Quantum computational universality of hypergraph states with Pauli-X and Z basis measurements

Yuki Takeuchi

NTT Communication Science Laboratories, NTT Corporation

Joint work with Tomoyuki Morimae (Kyoto)
and Masahito Hayashi (Nagoya, SUSTech, and NUS)

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Measurement-based quantum computation

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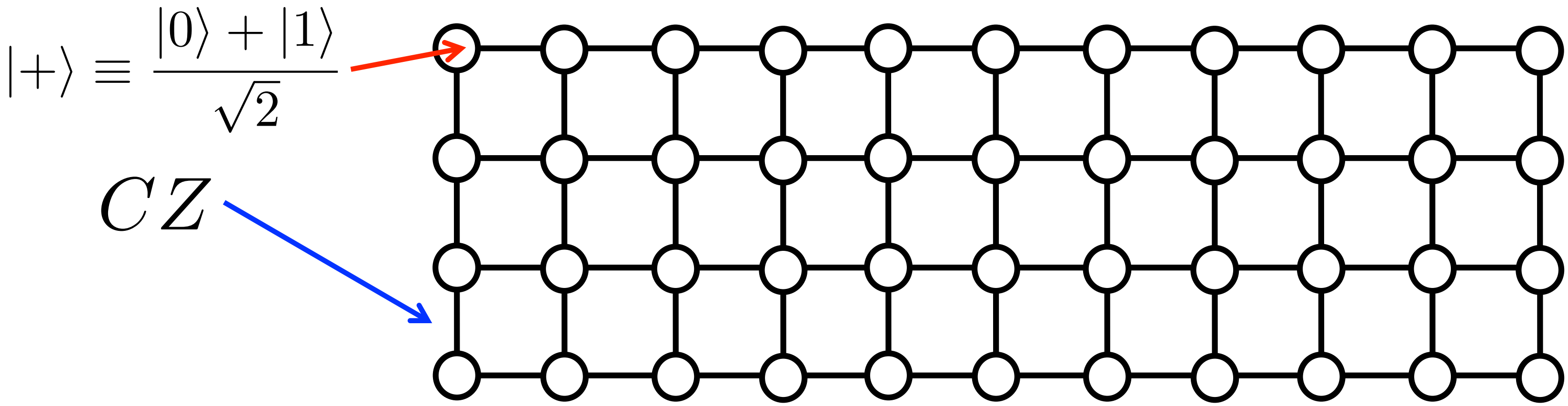

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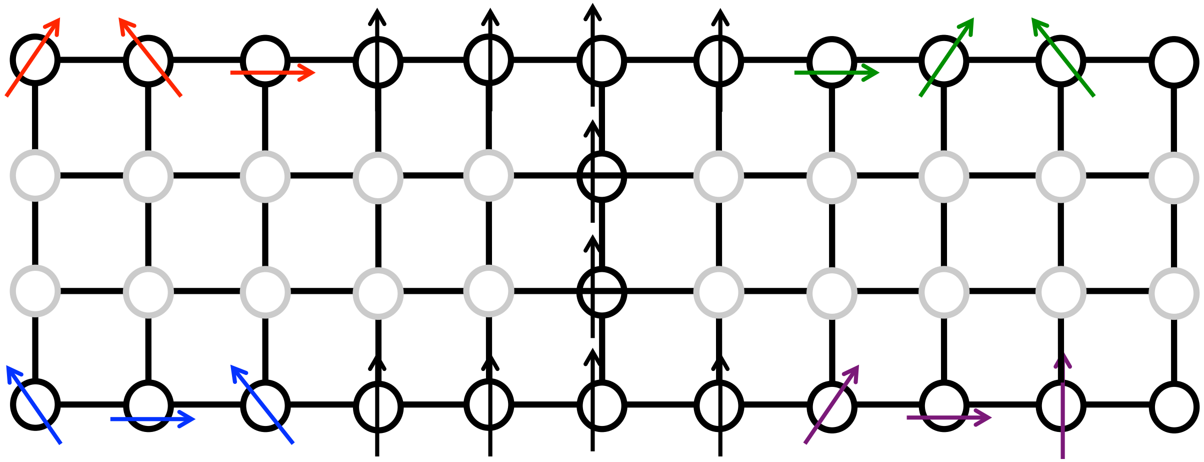

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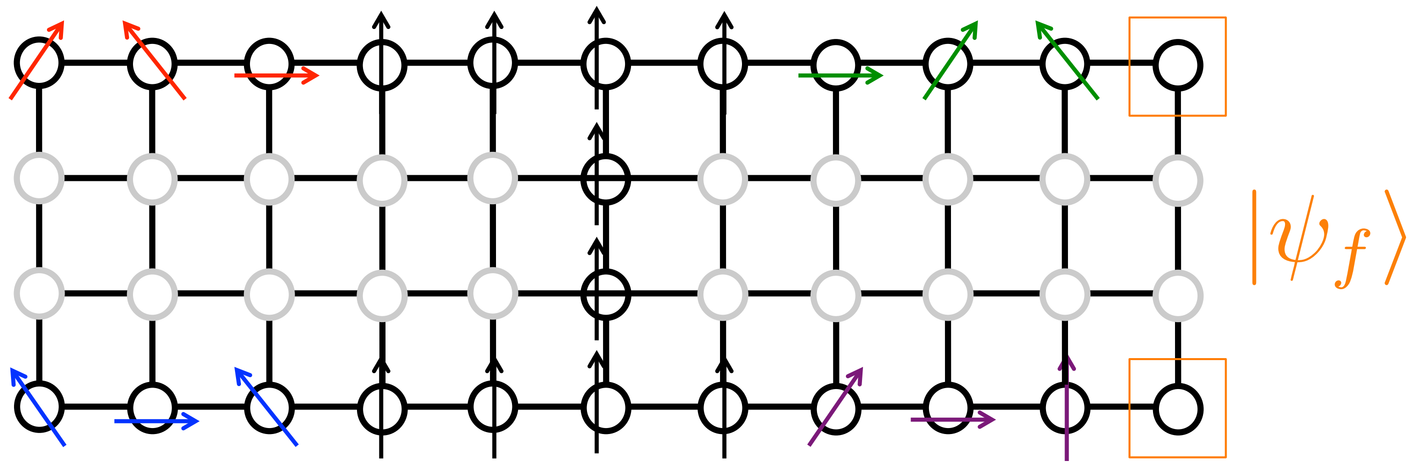

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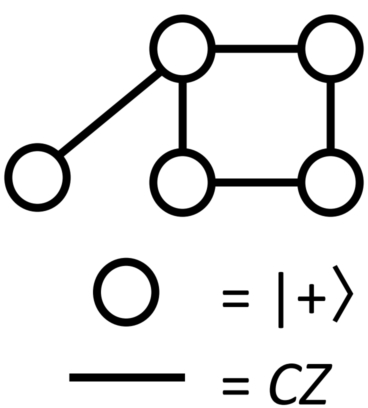
(A part of) our results:
We have constructed a universal resource state that only requires adaptive Pauli X and Z-basis measurements for universal MBQC.

Several universal resource states

Resource state	Measurement basis	Class
Cluster state [15]	$X, Y, TXT^\dagger$ [16]	graph state
Brickwork state [17]	$X, Y, TXT^\dagger$ [17]	graph state
Triangular lattice state [18]	X, Z, H, XHX [18]	graph state
Raussendorf-Harrington-Goyal (RHG) lattice [19, 20]	$X, Y, Z, TXT^\dagger$ [20, 21]	graph state
Decorated RHG lattice [22]	$X, Y, TXT^\dagger$ [22]	graph state
Affleck-Kennedy-Lieb-Tasaki (AKLT) state [23, 24]	qutrit bases [24]	matrix-product state
Union Jack state [25]	X, Y, Z [25]	hypergraph state
Three-uniform hypergraph state [26]	X, Y, Z [26]	hypergraph state
Mølmer-Sørensen graph state [27]	X, Z [27]	weighted graph state
Our state	X, Z	hypergraph state

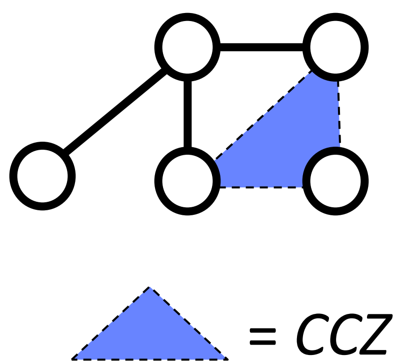
[Briegel *et al.* '01]

- Graph state



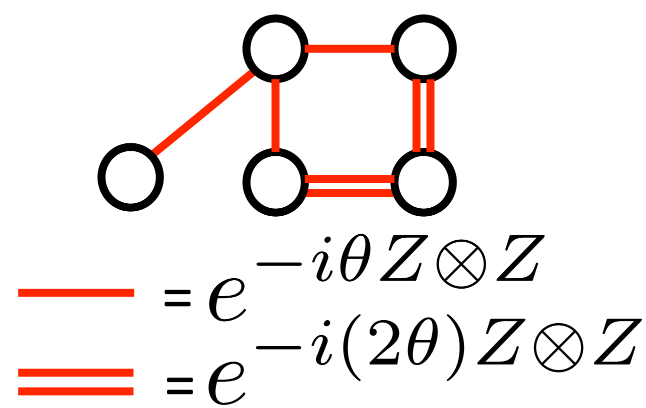
[Rossi *et al.* '13]

- Hypergraph state

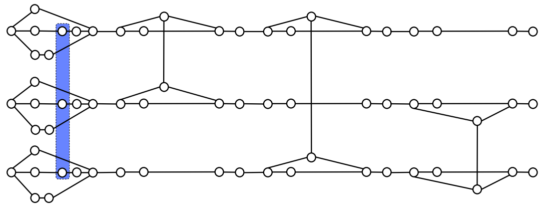
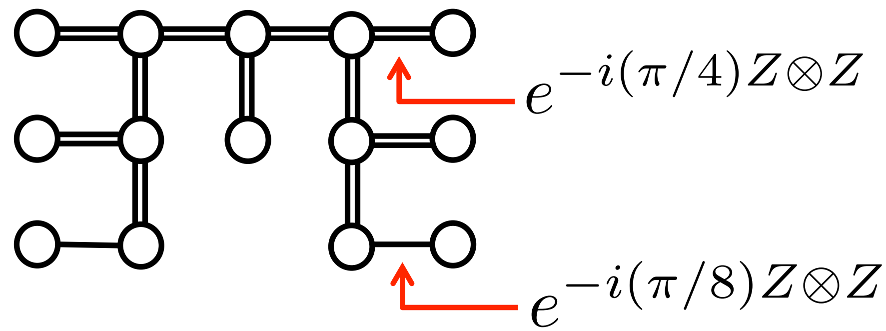


[Hartmann *et al.* '07]

- Weighted graph state



Hypergraph states vs. weighted graph states

	Our resource state	$\mathrm{M\o lmer-S\o rensen}$ state [Kissinger *et al.* ‘17]
		
Bases (Computation)	$\{X, Z\}$	$\{X, Z\}$
Class	Hypergraph state	Weighted graph state
Bases (Verification)	$\{X, Z\}$ [Morimae-Takeuchi-Hayashi ‘17, Miller-Sanders-Miyake ‘17, Takeuchi-Morimae ‘18, Zhu-Hayashi ‘18]	X + Other six measurement bases in the *x*-*y* plane of the Bloch sphere [Hayashi-Takeuchi ‘19]

Our contribution

Our result:

We have constructed, for the first time, a universal hypergraph state that achieves

- Universality
- Verifiability

at the same time with only Pauli X and Z-basis measurements.

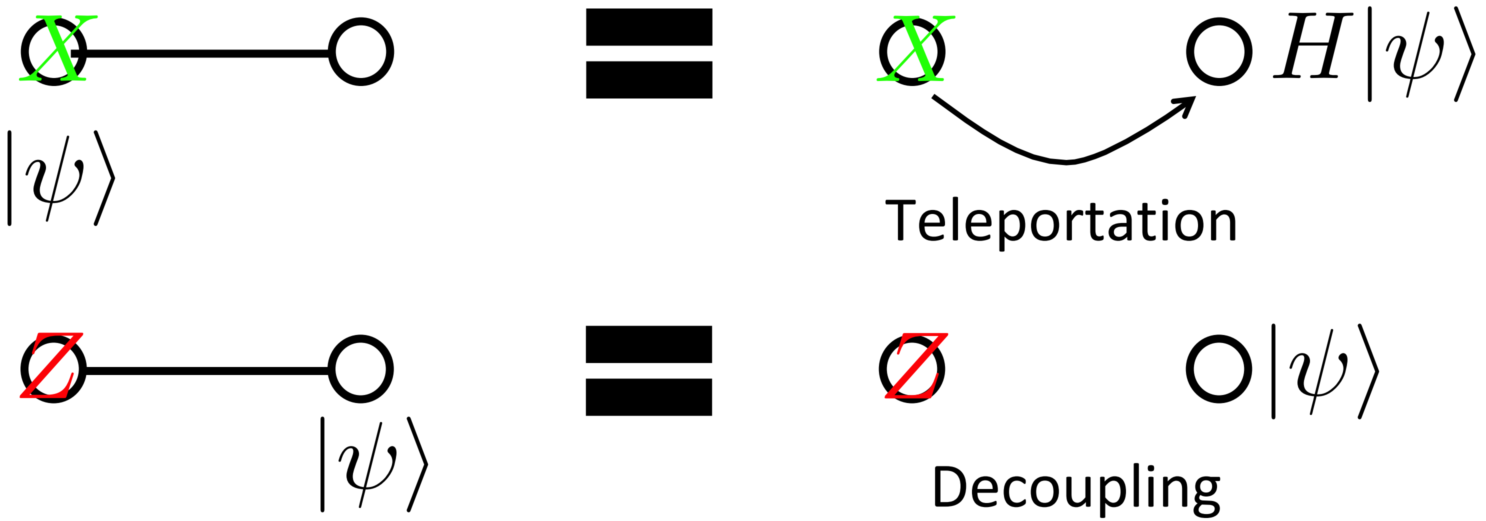
↑ This property cannot be achieved using graph states due to the Gottesman-Knill theorem.

In the rest of this presentation, I would like to explain (the idea of) how to construct our universal hypergraph state.

Our universal resource state

Idea of our construction

- Fact 1: Teleportation & Decoupling



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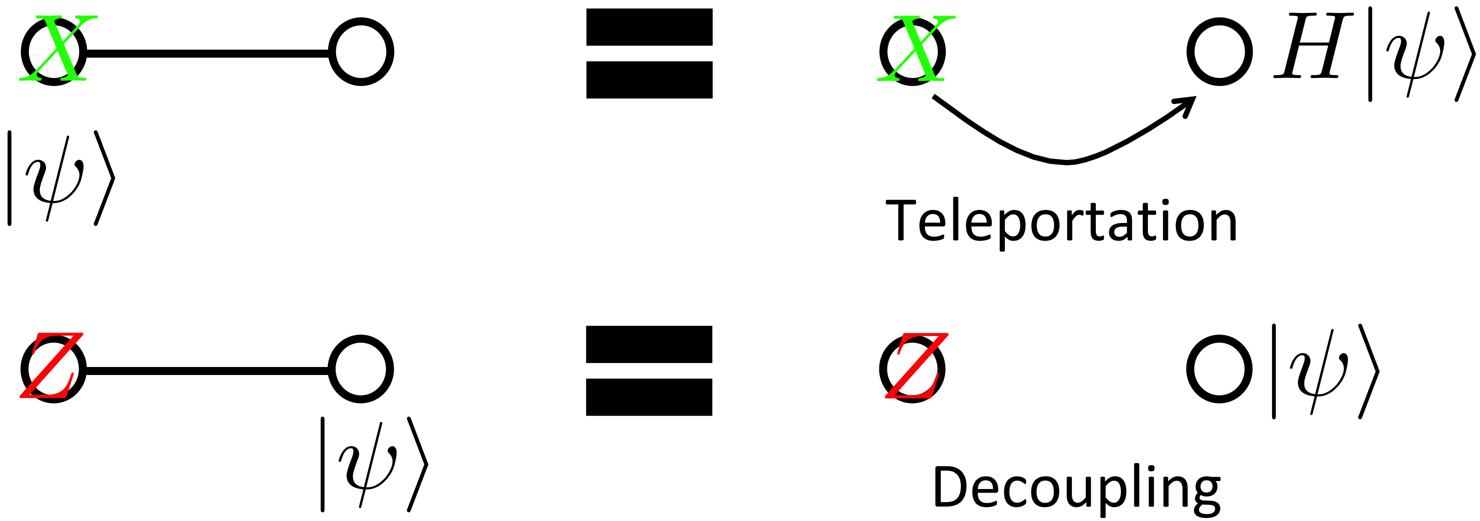

- Fact 2: Universal gate set

$$\{H, CCZ\}$$ [Shi '02]

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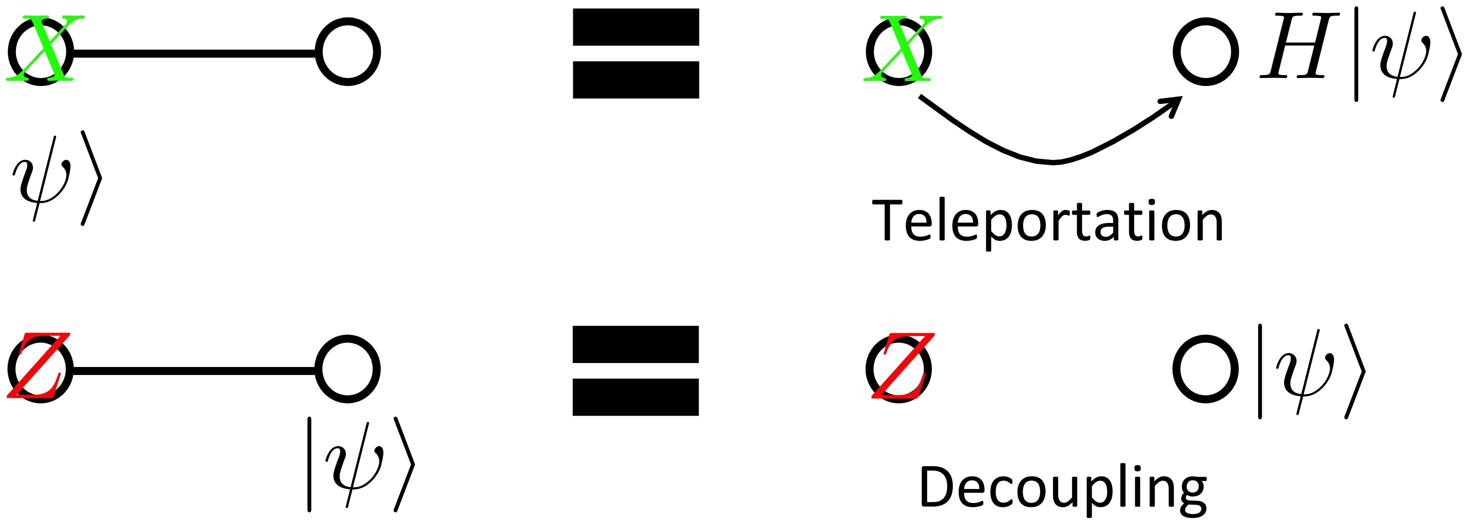


- Fact 2: Universal gate set

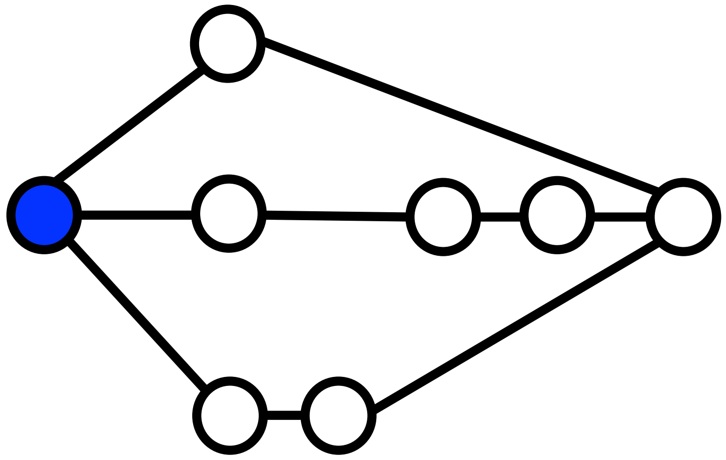
$$\{H, CCZ\}$$ [Shi '02]

Our goal: Using only fact 1, we construct a hypergraph state that realizes *H*, *CCZ*, and *I*.

Our universal resource state

<u>Idea of our construction</u>

- Identity gate



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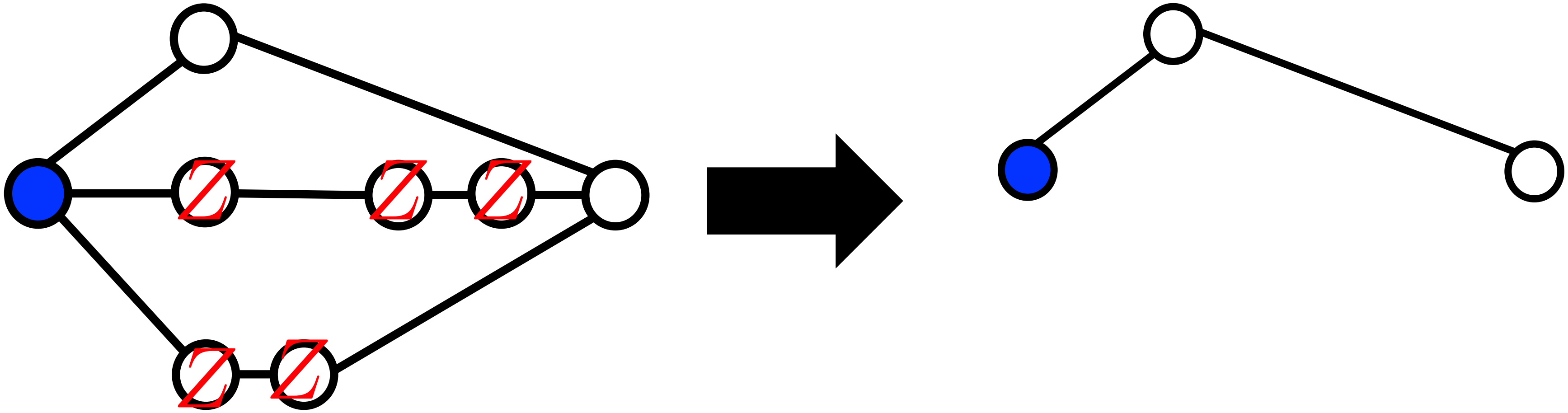

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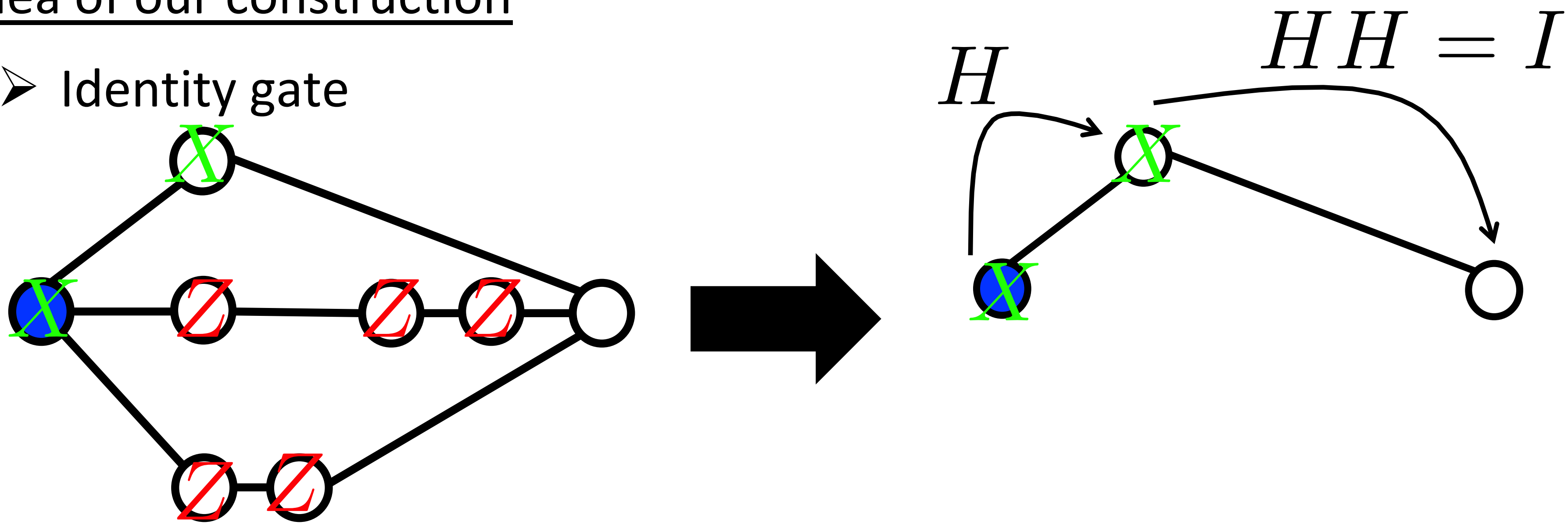

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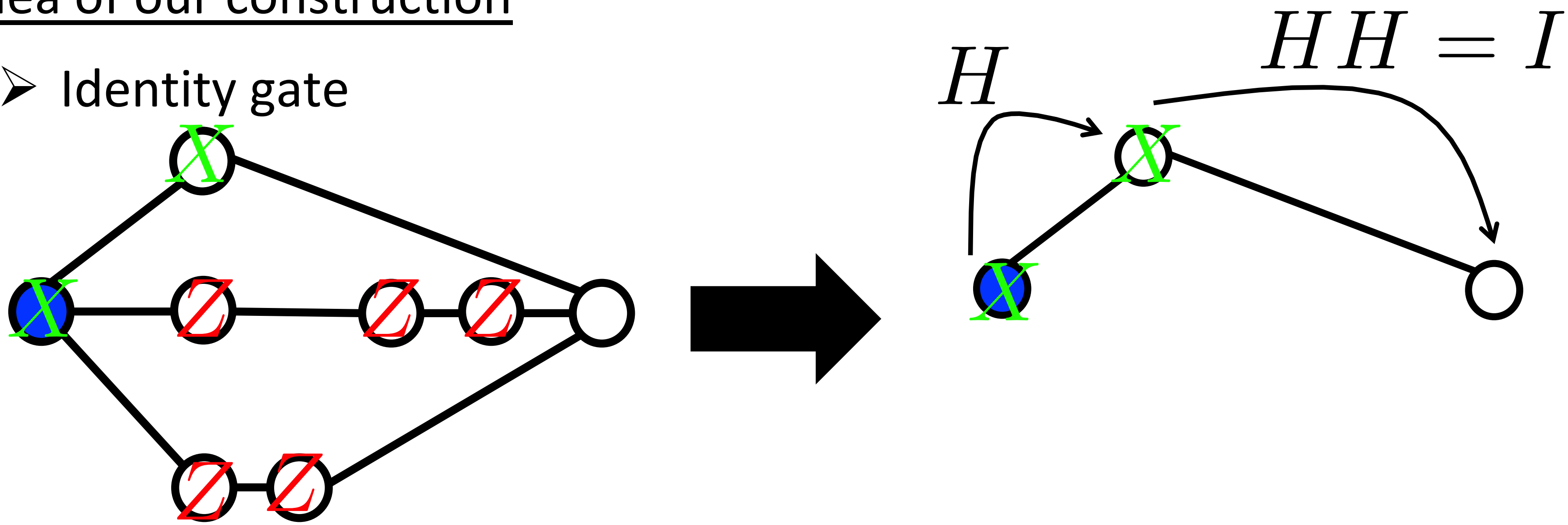
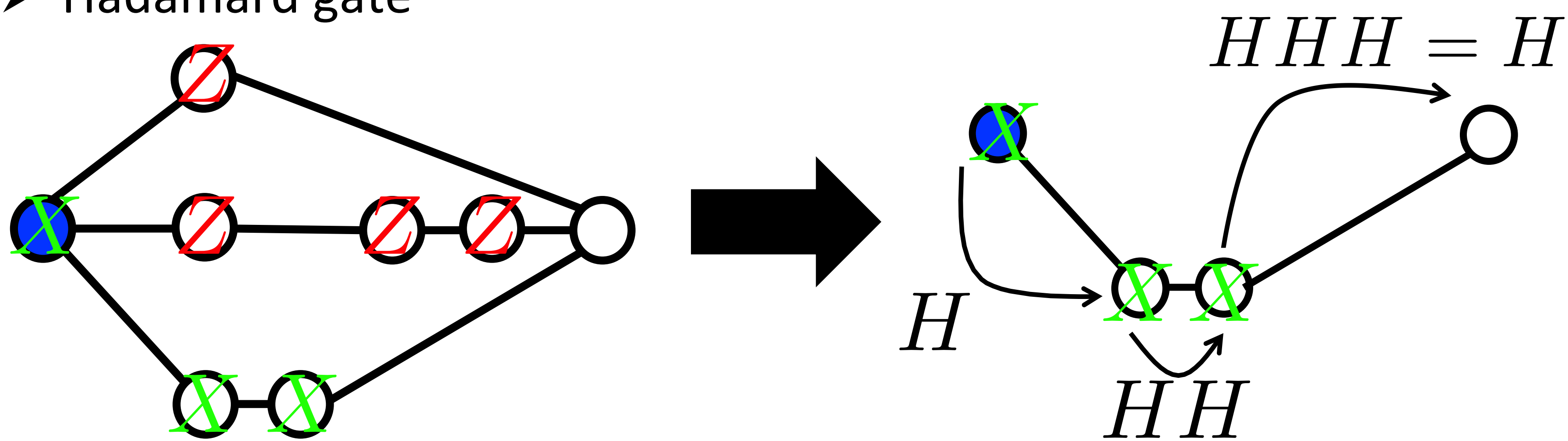


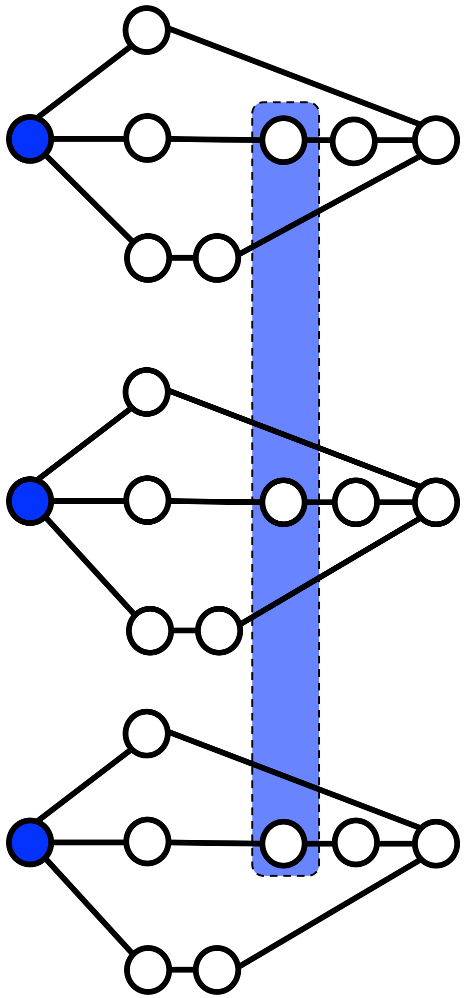
- Hadamard gate



Our universal resource state

Idea of our construction

- *CCZ* gate



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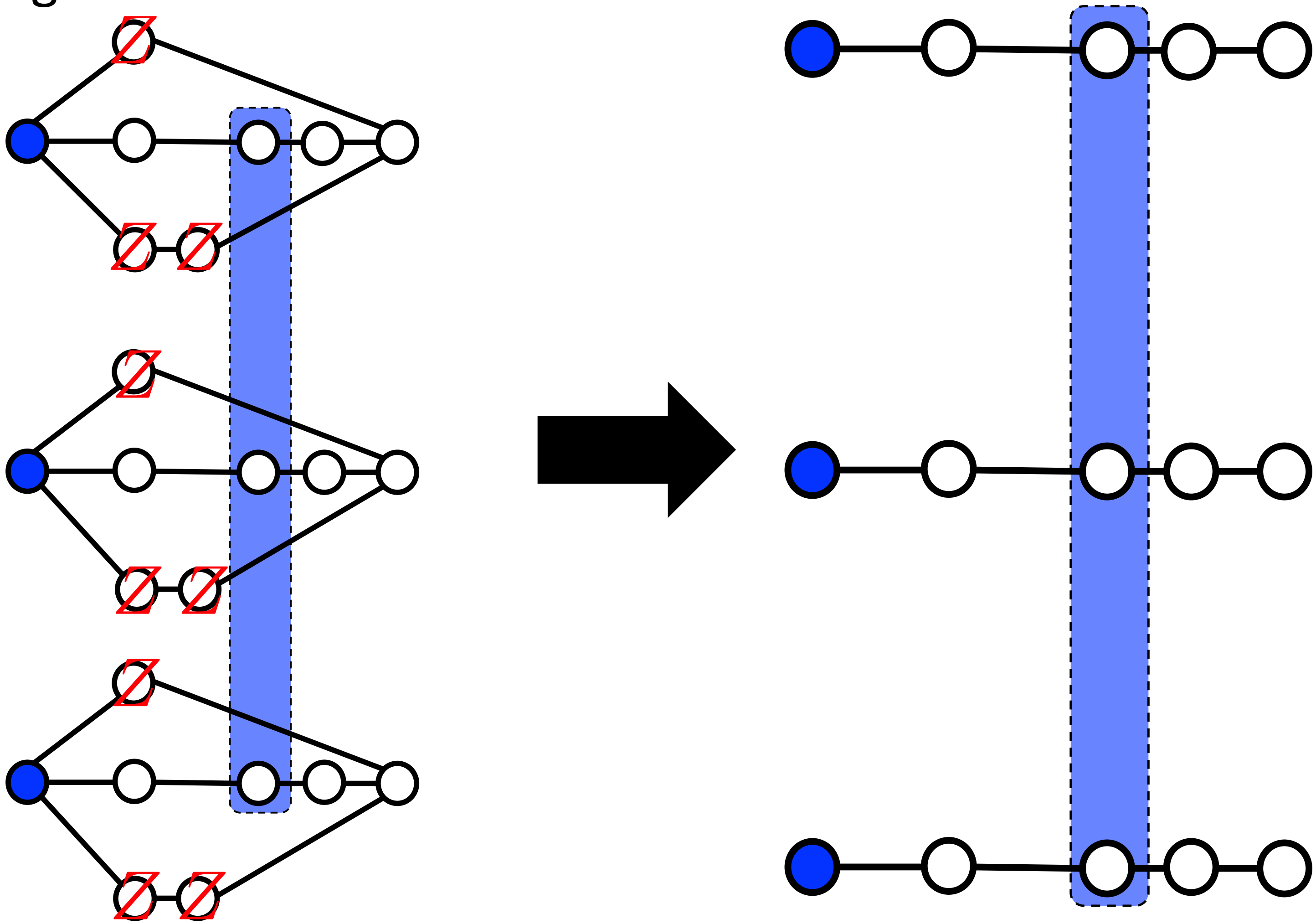

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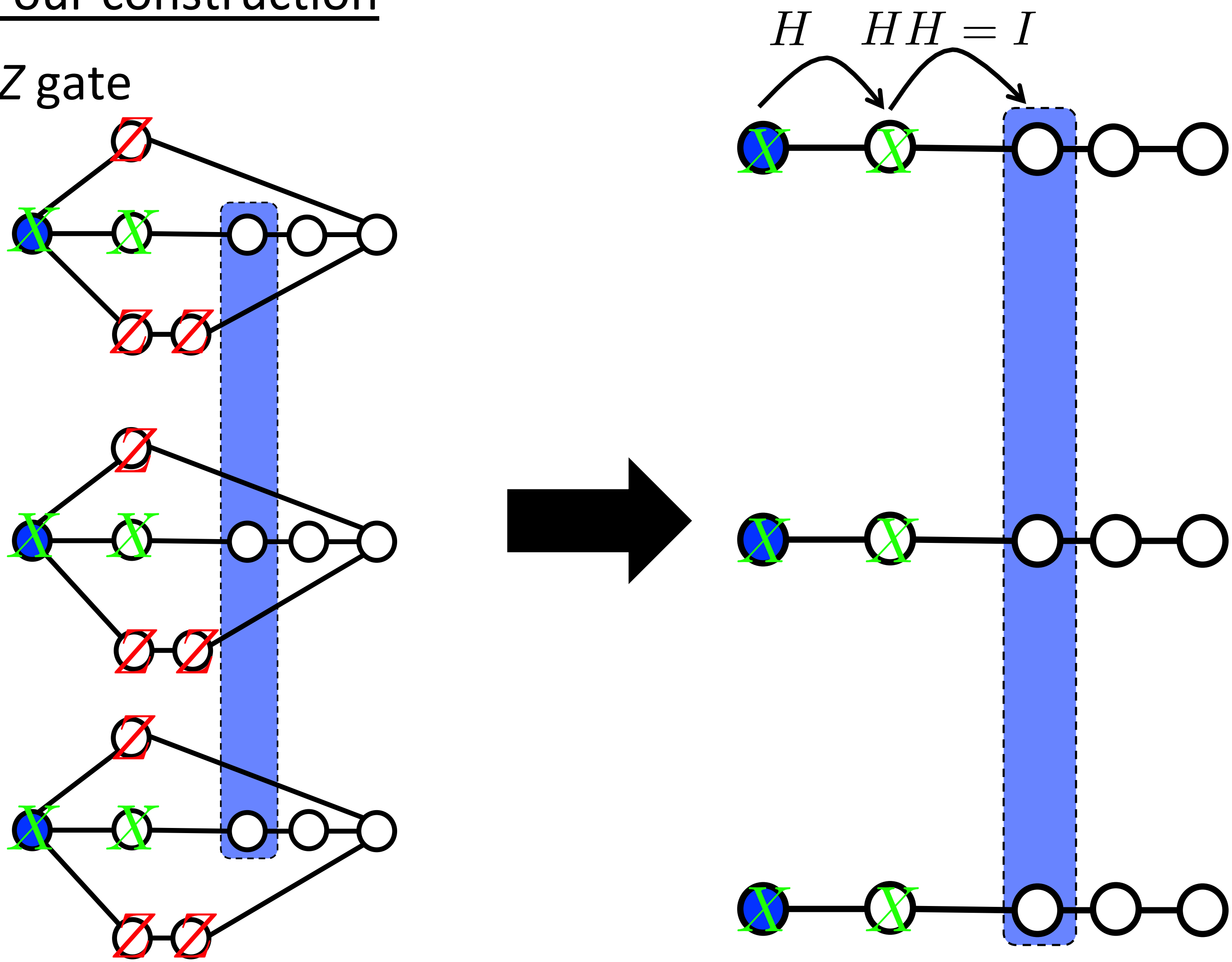
- *CCZ* gate



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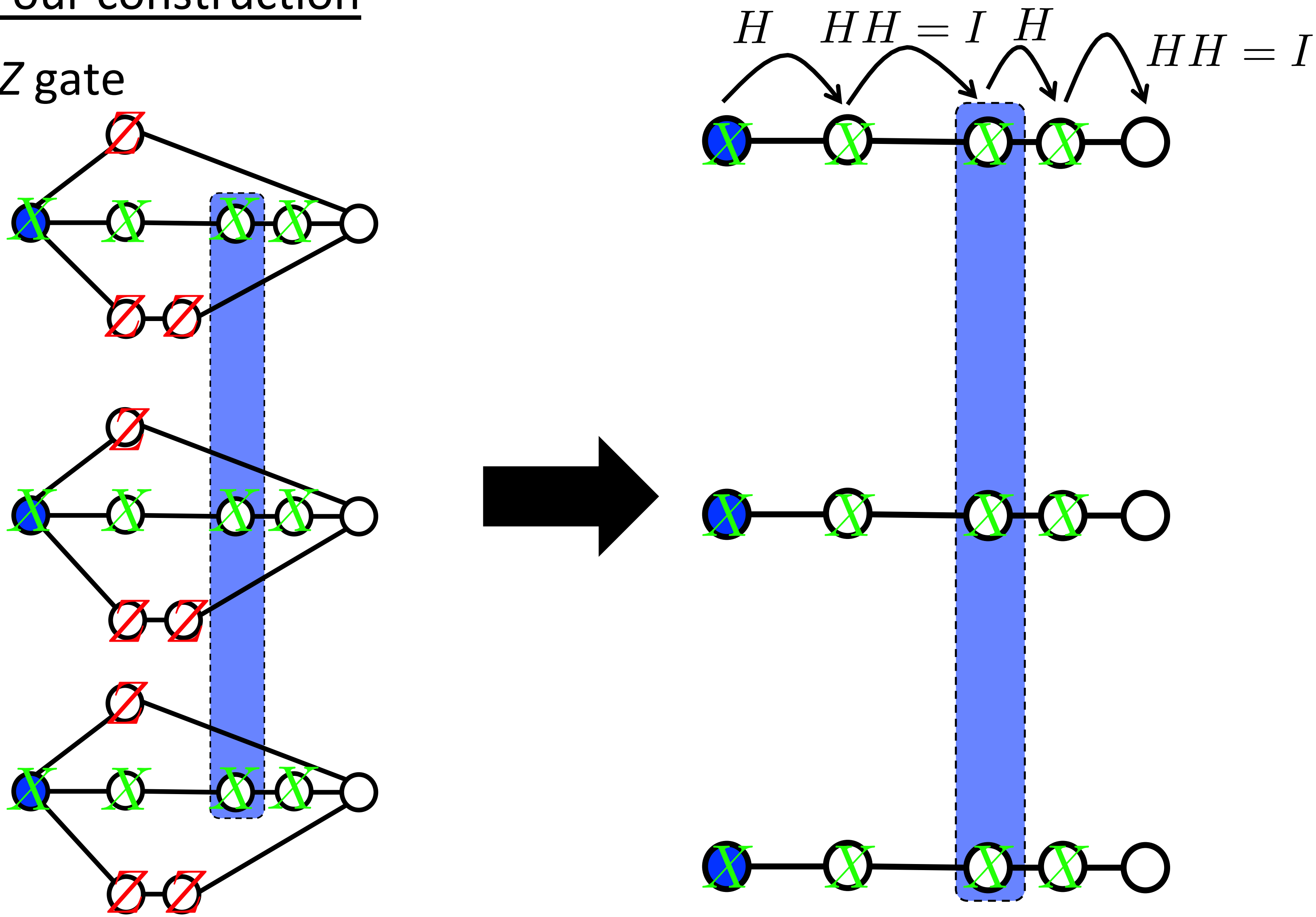

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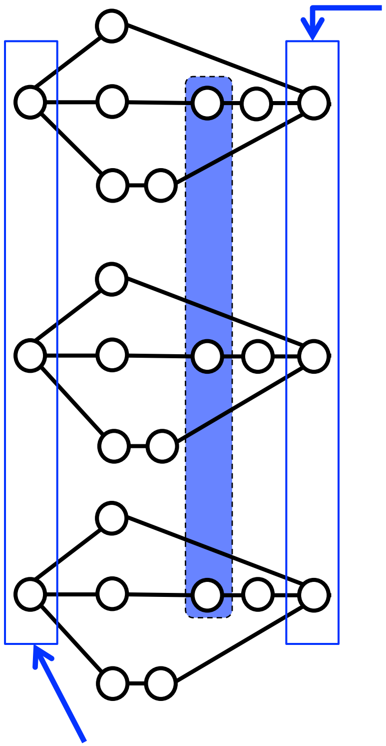
Idea of our construction

- One-depth quantum computing on three input qubits

Three output qubits



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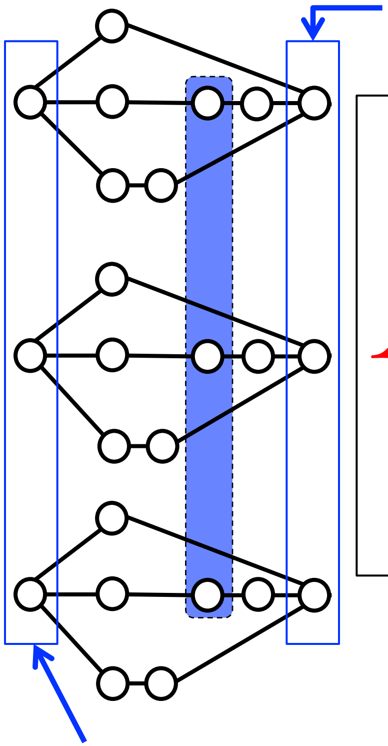

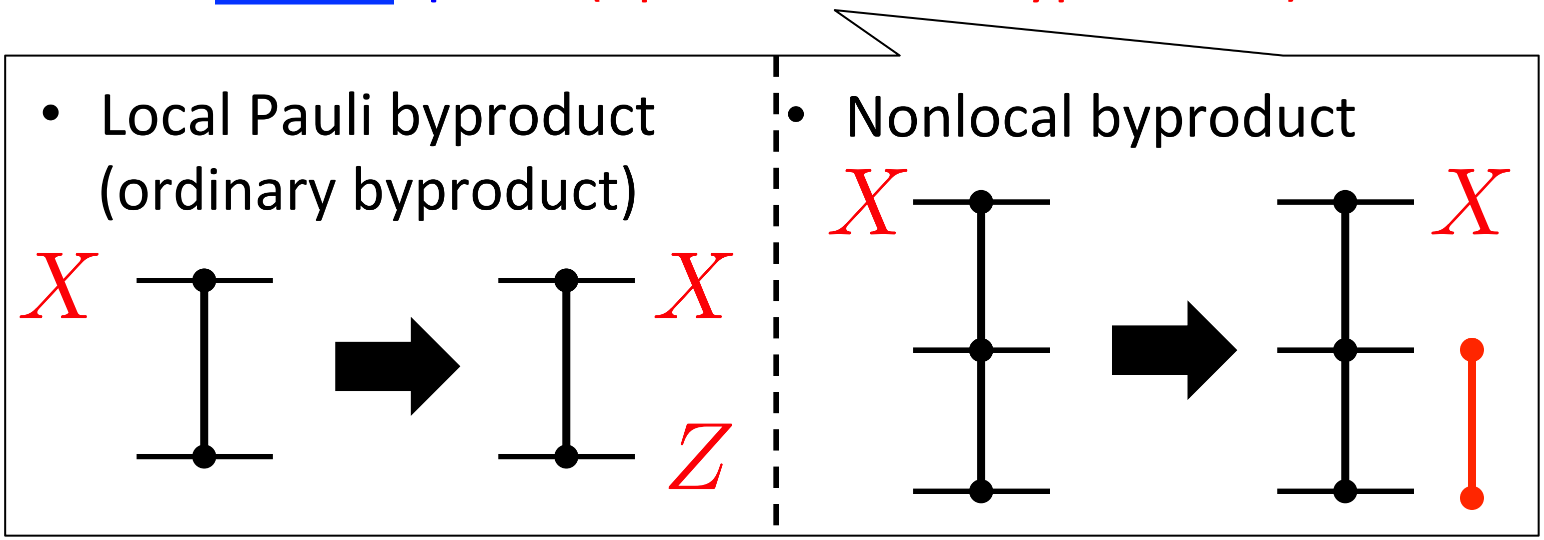

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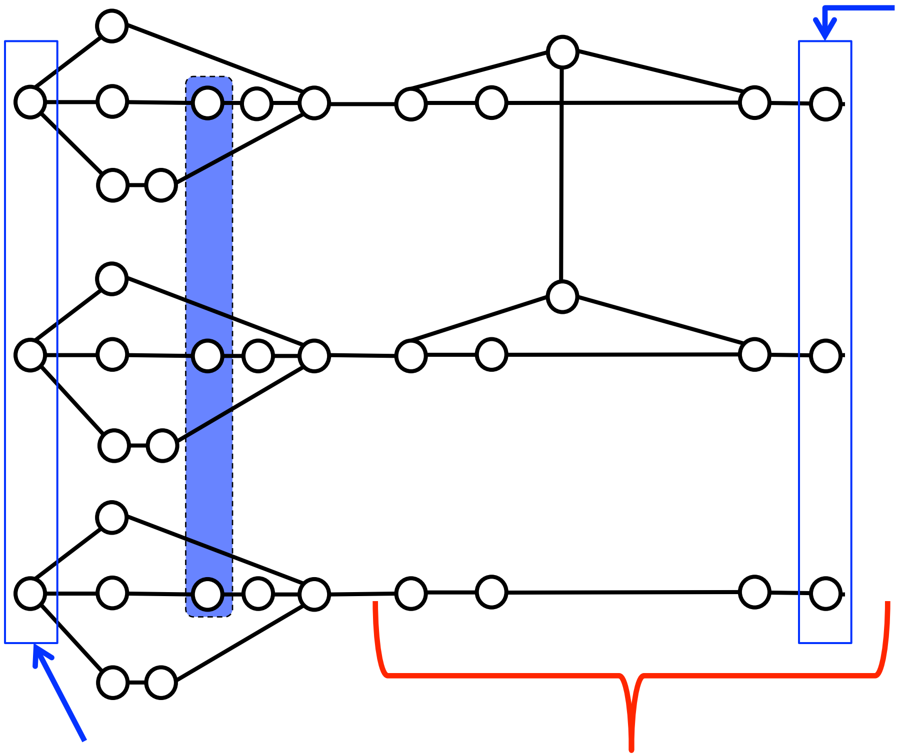

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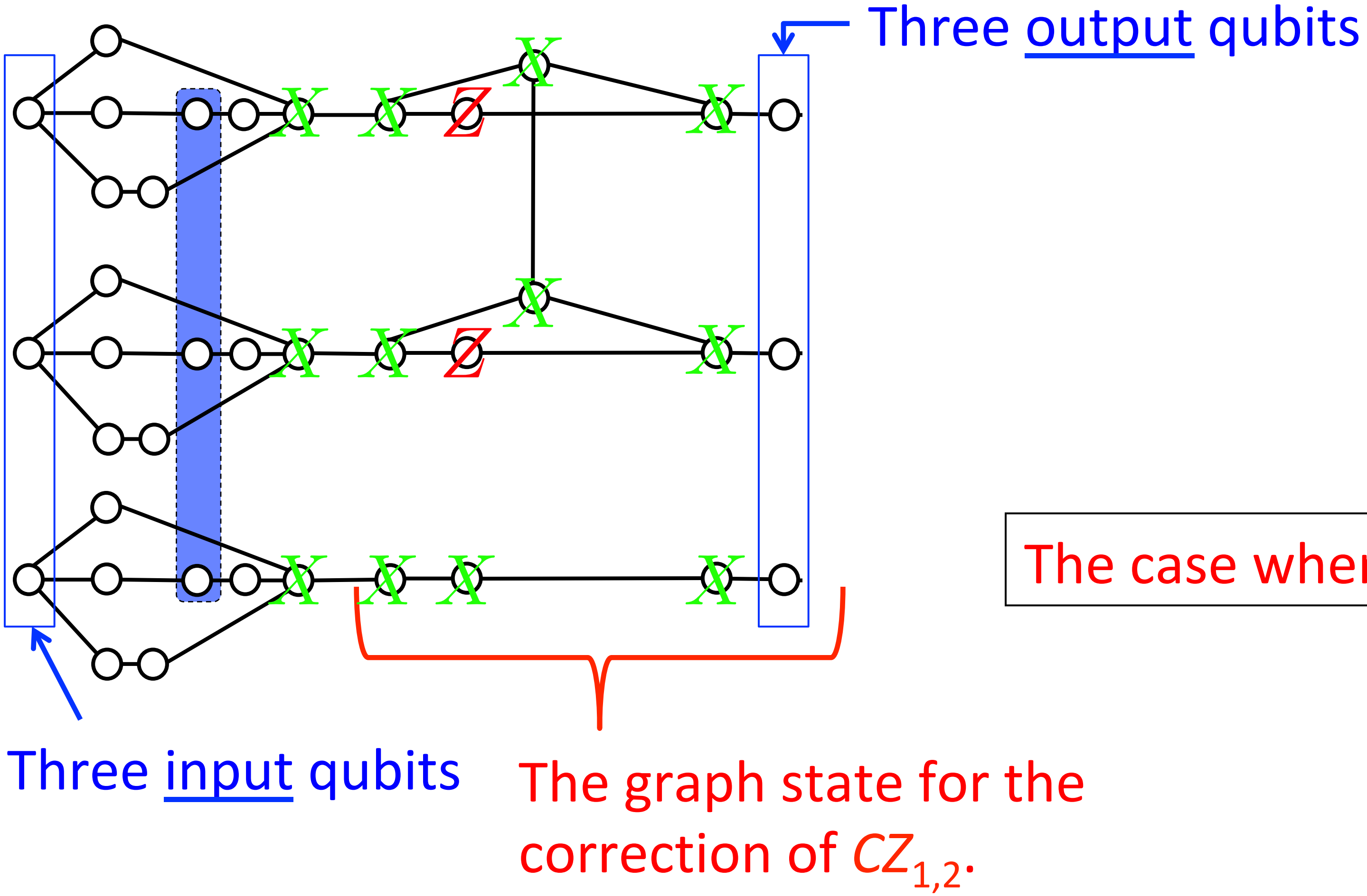

The case where $CZ_{1,2}$ occurs

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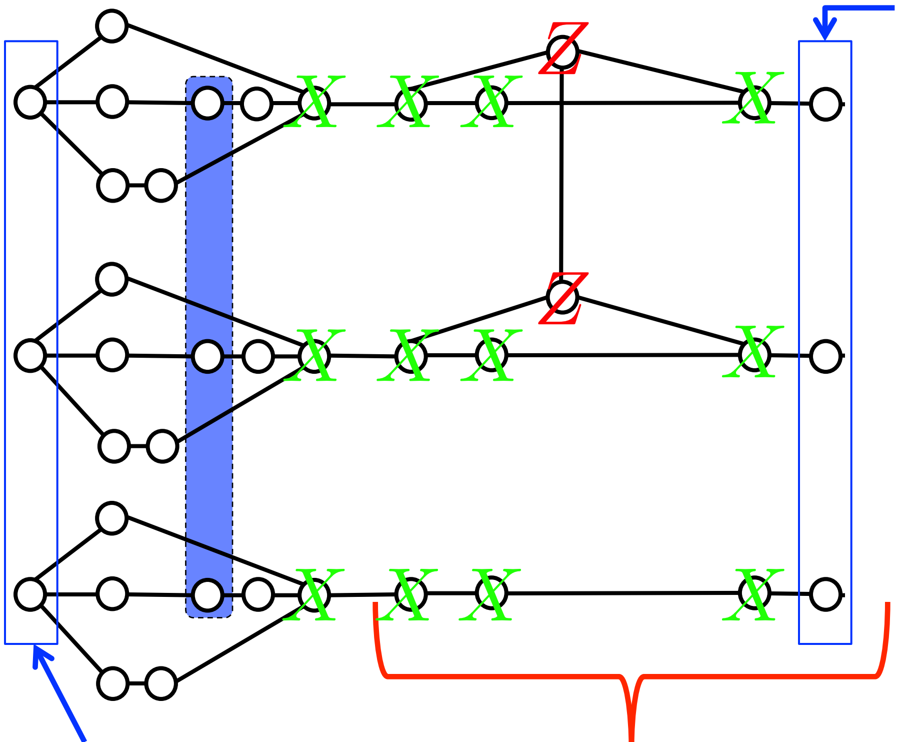

Three input qubits

The graph state for the correction of $CZ_{1,2}$.

The case where $CZ_{1,2}$ does not occur

Our universal resource state

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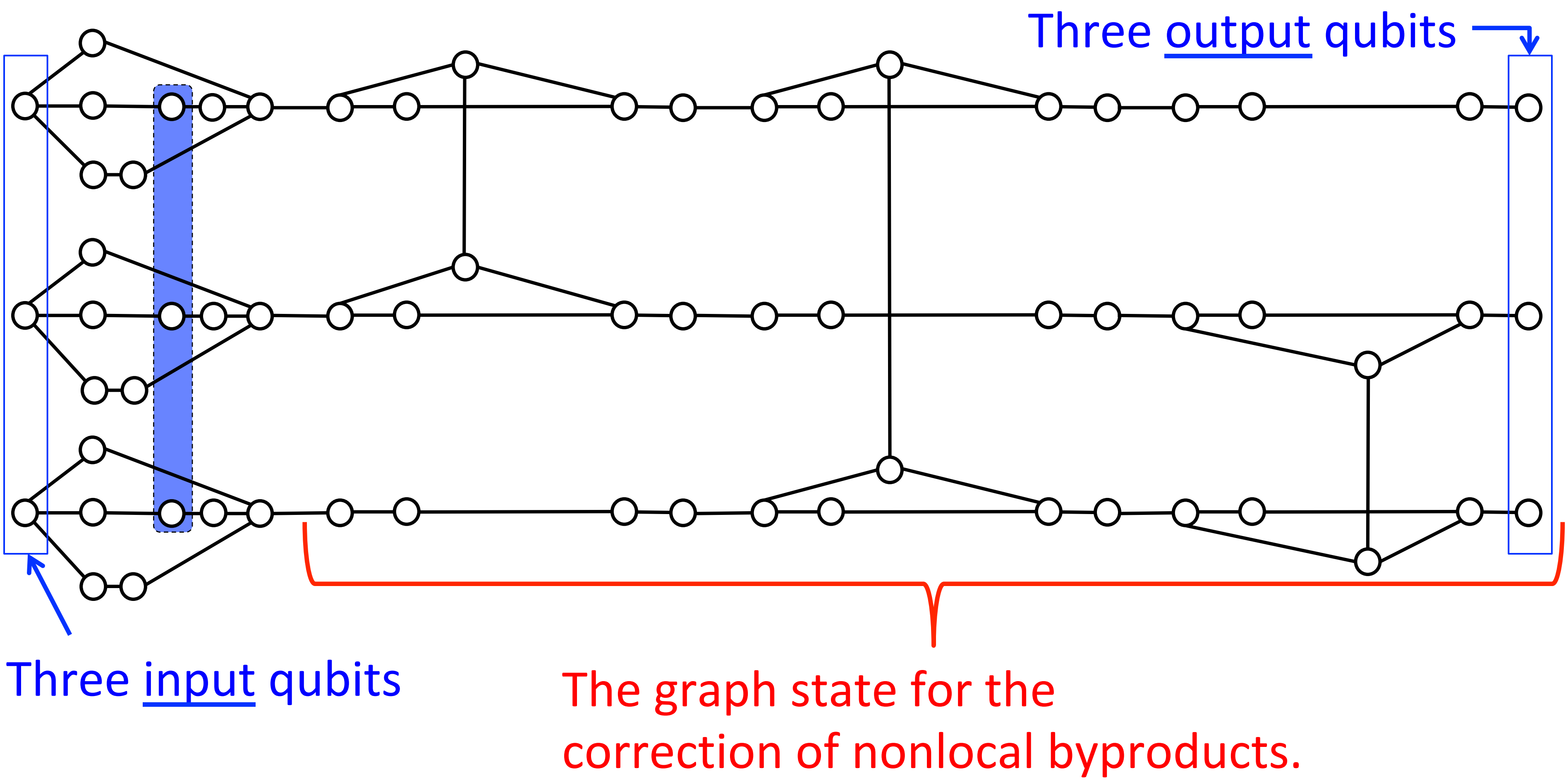

Conclusion

arXiv:1809.07552

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