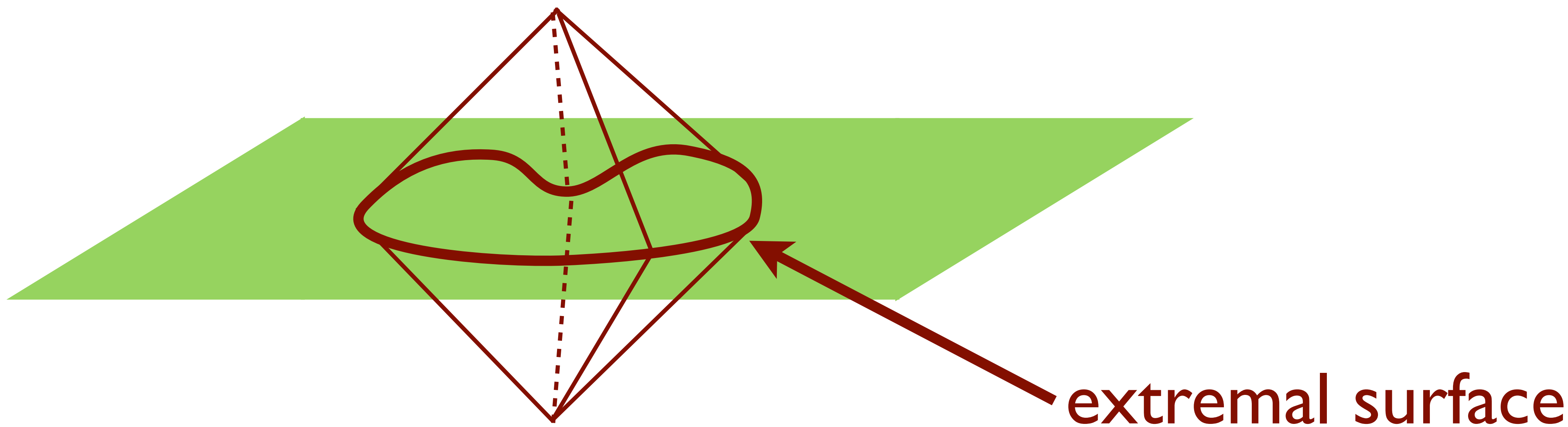
Superselection Sectors of Gravitational Subregions

Joan Camps
UCL

1810.01802
1905.10121

Main point

- Unlike for other systems, the quasilocal parts of the gravitational field are always* bounded by surfaces of stationary area



* I consider only pure gravity: GR without matter

Simple argument

JC 1905.10121

- Quasilocal part: degrees of freedom bounded by a surface σ
- To tell which dofs have been excited inside σ under a change of g , we need σ across $\{g\}$
- Quantumly: $\psi[g] \approx$ "δ" $[g - \bar{g}] + \cdots$
- A generic σ in $\bar{g}$ is described by $X^{\mu}_{\bar{g}}(\sigma)$. In $\{g\}$?
- It extends naturally if defined by its local geometry (otherwise it is not 'just a surface')

Symplectic form

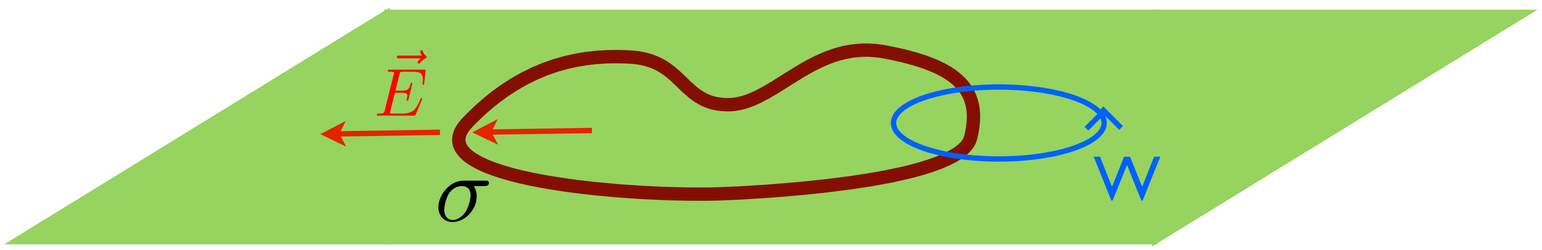
- Basic object in phase space (= space of initial data, space of solutions to the eoms)

$$W(\delta_1\phi, \delta_2\phi) = -W(\delta_2\phi, \delta_1\phi)$$

- A gauge, non-degree of freedom is such that it is a null direction and a symmetry of W
- Imaginary subregions do not introduce dofs
- We then demand: $W\left(\delta g, \pounds_\zeta \bar{g}\right) = \delta H_\zeta = 0$ locally

Symplectic photon

Casini, Huerta, Rosabal
Donnelly, Freidel
Casini, Huerta, Magan, Pontello
Soni, Trivedi
many others...



- For a gauge transformation $\delta_\varepsilon A = d\varepsilon$

$$W(\delta_\varepsilon A, \delta A) = \int_\sigma d^2x\, \varepsilon\, \delta E^\perp$$

- Recall algebraic conclusion

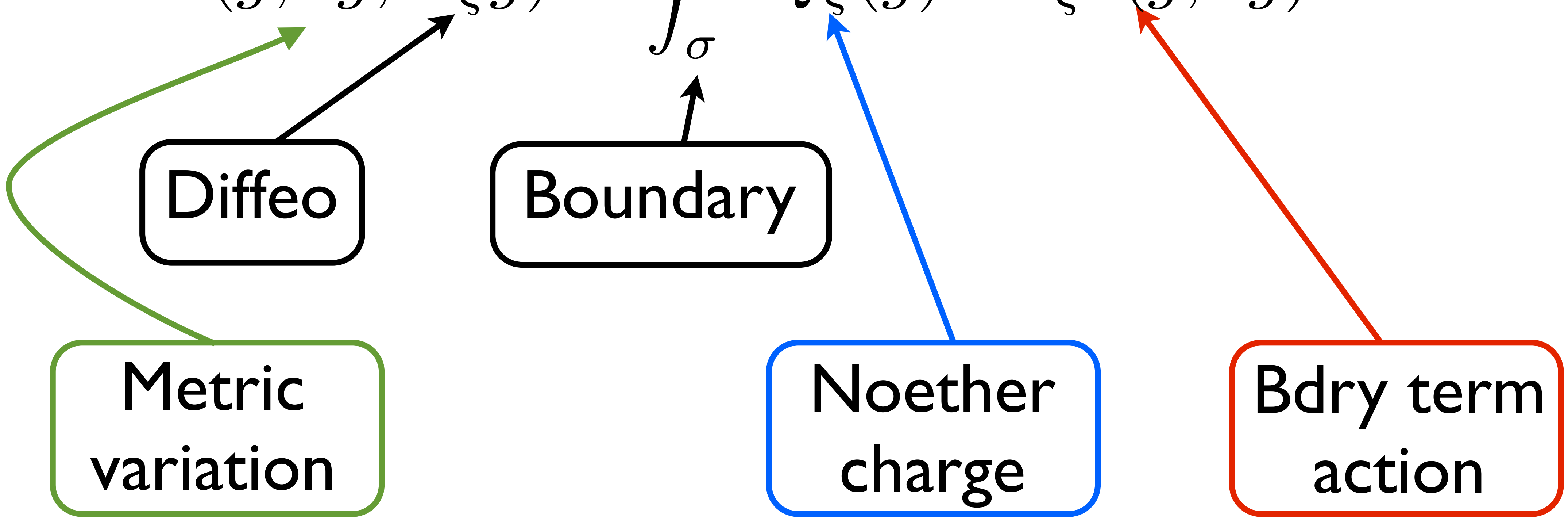
$$\rho = \bigoplus_{\{E_\sigma^\perp\}} p_{E_\sigma^\perp}\, \rho_{E_\sigma^\perp}$$

Symplectic graviton

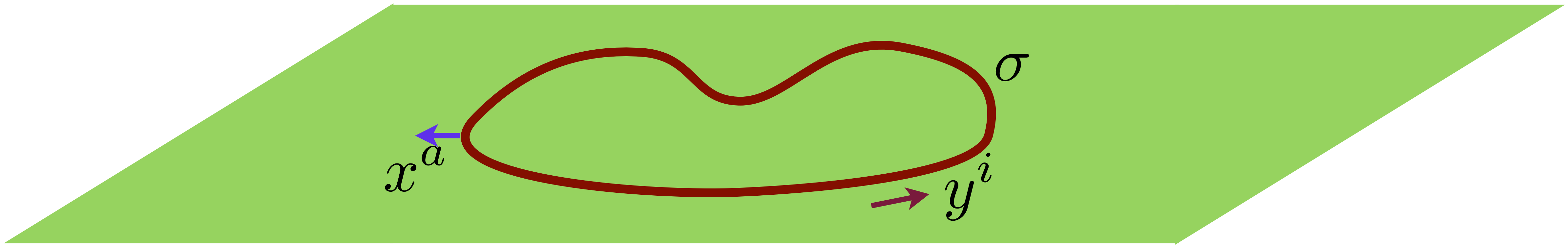
Hollands, Wald
Jafferis, Lewkowycz, Maldacena, Suh

- Symplectic form on a diffeomorphism

$$W(g;\delta g,\pounds_\zeta g)=\int_\sigma \delta Q_\zeta(g)-i_\zeta\theta(g;\delta g)$$



Notation



Induced metric

$$h_{ij} = e^{2\Omega} \bar{h}_{ij}$$

$$\det \bar{h} = 1$$

$$K_{ija} = \frac{1}{D-2} K_a h_{ij} + e^{2\Omega} \bar{K}_{ija}$$

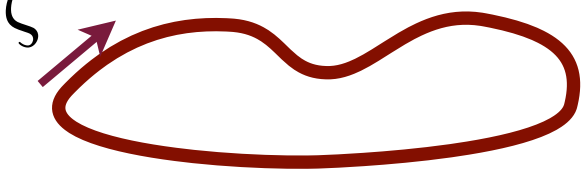
$$h^{ij} \bar{K}_{ija} = 0$$

Extrinsic curvature

D spacetime dimensions

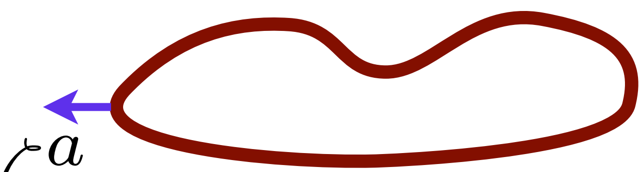
Types of non-trivial diffs

- Boundary diffs



- Boosts

- Translations



Ham	Null
Y	$\delta \bar{h}_{ij} = 0$
Y	$\delta A = 0$
???	???

$$W(g; \delta g, \pounds_{\zeta^a} g) = \frac{1}{16\pi} \int_\sigma \left[\frac{2}{D-2} \zeta^b \epsilon^a{}_b \, \delta(K_a \, \epsilon_h) \right.$$

$$\left. +2 \left(\frac{D-3}{D-2} \delta K_a + \frac{1}{2} \bar{K}^{ij}{}_a \, \delta \bar{h}_{ij} \right) \zeta^b \epsilon^a{}_b \, \epsilon_h \right]$$

Graviton bdry conds.

$$W(g;\delta g,\pounds_{\zeta^a} g) = \frac{1}{16\pi}\int_\sigma \Bigg[\frac{2}{D-2}\zeta^b \epsilon^a{}_b\,\delta(K_a\,\epsilon_h) + 2\left(\frac{D-3}{D-2}\delta K_a + \frac{1}{2}\bar{K}^{ij}{}_a\,\delta\bar{h}_{ij}\right)\zeta^b\epsilon^a{}_b\,\epsilon_h\Bigg]$$

- Most natural: fix $K^a = 0$ and $\bar{K}^{ij}{}_a\delta\bar{h}_{ij} = 0$
- Superselection sectors labelled by $\delta\bar{h}_{ij}$, such that

$$\bar{K}^{ij}{}_a\delta\bar{h}_{ij} = 0$$

- Examples: Biff surfaces, HRRT surfaces

Intepretation of BCs

$$\rho_\Sigma = \bigoplus_{\{\delta\bar{h}_{ij} | \bar{K}^{ija}\,\delta\bar{h}_{ij}=0\}} p_{\delta\bar{h}_{ij}}\,\rho_{\delta\bar{h}_{ij}}$$

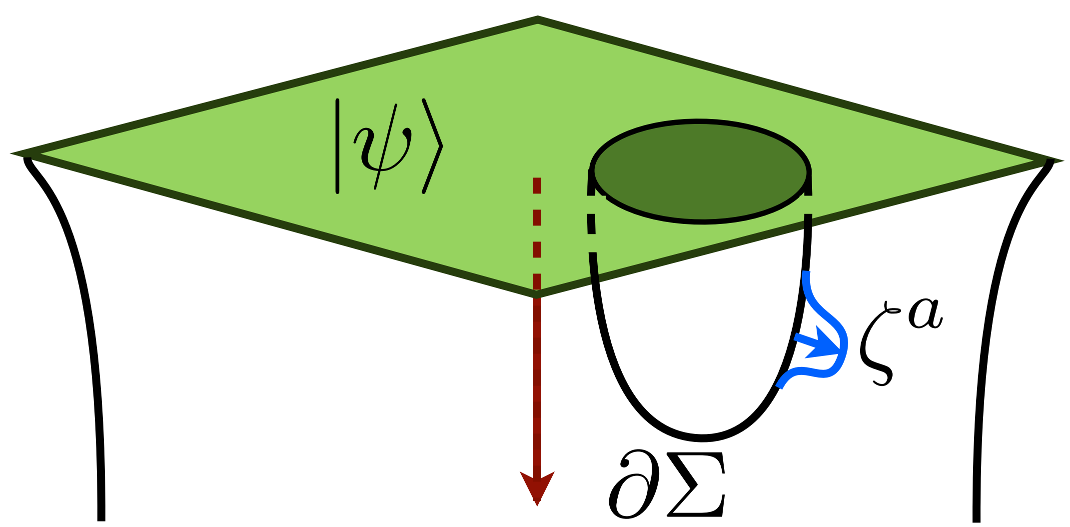
- $K^a = 0$ is a good subregion: It is diff-invariant
- $\bar{K}^{ij}{}_a \delta\bar{h}_{ij} = 0$ discards $\delta\bar{h}_{ij}$ achievable by a displacement (gauge transformation):

$$\delta\bar{h}_{ij} = 2\bar{K}_{ija}\zeta^a$$

- Locally infinitesim. deformable extremal surfaces?
- Jacobi fields?

Deformability of $\partial\Sigma$.

- Claim: Generically, there are inf. nearby extremal surfaces reachable with ***local*** translations



- Jacobi equation

$$\delta_\zeta K^a = -D^2\zeta^a + V^a{}_b\zeta^b = 0$$

- When

$$V^a{}_b = -\bar{K}_{ij}{}^a\bar{K}^{ij}{}_b \approx -\bar{K}^2\delta^a{}_b$$

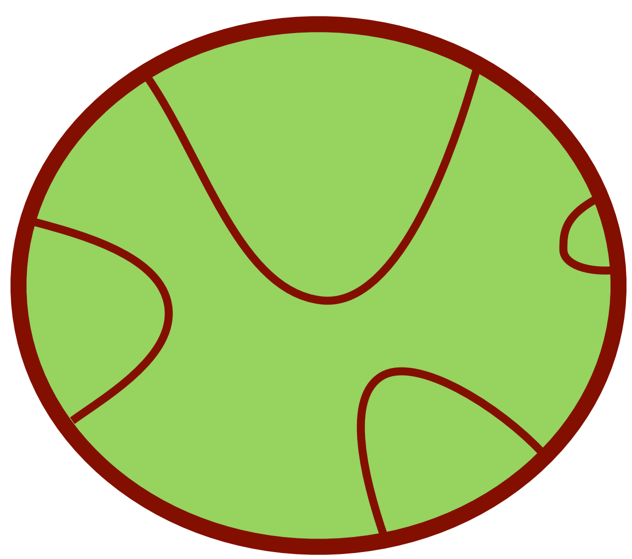
$$\boxed{\zeta^a = c^a\, J_0\left(\sqrt{\bar{K}^2}\,|y|\right)}$$

Recap: Graviton

- Principle: Subregions are imaginary separations: Do not introduce degrees of freedom to the system.
- Symplectic reduction: diffs of subregions should annihilate and be symmetries of W. Non-trivial.
- Boundaries of subregions are extremal surfaces.
- Superselection sectors labeled by $\bar{h}_{ij}$ on $\partial\Sigma$, such that $\bar{K}^{ij}{}_a \delta\bar{h}_{ij} = 0$. This discards $\delta\bar{h}_{ij} = \bar{K}_{ija}\zeta^a$, that take us to nearby extremal surfaces.

Outlook: (i) Holography

- The scarcity of subregions would explain why gravity is holographic from a bulk point of view
- We normally have $e^{N_{\rm dof}} \propto \mathrm{Vol}$, except for gravity
- If the correct identification is $N_{\rm dof} \propto N_{\rm subregions}$
- We then have, for gravity $N_{\rm dof} \propto N_{\partial}$, so $e^{N_{\rm dof}} \propto \mathrm{Area}$



(ii) Black Hole entropy

- With Euclidean methods, quantum corrections to BH entropy can be accounted for precisely for extremal BHs in string theory.
- Mismatch for Schwarzschild black hole:

$$S_{\text{Sen}} = \frac{A}{4G} + \frac{77}{90} \log \frac{A}{4G} + \cdots$$

- And LQG:

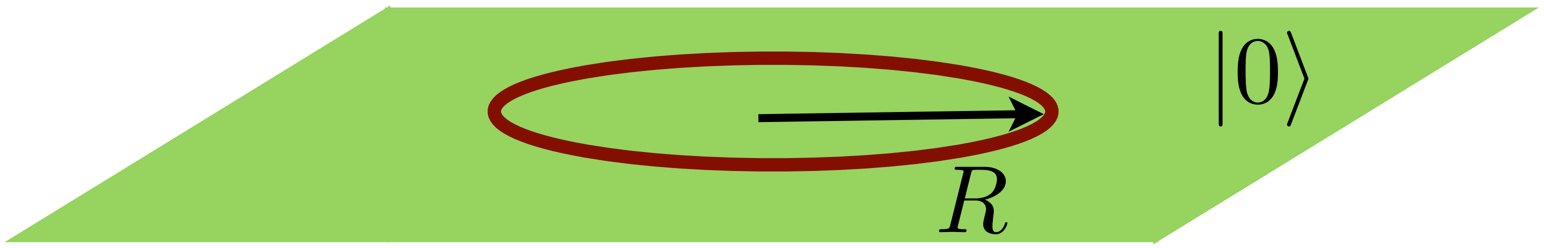
$$S_{\text{LQG}} = \frac{A}{4G} - \log \frac{A}{4G} + \cdots$$

- Real-time entanglement-across-horizon picture?

Summary

- Regions are subtle with gauge symmetry
- Important for, eg, Q corrections to BH entropy
- The phase space of a gravitational subregion is gauge invariant if the boundary is extremal surface
- Extremal surfaces are locally deformable?

Photon ambiguities



- Choice of algebra affects what you get
- EE of the photon, in the vacuum, across a sphere, with electric bcs

$$S_{\rm el} = \# \frac{A}{\epsilon^2} - \frac{31}{45} \log \frac{R}{\epsilon} + \dots$$

Donnelly, Wall

- With another prescription

$$S_{\rm M.I.} = \#' \frac{A}{\epsilon^2} - \frac{16}{45} \log \frac{R}{\epsilon} + \dots$$

Casini, Huerta

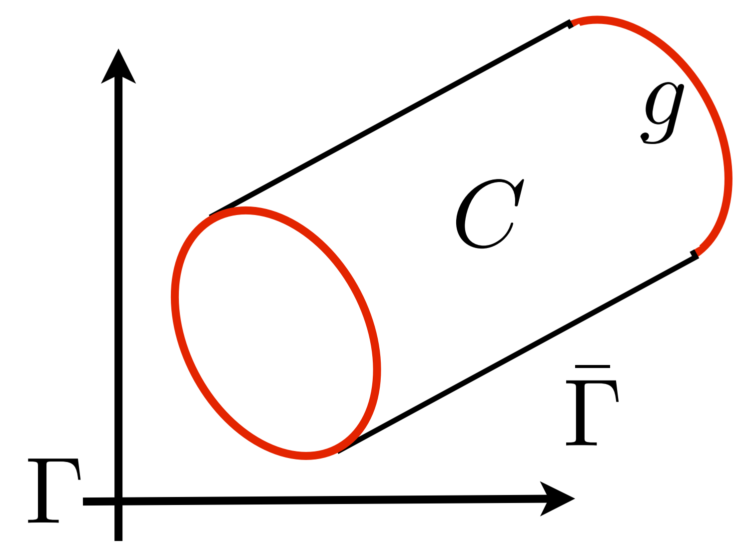
Symplectic reduction

Lee, Wald

- It is common to embed the physical phase of a gauge thy $\bar{\Gamma}$ in a larger space Γ , eg $\{A_i(x), E^j(x)\}$
- On a constraint surface C in Γ, eg $\nabla_i E^i = 0$, the symplectic form has null directions g

$$I_g W|_C = 0$$

- Can be dispensed via 'symplectic reduction' if this is also a symmetry: $L_g \, W|_C = 0$
- C : Fiber bundle over the physical $\bar{\Gamma}$



Comments

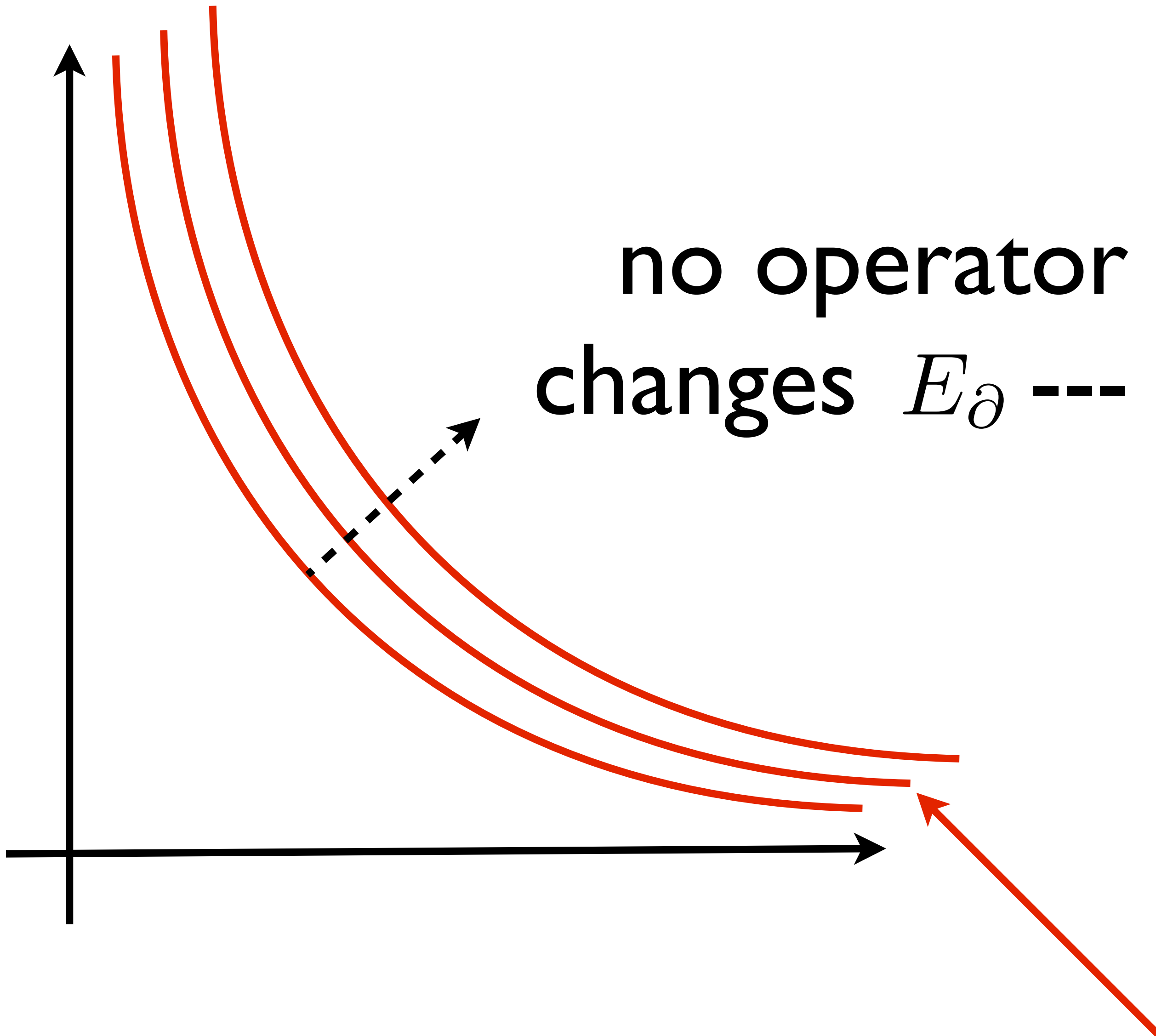
- Since closedness is preserved under restriction, one normally expects trivially

$$L_g \, W|_C = \delta I_g W + I_g \delta W = 0$$

- Failure of g to be hamiltonian on C, $I_g W \neq \delta H_g$ would indicate that restriction $W|_C$ has not been done properly

Cartoon

no operator in the algebra changes E_∂ --- there is no A_∂ .



$\delta E_\partial = 0$