Quantum information in scattering, IR divergences and asymptotic states

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based on 1706.03782, 1710.02531, 1803.02370 (w/ Carney, Chaurette, Semenoff), 1810.11477

Quantum information in scattering

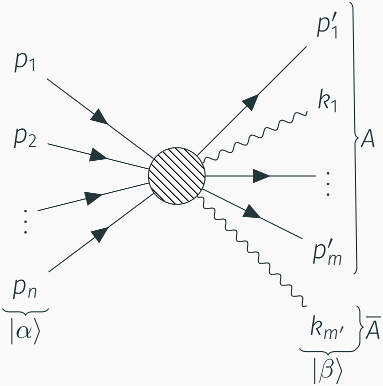
- Quantum information theory is well developed in position space, less so in momentum space.
- Black hole formation/evaporation can be treated as a scattering process.

- Focus on four dimensions: in theories with long range forces, IR divergences occur.
  - $\rightarrow$ Use as a guide to understand IR structure of theories with long range forces (QED, gravity) better.

- Decoherence, asymptotic states/Hilbert space, …

Entanglement entropy and scattering

[Carney et al. '16; Grignani, Semenoff '17]



$$S_{\text{EE}}(\rho_A) = -\text{tr}(\rho_A \log \rho_A), \qquad \text{Entanglement entropy,}$$

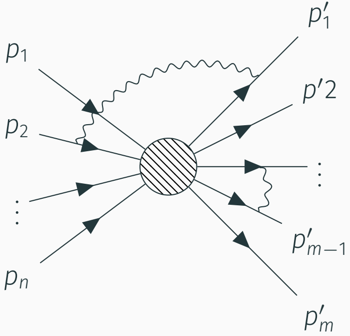
$$\rho_A = \text{tr}_{\overline{A}}\rho, \qquad \text{Reduced density matrix.}$$

Problem: infrared divergences

$$\int d|\mathbf{k}| \to \int_\lambda^\Lambda d|\mathbf{k}| + \int_\Lambda^\infty d|\mathbf{k}|$$

$$S_{\beta,\alpha} \to \left(\frac{\lambda}{\Lambda}\right)^{A_{\alpha,\beta}/2} S^\Lambda_{\beta,\alpha} \overset{\lambda\to 0}{\to} \delta_{\beta,\alpha}$$

$$\rho^{\text{out}} = S\rho^{\text{in}}S^\dagger$$



Inclusive formalism

[Bloch, Nordsieck ’37; Yennie, Frautschi, Suura ’61; Weinberg ’65]

$$\rho_{\beta\beta'} = \sum_{b \text{ soft}} \rho^{\text{out}}_{\beta b, \beta' b}$$

Need to trace out soft modes!

Dressed formalism

[Chung ’65; Faddeev,Kulish ’70; Carney, Chaurette, DN, Semenoff ’17]

$$|\mathsf{p}\rangle \to \|\mathsf{p}\rangle\!\rangle = W[f(\mathsf{p})]\ |\mathsf{p}\rangle \qquad \mathbb{S}_{\beta,\alpha} = \langle\!\langle\beta\|S\|\alpha\rangle\!\rangle = \text{finite}$$

Entanglement entropy for momentum eigenstates

[Carney, Chaurette, DN, Semenoff '17; Carney, Chaurette, DN, Semenoff '17]

Inclusive

$$\rho^{\text{out, incl}}_{\beta\beta'} \propto \lambda^{A_{\alpha,\beta}/2 + A_{\alpha,\beta'}/2 - A'_{\alpha,\beta\beta'}}$$

$$\rho^{\text{out,incl.}} = \begin{pmatrix} p_1 & 0 & \dots & 0 \\ 0 & & & \\ \vdots & & \ddots & 0 \\ 0 & & 0 & p_n \end{pmatrix}$$

$$S_{EE}(\text{soft}) = -\sum_{\beta} \left|S^{\Lambda}_{\beta,\alpha}\right|^2 \mathcal{G}_{\beta\alpha} \log\left(\left|S^{\Lambda}_{\beta,\alpha}\right|^2 \mathcal{G}_{\beta\alpha}\right)$$

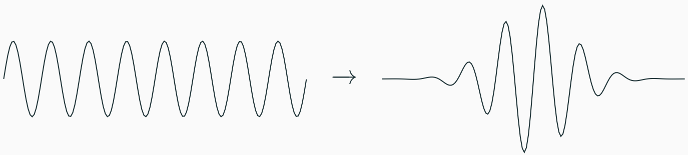
Dressed

$$S_{EE}(\text{soft}) = -\sum_{\beta} |\mathbb{S}_{\beta,\alpha}|^2 \log\left(|\mathbb{S}_{\beta,\alpha}|^2\right)$$

Inequivalence of inclusive and dressed formalism

[Carney, Chaurette, DN, Semenoff '18]

Use superpositions/wavepackets



$|\mathsf{p}\rangle$ $\qquad$ $|\psi\rangle = \sum_i f_i |\mathsf{p}_i\rangle$

Inclusive formalism

$$\rho_{\beta\beta}^{\text{out, incl.}} = \sum_i^N |f_i|^2 |S^\Lambda_{\beta,\alpha_i}|^2 \mathcal{G}_{\beta\beta,\alpha_i\alpha_i}$$

Dressed formalism

$$\rho_{\beta\beta}^{\text{out, incl.}} = \left| \sum_i^N f_i \mathbb{S}_{\beta,\alpha_i} \right|^2 .$$

Asymptotic Hilbert spaces for QED

[DN '18]

$$W[f(\mathrm{p},\mathrm{k})] = \exp\left(\sum_\ell \int \frac{d^3\mathrm{k}}{(2\pi)^3 2|\mathrm{k}|} f_\ell(\mathrm{p},\mathrm{k}) a_\ell^\dagger(\mathrm{k}) - h.c.\right) \qquad f_\ell(\mathrm{p},\mathrm{k}) \sim \frac{p\cdot\varepsilon_\ell}{p\cdot k}$$

Problems

- $W[f(\mathrm{p},\mathrm{k})]$ is not defined on Fock-space
  - $\rightarrow$ choose different representation
- $W[f(\mathrm{p}',\mathrm{k})]\,|\mathrm{p}'\rangle$, $W[f(\mathrm{p},\mathrm{k})]\,|\mathrm{p}\rangle$ live in unitarily inequivalent representations

Look at

- classical problem
- asymptotic Hamiltonain

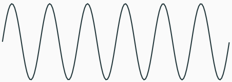
Asymptotic Hilbert spaces for QED

[DN ’18]

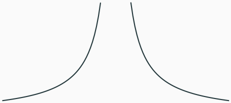
Definition of dressed states

$$\|\mathbf{p}\rangle\!\rangle \equiv W[f^{\mathrm{rad}}(\mathbf{p}) + f^{\mathrm{field}}(t,\mathbf{p})]\,|\mathbf{p}\rangle$$

Radiation: $k^0 = |\mathbf{k}|$



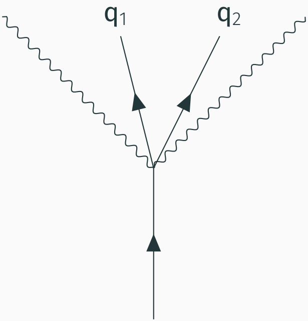
Field: $k^0 = \mathbf{k}\cdot\mathbf{v}$



- $W[f^{\mathrm{rad}}(\mathbf{p}') + f^{\mathrm{field}}(t,\mathbf{p}')]\,|\mathbf{p}'\rangle$ and $W[f^{\mathrm{rad}}(\mathbf{p}) + f^{\mathrm{field}}(t,\mathbf{p})]\,|\mathbf{p}\rangle$ live in the same representation

Time-controlled decoherence

[DN '18]



Elements of $\rho_{\mathbf{q}_1\mathbf{q}_2}$

$$\langle 0| \, W^{\mathrm{IR}\dagger}_{\mathbf{q}_2} W^{\mathrm{IR}}_{\mathbf{q}_1} \, |0\rangle = (\Lambda t)^{-A_1} e^{A_2(\Lambda, t)}$$

$$A_2(t,\Lambda) = -\frac{e^2}{2(2\pi)^3}\int d^2\Omega \frac{\mathbf{q}_1^{\perp}\mathbf{q}_2^{\perp}}{(q_1\cdot\hat{k})(q_2\cdot\hat{k})} \Big(\mathrm{Ci}(\Lambda t|(v_1 - v_2)\cdot\hat{k}|) - \log(\Lambda t|(v_1 - v_2)\cdot\hat{k}|) - \gamma - i\mathrm{Si}(\Lambda t(v_1 - v_2)\cdot\hat{k})\Big).$$

Properties of asymptotic Hilbert space(s)

- Full Hilbert space is non-separable [Kibble '68] and splits into subspaces which are
  - unitarily inequivalent representations of the canonical commutation relations.
  - scattering selection sectors. [Frohlich, Morchio, Strocchi '79]
  - eigenspaces of Q_ϵ. [Kapec, Perry, Raclariu, Strominger '17]
- Selection sectors transform non-trivially under Lorentz transformations $\to$ Choice of representation breaks Lorentz invariance. [Frohlich, Morchio, Strocchi '79]
- Normalized asymptotic states are *infraparticles*, i.e., cannot sit on the mass-shell [Schroer '63, Buchholz '86]

Decoherence condition

[Carney, Chaurette, DN, Semenoff ’17]

(De)coherence if

$$\rho^{\text{out, incl}}_{\beta\beta'} \propto \lambda^{A_{\alpha,\beta}/2 + A_{\alpha,\beta'}/2 - A'_{\alpha,\beta\beta'}} \qquad A_{\alpha,\beta}/2 + A_{\alpha,\beta'}/2 - A'_{\alpha,\beta\beta'} = 0$$

Define an infinity of currents

$$j_{\mathrm{v}} = \sum_i q^i a^{i\dagger}(\mathrm{p}_i(\mathrm{v}))a^i(\mathrm{p}_i(\mathrm{v}))$$

Decoherence condition

$$j_{\mathrm{v}} \left|\beta\right\rangle \sim j_{\mathrm{v}} \left|\beta'\right\rangle$$

Decoherence condition II

[Strominger '17, DN '19]

(De)coherence if

$$\rho_{\beta\beta'} \propto \lambda^{A_{\alpha,\beta}/2+A_{\alpha,\beta'}/2-A'_{\alpha,\beta\beta'}} \qquad A_{\alpha,\beta}/2 + A_{\alpha,\beta'}/2 - A'_{\alpha,\beta\beta'} = 0$$

Define an infinity of conserved charges $\mathcal{Q}^{\pm}_{\varepsilon}$

$$\mathcal{Q}^{+}_{\varepsilon} = \underbrace{\int_{\mathcal{I}^+} d\varepsilon \wedge \star F}_{\mathcal{Q}^{+}_{\varepsilon,S}} + \underbrace{\int_{\mathcal{I}^+_+} \varepsilon \star F}_{\mathcal{Q}^{+}_{\varepsilon,H}}$$

Equivalent decoherence condition

$$\mathcal{Q}^{+}_{\varepsilon,H} \left|\beta\right\rangle \sim \mathcal{Q}^{+}_{\varepsilon,H} \left|\beta'\right\rangle$$

Properties of asymptotic Hilbert space(s)

[Klauder, McKenna, Woods '65; Kibble '68]

Standard Hilbert space

$$\sum_\ell \int \frac{d^3\mathbf{k}}{(2\pi)^3 2|\mathbf{k}|} |f_\ell(\mathbf{k})|^2 < \infty$$

In the presence of charge a photon Hilbert space $\mathcal{H}_\otimes(g)$ is generated by acting with

$$\sum_\ell \int \frac{d^3\mathbf{k}}{(2\pi)^3 2|\mathbf{k}|} f_\ell(\mathbf{k}) a_\ell^\dagger(\mathbf{k})$$

on

$$|g\rangle = W[g]\,|0\rangle\,, \qquad a_\ell(\mathbf{k})\,|g\rangle = g_\ell(\mathbf{k},\ldots,t)\,|g\rangle$$

where

$$\sum_\ell \int \frac{d^3\mathbf{k}}{(2\pi)^3 2|\mathbf{k}|} \frac{1}{|\mathbf{k}|} |f_\ell(\mathbf{k})|^2 < \infty, \qquad \sum_\ell \int \frac{d^3\mathbf{k}}{(2\pi)^3 2|\mathbf{k}|} \frac{|\mathbf{k}|}{|\mathbf{k}|+1} |g_\ell(\mathbf{k})|^2 < \infty.$$

Summary and outlook

Summary

- definition of QI quantities in presence of IR divergences
- strong entanglement between hard and soft modes
- time-dependence of decoherence
- rich Hilbert space structure

Future directions and w.i.p.

- Extend results to asymptotically Minkowski gravity?
- Relations between results and asymptotic symmetries?
- Insight into flat-space holography? (Mink/QFT, S-matrix/QFT, flat space holographic renormalization)