Effective Field Theory of Large-c CFTs

Felix Haehl
UBC Vancouver (—> IAS)

Based on
1712.04963, 1808.02898 with **M. Rozali**,
and work in progress with **W. Reeves** & M. Rozali

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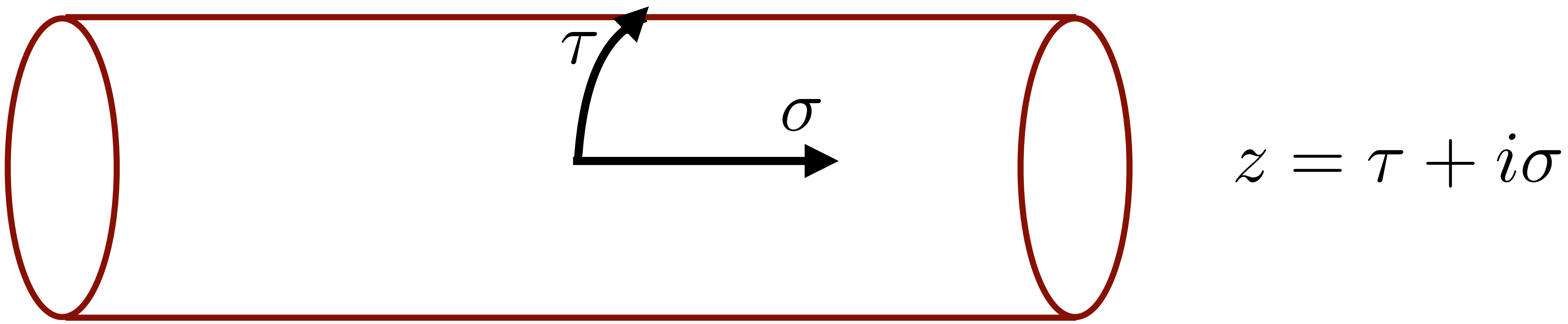
poster!

Introduction

Basic idea

- Consider 2d CFT at finite temperature



- Conf. symmetry $(z, \bar{z}) \to (f(z), \bar{f}(\bar{z}))$ is spontaneously broken
- I want to study the *Goldstone mode* associated with this effect
- In the "holographic regime" (large c etc.) there is a systematic *effective field theory* for this mode

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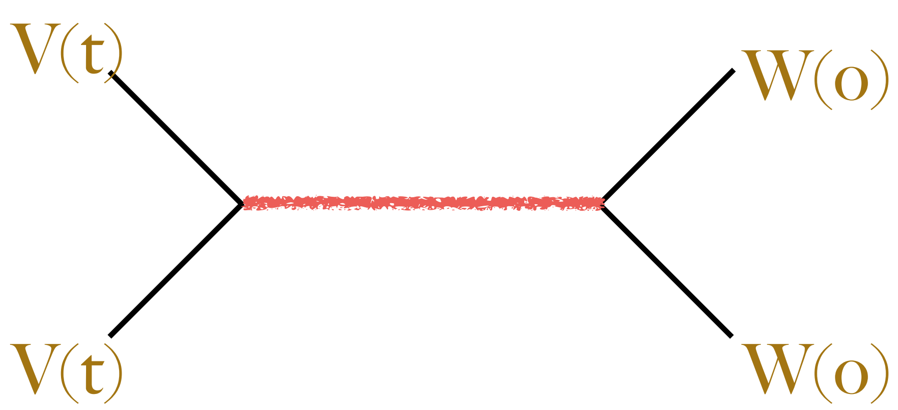
  - Describes universal physics of CFTs associated with *energy-momentum conservation* ("gravity")
  - For example: effective field theory description for universal aspects of…

    … OTOC observables, related to *quantum chaos*

    … *conformal blocks,* kinematic space operators, …



Basics

Reparametrization modes

- Consider 2d CFT at finite temperature and a small reparametrization $(z, \bar{z}) \to (z + \epsilon, \bar{z} + \bar{\epsilon})$

$$S_{CFT} \longrightarrow S_{CFT} + \int d^2z \;\left\{\bar{\partial}\epsilon \, T(z) + \partial\bar{\epsilon} \, \bar{T}(\bar{z})\right\}$$

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- For *conformal* transformations,

$$\bar{\partial}\epsilon = 0 = \partial\bar{\epsilon}$$

the associated conserved symmetry currents are $(J, \bar{J}) = (\epsilon T, \bar{\epsilon} \bar{T})$

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- Conformal symmetry is *spontaneously broken*

- Regard $(\epsilon, \bar{\epsilon})$ as the associated *Goldstone modes*

[Turiaci-Verlinde '16] [FH-Rozali '18]

- $(\epsilon, \bar{\epsilon})$ have an *effective action* determined by $\langle T_{\mu\nu} \cdots T_{\rho\sigma} \rangle$

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$$W_2 = \int d^2 z_1 \, d^2 z_2 \; \bar{\partial}\epsilon_1 \; \bar{\partial}\epsilon_2 \, \langle T(z_1) T(z_2) \rangle \; + \; (\text{anti-holo.})$$

fixed by conformal symmetry!
=> dynamics of $(\epsilon, \bar{\epsilon})$ is universal

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- The effective action is actually *local*

… because: $\bar{\partial}_1 \langle T(z_1) T(z_2) \rangle \sim \delta^{(2)}(z_1 - z_2)$

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$$W_2 = \frac{c\pi}{6} \int d\tau d\sigma\, \bar{\partial}\epsilon\, (\partial_\tau^3 + \partial_\tau)\epsilon \; + \; (\text{anti-holo.})$$

$(z = \tau + i\sigma)$

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- Euclidean propagator:

$$\langle \epsilon(\tau, \sigma)\epsilon(0,0) \rangle \sim \frac{1}{c} \, \sin^2\left(\frac{\tau + i\sigma}{2}\right) \log\left(1 - e^{-\text{sgn}(\sigma) i(\tau + i\sigma)}\right)$$

[FH-Rozali '18] [Cotler-Jensen '18]

- (Lorentzian "Schwinger-Keldysh" version is available)

Feynman rules

$$\langle \epsilon(\tau,\sigma)\epsilon(0,0)\rangle \sim \frac{1}{c}\ \sin^2\left(\frac{\tau+i\sigma}{2}\right)\log\left(1-e^{-\mathrm{sgn}(\sigma)i(\tau+i\sigma)}\right)$$

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- “Coupling” to pairs of other operators via reparametrization:

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle \ \longrightarrow\ [\partial f(x)\,\partial f(y)]^{\Delta}\ \langle \mathcal{O}(f(x))\,\mathcal{O}(f(y))\rangle \qquad f(x)=x+\epsilon(x)$$

$$= \langle \mathcal{O}(x)\mathcal{O}(Y)\rangle\left\{1+\ \Delta\left[\partial\epsilon(x)+\partial\epsilon(y)-\frac{\epsilon(x)-\epsilon(y)}{\tan\left(\frac{x-y}{2}\right)}\right]\ +\ (\text{anti-holo.})\right\}$$

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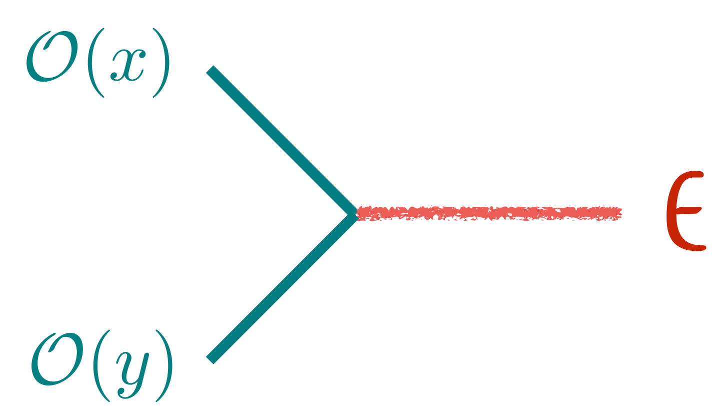

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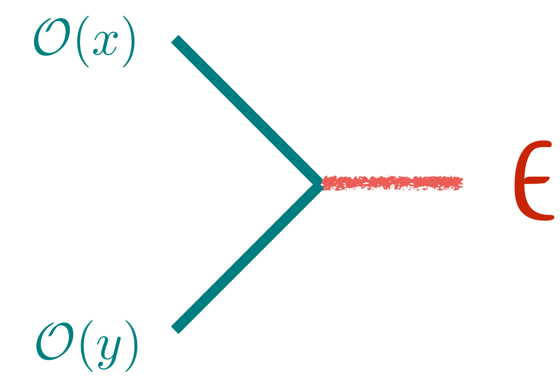

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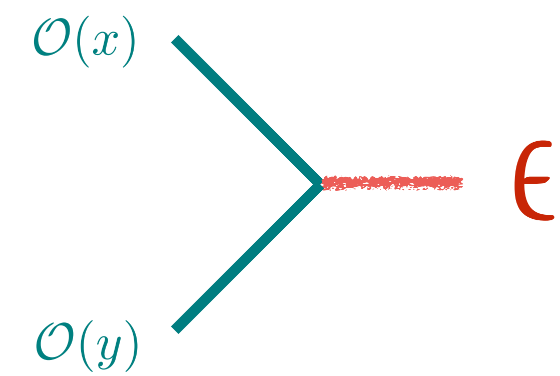

- “Feynman rules” for reparametrization Goldstone
- At *large c*, this gives a *systematic perturbation theory* of energy-momentum exchanges (“gravity channel”)

Applications

Out-of-time-order correlators

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Out-of-time-order correlators

- "Usual" QFT: time-ordered correlators (*TOCs*):

$$\langle W(t)W(t)V(0)V(0)\rangle_\beta \sim \langle WW\rangle\langle VV\rangle + \mathcal{O}(e^{-t/t_d})$$

dissipation time: $t_d \sim \frac{\beta}{2\pi}$

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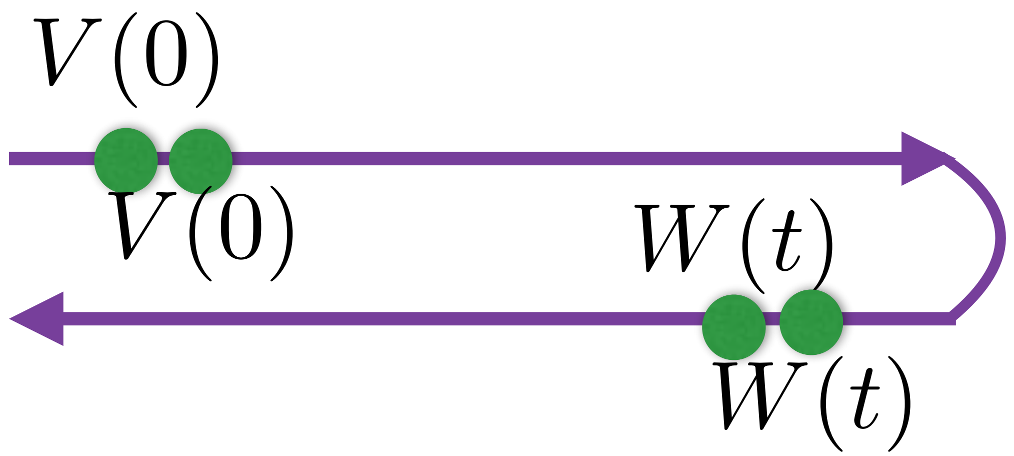

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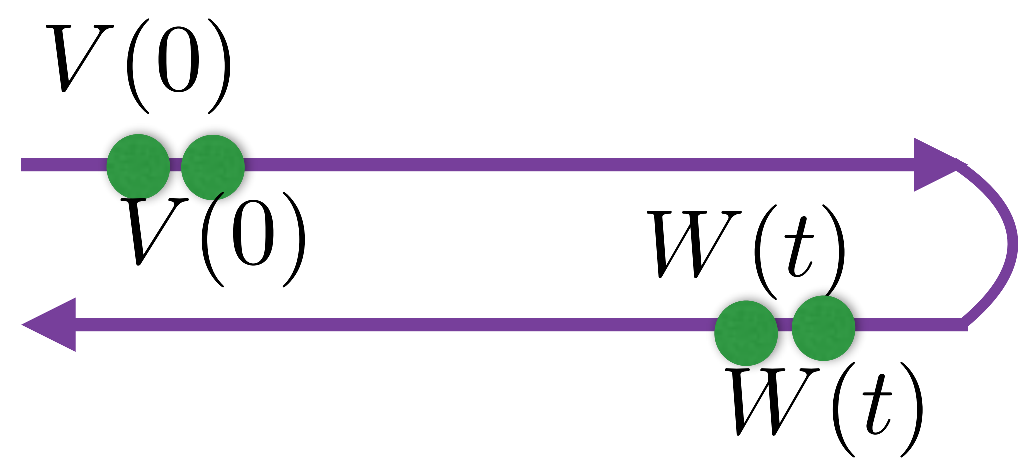

$$\langle W(t)V(0)W(t)V(0)\rangle_\beta$$
$$\sim \langle WW\rangle\langle VV\rangle \left[1 - \#\, e^{\lambda_L\,(t-t_*)}\right]$$

scrambling time: $t_* \sim \dfrac{\beta}{2\pi}\log N$

[Shenker-Stanford '13]
[Maldacena-Shenker-Stanford '15]
[Kitaev '15]
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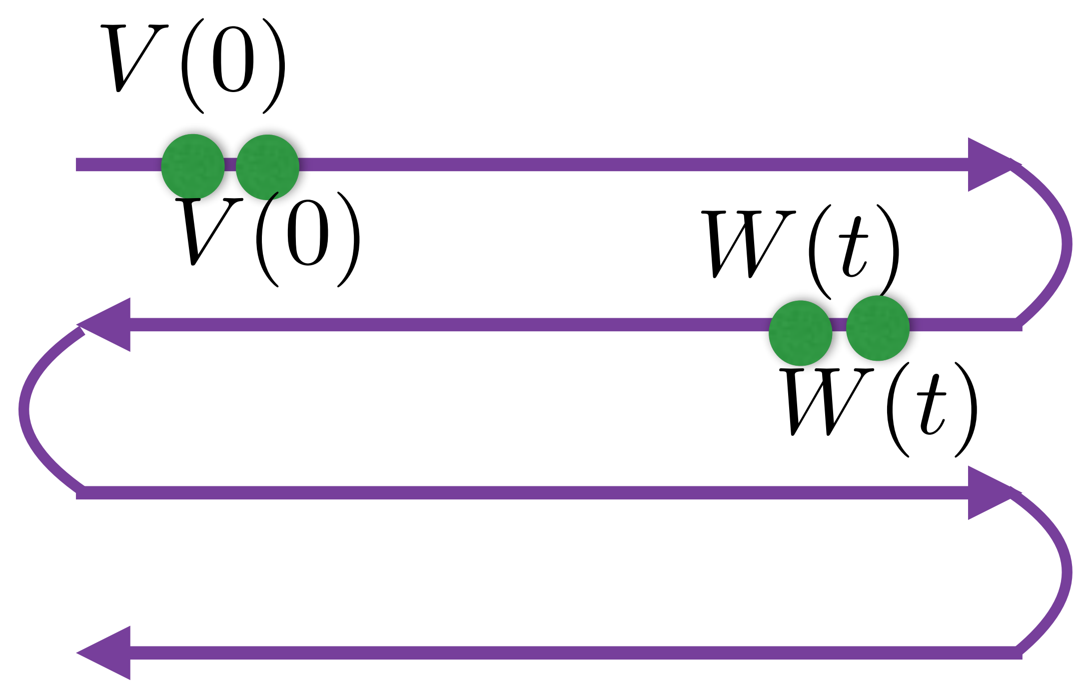

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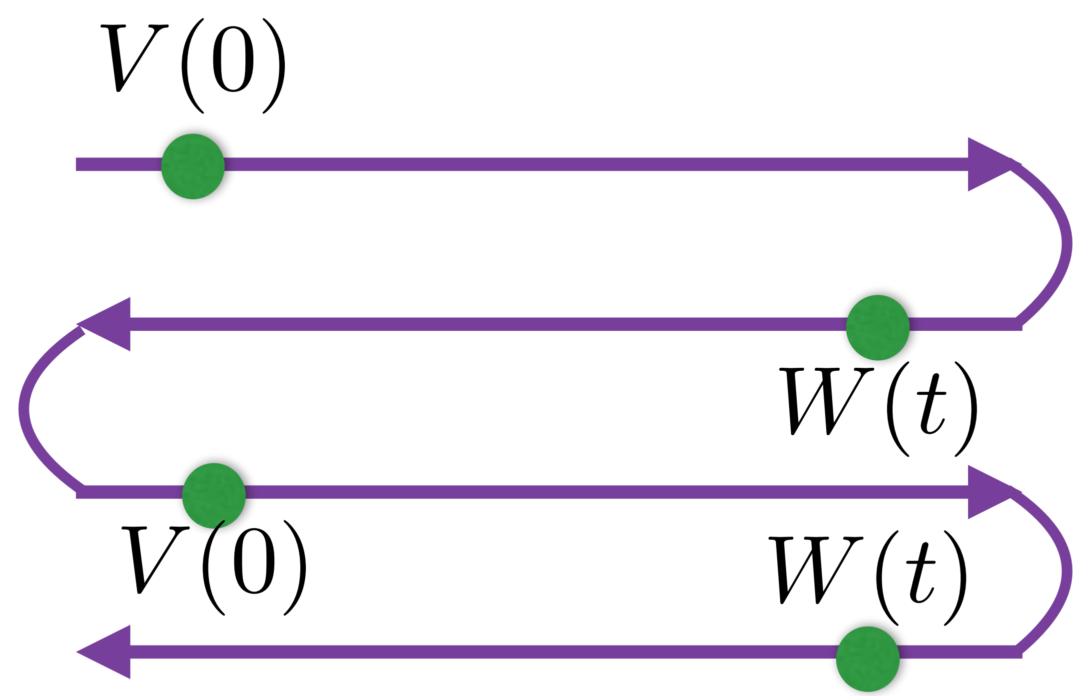

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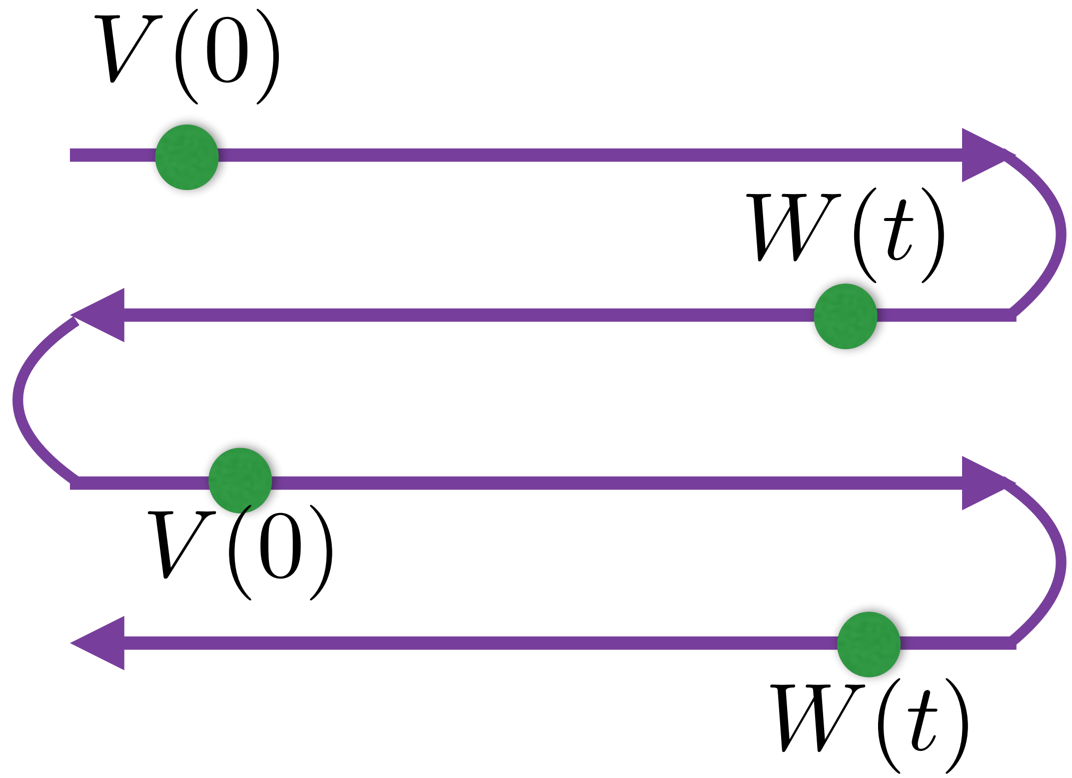

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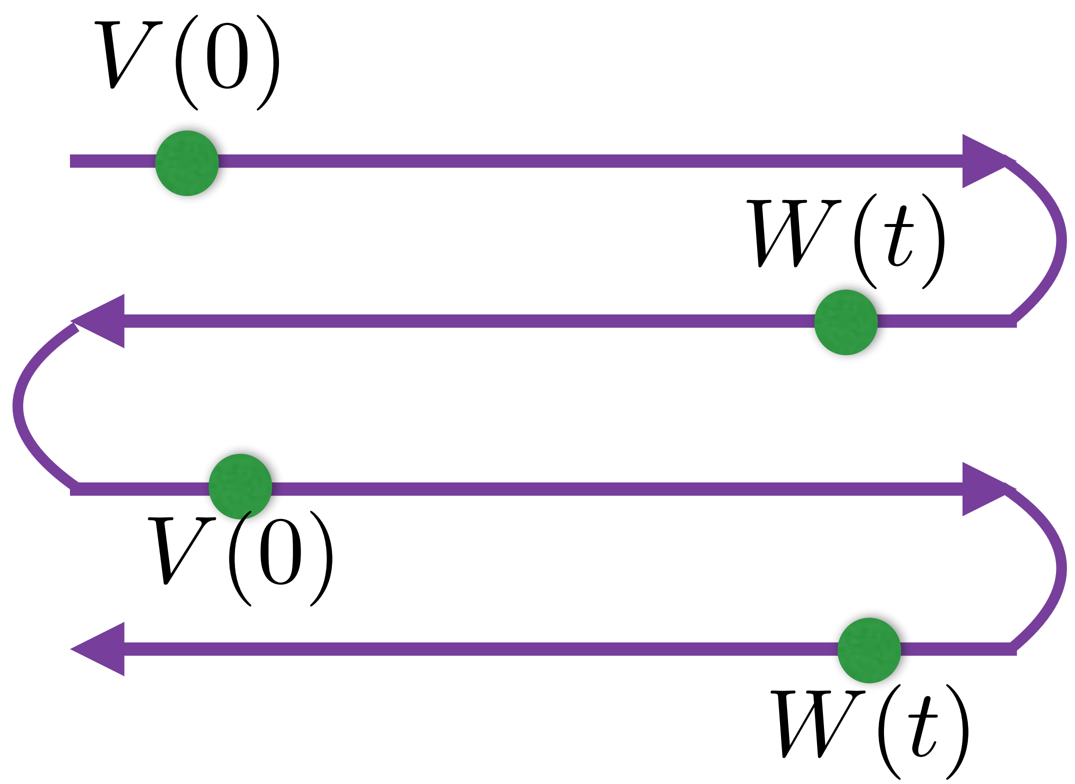

- $\lambda_L = 2\pi T$: Lyapunov growth is described by an exchange of the reparametrization mode:

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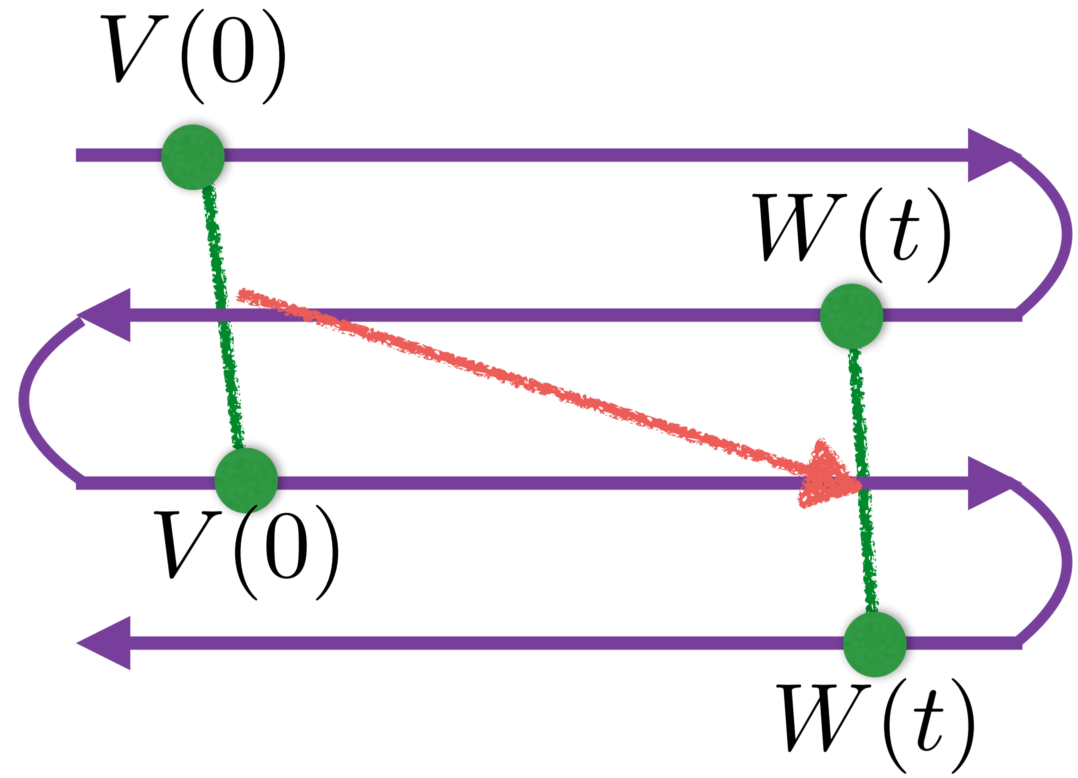

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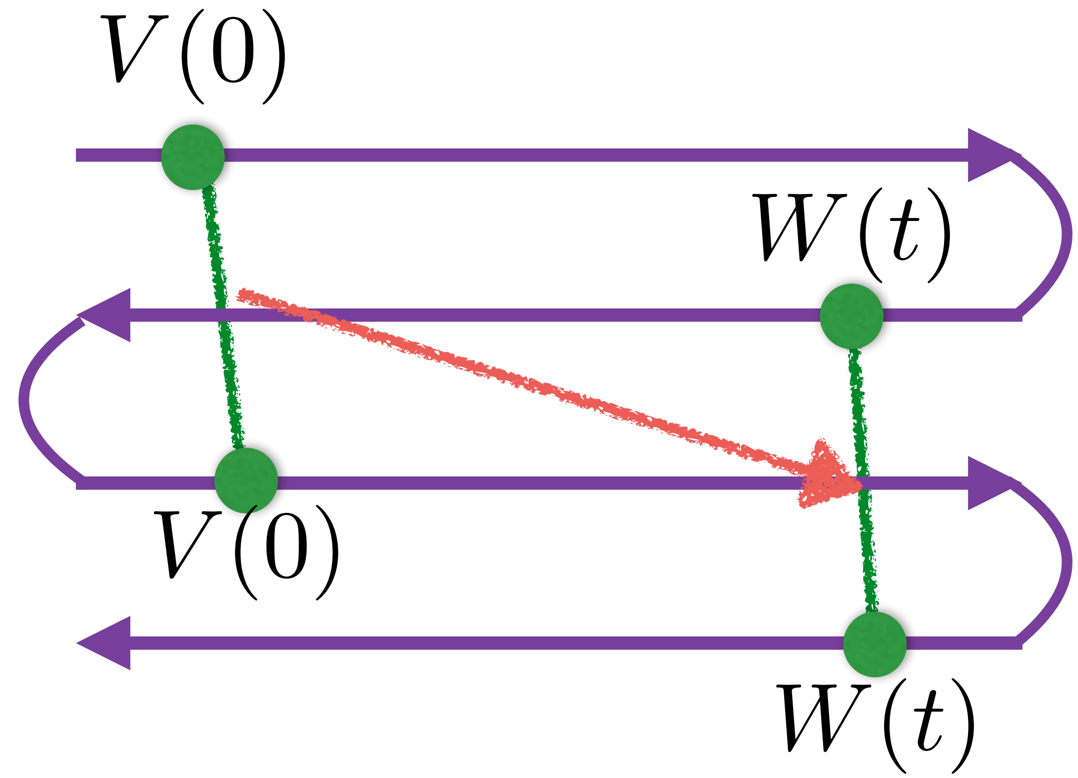

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- A *universal contribution to the OTOC*, described by the collective mode ϵ

[FH-Rozali '18]

[Blake-Lee-Liu '18]

2k-point OTOC

- Higher-point generalisation of OTOC: [FH-Rozali '17 '18]

$$F_{2k}(t_1, \ldots, t_k) = \frac{\left\langle V_1 [V_2, V_1] [V_3, V_2] [V_4, V_3] \;\; \cdots \;\; [V_k, V_{k-1}] V_k \right\rangle_\beta}{\langle V_1 V_1 \rangle \cdots \langle V_k V_k \rangle}$$

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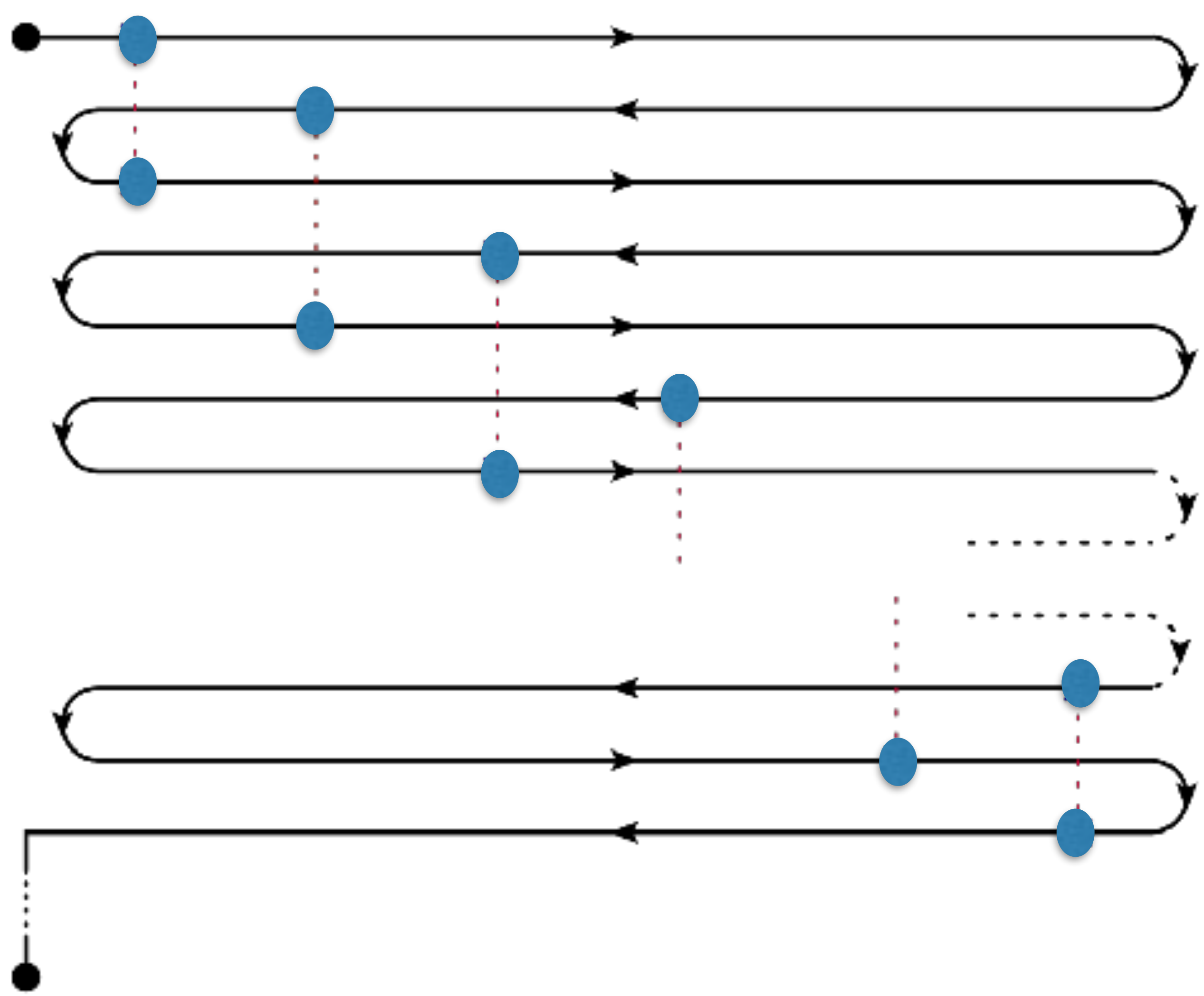
- *Maximally OTO*
- *Maximally "braided"* in Euclidean time



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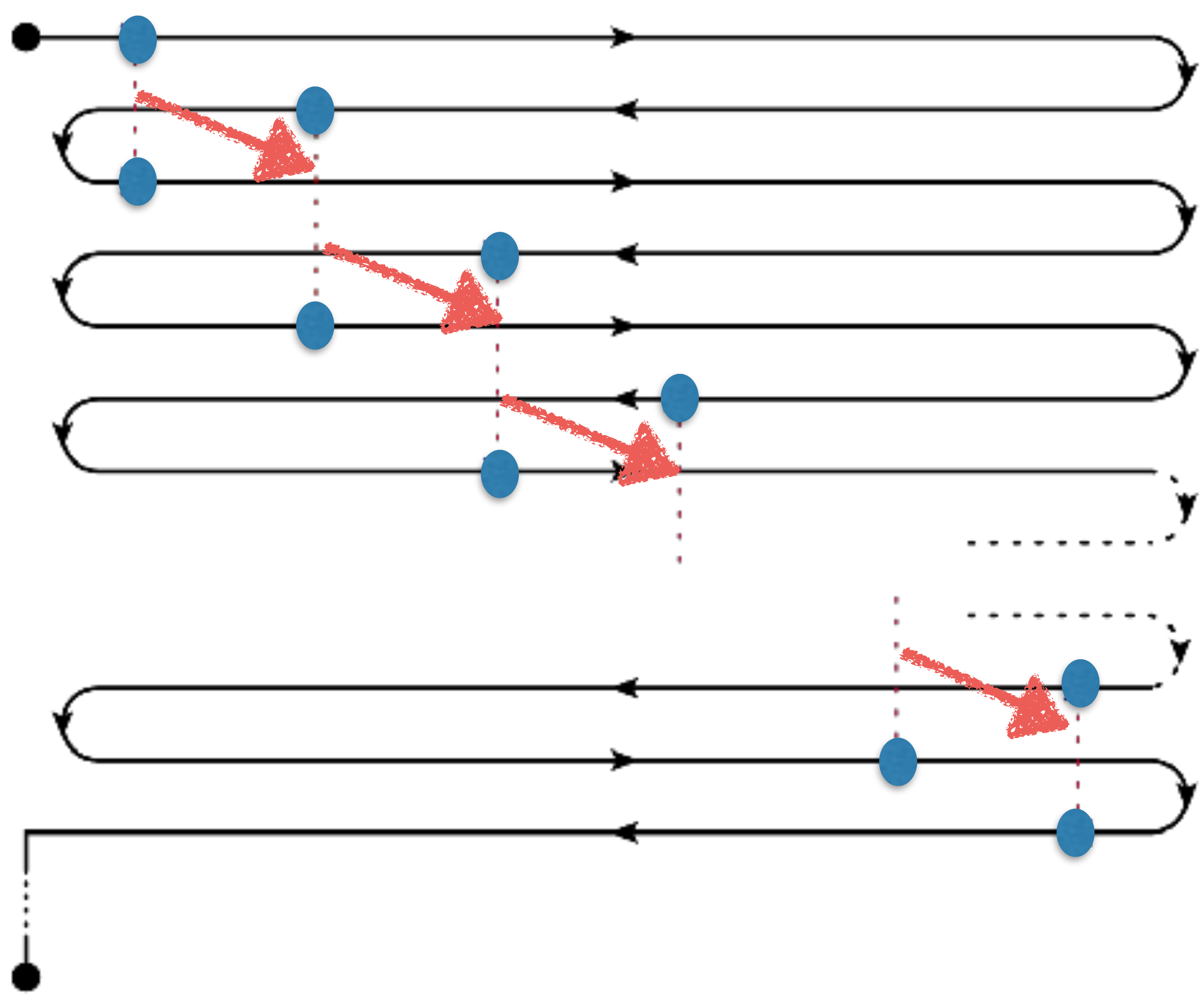
- *Maximally OTO*
- *Maximally "braided"* in Euclidean time
- Computation involves (k-1) ϵ-exchanges



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- Result:

$$F_{2k} \sim e^{\lambda_L(t-(k-1)t_*)} \qquad \text{with } t = t_1 - t_k$$

- *Hierarchy of time scales* associated with scrambling of quantum information

- Have calculated this in the *Schwarzian theory* and in maximally chaotic *2d CFTs* (—> additional spatial dependence)

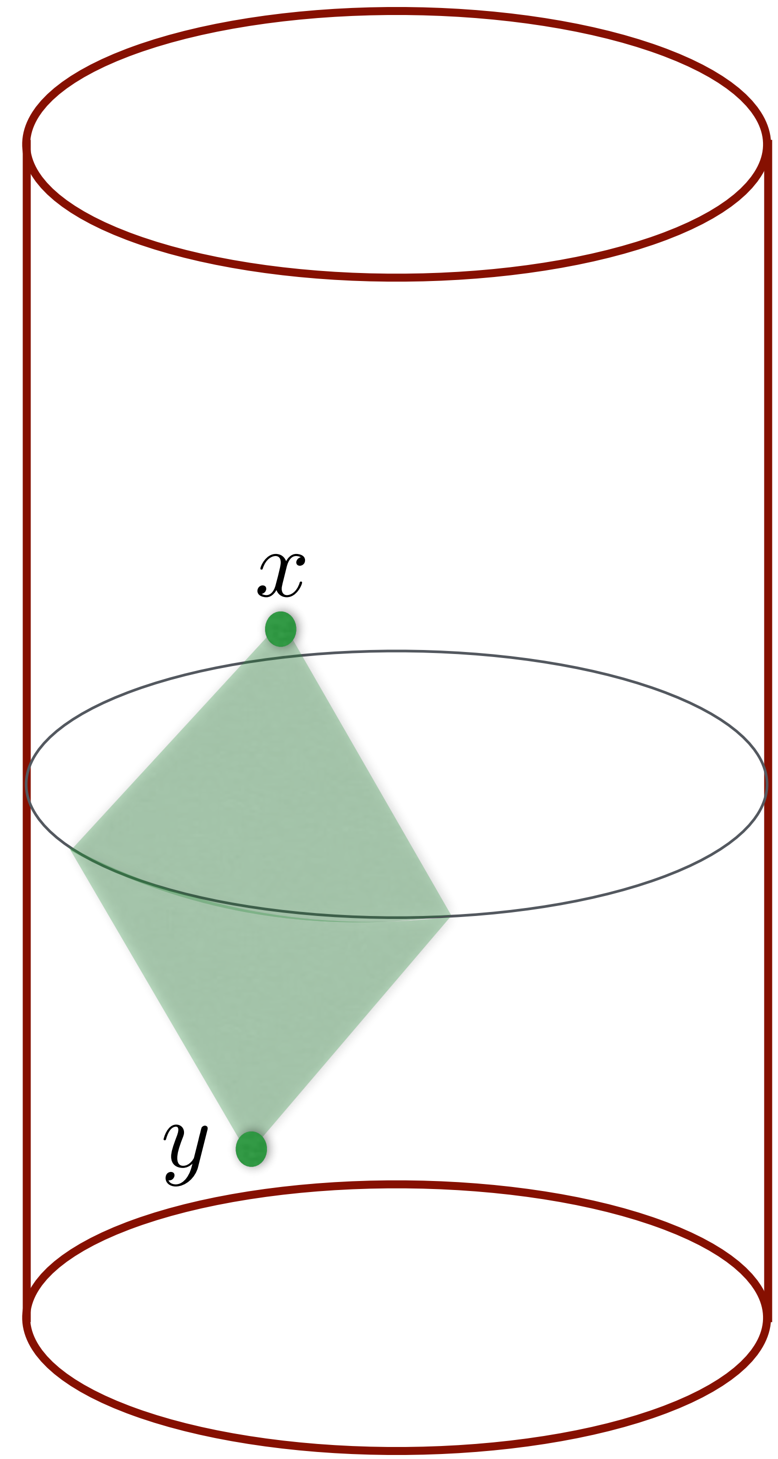
Kinematic space interpretation

Kinematic space

- *Kinematic space* is the space of timelike separated pairs of points (x^μ, y^μ) in the CFT:

  - = space of causal diamonds

    [Czech-Lamprou-McCandlish-Mosk-Sully '16]
    [de Boer-FH-Heller-Myers '16] ...



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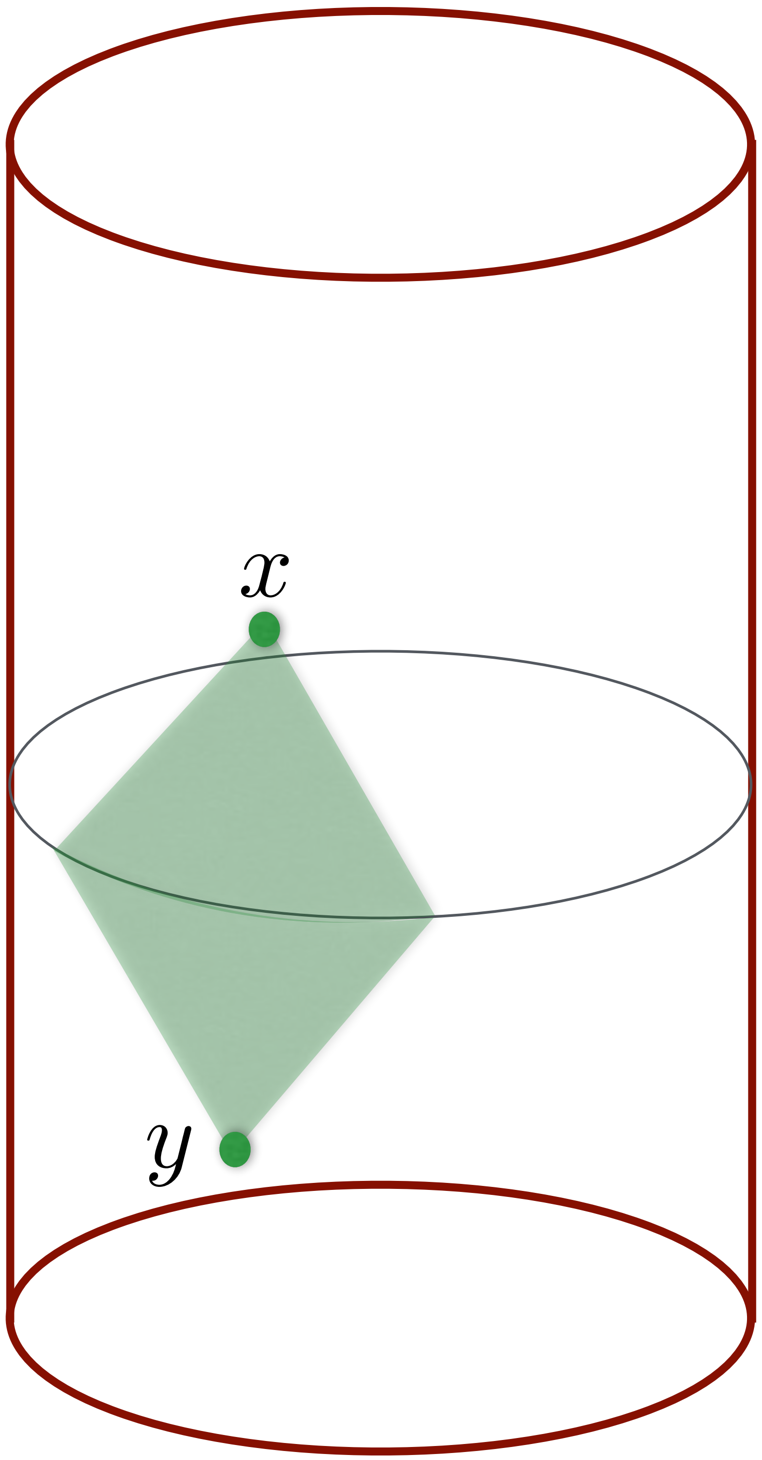

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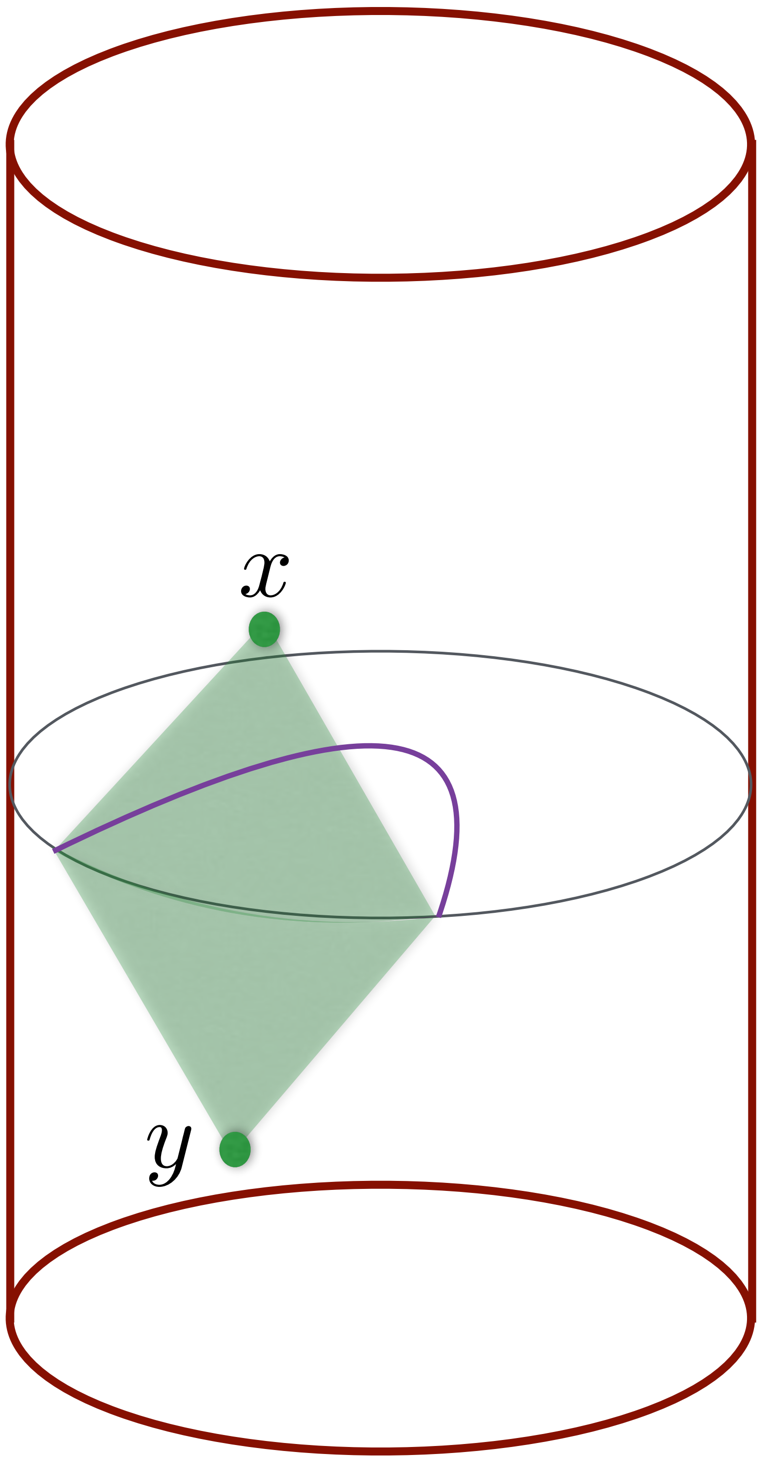

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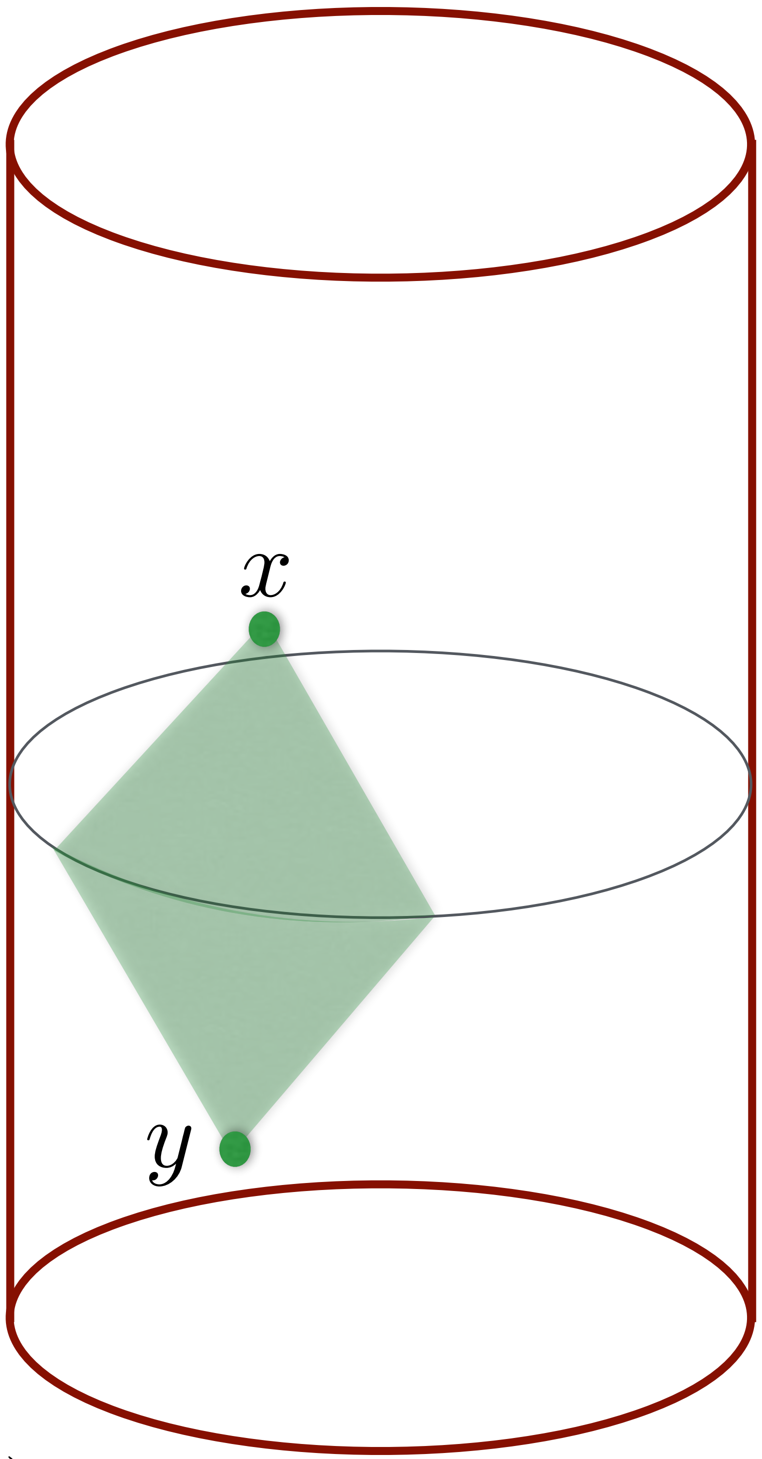
  - Operator product expansion:

$$\mathcal{O}(x)\mathcal{O}(y) = \sum_{\mathcal{O}_i} C_{\mathcal{O}\mathcal{O}\mathcal{O}_i} \left(1 + a_1\partial + a_2\partial^2 + \ldots\right) \mathcal{O}_i(x)$$



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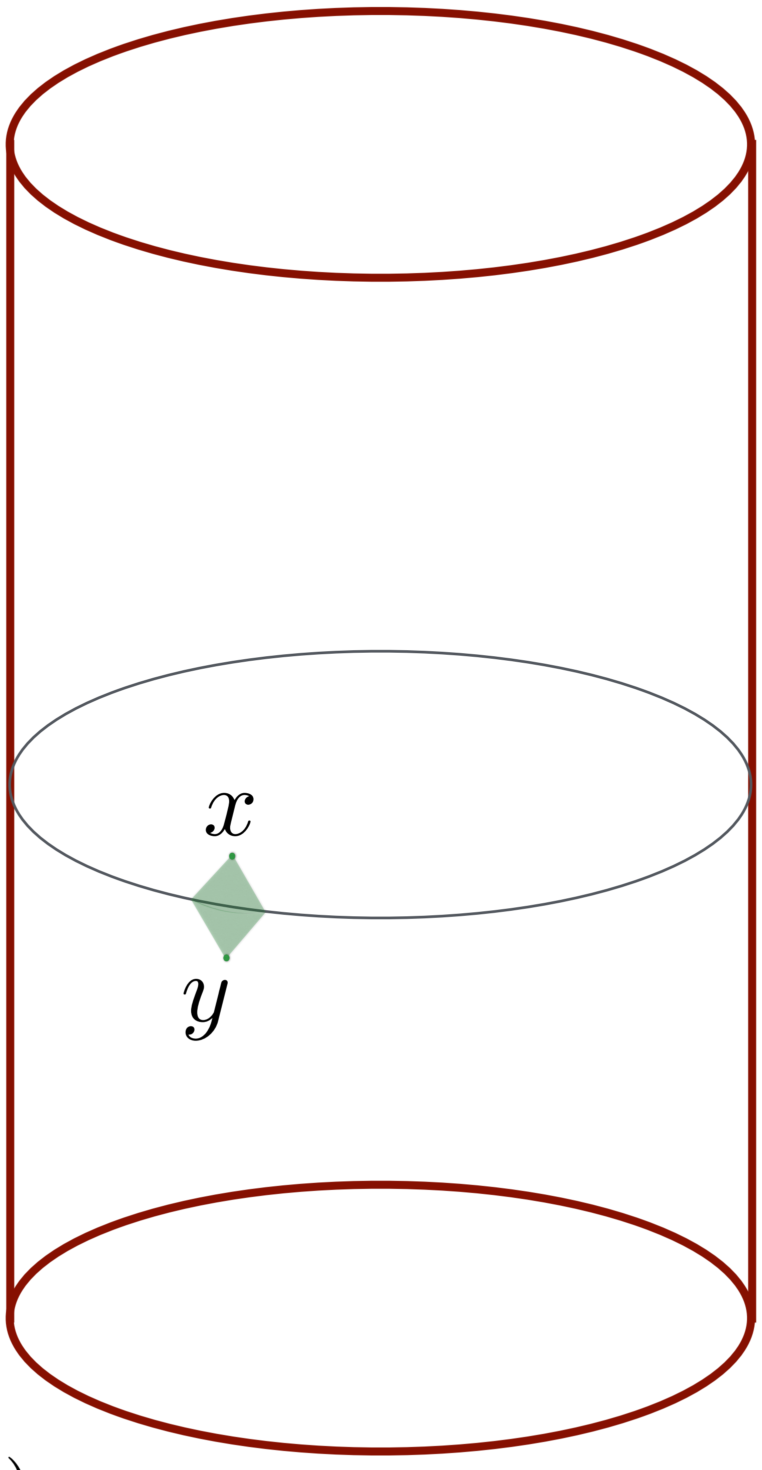

OPE blocks

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"OPE block"

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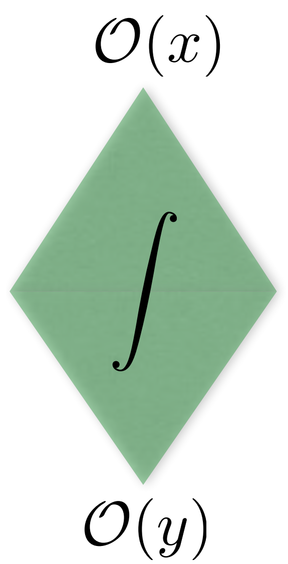
"OPE block"

- OPE block = field on kinematic space
- Smeared representation of OPE block

$$\mathfrak{B}_{\Delta_i}(x,y) = C_{\Delta_i} \int_{\Diamond(x,y)} d^d\xi \; I_{\Delta_i}(x,y;\xi)\; \mathcal{O}_i$$



- OPE block = field on kinematic space
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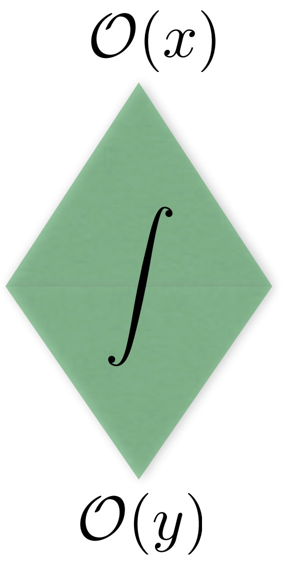
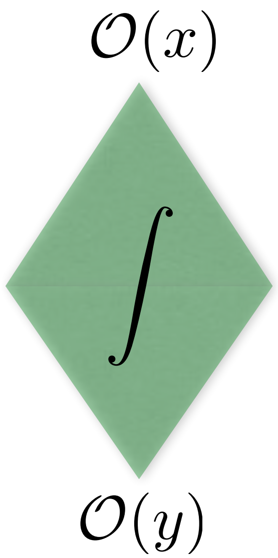


$$\sim \langle \mathcal{O}(x)\mathcal{O}(y)\widetilde{\mathcal{O}}_i(\xi)\rangle$$

"shadow operator" formalism

[Ferrara-Parisi '72]
[Dolan-Osborn '12]
[Simmons-Duffin '12]

- OPE block = field on kinematic space
- Smeared representation of OPE block for *stress tensor:*



$$\mathfrak{B}_T(x,y) = C_d \int_{\Diamond(x,y)} d^d\xi \; I_T(x,y;\xi)\; T(\xi)$$

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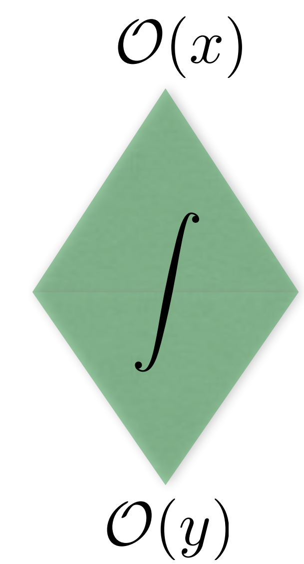
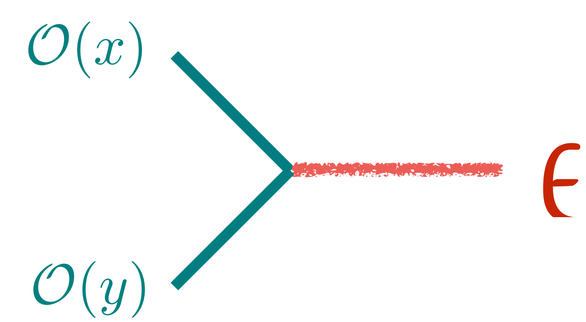


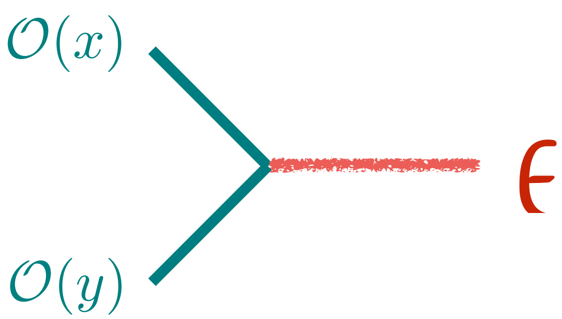
$$\mathfrak{B}_T(x,y) = C_d \int_{\Diamond(x,y)} d^d\xi \; I_T(x,y;\xi)\; T(\xi)$$

- Can show: this is equivalent to the coupling to our reparametrization mode!

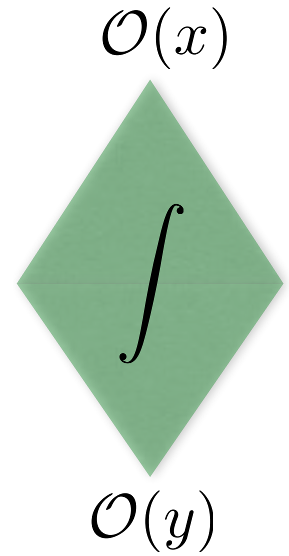
$$\mathcal{B}_\Delta^{(1)}(x,y) = \Delta \left[\frac{1}{d} \left(\partial_\mu \epsilon^\mu(x) + \partial_\mu \epsilon^\mu(y)\right) - 2\, \frac{\left(\epsilon(x) - \epsilon(y)\right)^\mu (x-y)_\mu}{(x-y)^2} \right]$$

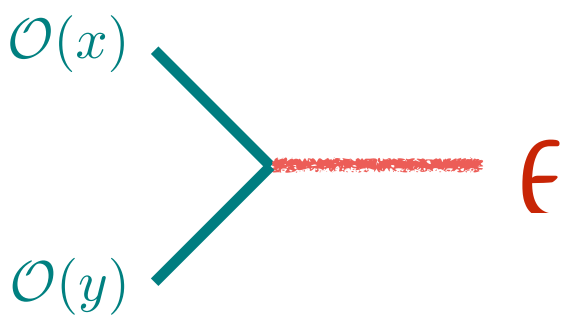


$$\boxed{\mathcal{B}_\Delta^{(1)}(x,y) \propto \mathfrak{B}_T(x,y)}$$



$$\mathcal{B}^{(1)}_{\Delta}(x,y) \propto \mathfrak{B}_T(x,y)$$





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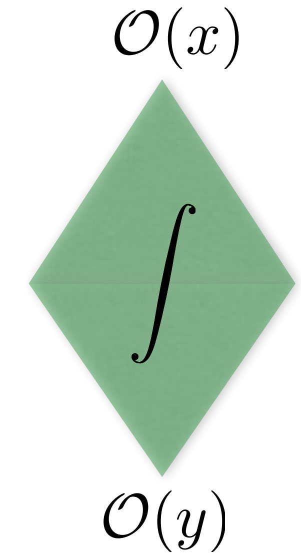
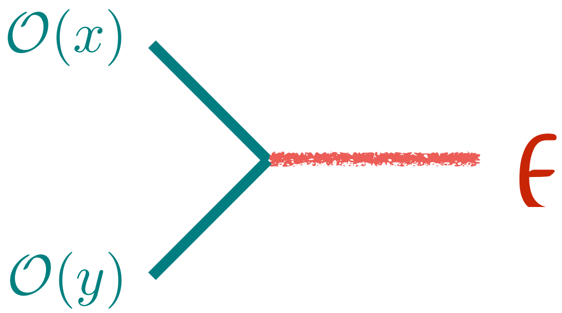


- For example, *stress tensor 4-point conformal block:*

$$\langle V(x_1)V(x_2)W(x_3)W(x_4)\rangle = \langle VV\rangle\langle WW\rangle \times C_{VVT}C_{WWT}\left\langle\, (T+\text{desc.})(T+\text{desc.})\right\rangle$$



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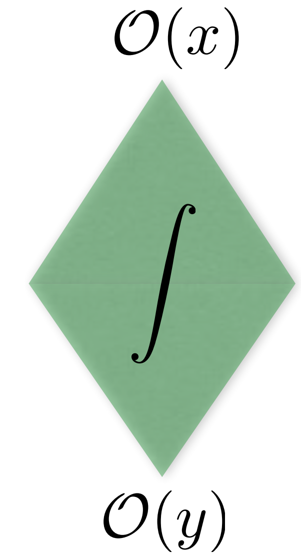
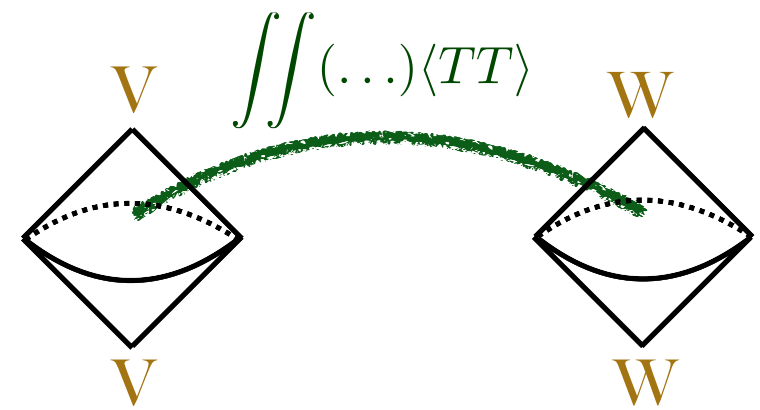
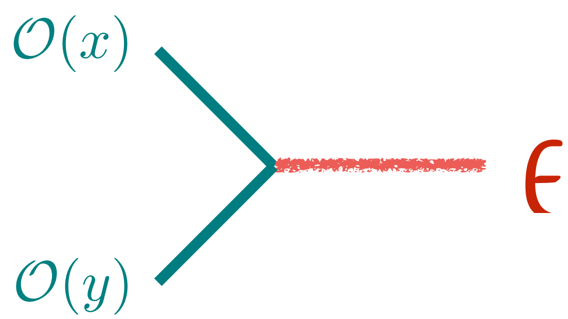


- For example, *stress tensor 4-point conformal block:*

$$\langle V(x_1)V(x_2)W(x_3)W(x_4)\rangle = \langle VV\rangle\langle WW\rangle \times [1 + \langle \mathfrak{B}_T(x_1,x_2)\mathfrak{B}_T(x_3,x_4)\rangle + \ldots]$$





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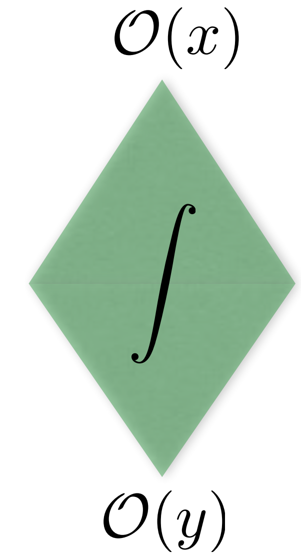

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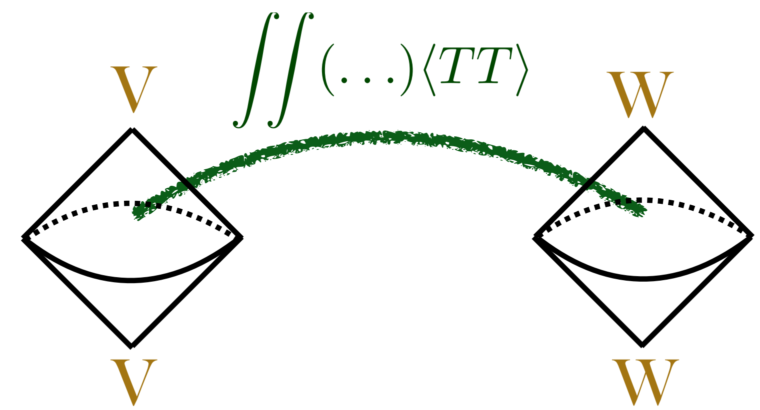

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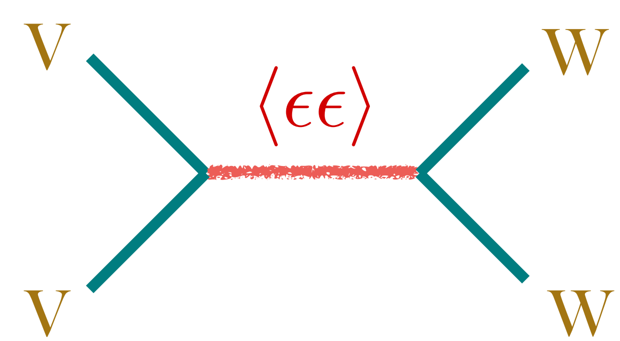
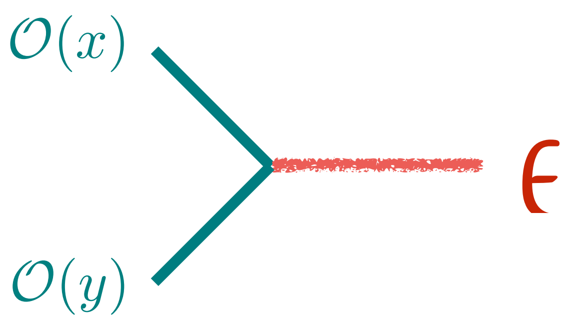
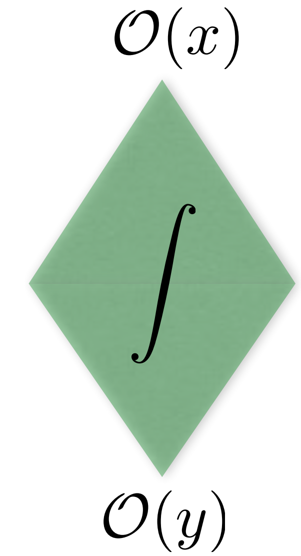


- A *local* reformulation of OPE block techniques



$$\mathcal{B}_{\Delta}^{(1)}(x,y) \propto \mathfrak{B}_T(x,y)$$



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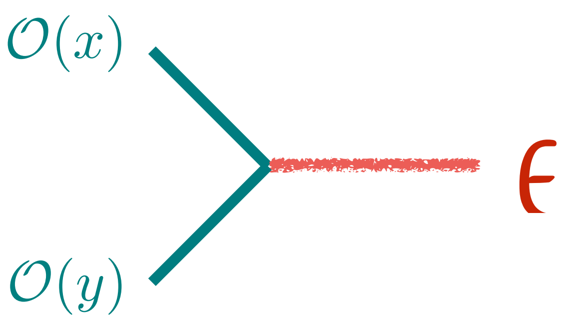

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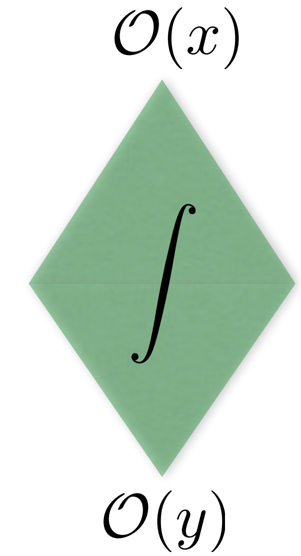
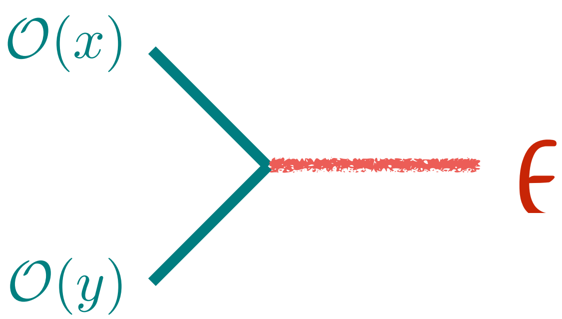


- A *local* reformulation of OPE block techniques
- No time for details… see [FH-Reeves-Rozali (to appear soon)] …
  - Basic idea: close connection between *reparametrization modes* and *shadow operators*
  - Proposal:

$$\partial_{(\mu}\epsilon_{\nu)} - \frac{1}{d}\,\eta_{\mu\nu}\,(\partial.\epsilon) \;\sim\; \widetilde{T}_{\mu\nu}$$

stress tensor "shadow"

—> boundary cond. on ϵ distinguishes block vs. shadow block



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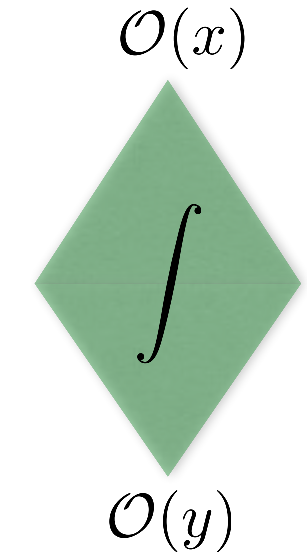

- No time for details… see [FH-Reeves-Rozali (to appear soon)] ...
  - Proposal: $$\partial_{(\mu}\epsilon_{\nu)} - \frac{1}{d}\eta_{\mu\nu}\,(\partial.\epsilon) \;\sim\; \widetilde{T}_{\mu\nu}$$
  - Seems to work in *higher dimensions*, as well: effective field theory <-> shadow operator formalism
  - *Conformal blocks* can be computed systematically from reparametrization mode perturbation theory [Cotler-Jensen ‘18]

Summary

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- Theory of reparametrization modes in CFTs similar to Schwarzian in d=1 (e.g. SYK)

  - Systematic effective field theory to study *OPE, shadow operators, conformal blocks, quantum chaos, etc.*

  - Example A: 2k-point OTOCs have *hierarchy of scrambling timescales* $t_*^{(k)} \sim (k-1) \times t_*$

  - Example B: *stress tensor OPE block* = coupling of bilocals (kinematic space fields) to reparametrization mode