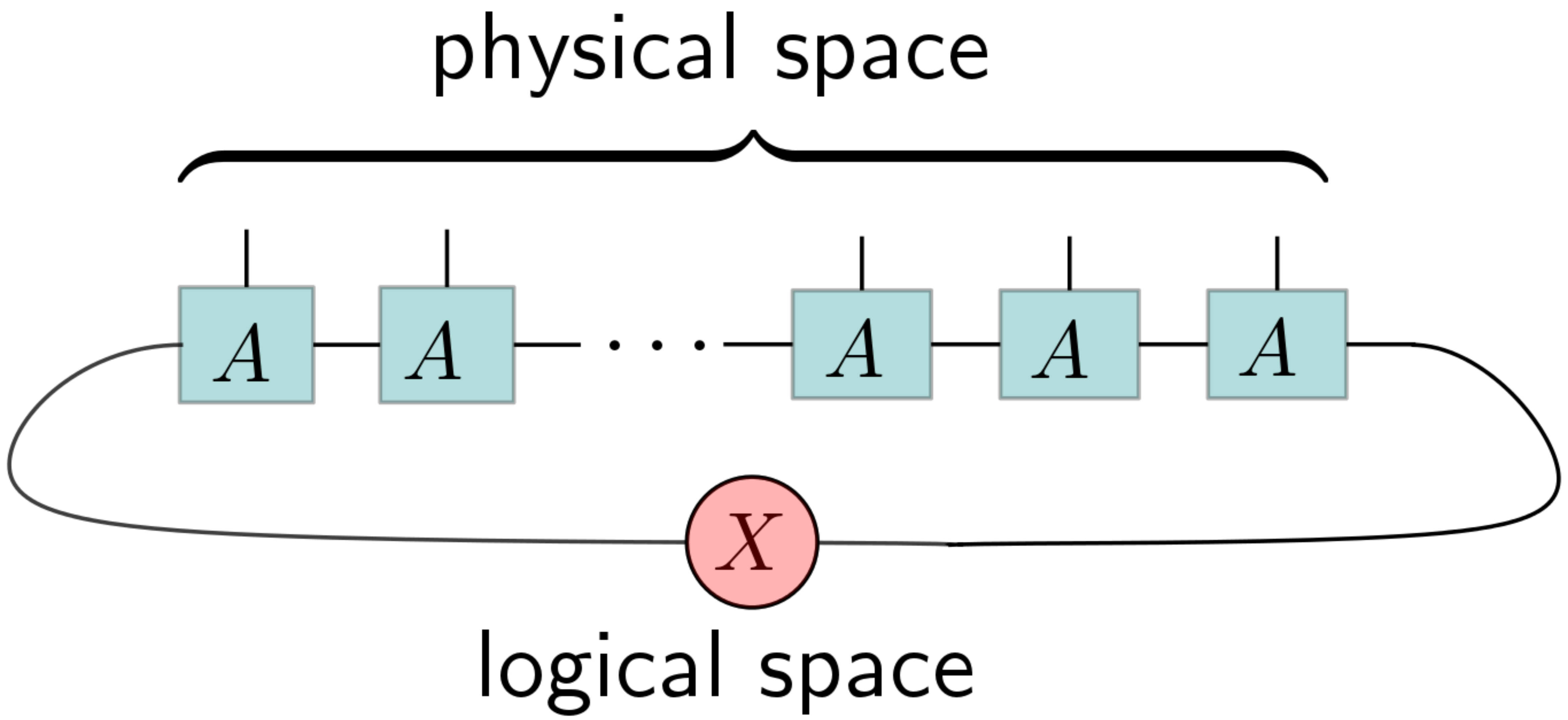
- An encoding of boundary space into bulk Hilbert space, the code space $\mathscr{C}$ is given by

$$span\{\,|\psi(A,N,X)\rangle\,|\,X \in \mathbb{M}_{DXD}\}$$

- It has the potential of creating a

$$(\epsilon = N^{-\nu}, \delta \to 0)[[N, k = \log D^2, d]] - AQEDC$$

- We want to understand whether above is possible with a nontrivial distance d, i.e., sth that scales with the system size.
- No-Go Theorem: $d \leq c \log D$



Encoding of boundary degrees of freedom into the bulk via an MPS network. The set of states spanned by varying the boundary tensor X, is the ground space of a local gapped hamiltonian with open boundary conditions.

No-Go theorem: No nontrivial QEDC in the ground space

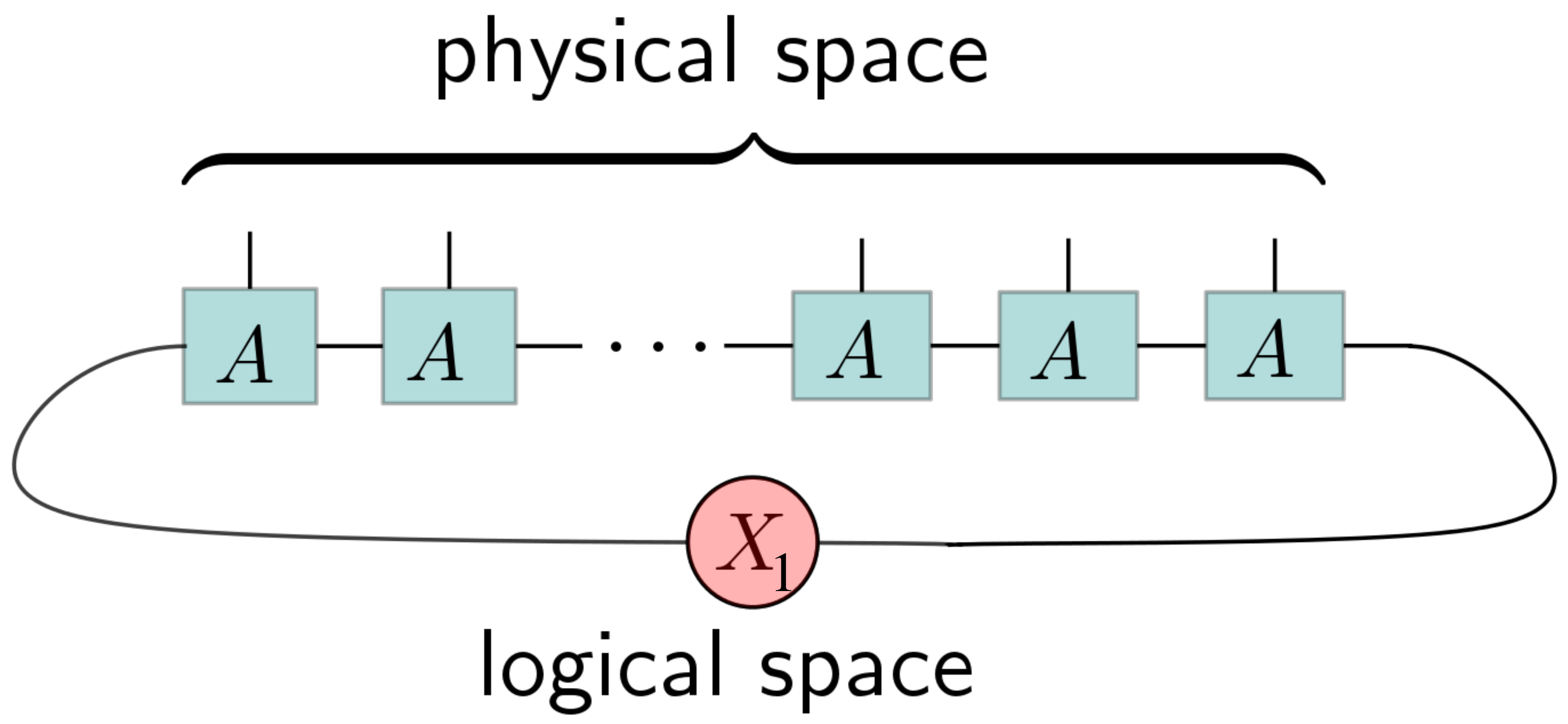
- The first condition that we have to satisfy is local indistinguishability! We want two orthogonal codevectors

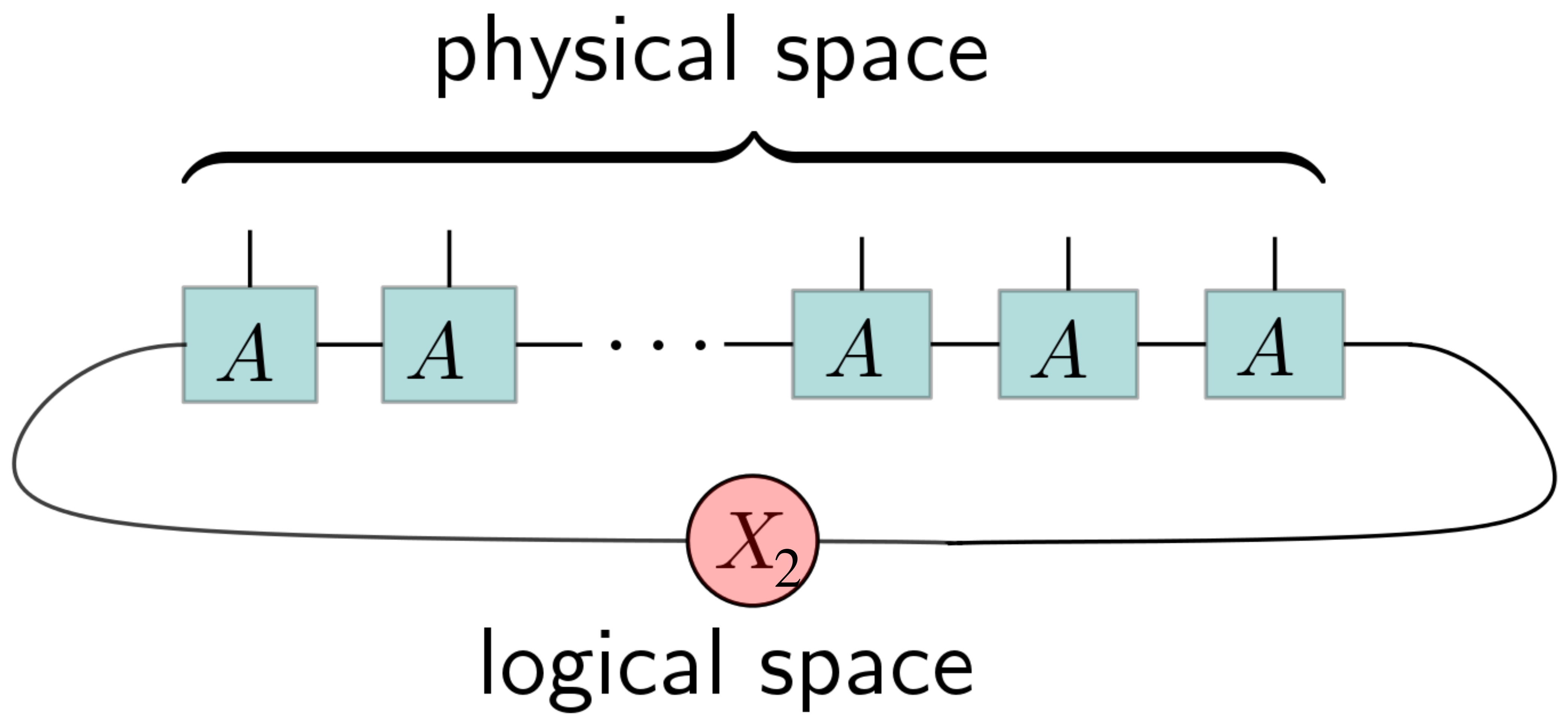
  $|\psi(A,N,X_1)\rangle$ and $|\psi(A,N,X_2)\rangle$

  to look locally the same.

- Due to exponential decay of correlations, they look very much the same in the bulk!

- However, in 1D (injectivity of MPS transfer matrix), this implies that most of the boundary information is encoded in the physical qubits close to the boundary! Hence if an error happens in a few of them we make a logical error!





Getting around the No-Go theorem

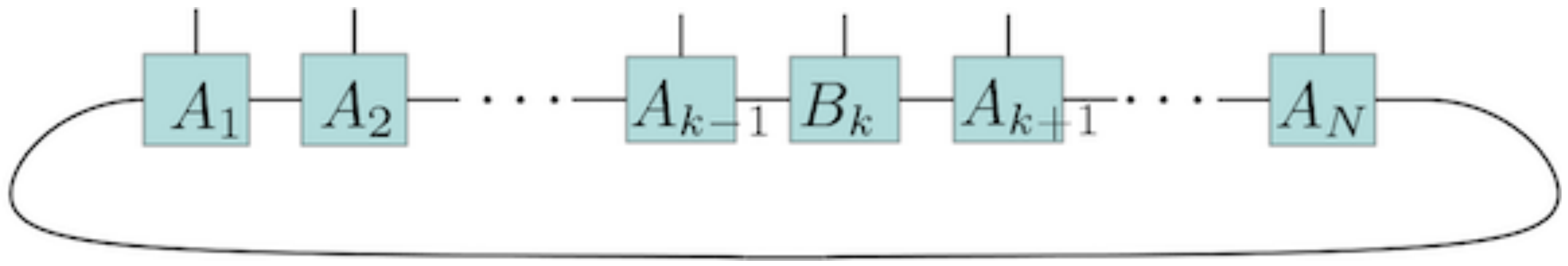
- No-Go theorem assumes:
  - Injectivity
  - The ground space MPS form with constant bond dimension

- Hence, we have to investigate the cases where we violate these assumptions:
  - Use an ansatz that accounts for superpositions of MPSs:

    Excitation ansatz: Represents momentum eigenstates faithfully!
  - Go non-injective: Construct higher excitations with Matrix Product Operators (MPO & Injective MPS —-> Noninjective MPS)

Excitation Ansatz

$$|\psi(A,N;B,p)\rangle = \sum_{k=1}^{N} e^{2\pi i p k/N}$$

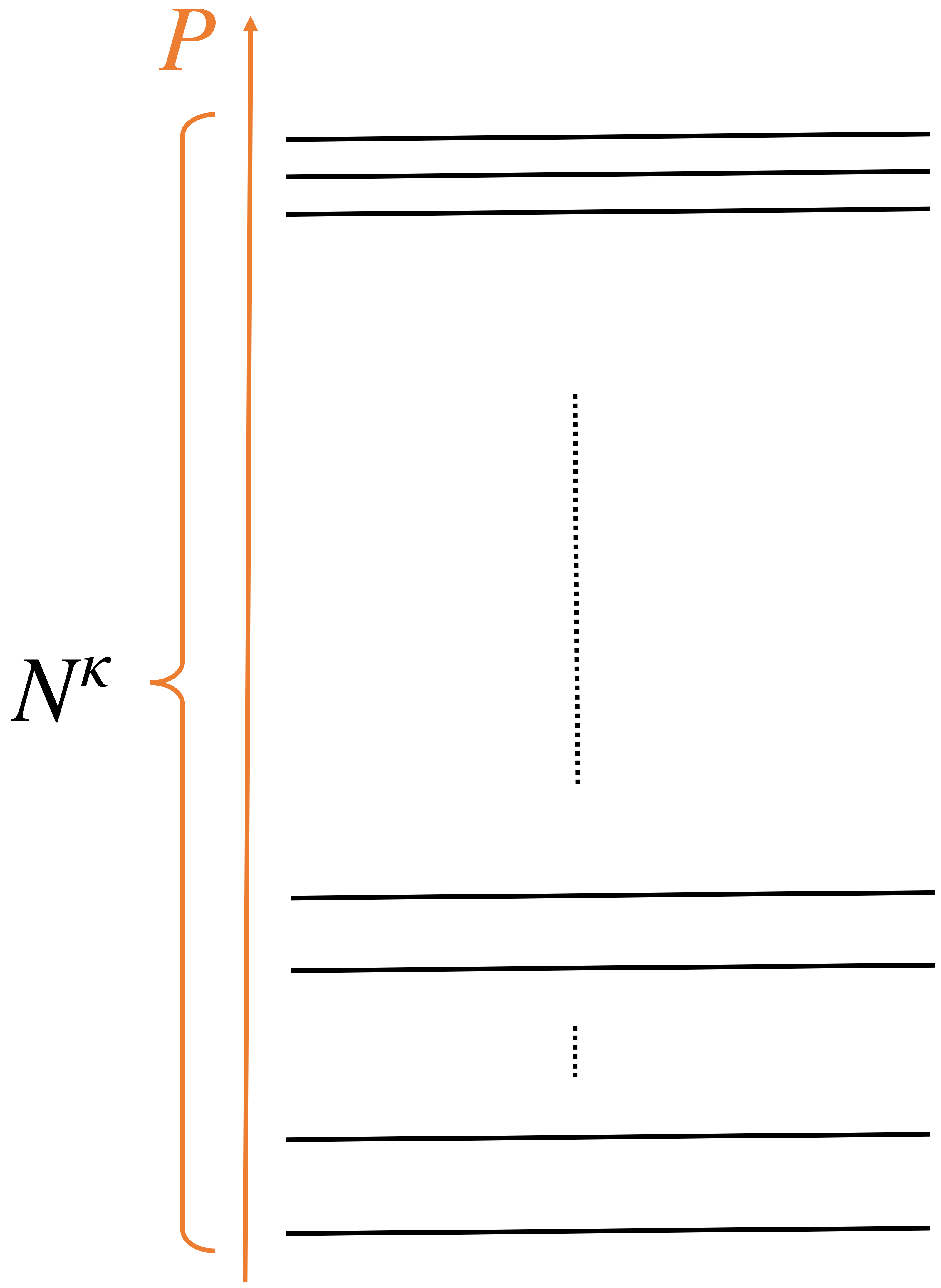


- Onb for the code subspace $\mathscr{C}$ is: $\{\,|\psi(A,N;B,p)\rangle\,|\,p\}$

- The goal is to figure out which set of momentum eigenstates can be packed into the code space with what parameters of the number of logical qubits= k and distance= d.

- Note that given a faithful MPS ground state, above type of states can faithfully represent single quasi-particle momentum eigenstates (after blocking). (~Haegeman, Michalakis, Nachtergaele, Osborne, Verstraete)

AQEDC at low energies of local gapped H



- Start from an injective MPS (hence there is a local gapped Hamiltonian):

Local tensor A

- Variationally construct a quasi-particle band:

Local tensor B

- This is an $(\delta, \epsilon)[[N, k, d]] - AQEDC$ with

$$k = \kappa \log N, d = N^{1-\nu}, \epsilon = \Theta(N^{-(\nu-(5\kappa+\Delta))}), \delta = N^{-\Delta}$$

- Intuitively: Packing more states = Ability of constructing localized wave packets!

Status of recent physical codes

- General low energy eigenspace of CFTs?
- Iff conditions for matrix elements in the code space vs. correctability/detectability
- Decay of energy gap vs. code parameters
- Next two slides for further applications of MPS and QECC